Strategic manipulation induced by moral hazard: the "aim for rejection" problem in Japanese daycare allocations

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Abstract

This paper focuses on a particular type of strategic manipulation observed in Japanese daycare allocations. Some parents intentionally apply to a highly demanded daycare and are rejected, because a certificate indicating that they were rejected is required to extend the parental leave period. This behavior, which we term "aim for rejection" behavior, induces several negative effects such as efficiency loss. To fix this problem, the Japanese government has proposed modifying the priorities over parents depending on the intensity of their desire to secure a slot. Using game-theoretic models, we analyze and propose a solution to this problem. First, using a two-sided matching model, we show that even after the modification of priorities, the "aim for rejection" problem persists. Second, using a two-stage bargaining game model, we demonstrate that a parental leave policy adopted in Germany is more efficient than the Japanese policy. Our results indicate that game theory is a useful tool for designing a social security system.

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Keywords: daycare allocation, two-sided matching, bargaining, ultimatum game, moral hazard

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1 Introduction

Daycare slots in Japan are rationed via an allocation rule. Over the past 10 years, the demand for daycare slots in Japan has increased sharply.[1] This increase entails fierce competition for daycare slots, which in turn has engendered criticisms of the government's allocation rule. In particular, strategic manipulation and a lack of fairness have become highly contentious issues. For example, some mothers give birth by cesarean section in an attempt to enter the allocation system earlier and increase the probability of securing a slot.[2] It is reported that high-income parents are more likely to obtain a slot and are thus treated favorably.[3]

In 2018, several local governments reported observing a particular type of strategic manipulation in daycare allocations, which we term the "aim for rejection" problem (henceforth, the AR problem). Some parents *intentionally* apply to a highly demanded daycare and are rejected, because a certificate indicating that they were rejected is required to extend the parental leave period. Local governments have also reported negative effects induced by this behavior, e.g., a lack of efficiency. In response to these reports, the central government has proposed the following solution: ask parents how intense their desire to secure a slot is and lower their priority if the intensity is low.

This AR behavior is induced not by a poorly designed allocation rule but by moral hazard. The original intention of the parental leave policy was to support parents who *tried* to secure a daycare slot but were unable to do so (the details of the law will be explained later). However, we cannot observe parents' effort; indeed, some parents did not try to secure a place but *tried to be rejected*.

The aim of this paper is to analyze and propose a solution to the AR problem using game-theoretic models. Our analysis consists of two parts. First, using a two-sided matching model, we test the effectiveness of the central government's proposal. Second, using a two-stage bargaining game model, we show that a parental leave policy adopted in Germany is more efficient than the Japanese policy.

In the first part, we construct a two-sided matching model tailored to the target situation. In this model, parents have a preference relation over three elements: daycare, being unmatched without a certificate, and being unmatched with a certificate. We search for an allocation rule that satisfies a new property, rejection-proofness. This property states that an agent cannot be better off by misrepresenting her preference to attempt to be rejected. We prove that there is no rule that satisfies individual rationality and rejection-proofness.

[1] This is reflected in the percentage of children in daycare relative to all children, which for children aged 1-2 was 28.5% in 2009 but 41.1 % in 2016 (Maeda 2017).

[2] Maeda (2017), p.12.

[3] Suzuki (2018), p.75.

This result demonstrates the limitation of the central government's proposal.

Faced with this impossibility result, in the second part, we consider an alternative way to overcome the moral hazard problem. As a promising solution, we focus on the parental leave policy adopted in Germany. It allows parents to choose full- or part-time leave, and the central government supplements a proportion of the loss in income. This policy has two advantages from an economic perspective. First, parents are required to take *observable* actions rather than *unobservable* actions. Hence, asymmetric information is eliminated. Second, it encourages employers and employees to bargain over better working conditions during the parental leave period, because monetary support is provided under any working contract. To provide theoretical support for the effectiveness of the German policy, we construct a two-stage bargaining game model. An employee and an employer determine a contract (wage and hours worked) during parental leave following the procedure of the ultimatum game. We derive a subgame perfect equilibrium and show that the German policy improves the welfare of both the employee and the employer, as well as decreases government expenditures, compared to the Japanese policy.

Related literature

The AR problem is related to several streams of research in economics. Most notably, the problem involves a matching/allocation problem that is discussed in the market design literature. Since the work of Gale and Shapley (1962), market designers have had considerable success in developing real matching/allocation rules (see Roth (2015)). In a recent study, Okumura (2018) adopts a market design approach to the daycare allocation problem in Japan. He introduces a new fairness property that eliminates envy between children in different age groups and finds an algorithm that produces a matching satisfying the property. Kamada and Kojima (2018) define the notion of a fair matching and apply it to Japanese daycare allocation. They demonstrate that the number of children matched to a daycare place can be improved by allowing flexibility in seat allocation across different ages.

Moral hazard is a major research subject in information economics. For a survey in the context of insurance economics, see Dionne and Harrington (1992). Among many variations of the principle-agent model, Shavell's (1979) two-state model is well suited to the government-parent relationship we consider. A parent faces two states, "secure a daycare slot" or not, with some probabilities that depend on effort. Shavell (1979) showed the effectiveness of two solutions to morel hazard, i.e., (i) incomplete coverage against loss and (ii) "observation" by the insurer of the care taken to prevent loss. The Japanese government proposes overcoming the moral hazard problem *indirectly* by modifying an allocation rule. We highlight the limitations of this idea and instead focus on a more direct solution, i.e., to make parents take *observable* actions.

The bargaining problem also has a long history in the game theory literature. Previous studies adopt axiomatic (Nash 1950), non-cooperative (Rubinstein 1982), and experimental approaches (Roth 1995). In the typical employer-employee relationship, an employer proposes a work plan and then an employee determines whether to accept it. To capture this, we employ a variant of the ultimatum game tailored to our setting. For an equilibrium analysis of the ultimatum game, see, for example, Chapter 6 of Osborne (2004).

Some researchers statistically analyze the effects of parental leave policy on female labor force participation. See Ruhm (1998) or Bergemann and Riphahn (2010), among others.

The reminder of this paper is organized as follows. Section 2 describes background and institutional details. Section 3 presents the market design approach. Section 4 presents the bargaining game approach. Section 5 provides a discussion of the theretical analyses. Section 6 concludes this paper.

2 Background and institutional details

2.1 Unemployment insurance and parental leave benefit

We explain basic aspects of the parental leave policy in Japan.[4]

In Japan, unemployment insurance is compulsory. The employer and employee pay a tiny fraction of the employee's salary to the government. The central government uses the premium to improve worker welfare.

Unemployment insurance is the source of parental leave benefits. Parental leave is guaranteed by law and involves job protection rights, but its length is not fixed; in essence, every parent with a child less than 1 year old can apply for a parental leave. "If a parent hopes to secure and applies for a daycare slot but cannot secure childcare services for the time being,"[5] she is allowed to extend her parental leave period for one year (i,e., until her child turns 2 years old). The payment is 67% of the pre-birth net income for the first 6 months and 50% for the remaining period. Parents are not allowed to work regularly during the leave period.[6]

2.2 Daycare allocation problem

We enumerate basic aspects of the problem below, where quoted sentences are from Kamada and Kojima (2018).

[4]We cite data on Japanese parental leave policy summarized in International Network on Leave Policies and Research (2019).

[5]Ordinance for Enforcement of the Act on Childcare Leave, Caregiver Leave, and Other Measures for the Welfare of Workers Caring for Children or Other Family Members, Article 5-8 (No. 66 of June 30, 2017).

[6]This point is stated in Ministry of Internal Affairs and Communications (2019).

- Local governments manage the daycare allocation problem in each municipality.
- Daycare places are rationed via an allocation rule, which takes as inputs the parents' preferences and their priorities.
- Priorities are "determined by applicant characteristics such as whether parents have full-time jobs and whether the parent is a single parent."
- The most popular allocation rules are "versions of serial dictatorship and the 'Boston' mechanism."

2.3 Report on the AR problem

Since 2014, the Cabinet Office of Japan has gathered reform plans from local governments in an attempt to better understand the problems local governments face. In 2018, several local governments reported the AR problem. In Sections 2.3 and 2.4, we cite the question-and-answer session between the central government and local governments.

The local governments' argument is summarized as follows:

- To confirm that parents attempted to secure a daycare place, the central government requires them to submit a certificate that they were rejected by a daycare.
- However, some parents who do not want to return to work intentionally apply to a highly demanded daycare and obtain a certificate that they were rejected. The number of such parents is not negligible.

Furthermore, the local governments cited three negative effects of applications seeking rejection.

1. They impose unnecessary office work on local governments.
2. They give rise to an unfair allocation; some parents would have been able to secure a place had such applications not been submitted.
3. They make it difficult to gather accurate data on the number of parents who truly want to secure a daycare slot.[7]

Given these points, the local governments proposed that parents be allowed to extend parental leave without a certificate that they had been rejected.

[7]This is a serious problem in view of the recent changes to childcare systems. The central government decided to make daycares free of charge in 2019, but its effect on daycare demand cannot be evaluated without accurate data.

2.4 Response from the central government

The central government did not agree to cease requiring a certificate. The parental leave policy allows the leave period to be extended to help parents who would otherwise have no choice but to quit their jobs. Namely, the original intention of the policy is to *help parents return to work*, and granting an extension is considered exceptional treatment. The central government offered the following argument: "A certificate of having been rejected is issued for certain when a parent attempts to secure a slot but is rejected. Hence, it is rational to utilize it" (Cabinet Office of Japan 2019).

The central government instead proposed to modify the priorities over parents based on the intensity of their desire to secure a slot. To assess this intensity, the government proposed to create a checklist with the following yes-no questions:

☐ *I want to return to work.*

☐ *I can extend the parental leave period if I'm rejected.*

The answers to these questions are used to evaluate the intensity of parents' desire to secure a place, which is then reflected in their priority. Below, we argue against this idea and propose a solution to the AR problem.

Remark 1. In addition to the above proposal, the central government also proposed "monitoring" the parents' behavior.

The daycare allocation process consists of (at least) two rounds. Hence, if a parent aiming for rejection is accepted in Round 1, then she has an incentive to decline the acceptance and go to Round 2. If such an action is taken, then it is disclosed on the application form.

Although the principle-agent model indicates that monitoring effort can be effective, the government's proposal does not reflect these findings and has two drawbacks. First, monitoring *behavior* is not directly related to monitoring *effort*. A parent might decline acceptance in Round 1 not because she wants to be rejected in Round 2 but for some other reason. Second, disclosure is only a "threat" and does not immediately imply that she cannot extend her leave period. This idea requires parents to have morals but does not take their incentives into account. ∎

2.5 Questionnaire survey on the AR problem

Here we cite a questionnaire survey on working mothers' way of thinking about the AR problem, in an attempt to better understand the participants' cognitive aspect. This survey was conducted by a private company in Japan (b-style Inc. 2019) from January 30 to February 6 in 2019, gathering responses from 650 working mothers through the internet.

The first question is, "Do you want to extend the leave period when you take parental leave?" Of the respondents, 55.6 % answered "yes", while 13.5 % answered "no". This result shows a relatively strong incentive for mothers to be rejected. The second question is, "How do you feel about the 'aim for rejection' problem?" Of the respondents, 60.2 % answered that the root problem is the design of the leave policy. The director of the survey offered the following quote: "Many mothers answer in the free comment box that the central government's proposal won't solve the actual cause of the problem." In the next section, we demonstrate that these working mothers' intuition is correct.

3 Market design approach

Using a two-sided matching model, we evaluate the government's proposal in Section 2.4. We construct a model tailored to the target situation and introduce axioms that are essential for fixing the AR problem. We prove an impossibility theorem.

We follow the notations of Kamada and Kojima (2018). Let I be a set of **parents** and D be a set of **daycares**. Each daycare d has a **quota** $q_d \in \mathbb{Z}$, $q_d > 0$, representing the maximum number of slots it may fill.[8] We fix $(I, D, (q_d)_{d\in D})$ in the remainder of this section.

In our model, being unmatched has two states, $\emptyset$ and $\widehat{\emptyset}$, where $\emptyset$ represents being unmatched without a certificate, while $\widehat{\emptyset}$ indicates that one is unmatched but has a certificate. Let $\overline{\mathcal{R}}$ denote the set of all preference relations over $D\cup\{\emptyset,\widehat{\emptyset}\}$ with a generic element $\succ\in\overline{\mathcal{R}}$.

Let $\mathcal{R}_i \subseteq \overline{\mathcal{R}}$ denote the domain of preference relations that an agent i can submit. We assume that i can at least rank $\widehat{\emptyset}$ as her first choice; formally,

$$\mathcal{R}_i \supseteq \widehat{\mathcal{R}} \equiv \big\{\succ\in\overline{\mathcal{R}} : \widehat{\emptyset}\succ d \text{ for all } d\in D\cup\{\emptyset\} \text{ and } d\succ\emptyset \text{ for some } d\in D\big\}.$$

This domain is rich enough to discuss the central government's proposal. In this proposal, parents can submit a preference relation over $D\cup\{\emptyset\}$ and convey their strong desire for securing a slot by answering the yes-no questions (see Section 2.4). We denote by $\succ_I\equiv(\succ_i)_{i\in I}\in\times_{i\in N}\mathcal{R}_i$ the profile of all parents' preferences.

A **matching** μ is a mapping that satisfies (i) $\mu_i\in D\cup\{\emptyset,\widehat{\emptyset}\}$ for all $i\in I$, (ii) $\mu_d\subseteq I$ for all $d\in D$, and (iii) for any $i\in I$ and $d\in D$, $\mu_i=d$ if and only if $i\in\mu_d$. That is, a matching specifies who is assigned to which daycare (if any) and whether a rejected parent obtains a certificate.

A **mechanism** φ is a function that maps preference profiles $\succ_I\in\times_{i\in I}\mathcal{R}_i$ to matchings. The matching under φ at parents' preference profile $\succ_I$ is denoted $\varphi(\succ_I)$, and parent i's

[8] We do not specify priorities over parents here. The central government has proposed modifying priorities, which is *one* idea to modify an allocation rule. Below, we prove the impossibility of finding a desirable allocation rule, thereby demonstrating the impossibility of modifying priorities.

match is denoted by $\varphi_i(\succ_I)$ for each $i \in I$. Let $\varphi_d(\succ_I)$ denote the set of parents who are matched to daycare d. We assume as primitives that a mechanism φ satisfies the following conditions: for any $\succ_I$ and $i \in I$,

(P-i) $|\varphi_d(\succ_I)| \leq q_d$ for all $d \in D$

(P-ii) $\varphi_i(\succ_I) = \emptyset \Longrightarrow \emptyset \succ_i d$ for all $d \in D$.

(P-iii) $\varphi_i(\succ_I) = \widehat{\emptyset} \Longrightarrow \exists\, d \in D$ s.t. $d \succ_i \emptyset$ and $|\varphi_d(\succ_I)| = q_d$.

(P-i) means that a mechanism cannot assign more parents to a daycare than available. (P-ii) means that a parent obtains neither a daycare slot nor a certificate only if she does not apply to any daycare. (P-iii) means that, for a parent i to obtain a certificate, she has to apply to at least one daycare and be rejected due to capacity constraints.

A mechanism φ is said to be **individually rational (IR)** if, for any preference profile $\succ_I$ and $i \in I$, we have

$$\varphi_i(\succ_I) \succ_i \emptyset \text{ or } \varphi_i(\succ_I) = \emptyset.$$

A mechanism φ is said to be **rejection-proof (RP)** if there do not exist a preference profile $\succ_I$, a parent $i \in I$ and a preference $\succ_i' \in \mathcal{R}_i$ such that

$$\varphi_i(\succ_i', \succ_{I\setminus\{i\}}) = \widehat{\emptyset} \text{ and } \widehat{\emptyset} \succ_i \varphi_i(\succ_I).$$

Assuming that $\succ_i$ is i's true preference, rejection-proofness requires that i cannot be better off by misrepresenting her report in an attempt to be rejected.

We are in a position to prove our main result.

Proposition 1. *Suppose that $|D| \geq 2$ and there exists $d' \in D$ such that $|I| > q_{d'}$. Then, there does not exist a mechanism that satisfies IR and RP.*

Proof. For each $d \in D$, let $\succ^d \in \widehat{\mathcal{R}}$ denote a preference relation such that the first choice is $\widehat{\emptyset}$, the second choice is d, and the other daycares are not acceptable, i.e., $\emptyset \succ^d d'$ for all $d' \in D\setminus\{d\}$.

Let $d' \in D$ be a daycare such that $|I| > q_{d'}$. Let $\succ_I$ be a preference profile such that

$$\succ_i = \succ^{d'} \text{ for all } i \in I.$$

By IR,

$$\varphi_i(\succ_I) \in \{d', \widehat{\emptyset}, \emptyset\} \text{ for all } i \in I. \tag{1}$$

By (P-ii),

$$\varphi_i(\succ_I) \neq \emptyset \text{ for all } i \in I. \tag{2}$$

(1) and (2) imply

$$\varphi_i(\succ_I) \in \{d', \hat{\emptyset}\} \text{ for all } i \in I.$$

Together with (P-i) and $|I| > q_{d'}$, there exists at least one agent $i' \in I$ such that

$$\varphi_{i'}(\succ_I) = \widehat{\emptyset}. \tag{3}$$

Let $d'' \in D \backslash \{d'\}$ and consider the preference profile $(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}})$. By IR,

$$\varphi_i(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) \neq d'' \text{ for all } i \in I\backslash\{i'\}.$$

This condition implies $\varphi_{d''}(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) \subseteq \{i'\}$. Together with (P-iii),

$$\varphi_{i'}(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) \neq \widehat{\emptyset}. \tag{4}$$

By IR,

$$\varphi_{i'}(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) \in \{d'', \widehat{\emptyset}, \emptyset\}. \tag{5}$$

By (P-ii),

$$\varphi_{i'}(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) \neq \emptyset. \tag{6}$$

By (4)-(6),

$$\varphi_{i'}(\succ_{i'}^{d''}, \succ_{I\backslash\{i'\}}) = d''. \tag{7}$$

(3) and (7) contradict RP. □

Proposition 1 means that, as long as we maintain the policy of requiring a certificate, we cannot eliminate the AR behavior. Hence, we need a more drastic reform, which is considered in the next section.

4 Bargaining game approach

The central government required a certificate of having been rejected to encourage parents to make efforts to return to work. However, as suggested in Section 3, this policy induces the AR behavior (regardless of the design of the allocation rule) and cannot solve the problems local governments face. As an alternative way to incentivize parents to return to work, we propose to *change the subsidy provision system.* This proposal is inspired by the parental leave policy in Germany, which is intended to encourage parents and their employers to *bargain* over a better working contract. Using a bargaining game model, we demonstrate the advantages of this policy.

4.1 Parental leave policy in Germany

Here, we explain basic aspects of the parental leave policy in Germany.[9] A parent can choose full- or part-time leave. In the full-time case, the parent receives 65 % of the previous year's net earnings for 12 months. In the part-time case, the parent is compensated for 65 % of the loss in income for 24 months. For example, if a parent "worked 40 hours weekly before taking Paretal leave, and continues working 30 hours thereafter, s/he receives 65 per cent of the margin between the present and the former income in addition to her/his employment income" (International Network on Leave Policies and Research 2019).

This policy has two advantages. First, parents take *observable* actions rather than *unobservable* actions. This is consistent with the lesson from the principle-agent model that controlling unobservable actions yields efficiency loss. Second, it encourages employers and employees to bargain over better working conditions during the parental leave period. In the Japanese case, the central government provides monetary support only if a parent does not return to work. By contrast, in the German case, such support is provided under any contract. Hence, employers/employees attempt to reach a better contract, which would help improve efficiency.

4.2 Baseline model

The purpose of the remaining sections is to provide theoretical support for the greater effectiveness of the German policy discussed in Section 4.1 compared with the Japanese policy. We first present the baseline model in which we do not consider parental leave benefit. Although extremely simple, this model proves useful for highlighting the difference between the two policies.

[9]We cite data on German parental leave policy summarized in International Network on Leave Policies and Research (2019).

There are two players, an employer (player 1, "he") and an employee (=a parent, player 2, "she"). Player 2 has a fixed stock of time $\overline{\ell}$ that can be divided between labor supply and leisure time (spent raising her child). During the parental leave period, a parent can receive a wage of between 0 and $\overline{w}$, where $\overline{w}$ denotes the pre-birth net income. We call a pair (ℓ, w) satisfying $0 \leq \ell \leq \overline{\ell}$ and $0 \leq w \leq \overline{w}$ a **contract.**

Player 1 has a utility function U_1 over contracts, i.e., $U_1 : [0, \overline{\ell}] \times [0, \overline{w}] \to \mathbb{R}$.[10] We assume that U_1 is a linear function, with the following justification: an employer typically negotiates contracts with many employees, and a contract with one particular employee forms a small part of the overall contract. A linear function is a good approximation of this small part. We assume

$$U_1(\ell, w) = c - a\ell - bw, \text{ where } a > 0, b > 0, c > 0.$$

Note that player 1's indifference curve has the slope a/b.

Player 2 also has a utility function U_2 over contracts. As is standard in labor supply models (see, for example, Section 15.3 of Hindriks and Myles (2006)), we assume that $U_2(\cdot, \cdot)$ is a differentiable, strictly-monotonic and quasi-concave function, where quasi-concavity represents diminishing marginal rate of substitution.

Players 1 and 2 negotiate a contract following the procedure of the ultimatum game. First, player 1 offers player 2 a contract, (ℓ, w). If 2 accepts this offer, then the proposed contract is implemented. If 2 rejects the offer, then she does not return to work, and contract $(\overline{\ell}, 0)$ is implemented. We call the outcome when 2 rejects 1's offer the **outside option**.

4.3 Japanese case

In the Japanese case, a parent who rejects her employer's offer seeks to be rejected in the daycare allocation problem. If she is rejected, she enjoys the full leisure time $\overline{\ell}$ and the subsidized salary $\alpha\overline{w}$, where α denotes the income discount rate in the parental leave benefit.[11]

Let p denote the probability that a parent is rejected by a daycare. In reality, p appears to be close to 1 for the following reasons:

- Parents can ask which daycares are popular at a municipal office.
- Some local governments publicly announce past data on daycare applications. The

[10] Note that $\ell \in [0, \overline{\ell}]$ represents the amount of leisure time spent by a parent. As this amount decreases, hours worked $\overline{\ell} - \ell$ increases, which is desirable for player 1.

[11] We fix the income discount rate α throughout Sections 4.3 and 4.4. This is because our focus is not on identifying the appropriate value of α but on how to provide parental leave benefit.

ratio of daycare seekers to daycare places is very high in some daycares.[12]

Local governments also argue that "it is easy for parents to be rejected" (Cabinet Office of Japan 2019). From these points, we assume for simplicity that $p = 1$. With respect to welfare analysis, imposing this assumption does not differ substantially from the case in which p is not equal to 1 but sufficiently high. This means that, when player 2 rejects 1's offer, she consumes $(\overline{\ell}, \alpha\overline{w})$ for certain.

In sum, we obtain the following game tree:

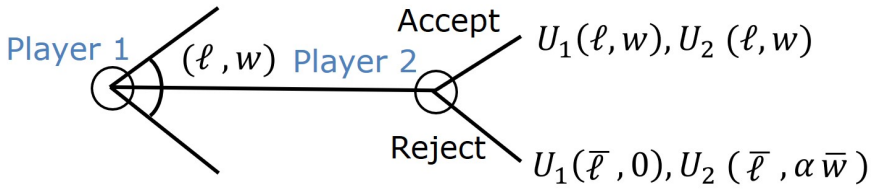
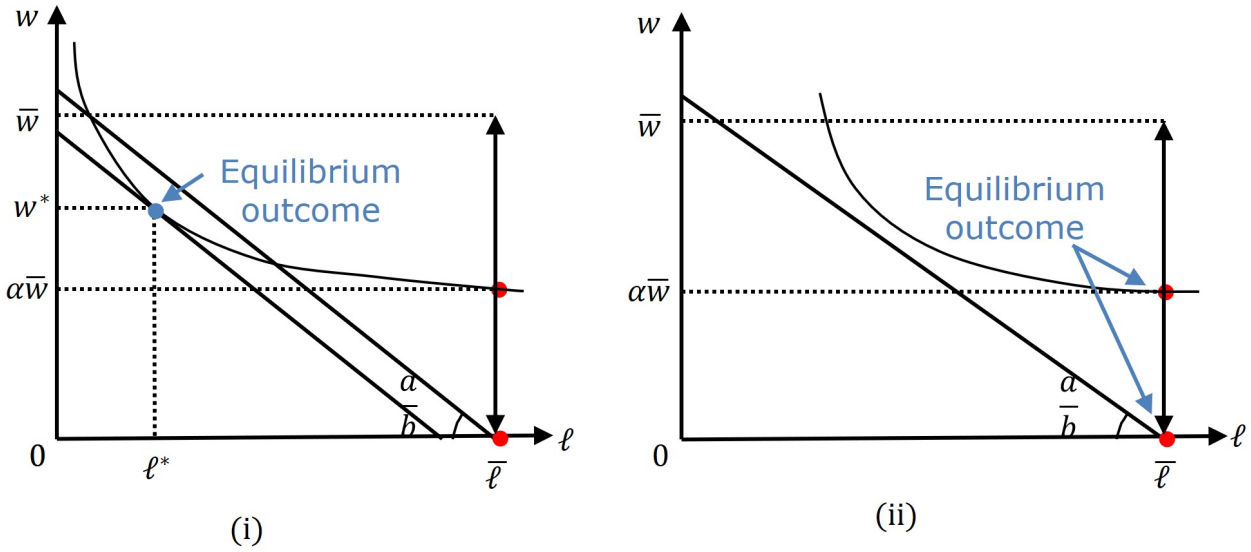


Fig. 1. Game tree for the Japanese case

A subgame perfect equilibrium can be derived in the same way as in the ultimatum game. Two indifference curves passing through the outcome at the outside option (i.e., 1's indifference curve passing through $(\overline{\ell}, 0)$ and 2's indifference curve passing through $(\overline{\ell}, \alpha, \overline{w})$) play a key role. We consider two cases: (i) the two curves intersect, and (ii) they do not intersect. They are illustrated in Fig. 2.[13]



[12]For example, at a few daycares in Musashino City in Tokyo, the applicants-to-capacity ratio has been more than 10 for the past 5 years (Musashino City 2019).

[13]We remark that 1's utility increases in the lower-left direction and 2's utility increases in the upper-right direction.

Fig. 2. Comparison of the two cases and equilibrium outcomes

The equilibrium outcomes are described as follows:

Case (i): Player 1 offers (ℓ^*, w^*), which lies on the boundary of player 2's indifference curve and the line with slope a/b. Player 2 accepts the offer.

Case (ii): Player 1 offers a contract below 2's indifference curve. Player 2 rejects the offer and the outside option is implemented.

In case (i), the two players voluntarily negotiate over a contract without government support and a Pareto optimal outcome is realized. In case (ii), the two players cannot reach an agreement. In the sequel, we confine our attention to the latter case (ii) and show that there is efficiency loss. Adopting the German policy can lead to a Pareto improvement.

4.4 German case

In the German case, when 2 accepts 1's offer (ℓ, w), her monetary outcome is w plus the benefit from the government, a proportion α of the loss in income. Namely, player 2's overall wage is $w + \alpha(\overline{w} - w)$. If 2 rejects the offer, then she does not return to work. Player 1 consumes $(\bar{\ell}, 0)$, while player 2 consumes $(\bar{\ell}, \alpha\overline{w})$ (i.e., the parent enjoys the full leisure time and receives the parental leave benefit). The game tree is illustrated in Fig. 3.

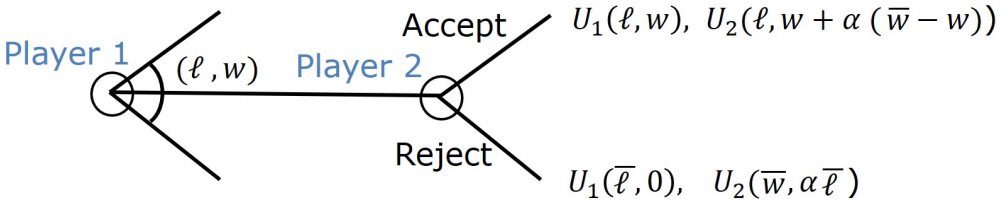


Fig. 3. Game tree in the German case

We derive a subgame perfect equilibrium. In the subgame that follows an offer (ℓ, w) for which $U_2(\ell, w) \geq U_2(\bar{\ell}, \alpha\overline{w})$, player 2's optimal action is to accept. Otherwise she rejects the offer.

Under the constraint that 2 accepts 1's offer, player 1 faces the following problem:

$$\max_{0 \leq \ell \leq \bar{\ell}, 0 \leq w \leq \overline{w}} U_1(\ell, w) \text{ s.t. } U_2(\ell, w) \geq U_2(\bar{\ell}, \alpha\overline{w}). \tag{8}$$

By the standard argument of constrained optimization, a solution (ℓ^*, w^*) satisfies[14]

$$\frac{a}{b} = \frac{1}{(1-\alpha)} \cdot \frac{\partial U_2(\ell^*, w^* + \alpha(\overline{w} - w^*))/\partial \ell}{\partial U_2(\ell^*, w^* + \alpha(\overline{w} - w^*))/\partial w}.$$

Note that the right-hand side represents $1/(1-\alpha)$ times 2's marginal rate of substitution. The analysis here is illustrated in Fig. 4.

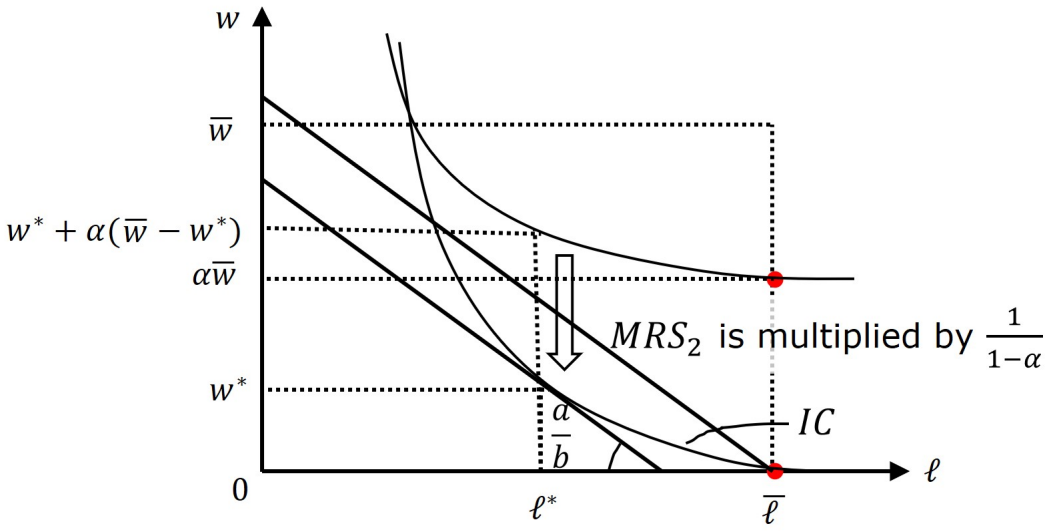


Fig. 4. Equilibrium analysis

A solution (ℓ^*, w^*) to maximization problem (8) lies on two curves: (i) the curve (depicted as IC) in which the MRS is multiplied by $1/(1-\alpha)$ from 2's original indifference curve, and (ii) the tangent line with slope a/b.

As IC passes through $(\overline{\ell}, 0)$, 1's utility is higher at (ℓ^*, w^*) than at $(\overline{\ell}, 0)$. Hence, 1's optimal strategy is to choose (ℓ^*, w^*) rather than to let 2 reject his offer.

4.5 Comparison and intuition

The following figure describes a comparison of the equilibrium outcomes under the two policies.

[14]For simplicity, we confine our attention to cases in which interior solutions exist.

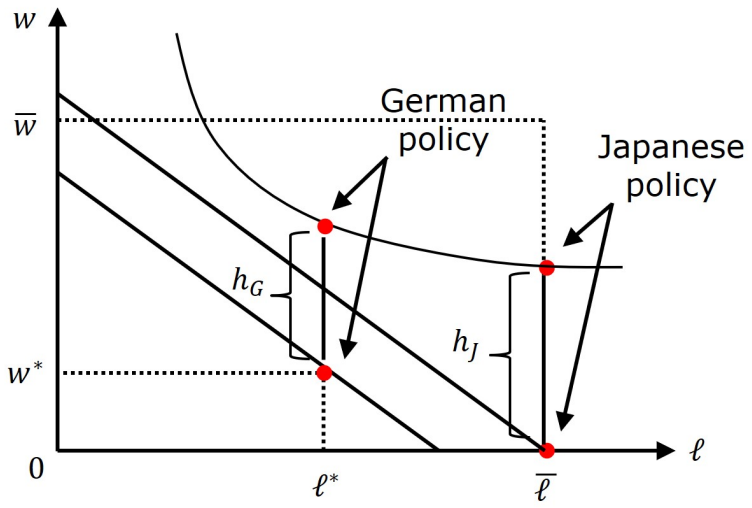


Fig. 5. Comparison of the Japanese and German policies

Player 2's utilities are the same in both cases. However, the German policy has the following three advantages over the Japanese policy:

1. Player 1's utility is higher.
2. Player 2 partly returns to work, which is desirable from the perspective of the original intention of the parental leave policy.
3. The government's expenditure h_G is smaller than h_J.[15]

From these points, we conclude that the German policy outperforms the Japanese policy. The intuition for this result can be described as follows:

- An employer can choose $(\overline{\ell}, 0)$ but chooses a different contract (ℓ^*, w^*), meaning that his utility increases.
- This choice is made while incorporating the employee's incentives.[16]
- A Pareto-improving contract is reached.
- Moreover, as the employer pays wages, government expedinture can be reduced.

[15]This follows from $h_G = \alpha(\overline{w} - w^*) \leq \alpha \cdot \overline{w} = h_J$.

[16]This point is reflected in the derivation of a subgame perfect equilibrium. Player 1 chooses his strategy in such a way that player 2 accepts his offer.

5 Discussion

5.1 Effect of changing the subsidy system on daycare allocation

In Section 4, we discussed the effect of changing the subsidy provision system on the bargaining outcome. Here, we discuss its effect on daycare allocation. As proven by Gale and Sotomayor (1985), at the parent-optimal stable matching (i.e., the outcome of the serial dictatorship algorithm), reducing the set of participating parents always benefits the remaining parents. Hence, if a parent changes her action from aiming for rejection to returning to work in the bargaining stage, then this change benefits all the parents in the allocation stage. This viewpoint supports the desirability of the policy in Germany.

5.2 Remark on quasi-concavity

In Section 4, we assumed that parents have quasi-concave utility functions. One might argue that this is a strong assumption in describing parents' preferences; if the parent has too little leisure time, then she needs to find alternative childcare (e.g., ask the grandparents to watch her child), which induces a discontinuous point in the indifference curve.

We emphasize that, in deriving a subgame perfect equilibrium, quasi-concavity does not play an essential role. The key points are as follows: (i) with government support, the set of acceptable contracts for player 2 expands and (ii) the intersection with player 1's indifference curve (equivalently, the equilibrium outcome) moves in the lower-left direction. Quasi-concavity is imposed to characterize an equilibrium mathematically, but the main message of the analysis (see Section 4.5) remains valid without the assumption.

5.3 Remark on the reduction in government expenditure

In Section 4.5, we argued that government expenditure can be reduced. This result crucially relies on confining our attention to case (ii) (see Section 4.3). In case (i), adopting the German policy might *increase* government expenditure. One might ask how to detect parents facing case (ii) in reality. This is a difficult question, and we do not yet have a clear answer. However, we believe that there is value in showing that there is efficiency loss in the target social problem. It might be possible to achieve a Pareto-improving outcome, which is valuable guidance for policy makers.

6 Conclusion

A matching market should consist of agents who want to obtain/provide objects; otherwise a matchmaker wastes time handling "dummy" agents who have negative effects on "true" agents. This paper reports an instance in which this principle is undermined by moral hazard. The problem was rooted in the central government's attempt to control unobservable actions. We highlight the limitations of the Japanese government's proposal and the advantages of the parental leave policy adopted in Germany. The key idea is (i) to make parents face observable actions and (ii) to encourage employers/employees to negotiate over better working conditions via monetary support.

In the 21st century, many developed countries, including Japan, face rapidly aging societies. Hence, it is urgent to introduce social security systems that support parents having a child. Such systems must be fair, efficient, and incentive-compatible. Game theory has analyzed these issues in various contexts and presents a useful tool for designing a desirable system.

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References

b-style Inc., "Survey on the way of thinking about the 'aim for rejection' behavior (in Japanese)." `https://www.bstylegroup.co.jp/news/shufu-job/news-16267/`, accessed on 2019-4-20.

Bergemann, Annette and Regina T Riphahn, "Female labour supply and parental leave benefits–the causal effect of paying higher transfers for a shorter period of time," *Applied Economics Letters*, 2010, *18* (1), 17–20.

Cabinet Office of Japan, "Considering proposals from local governments, 85th meeting (in Japanese)." `https://www.cao.go.jp/bunken-suishin/kaigi/kaigikaisai/teianbukai85gijishidai.html`, accessed on 2019-1-26.

Dionne, Georges and Scott E Harrington, "An introduction to insurance economics," in "Foundations of Insurance Economics," Springer, 1992, pp. 1–48.

Gale, David and Lloyd S Shapley, "College admissions and the stability of marriage," *The American Mathematical Monthly*, 1962, *69* (1), 9–15.

_ and Marilda Sotomayor, "Some remarks on the stable matching problem," *Discrete Applied Mathematics*, 1985, *11* (3), 223–232.

Hindriks, Jean and Gareth D Myles, *Intermediate public economics*, Vol. 2, MIT press Cambridge, 2006.

International Network on Leave Policies and Research, "Country reports on leave policies." `https://www.leavenetwork.org/leave-policies-research/country-reports/`, accessed on 2019-1-26.

Kamada, Yuichiro and Fuhito Kojima, "Fair Matching Under Constraints: Theory and Applications," *Working paper*, 2018.

Maeda, Masako, *Daycare problem (in Japanese)*, Chuokoron-Shinsha Publishing Company, 2017.

Ministry of Internal Affairs and Communications, "Working during the leave period (in Japanese)." `http://www.soumu.go.jp/schresult.html?q=%E8%82%B2%E4%BC%91`, accessed on 2019-2-25.

Musashino City, "Past data on daycare applications (in Japanese)." `http://www.city.musashino.lg.jp/kurashi_guide/hoikuen/1006830/1006850/1018128.html`, accessed on 2019-1-26.

Nash, John F, "The bargaining problem," *Econometrica*, 1950, *18*, 155–162.

Okumura, Yasunori, "School choice with general constraints: A market design approach for the nursery school waiting list problem in Japan," *Japanese Economic Review*, 2018. doi:10.1111/jere.12212.

Osborne, Martin J, *An Introduction to Game Theory*, Vol. 3, Oxford University Press, 2004.

Roth, Alvin E, "Bargaining experiments," in "Handbook of Experimental Economics," Princeton University Press, 1995, pp. 253–348.

_ , *Who Gets What–and Why: The New Economics of Matchmaking and Market Design*, Houghton Mifflin Harcourt, 2015.

Rubinstein, Ariel, "Perfect equilibrium in a bargaining model," *Econometrica*, 1982, *50*, 97–109.

Ruhm, Christopher J, "The economic consequences of parental leave mandates: Lessons from Europe," *The Quarterly Journal of Economics*, 1998, *113* (1), 285–317.

Shavell, Steven, "On moral hazard and insurance," in "Foundations of Insurance Economics," Springer, 1979, pp. 280–301.

Suzuki, Wataru, *An economist tries to reduce the daycare waitlist to zero (in Japanese)*, Shincho-sha Publishing Company., 2018.