Sharpening the Arithmetic of Active Management

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Abstract.

I challenge Sharpe's (1991) famous equality that "before costs, the return on the average actively managed dollar will equal the return on the average passively managed dollar." This equality is based on the implicit assumption that the market portfolio never changes, which does not hold in the real world because new shares are issued, others are repurchased, and indices are reconstituted so even "passive" investors must regularly trade. Therefore, active managers can be worth positive fees in aggregate, allowing them to play an important role in the economy: helping allocate resources efficiently. Passive investing also plays a useful economic role: creating low-cost access to markets.

Keywords. Active investing; passive investing; market efficiency; asset management; asset pricing; index funds; portfolio choice.

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Sharpe's (1991) famous "arithmetic of active management" states that

> "it *must* be the case that
> (1) before costs, the return on the average actively managed dollar will equal the return on the average passively managed dollar, and
> (2) after costs, the return on the average actively managed dollar will be less…
> These assertions will hold for *any* time period. Moreover, they depend *only* on the laws of addition, subtraction, multiplication and division. Nothing else is required." [Emphasis in original]

Sharpe's arithmetic has been invoked by Warren Buffett,[1] is often stated as incontrovertible fact by speakers at conferences followed by a triumphant "QED!", and is cited as proof that active management is "doomed" in aggregate (French 2008). If active management is doomed in aggregate, then so is our market-based financial system because we need someone to make prices informative. However, we may avoid doom based on my arithmetic.

Sharpe's powerful insight is that one active investor's gain is another active investor's loss, which aggregates to zero for all active investors. This useful insight is correct when considering a fixed set of securities over a single time period, but, in the real world, the set of securities in the market changes over time.

Sharpe's argument thus abstracts from a key aspect of addition and subtraction; namely the addition of new firms and shares and the subtraction of disappearing ones. Although seemingly minor, the market portfolio does change over time such that even "passive" investors must regularly trade, for instance in connection with issuances, share repurchases, index inclusion and deletions, and so on. Whenever passive investors trade in order to maintain their market-weighted portfolios, they may trade at less favorable prices than active managers, which breaks Sharpe's equality.

This turnover of the market portfolio is important for two reasons. First, the changes of the market portfolio may be large enough that active managers in aggregate can add modest, yet noticeable, returns relative to passive investors. Second, the issuance of securities is at the heart of a market-based economy: capital markets are about raising capital. When we put these reasons together, we see that active management can be worth positive fees in aggregate, which in turn allows active managers to provide an important, beneficial role in the economy — helping to raise capital and allocate resources efficiently.

Sharpe (1991 and 2013) is fighting a good fight in pointing out the importance of fees and the flaws of many arguments by self-interested active managers. I think that low-cost index funds is one of the most investor-friendly inventions in finance and this paper should not be used as an excuse by active managers who charge high fees while adding little or no value.

Nonetheless, we need the right arithmetic and all the assumptions on the table. My arithmetic shows that active management *can* add value in aggregate, but whether it *actually* does, and how much, are empirical questions. Based on realistic arithmetic, we need to empirically evaluate the costs vs. benefits of active management. Investors should understand how fees diminish performance, but we shouldn't expect to be able to allocate global capital in a market-based system without any active management.

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[1] Berkshire Hathaway Inc., 2016 annual report, p. 24-25.

Sharpe's hidden assumption: A market without addition or subtraction

Sharpe's active management arithmetic, in its beautiful simplicity, is that

> "Each passive manager will obtain precisely the market return, before costs. From this, it follows (as the night from the day) that the return on the average actively managed dollar must equal the market return. Why? Because the market return must equal a weighted average of the returns on the passive and active segments of the market. If the first two returns are the same, the third must be also."

Sharpe's argument relies on the notion of a "passive investor," but what does this really mean? Two definitions seem plausible:

i) an investor who holds all securities in their market-capitalization weights;
ii) an investor who never trades.

Sharpe defines a passive investor based on i), but people tend to make the implicit assumption that i) and ii) are equivalent. Indeed, in a world with a fixed set of securities (i.e., no issuance or repurchases), part i) does (sort of) imply part ii); that is, if you start with market cap weights, your portfolio remains market-cap weighted no matter how prices change (which is a helpful property of market-cap weights).

But – as anyone who has tried this in practice knows – you first need to buy your portfolio, and you eventually need to sell it. Furthermore, holding a market-cap weighted portfolio does require trading because securities come in and out of the market as shares are issued, shares are repurchased, and indices reconstituted.

If you read Sharpe's argument carefully, he states that his conclusion holds for "any time period." Strictly speaking, if passive investors somehow magically arrive at the beginning of any time period with all securities at their market-cap weights bought at the current mid-quote, then his conclusion follows and is correct. But, real life is more than one time period and it's important to note that "any time period" is not the same as "over all time periods." Passive investors must trade to achieve their market-cap weights. Sharpe's equality identity breaks down when we take this trading activity into account. Indeed, if passive investors on average buy at a premium and sell at a discount relative to active investors, then active investors can outperform passive before fees. The appendix contains a simple equilibrium model in which active managers outperform passive before fees, while the after-fee performance difference naturally depends on the level of fees vs. the value added.

So the implicit assumption lies in Sharpe's definition of passive and the abstraction from trading. Sharpe discusses trading in a footnote[2] to his paper and, while the footnote is precise and helps to clarify the issue, others have seemingly missed this important point and incorrectly interpreted his result as a truth that relies on no assumptions other than the laws of arithmetic. Some researchers have even

[2] Footnote 4 in Sharpe (1991) states: "We assume here that passive managers purchase their securities before the beginning of the period in question and do not sell them until after the period ends. When passive managers do buy or sell, they may have to trade with active managers; at such times, the active managers may gain from the passive managers, because of the active managers' willingness to provide desired liquidity (at a price)."

claimed that his argument "always" holds even for subsets of the market and trading strategies with larger turnover. For instance, Fama and French (2009) say "The same arguments apply whatever one takes to be the market, for example, value stocks, growth stocks … active investors can only win at the expense of other active investors. In short, active investing in any sector is always a zero sum game." Here, the potential error is clearly larger than in the case of a market-weighted equity index because of the larger turnover required to track a value or growth strategy. Indeed, "passive" value investors may lose from their trading as stocks regularly switch between being classified as value or growth.

How active managers can outperform: market is *not* buy-and-hold

I first provide some conceptual examples of why active management can outperform due to the necessary trading of passive investors. Then, in the next sections, I discuss the magnitude of this passive trading in the real world and the resulting performance impact. The appendix contains a simple equilibrium model in which active managers beat the market in aggregate.

Example 0: Who are the "active managers"?

Most active managers typically suggest that they add value by selecting good securities that outperform the market, not necessarily exploiting price moves related to changes in the market portfolio. Sharpe's powerful argument shows that the activity of selecting good securities within a fixed set of securities is a zero-sum game so not all managers can win this game. While some managers do, in fact, emphasize their value added in connection with changes in the market (e.g., the so-called "event driven" hedge funds),[3] let's nevertheless first dig further into Sharpe's zero-sum case.

The most obvious reason that "informed active managers" can outperform in aggregate is that they trade against "non-informational investors" who are motivated by liquidity needs, institutional constraints, hedging, or are influenced by behavioral biases. This argument is ruled out, however, by Sharpe's definition of "active managers" as everyone who is simply not passive. In other words, Sharpe's approach groups together informed investors along with all these non-informational traders. For example, if naïve investors buy glamour stocks and informed active managers benefit from value stocks, then these gains do not "count" because Sharpe treats all these investors as one group. Indeed, since the loss by naïve investors equals the gain by informed managers, the net profit of the entire group is zero. Similarly, if leverage constrained investors tilt toward risky stocks and less constrained managers profit from betting against beta, this is counted as a net of zero. If a central bank intervenes in the FX or bond markets for purposes of managing the macroeconomy and informed asset managers profit as a result, this is also counted as a zero. If pension funds hedge their asset-liability mismatch and fixed-income traders profit from providing liquidity, this is a net zero according to Sharpe's definition.

The main point here is that active managers may systematically profit from other non-passive investors with special motives to trade. This part of the debate is well-known and recognized by Sharpe (1991). It depends on semantics and the measurement on the relative importance of different types of

[3] Event driven hedge funds specialize in trading around corporate events such as mergers, new issues, SEOs, spin-offs, etc.

non-passive investors. Therefore, I will not go into this debate and leave this example at number "0" as Sharpe will rightly say that it doesn't count by his method.

Instead, let us focus on the indisputable cases where even passive investors must trade and all non-passive investors can win in aggregate. The idea is simple: since passive investors must trade to hold the market, nothing ensures that they trade at the same prices as active investors, and, therefore, nothing ensures that the two groups get the same return.

Example 1: IPOs, SEOs, and share repurchases

Suppose that we define the market portfolio as all securities traded on any exchange in the world. Recall that, each year, many new securities are listed while some other securities are delisted. For example, firms that go public sell their shares in an initial public offering (IPO) before they are floated on the exchange.

Suppose first that passive investors do not participate in the primary market (i.e., the market for new shares like the IPO). Research has shown that IPO securities are, on average, sold at a discount in the IPO relative to the price in the secondary market when the shares start trading on the exchange.[4] This means that informed investors can buy the new shares cheaply in the IPO and then sell some of the shares in the secondary market to other (passive) investors at a premium. In this case, clearly the group of informed, active investors can outperform the group of non-informed, passive investors before fees.

Some may ask, what happens if passive investors participate in the IPO? In this case, they will ask for the same fraction of the shares in any IPO. For example, if half of the investors are passive, they would ask for half of the shares being issued—let's keep this assumption for clarity. Active investors will seek out shares in the IPOs that they deem to be priced cheaply based on their security analysis, while avoiding those in overpriced IPOs. Hence, an underpriced IPO will be oversubscribed and, therefore, all investors (including the passive ones) will get fewer shares than they asked for. When the price jumps up in the secondary market, the passive investors, in seeking to get to their market-cap weighting, must buy the additional shares at a price above the IPO value, again losing to the informed active investors. For an overpriced IPO, the passive investors end up with all of the shares they asked for while the firm (or its bank) retains some of the shares they planned to issue, and these shares will later be sold to the active investors at a discount in the secondary market following a decline in price to fundamental value once listed on the exchange. Again, the active investors obtain the shares at a more favorable price than the passive investors.

In addition to IPOs, a passive investor must also trade in connection with seasoned equity offerings (SEOs) and share repurchases. Here, similar arguments apply, and passive investors face a cost due to adverse selection while active management can outperform.

Some skeptics may question whether active investors can systematically profit from IPOs and other changes in the market portfolio. The general point is that passive investors are not *guaranteed* the same IPO performance as the group of active investors since they trade at different prices and quantities, thereby breaking Sharpe's equality. Once the equality is broken, is it so hard to believe that those who

[4] Ljungqvist (2005) documents an average IPO underpricing of 10-20%. The IPO underpricing is needed to compensate passive investors from adverse selection and give active investors an incentive to take part in the underwriting process and spend resources in determining the value of the securities (Rock 1986).

spend resources collecting information are compensated for their costs in the form of better prices as theory and evidence suggests?

Example 1b: What happens if everyone is passive?

To level the playing field for passive investors, can't we simply ban active management from IPOs? Well, even if we could, this would not work. To see why, suppose that only passive investors could participate in IPOs. Then they would get 100% of the shares and, as discussed above, they would lose to active investors if the passive needed to sell part of the shares in the secondary market.

What if we ensured that everyone was passive in both the primary and secondary markets? In this case, all shares would be bought in every IPO at whatever the offer price because no investor would perform security analysis and every investor would simply request his fraction of shares (the same fraction that he or she owned of the rest of the market). This indiscriminate buying might initially lead to a fantastic IPO boom at high prices as most anybody could take a company public at any price. Ultimately, when many of these new, "opportunistic" companies go bankrupt, the confidence in the financial system would quickly vanish as investors would exit from the market, leading to a collapse in security prices, a complete halt in new issues, even for good companies. The economy would then come to a grinding halt.

In summary, in order for investors to be willing to buy new securities, these securities need to be sold at fair prices. To set fair prices, some investors must be active and collect information about the securities. Hence, when we take into account that capital markets are also about raising capital, we see that informational efficiency, which requires active investors, can have a significant positive impact on the real economy.

Example 2: Indices

Suppose instead we define the market portfolio as some index, e.g., the S&P 500 stock market index, or the MSCI World stock market index, or a combination of indices across global equities and global bonds. In this case, the market portfolio also clearly has turnover since securities are added to, and deleted from, the index in so-called index reconstitutions (in addition to share issuance and repurchases as discussed above).

When a security is added to an index, index investors simultaneously buy large numbers of shares, pushing up the price. Conversely, when securities are deleted from an index, index investors sell those securities, resulting in price drops. These price moves translate into costs for index investors and profit opportunities for active managers. For example, when a security is added to an index, active investors can buy the security ahead of the rebalancing date (or, they already own it) and then sell them to the passive investors at a higher price when the security is added. Likewise, when a security is known to be deleted from the index, active investors can short sell the security when this fact is known, and then cover the short at a lower expected price when the index deletion actually happens. These effects mean that active investors have a chance to outperform indices.

We see that even a passive investor who perfectly tracks the performance of an index incurs an implicit cost – since the cost is built into the index itself. Indeed, the index is *defined* as buying added securities at high prices and selling deleted ones at low prices.

Example 3: What "market"? Passive investors only own a subset of assets

Another issue is that no one, not even Sharpe, actually knowns what the market portfolio is in practice. Indeed, Sharpe's Nobel lecture states that: "no financial futures contract corresponds to the overall market portfolio." Hence, even those who seek to follow Sharpe's advice and buy the market portfolio would probably have differing interpretations of what constitutes the market portfolio and this interpretation would probably change over time. Some people might only buy domestic stocks, others may buy various indices of global stocks, while others could focus on stocks and bonds in only developed markets, while yet others may include emerging markets, corporate bonds, and so on. These differing interpretations on what defines the market portfolio means that the resulting portfolios need not add up to the true market portfolio so the residual creates an opportunity for active managers.

As a simple example, if passive investors only buy stocks included the S&P500, then these stocks may become expensive, and active investors who also hold the non-included stocks may earn higher average returns as a result.

Further, since the market portfolio is the portfolio of all investible assets, it should also include private equity, venture capital, real estate, among other private assets. However, you cannot be a passive investor in the private markets because you cannot demand to co-invest in every private deal at the same terms as other investors. Therefore, passive and active investors clearly obtain different outcomes when we include all the private assets.

Note that examples 1-3 are related since the typical life-cycle of a successful firm is to start is a private firm, later be listed on an exchange in an IPO, later yet be added to an index (and possibly issue other corporate securities, participate in mergers, spin-offs, etc.). While passive investors only hold the stock when it is added to the index (to keep their strategy simple and avoid complications with respect to private firms and IPOs), active investors may benefit from participating in the full life-cycle.

Example 4: Rebalancing and market timing

Passive investors must decide at each time period how much to invest in risk-free securities vs. the market portfolio of risky assets. How should they make this choice? An investor's allocation to the market should be based on his risk tolerance and perceived risk-vs-reward (Sharpe ratio) of the market – so the portfolio allocation is an active choice even for the passive!

Hence, passive investors must rebalance their portfolio allocation over time as their risk preference change, the market's risk-return profile changes, and other reasons. Indeed, they must initially buy their portfolio and will eventually need to sell it. Along the way, they need to decide whether to re-invest dividends or sell some fraction of their portfolio to pay for a new house, a car, or other expenses.[5]

Such rebalancing by passive investors involves buying or selling all securities in proportion to their respective benchmark weights. If there are an equal number of passive buyers vs. sellers at a given time, these portfolio trades could occur at little or no cost to both parties. However, there will likely be times with more sellers than buyers and vice versa and, therefore, passive investors will buy or sell in aggregate. At such times, someone must take the other side of the aggregate trade by the passive, and

[5] Furthermore, the passive trades in connection with IPOs and index reconstitutions discussed above are more complex than many investors realize. When a passive investor buys shares in an IPO, where does the money for these shares come from? If he does not want to use cash, he needs to sell parts of all his other security holdings.

this would need to be the group of active investors. Prices must necessarily respond, which translates into a transaction cost for passive investors and a corresponding trading gain for active investors.

The rebalancing costs of passive investors can arise at different time scales. At a high frequency, there may be costs associated with trades that create intra-day imbalances. At the other end of the spectrum, there may be years when passive investors herd into risky assets, potentially making (their preferred version of) the market portfolio overvalued. In other years, passive investors may panic and move toward risk-free securities, potentially making the market cheap. Active investors doing the opposite (since market must clear) may then benefit from buying the market when it is cheap and selling when it is high (a form of market timing). Hence, even if active and passive investors have similar returns on their risky investments in each year, active investors could over time realize higher dollar-weighted returns if their market-timing decisions are better in aggregate.[6]

Active managers also incur transaction costs. Given that active managers trade more than passive ones, they may in fact incur larger transaction costs. However, transaction costs consist both of pure commissions/fees, which are a drag on the whole universe of investors, and market impact costs, which are zero-sum. In other words, one active investor's market impact cost may be another active investor's trading profit. Hence, the aggregate effect of rebalancing on active investors is the sum of (i) minus the loss associated with commissions, (ii) plus zero coming from the zero-sum market impact-game among active, (iii) plus the potential profits from providing liquidity when passive investors trade.

The magnitude of trading required by "passive" investors

The next question is whether the arguments above are mostly theoretical or have a significant practical relevance. In other words, is Sharpe's assumption so close to being true that we can simply ignore the "error term"?

To address this issue, let us first consider the turnover of passive investing before we turn to the return implications. As a simple first look, suppose that you owned all stocks at the major US exchanges in 1926, i.e., all common stocks included the Center for Research in Security Prices (CRSP) database. Further, suppose that you stayed passive in the *inactive* sense of the word (part ii above), not Sharpe's sense (i above), meaning that you never participated in any IPOs, SEOs, or share repurchases. Then figure 1 shows how this portfolio evolved as a fraction of the market portfolio of all listed stocks.[7] As seen in the figure, the investor who never trades gradually owns a smaller and smaller fraction of the market because he never buys the new shares, and, over time, this effect is quite large. In just 10 years, the inactive investor's portfolio has dropped to about 60% of the market.

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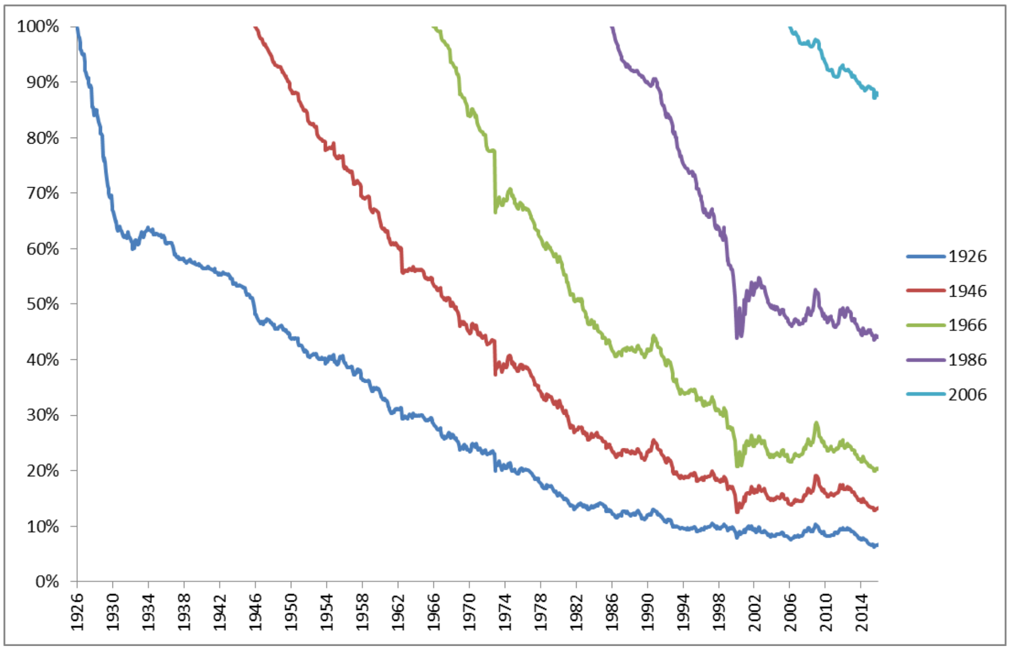
[6] If passive investors only hold publicly traded securities and these securities are mispriced, passive investors may also be exploited through firms' decisions to list on exchanges or delist. That is, the effects discussed in example 1 may be exacerbated by endogenous decisions to add and subtract securities from the public market at opportune times.

[7] This figure reports the total market value the buy-and-hold strategy as a fraction of the total market value of all shares, adjusting for stock splits by assuming that the passive investor is treated like other investors in any stock split. While some readers might be surprised that doing nothing is not enough to be passive, others may be surprised that the investor from 1926 continues to hold as much as 10% of the market today – this is due to old giants like Standard Oil, GE, Chevron, and Coca Cola.

Figure 1. An inactive investor is different from Sharpe's "passive" investor. This figure considers what happens to an investor who starts off with the market portfolio but never trades after that. The solid blue line shows an investor who bought the entire US stock market in 1926 and did not participate in any IPOs, SEOs, or share repurchases and did not reinvest any dividends, showing the resulting fraction of the market that is owned over time. We see that the investor gradually owns a smaller and smaller fraction of the market because he does not buy shares in the new firms and in the equity offerings of old firms. Already after 10 years of not trading, the inactive investor only owns about 60% of the market. The other lines show the same for investors who start in 1946, 1966, 1986, and 2006.



As the flipside of this issue, we can also focus on Sharpe's definition of a passive investor and ask how much trading is required to continue to own all the securities in the same market-cap weights. For example, how much trading is required to own 1% of all securities day in and day out, taking into account securities that are added and deleted? Said differently, how much trading is required to be "passive" in the sense of Sharpe? This question is answered in figure 2.[8] The figure shows the turnover for different definitions of the market portfolio.

Looking at all equities in CRSP from 1926 to 2015 in panel A, the average annual turnover is 7.6% per year. This turnover is computed as the sum of absolute changes in shares outstanding as a percentage of the total market value in the previous month. Of this turnover, 5.3% consists of new listings (e.g., IPOs),

[8] The CRSP turnover is calculated from the CRSP database 1926-2015 for US common stocks (share codes 10 and 11). The fixed-income issuance is calculated as annual issuance divided by bonds outstanding, averaged over 1996 to 2015, based on data from the Securities Industry and Financial Markets Association http://www.sifma.org/research/statistics.aspx. The S&P500 and Russell 2000 numbers are from 1990-2005 by Petajisto (2011), supplemented by SEO and repurchase data on S&P500 from CRSP over the same period. The BAML numbers are calculated based on the BAML database 2000-2016.

SEOs, de-listings, and share repurchases. The remaining “other” category of 2.4% includes mergers that may not require trading.

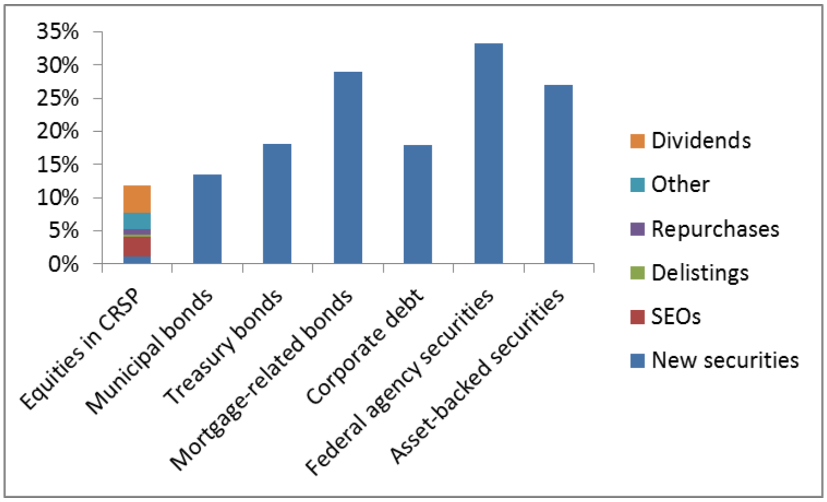
The figure also shows the 4.2% dividend rate, which leads to trading for investors who re-invest all proceeds. Further, the turnover number used is the more conservative so-called “one-sided” turnover, meaning that it assumes that the investor buys new shares with cash and invests the proceeds from repurchases in cash. If instead we assumed that the investor would sell shares to finance the purchase of new shares and buy shares with proceeds from sales, then we would get a “two-sided turnover,” which could be as high as double the 5.3% one-sided turnover number reported, but, since cash inflows and outflows can sometimes be netted, the true two-sided turnover would be somewhere in between.

While equities have the nice property of being perpetual, bonds have finite maturity. Hence, new bonds are continually issued, which makes it all the more obvious that trading is required as also seen in figure 2, panel A. For example, even if the set of firms doesn’t change, firms constantly issue new corporate bonds and other types of securities. Likewise, government bonds are continually issued, as are mortgage bonds, municipal bonds, and so on. Holding the market portfolio of all these fixed-income securities requires a turnover of 20% to buy new issues (which basically means that bonds have an average maturity around 5 years such that one fifths mature each year).

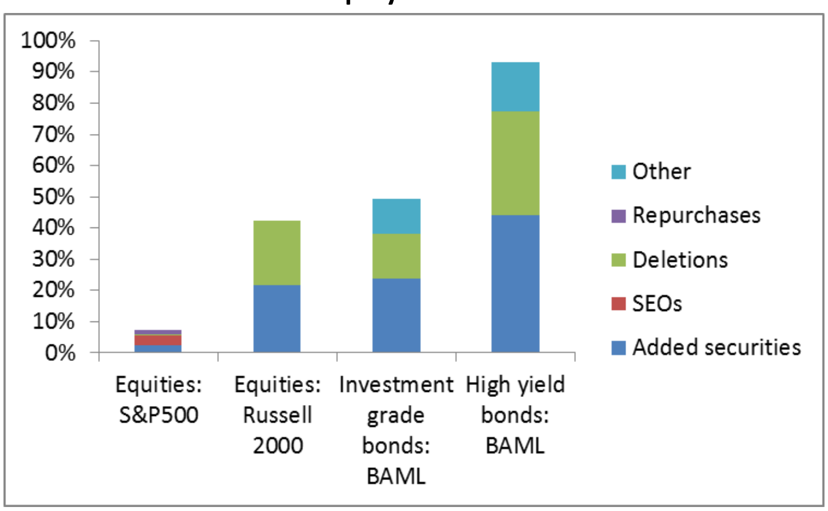
Figure 2, panel B shows the turnover of different major equity and bond indices. As seen in the figure, the turnover of the indices is large due to various types of additions and deletions from the index and because tracking an index requires a two-sided turnover as discussed above. The Russell 2000 index has a much larger turnover than the S&P500 because stocks are both added when they are large enough and deleted when they become too large (and are moved to the Russell large-cap index). The bond indices have even larger turnover because of the large issuance seen in panel A and because of rating changes, maturity dropping below 1 year, defaults, and other events.

Figure 2: Trading by a "passive" investor in the sense of Sharpe. This figure shows the average yearly turnover for a "passive" investor who keeps market-cap weights in a given equity or bond investment universe. Panel A shows the turnover for all US listed stocks included in CRSP and the US municipal bonds, Treasury bonds, mortgage-related bonds, corporate debt, federal agency securities, and asset-backed securities. Panel B shows the turnover for equity indices (S&P500 and Russell 2000) and corporate bond indices (BAML investment grade and high yield indices).

Panel A: Equities and bonds



Panel B: Equity and bond indices



Some index funds try to limit their turnover costs in various ways, e.g. bond funds often hold only a subset of the index. Nevertheless, when looking at the reported turnover rates of actual index funds, we do see meaningful trading. For example, S&P500 index funds often report turnover rates above 3%. The reported turnover for a U.S. mutual fund is the *lesser* of their purchases and sales, excluding the trading of derivatives and in-kind transactions. Hence, the actual trading is at least twice that number, and it could be significantly more, sometimes larger than 10% even for some of the best run funds. Bond index funds routinely report very high turnover numbers, sometimes in the hundreds of percent.[9]

Lastly, let us not forget that the true turnover of passive investors is likely to be higher than the numbers in figure 2 for several reasons. First, investors must buy and sell when they decide to save or consume or change their risk aversion. Second, investors tend to change their definition of the market. Emerging markets are added and deleted, frontier markets open up and are closed down (e.g., due to relaxing or tightening capital controls), and countries and asset classes are added and deleted as they are deemed appropriate or inappropriate investments for various reasons (e.g., the risk of a war or systemic risk in the financial system), and, again, active may benefit from all these changes. Third, passive investors often trade to hedge FX exposures and they trade to roll over futures contracts and other derivatives.

How large is the hidden cost of passive investing?

We have seen that passive investing entails a non-trivial amount of trading. We next consider the cost associated with the required trading by passive investors or, equivalently, the expected outperformance of active management resulting from that trading.

Starting with equity issuance and repurchases as discussed in example 1 above, note that IPOs have been underpriced by 10-20% on average in the U.S. over long time periods as well as in a number of other countries (e.g., Ljungqvist 2005). Given the new listings of 1.2% per year reported in figure 2, these numbers translate into a performance difference of passive vs. an informed participant in the new listings of about 1.2% times 15%, i.e. about 18bps (although not all of these new listings in CRSP are IPOs). Similarly, if SEOs are underpriced by about 2% on average, then these give rise to a performance effect of about 6bps. Share purchases are harder to evaluate, but they are a smaller group and passive investors may more easily avoid adverse selection. As seen in figure 2, the trading of bonds required by passive investors is substantial, which adds to their costs. Corporate bonds also have IPO underpricing, with an average magnitude of 0.47% for high yield and 0.02% for investment grade (Cai, Helwege, and Warga 2007).

Most passive investors actually follow specific indices, as discussed in example 2 above. Stock price moves around index reconstitutions imply costs for passive investors and Petajisto (2011) reports that for "additions to the S&P 500 and Russell 2000, we find that the price impact from announcement to effective day has averaged +8.8% and +4.7%, respectively, and −15.1% and −4.6% for deletions." Based on these price moves and the turnover reported in figure 2, Petajisto (2011) estimates the lower bound of "the index turnover cost" to be "21–28 bp annually for the S&P 500 and 38–77 bp annually for the Russell 2000." Note that this cost is embedded in the index itself (so it is separate from whether a given passive index fund delivers the same return as the index). Said differently, if you pay a manager 10bp to track the S&P500, you could be paying an explicit 10bp on top of an implicit cost of more than 25bp.

[9] These turnover numbers can be computed by looking at the separate numbers for purchases and sales in the footnotes to the annual reports. The turnover could be higher still because certain types of trades are not included in these statistics, for example derivatives trading.

Further, index funds often try to recover their costs via securities lending, but this revenue can also be earned by active managers.

The cost of passive investing in fixed-income securities is also non-trivial. Index inclusion or exclusions move prices from 0.20% to 5% depending on the type of event (Dick-Nielsen 2012) and, as seen in figure 2.B, the turnover is large so this can lead to a meaningful effect on returns.

In addition to these costs, passive investors incur rebalancing costs when they seek to simultaneously buy or sell as discussed in example 4, as do active investors. An exhaustive analysis of the aggregate costs of passive investing is beyond the scope of this paper, and such an analysis should also consider the magnitude of the gains to active relative to their portfolio size (see the model in appendix). I simply note that the preliminary findings suggest that active management can, in aggregate, add a modest, but non-trivial, amount of value relative to passive investing, before fees. Further, even if the cost of passive investing is modest in today's market, this cost would grow if most investors became passive. Nevertheless, the current value added for active managers may be below their typical asset management fees, which would lead the average active manager to underperform after fees and expenses as we discuss next.

Evidence on the performance of active managers

We next turn to the empirical evidence on performance. Active mutual funds have, on average, performed similar to the market before fees while underperforming after fees (Fama and French 2010), or outperformed before fees while performing similar to the market after fees (Berk and van Binsbergen 2015), depending on the sample and benchmark. Hence, active mutual fund fees have been so large on average that they consumed all of, or more than, the managers' alpha, so the average retail investor would have been well served by following Sharpe's advice to use low-cost passive investing. At the same time, institutional asset managers "outperformed their benchmarks by 96 basis points per year before fees, and by 49 basis points after fees" (Gerakos, Linnainmaa, and Morse 2016) so they have benefitted from active investing in aggregate according to this study.

Hence, the average manager across both segments may outperform slightly before fees, but finding a manager that can be expected to outperform after fees is difficult. This evidence suggests that Sharpe's equality could be violated, but, nevertheless, also supports his idea that low-cost index funds are extremely useful, especially for individual investors.

More broadly, the evidence is consistent with the model of Garleanu and Pedersen (2015), namely that investors looking for good asset managers face a similar problem to asset managers looking for good securities. In both cases, you can only hope to outperform if you spend resources locating a good manager or good security. Naturally, large institutional investors can better afford spending resources on a manager selection team. So, it is not a surprise that institutional investors have been more successful in their active management than smaller investors.

It is time to sharpen the debate on active vs. passive management.[10] Some commentators have interpreted Sharpe's equality as a law that holds without assumptions and called for everyone to be

[10] This debate has been intense ever since passive investing was introduced, see for example Langbein and Posner (1977).

passive, while active investors have an incentive to "talk up" their profits and some have even claimed that "passive investing is worse than Marxism."[11] Instead, we should find a common ground by first recognizing that both active and passive management can have value and then attempting to empirically evaluate their costs vs. benefits.

Do we need all the trading?

We have seen that active management plays a crucial role for firms' ability to efficiently finance new projects by issuing new securities. However, some people might still argue that most of the active trading is useless. The argument goes something like this: "sure, trading can be good when it helps firms raise capital to make real investments, e.g. to build a factory that produces medicine, but this is only about *x*% (see figure 2) so the majority of active trading is useless." This argument might be correct if we assume that, despite no trading in the secondary markets, (a) firms could still issue new securities at the same prices, and (b) the liquidity needs of passive investors were still met. But these assumptions run counter to reason and a lot of research.

Active management makes secondary market prices more informative and investors value the ability to trade in liquid markets (Acharya and Pedersen 2005). As a result, a firm's cost of issuing new securities is lower when secondary markets are more liquid (Butler, Grullon, and Weston 2005; Andrew and Pagano 2006).

In other words, if we eliminate all the "extraneous" trading of active management, then firms' cost of issuing securities increases. For instance, the pharmaceutical may no longer build the new factory and produce the medicine if the cost of raising capital increases. To see why at a more practical level, suppose that you were buying a house. Would you be more comfortable bidding a large amount of money if you knew that you could easily sell the house later? Surely, the answer is "yes" for most people, so liquidity matters. Would you be more comfortable bidding a large amount of money if you knew the prices of a lot of comparable houses and knew that these prices had been set efficiently? Again, surely "yes," efficient and transparent secondary markets matter.

If most investors were passive, the liquidity in individual securities would vanish as investors would only trade overall indices. The collapse in liquidity and the lack of active management would make prices much less informative. When the secondary market is illiquid and uninformative, buying in the primary market becomes much riskier, which in turn increases firms' cost of finance.

So, yes we need more trading than the trading in the primary markets, even if we may not need as much turnover and as much active management as we have today as more and more investors realize the benefits of low-cost passive investing.[12]

[11] "The Silent Road to Serfdom: Why Passive Investing is Worse Than Marxism," whitepaper by Sanford C. Bernstein & Co., LLC., 2016.

[12] Pastor and Stambaugh (2012) study the size of the active management industry.

Conclusion: the future of asset management

Sharpe's arithmetic has profound implications for the future of active management: Since active managers lose after fees, investors should move money out of active and into passive. Where does this process end? One would think that less active management means less efficient markets, which makes profiting from active management easier, restoring equilibrium (as in "normal" supply/demand economics). Not so, according to Sharpe's arithmetic. Sharpe's arithmetic means that active managers lose regardless of how inefficient the market is. So, eventually all capital should leave the doomed active management industry, spelling doom for capital markets, according to Sharpe's arithmetic.

My arithmetic paints a less gloomy picture. As I have shown, active managers can profit in aggregate, and more so if little capital is allocated to active. Therefore, the future of active management is not doom. At the same time, the historical record of average active manager returns after fees is not impressive, so more and more investors will surely recognize the benefits of passive investing. Hence, I expect that the fraction of passive investors will continue to grow, but converge to a number less than 100%.

More broadly, the capital market is not a zero-sum game – it is a *positive-sum game*. Firms benefit from access to capital markets, passive investors benefit from low-cost access to investing in diversified markets, and active managers benefit from their information collection efforts through potentially even higher investment returns before costs. Active management is socially valuable when it helps finance promising new firms, which increases the collective wealth.

Said differently, much of financial economics considers what is called a "pure exchange economy" (Lucas 1978) which basically means that the set of companies and what they produce are taken as given so capital markets are used only for risk sharing – i.e., a big insurance scheme – which works best if no one is informed. However, in the real world, capital markets are also used for *raising capital* and this function, which requires active investors, is crucial for economic growth.

In summary, active and passive investors can co-exist in an efficiently inefficient market equilibrium where passive investors benefit from low costs and active investors benefit from their efforts in making markets close to efficient.[13]

[13] Small investors likely perform better with low-cost passive investing while large investors are more likely to benefit from being active. The marginal investor should be indifferent between passive investing vs. spending resources finding an active manager who is worth more than his fees (see the formal model of asset management by Garleanu and Pedersen 2015 and the practical examples of Pedersen 2015).

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Appendix.

Stylized model: passive match the "market" and active beat it

This section seeks to present the simplest possible model in which the market portfolio changes over time, as it does in the real world. For this, we consider two kinds of risky securities: those included in passive investors' definition of the "market portfolio" (denoted by i for included) and those that are not included (denoted by n). For example, the included securities can be (a) the members of an index such as the S&P500 or (b) stocks traded on a major exchange. Non-included stocks in interpretation (a) are those outside the S&P500 index, and, in interpretation (b), they are firms that are yet to list on the major exchange.

Securities switch between being included in the index or not. Specifically, a fraction s^u of non-included securities "switch up" to being included at the end of each time period. For example, we can imagine that these securities are added to a stock index (or, in the alternative interpretation, they are listed on the exchange). Likewise, a fraction of the included securities are deleted from the market at the end of each time period, denoted s^d for switching down.

Included securities have a price premium of *x* relative to securities that are not included (as seen in the equilibrium model below and empirically by e.g., Petajisto, 2011). That is, when a security is included its price will be $(1+x)$ times what it would have been outside the index. For simplicity, we assume that all securities have similar risk characteristics such that the only difference between them is whether they are included or not. The return of included returns between time *t* and *t*+1 is then

$$r_{t+1}^i = \frac{D_{t+1}^i + \left(1-s^d\right)P_{t+1}^i + s^d P_{t+1}^i/(1+x)}{P_t^i} - 1$$

Here, the first term is the dividend, the second term is the value of the shares that remain included in the market, and the third term is the value of shares deleted from the market. We can write the expected return in terms of the dividend yield $\delta_t = \frac{E_t(D_{t+1}^i)}{P_t^i}$ and the expected price appreciation $\mu_t = \frac{E_t(P_{t+1}^i)}{P_t^i}$:

$$E_t(r_{t+1}^i) = \delta_t + (\mu_t - 1) - s^d \mu_t \frac{x}{1+x}$$

In other words, the expected return on included securities is the return in the absence of deletions (the dividend yield plus the price appreciation), minus the negative impact of deletions (which depends on the fraction of deletions s^d and the price drop x). Similarly, the return on non-included securities is

$$r_{t+1}^n = \frac{D_{t+1}^n + (1-s^u)P_{t+1}^n + s^u P_{t+1}^n(1+x)}{P_t^n} - 1$$

so the expected return is

$$E_t(r_{t+1}^n) = (1+x)\delta_t + (\mu_t - 1) + s^u x \mu_t$$

Here, the first term means that non-included securities earn a higher dividend yield because they have a lower price for a given level of dividends. The second term states that non-included securities earn the same relative price appreciation. The third term says that non-included securities enjoy a benefit from the price increases associated with inclusions in the market. Hence, this simple model can capture the idea that IPO's increase in value upon listing and stock prices increase when added to a major index (even though we do not consider information asymmetries and details of the IPO process).

The return difference between non-included and included securities is therefore

$$E_t(r_{t+1}^n - r_{t+1}^i) = x\left(\delta_t + \frac{s^d \mu_t}{1+x} + s^u \mu_t\right)$$

which can be written more simply by using $\mu_{t+1} \cong 1$ (which holds exactly in the steady state equilibrium derived below):

$$E_t(r_{t+1}^n - r_{t+1}^i) \cong x\left(\delta_t + \frac{s^d}{1+x} + s^u\right)$$

We see that the non-included securities earn higher returns because of increased dividend yields, inclusion effects, and deletion effects. The inclusion/deletion (or issuance/repurchase) effects correspond to my examples 1 and 2 in the body of the paper, while the dividend-yield effect corresponds to my example 3 (i.e., it arises because passive investors don't hold the full market).[14]

For example, if the price premium of included securities is 10%, the dividend yield is 3%, a fraction of 1% of included shares are deleted from the index, and 2% of non-included shares get included, then we have an approximate return difference of 10%(3%+1%+2%)=0.60%. More broadly, we see that the return difference is greater when the market portfolio has more turnover (larger $s^d + s^u$) and when the price premium x is greater.

We can also consider an investor a who holds both included and non-included securities, say with a fraction of non-included being f. The return difference between this "active" investor who holds both kinds of securities and a "passive" investor who only holds the included securities is

$$E_t(r_{t+1}^a - r_{t+1}^i) = f E_t(r_{t+1}^n - r_{t+1}^i) \cong f x\left(\delta_t + \frac{s^d}{1+x} + s^u\right)$$

Suppose as before that the price premium is $x = 10\%$, the dividend yield is $\delta_{t+1} = 3\%$, and $s^d = 1\%$. Suppose that the active investor only invests $f = 2\%$ of his capital in non-included securities, but he

[14] My example 4 in the body of the paper corresponds to a time-varying premium x (and a corresponding time-varying θ in the equilibrium model below), leading to additional dynamic effects not considered here.

does so very selectively by trying only to buy those that are about to be included. If all of the non-included securities that the active investor buys in fact get included then $s^u = 100\%$, and, in that case, the return difference is $2\% \times 10\% \times (3\% + 1\% + 100\%) = 0.21\%$.

As seen below, we can construct an equilibrium in which passive investors only hold included stocks and active investors hold the remaining included stocks as well as the non-included stocks. Included stocks are more expensive due to greater demand for these stocks, leading to more risk sharing. Passive investors earn the return of their definition of the "market," while active investors are expected to beat the market as a result of the effects discussed above (before fees). The extent to which active investors beat the market increases in the number of passive investors and decreases in the breath of the market index.

In principle, an active investor could also add value by avoiding to hold the included securities that are about to get deleted, which would further raise his return by approximately $(1-f)xs^d$ (i.e., the numerical example, this is $98\% \times 10\% \times 1\% = 0.10\%$). Note, however, that such trades that occur before the deletion are not consistent with the equilibrium derived below in which passive only trade upon deletion (i.e., such trades would be zero-sum among the active investors).

Equilibrium

To show how the above insights can arise in equilibrium, we consider an economy in which a fraction I of all securities are included in the "market." Passive investors buy a fraction θ of all shares included in the "market" and the supply of shares of each security is normalized to 1. Active investors choose their portfolio π (which is a vector of positions in included and non-included securities) in order to maximize their mean-variance preferences

$$\max_\pi \pi'\left(E_t\left(D_{t+1} + P_{t+1}\right) - (1+r^f)P_t \right) - \frac{\gamma}{2}\pi'\pi$$

where r^f is the risk-free rate and γ can be thought of as the variance of each security's return multiplied by investors' risk aversion parameter.[15] The solution is

$$\pi = \frac{1}{\gamma}\left(E_t\left(D_{t+1} + P_{t+1}\right) - (1+r^f)P_t \right)$$

In equilibrium, active investors must choose a position of $\pi^i = 1-\theta$ for included securities and $\pi^n = 1$ for non-included securities. Hence, in a steady-state equilibrium, we have two equations for the two unknowns, namely the prices of included securities (P^i) and non-included securities (P^n):

[15] We assume that securities have iid dividend risk, consider a steady-state equilibrium with constant prices, and assume that risk associated with inclusions and deletions is fully diversifiable. Hence, the variance of a diversified portfolio is $\mathrm{var}_t\left(\pi'\left(D_{t+1} + P_{t+1} - (1+r^f)P_t \right)\right) = \sigma^2\pi'\pi$, that is, the variance from dividend risk σ^2 times the squared portfolio weights. In other words, each period a fraction s^d of securities are deleted from the market for sure so that there is no aggregate deletion risk, and similarly for additions.

$$\left(1+r^f\right)P^i \ = D + P^i \ - s^d\left(P^i \ - P^n\right) - \gamma(1-\theta)$$
$$\left(1+r^f\right)P^n = D + P^n + s^u\left(P^i \ - P^n\right) - \gamma$$

where the expected dividend $D = E(D_{t+1})$ is the same for all securities. Combining these equations, we see that the equilibrium price difference $\Delta P = P^i \ - P^n$ is given by

$$\Delta P = \frac{\gamma\theta}{r^f + s^d + s^u}$$

and the equilibrium prices are

$$P^i \ = \frac{D - \gamma(1-\theta) - s^d \Delta P}{r^f}$$
$$P^n = \frac{D - \gamma + s^u \Delta P}{r^f}$$

Intuitively, we see that the price difference ΔP increases in the amount θ bought by the passive.

From these equilibrium prices, we can compute all of the numbers discussed above. For example, the price premium is $x = \Delta P / P^n$. Further, since active investors hold all of the non-included stocks and $1-\theta$ of the non-included, the fraction of non-included stocks in their portfolio is $\frac{1-I}{1-I+I(1-\theta)}$ and, in a value-weighted sense, it is $= \frac{(1-I)P^n}{(1-I)P^n + I(1-\theta)P^i} = \frac{1-I}{1-I+I(1-\theta)(1+x)}$. Lastly, we note that, in a steady state where the same number of firms move up and down each period, we have $Is^d = (1-I)s^u$, that is, $s^u = I/(1-I)s^d$.

We are ready to consider a numerical example for illustration. We normalize the expected dividend to be $D = 1$, the risk-free rate is $r^f = 2\%$, half the securities are included $I = 50\%$, the fraction of deletions is $s^d = 2\%$, the fraction of non-included that are added is s^u =2%, passive investors buy $\theta = 40\%$ of the included shares,[16] and the active investors have a risk aversion corresponding to $\gamma = 0.5$ (chosen to have a reasonable dividend yield of around 3%).

Based on these inputs, we can compute the price of included securities to be $P^i \ = 31.7$ and the price of non-included securities to be $P^n = 28.3$. Hence, the dividend yield of included securities is $\delta = 3.2\%$ and the price premium is $x = 12\%$. The expected return difference for non-included vs. included stocks is $E_t\left(r_{t+1}^n - r_{t+1}^i\right) = 0.82\%$. Given that the active investors hold $f = 60\%$ of assets in non-included securities, the excess return of active relative to passive is $E_t\left(r_{t+1}^a - r_{t+1}^i\right) = 0.49\%$ (before fees).

We can consider the effect of changing different parameters. First, if the fraction of included securities is higher, e.g., $I = 80\%$, then the return difference between included and non-included securities remains the same.[17] Nevertheless, the return difference between active and passive is smaller,

[16] See, e.g., Morningstar 2016 Global Asset Flows Report (exhibit 10) for an estimate of the size of passive.

[17] The choice of I depends on the setting. In U.S. exchange-traded equities, S&P500 stocks constitute about 80% of the market value of listed U.S. stocks, but less historically, and less if you assume that active investors can also hold some foreign equities, convertibles, private equities, and other assets.

$E_t\left(r_{t+1}^a - r_{t+1}^i\right) = 0.22\%$, because active investors now hold a smaller fraction f of their portfolio in non-included securities.

Second, Figure 3 Panel A shows how the return difference increases with the size of the passive investors. We see that, if more investors are passive, the price difference between included and non-included securities is larger, and active investors can add more value from changes in the market portfolio. Panel B shows how the return difference decreases with the size of active investors. Intuitively, having more active investors means that the price differences between included and non-included securities diminish, leading to lower return differences.

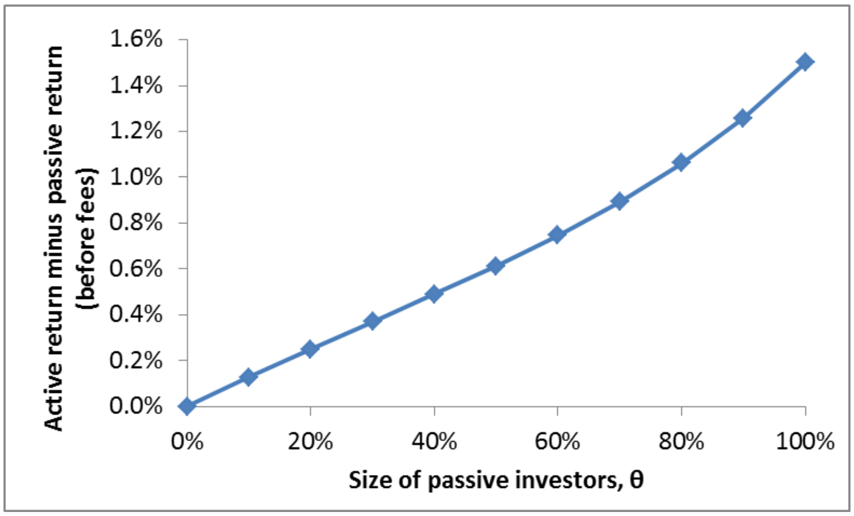
One could try to determine the equilibrium size of active vs. passive management based on their relative costs and benefits. That is, rather than separately varying the sizes of active and passive as in Figure 3, we could try to determine these endogenously based on an entry decision. See Garleanu and Pedersen (2015) for such an equilibrium level of active management in the context of a more sophisticated model with private information, search, and equilibrium asset management fees and see Petajisto (2009) for theory and evidence for how active management leads to downward sloping demand curves for stocks.

Finally, I note that the numbers above should only be viewed as illustrative examples. Future research should analyze realistic calibrations, considering different extensions that take into account the complexities of real markets. First, I have assumed here that the price impact at inclusion and deletion events are permanent price changes. One could make a similar calculation if the price impact is (partly) temporary.[18] Second, while I have assumed that all passive investors use the same index, the model can be extended to capture multiple different indices. Third, I have ignored seasoned equity offerings, repurchases, rebalancing trades, new firms, bankruptcies, private information, noise trading, and many other effects where active investors could in principle add value (these complexities are part of the reason that passive investors choose to track a simple index and why such indices must change over time). Fourth, I have assumed a simple structure for risk (constant prices, iid. dividend risk, and diversifiable inclusion/deletion risk), but in the real world discount rates are time-varying, adding significant risk due to such things as changing price levels. Fifth, Petajisto (2009) argues that, since some active investors are specialized in certain trades, the risk aversion implicit in the aggregate equity premium (e.g., the dividend yield) is lower than the risk aversion implicit in the slope of demand curves (e.g., both active and passive investors hold equities, but only active investors provide liquidity when a security leaves an index).

[18] If the price impact of inclusion/deletion is temporary, then included securities continue to suffer from a low return due to the price drop association with deletions, non-included securities continue to benefit from a price jump upon inclusion, but several other effects arise: newly included securities have a low return as their recent price increase reverses; recently deleted securities have a high return as their price drop reverses; while the dividend-yield effect discussed above disappears if price levels are equal across the two groups.

Figure 3. How much can active beat the market? This figure plots the expected return to the active investors in excess of the expected return of the securities included in the "market" portfolio, $E_t\left(r_{t+1}^a - r_{t+1}^i\right)$, based on the numerical example discussed in the text. Given the base-case parameters in the text, Panel A varies the fraction θ bought by passive investors and Panel B varies the size of the active investors by varying their risk tolerance $1/\gamma$ (more active investors means a smaller aggregate absolute risk aversion).

Panel A: Active minus passive return *vs.* the size of passive investing



Panel B: Active minus passive return *vs.* the size of active investing