

# Shephard's Lemma and Nonlinear Partial Differential Equations

Yuhki Hosoya<sup>\*†</sup>

Department of Economics, Kanto-Gakuin University  
1-50-1 Mutsuura-higashi, Kanazawa-ku, Yokohama-shi,  
Kanagawa 236-8501, Japan.

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## Abstract

This paper studies a reverse calculation method for consumer's preference from demand function that is not necessarily differentiable but locally Lipschitz. For this purpose, we treat existence results of both local and global solution of a nonlinear partial differential equation, which is called Shephard's lemma in economics. Our solution of this PDE is the classical solution, and our local existence result is a "necessary and sufficient" result.

**Keywords:** demand function, integrability, income-Lipschitzian, expenditure function, Shephard's lemma.

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<sup>\*</sup>1-50 1601 Miyamachi, Fuchu-shi, Tokyo 183-0023, Japan.

<sup>†</sup>TEL: +81-90-5525-5142, E-mail: hosoya(at)kanto-gakuin.ac.jp