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金融ネットワークのモデルと実データ解析

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KOBE UNIVERSITY

科研費
KAKENHI

自己紹介

もともとの専門：マクロ経済学（金融政策）

最近の研究：複雑ネットワーク、特に金融・社会ネットワーク

学歴：経済学博士(名古屋大)

これまでの共同研究者:

物理系:

増田直紀（ブリストル大）

高口太朗 (LINE Corp.)

Alain Barrat (CNRS)

応用数学：

Charles Brummitt

谷口隆晴 (神戸大、さきがけ)

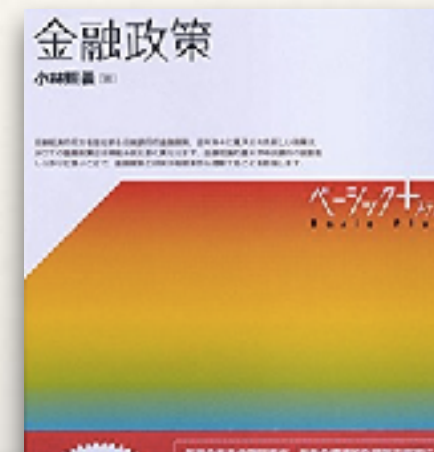
計算機科学(CS):

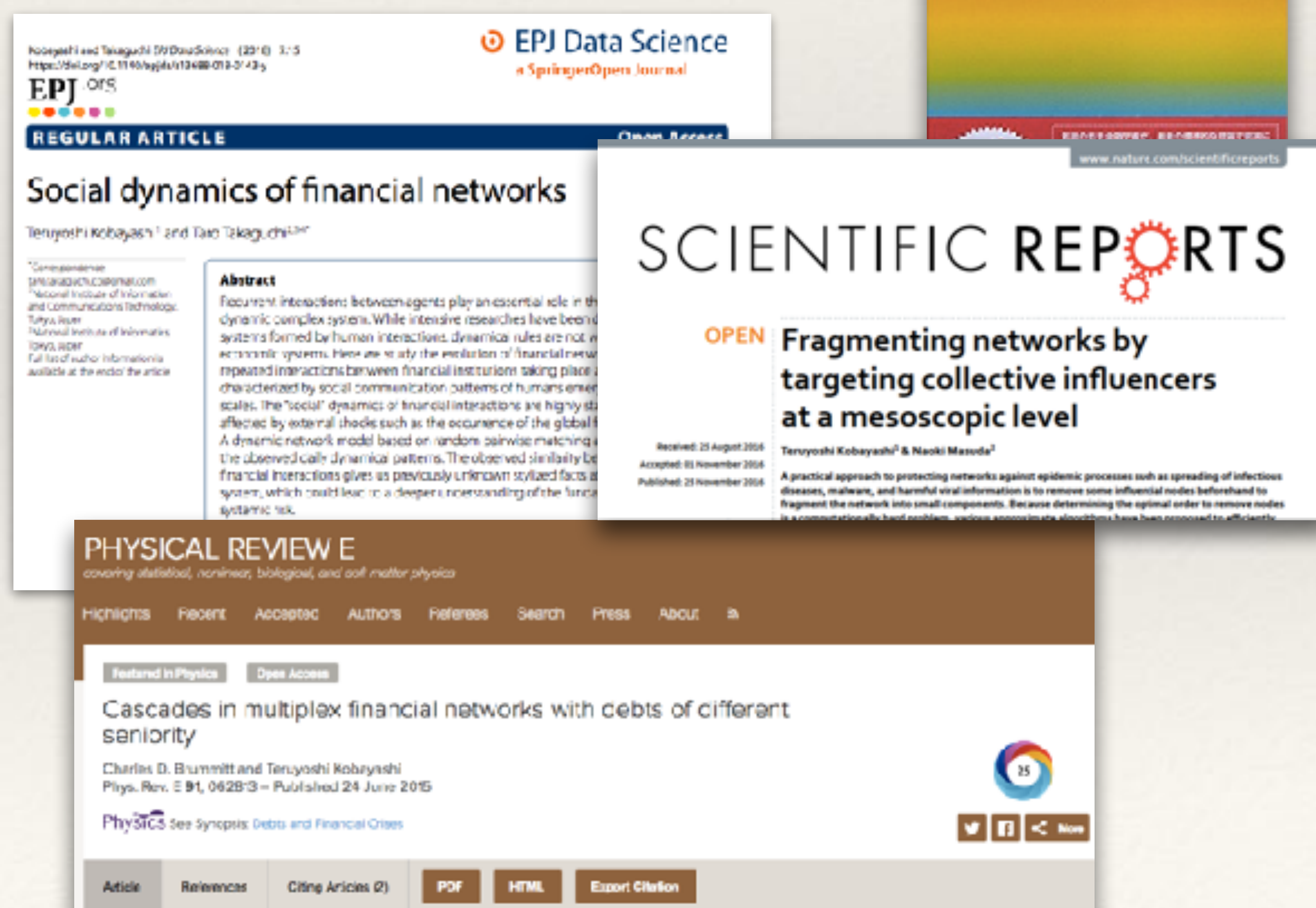
Emilio Ferrara (USC)

Anna Sapienza (USC)

経済学：

武藤一朗(日本銀行)　など



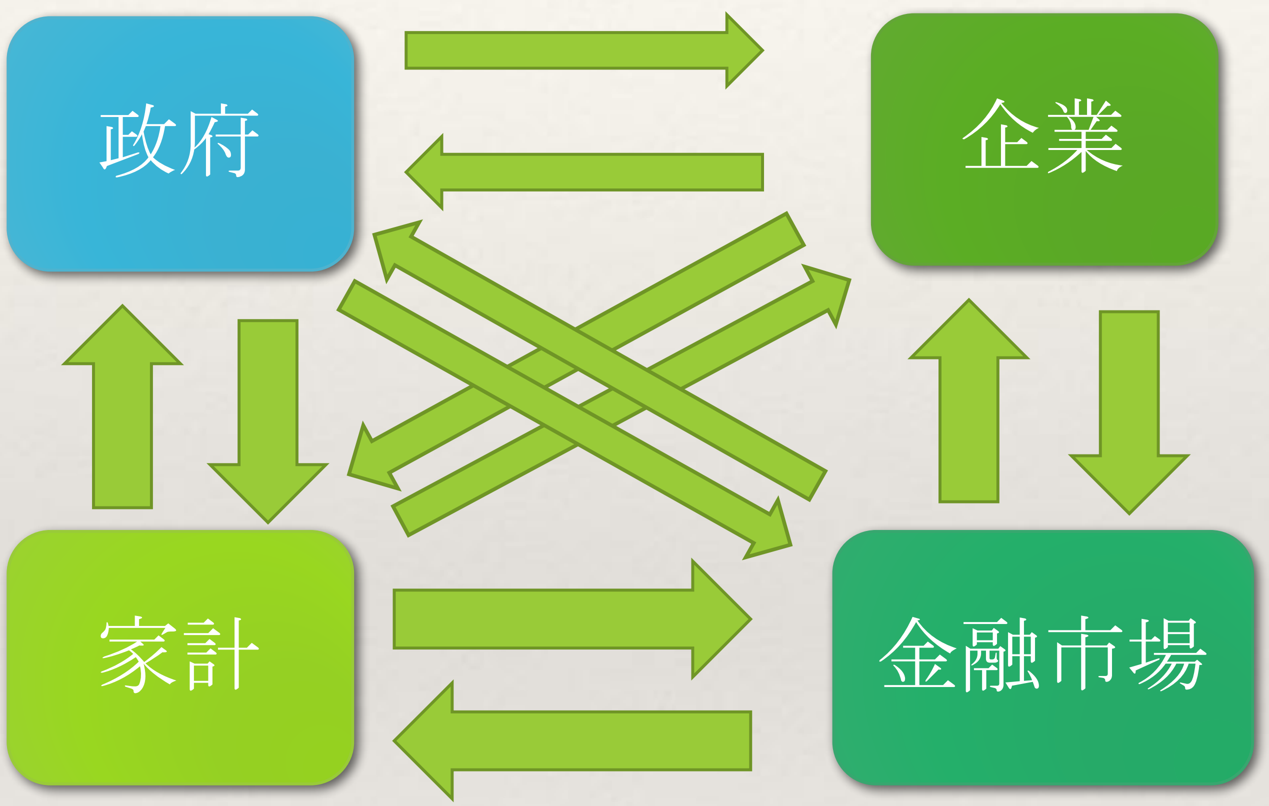


自己紹介

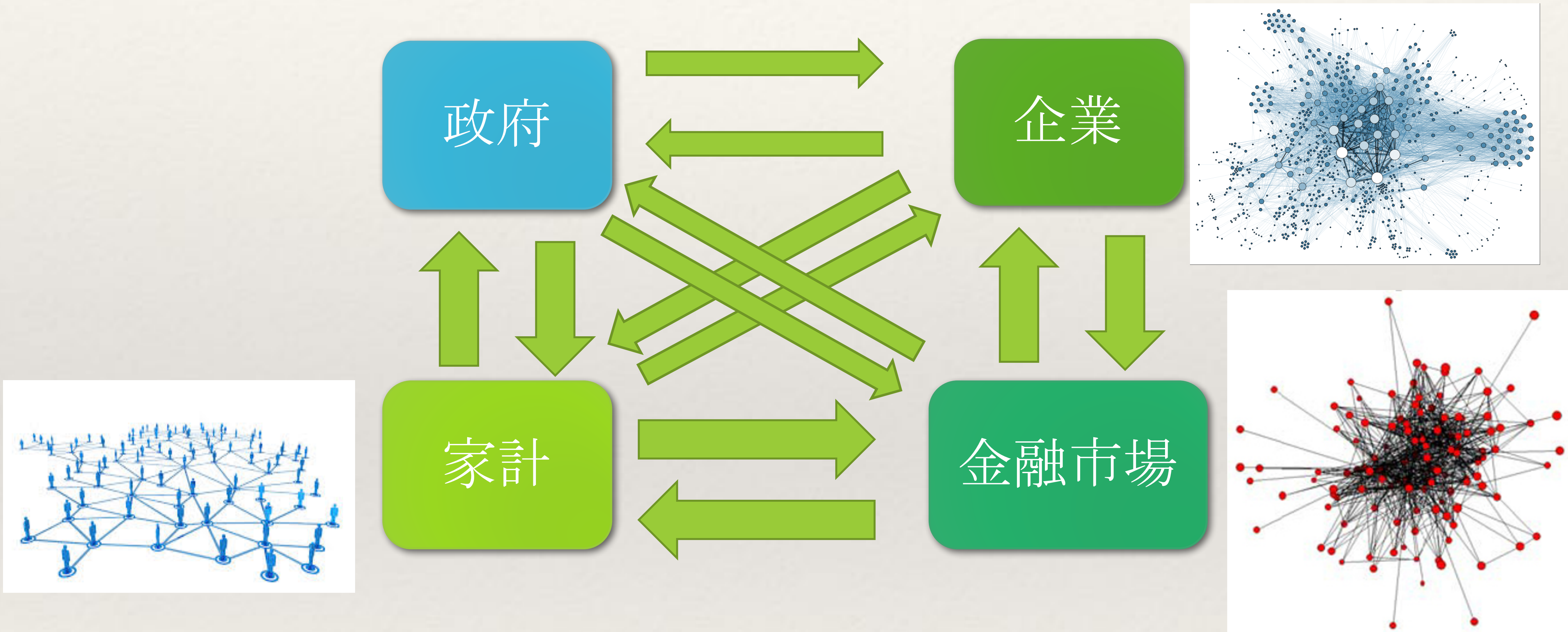
研究の基本スタンス

とりあえず経済学のモデルを忘れる

マクロ経済学の概念図



マクロ経済学の概念図



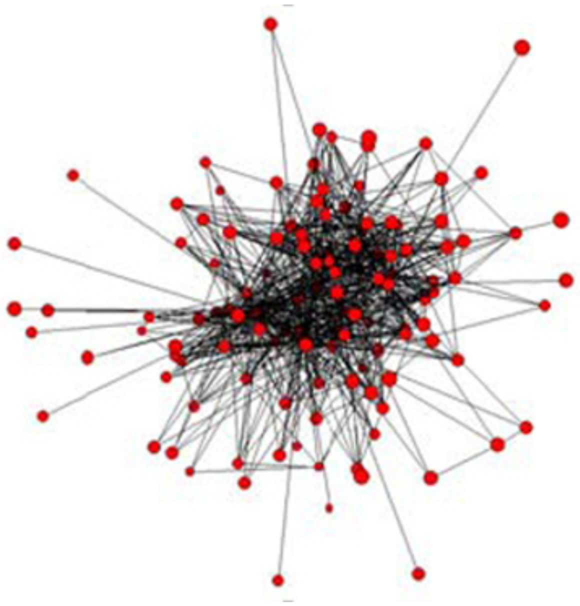
これまでの論文テーマ(経済学除く)

- 金融破綻/情報カスケードモデルの構築 (EPJB 2013, PRE 2015)
- コミュニティー構造がある場合における重要ノードの同定 (Sci Rep 2016)
- 重要なノード組の同定 (国民経済雑誌 2016)
- 銀行間取引ネットワークの実データ解析 (EPJ Data Science 2018)
- 非負値テンソル分解(NTF)を用いた銀行取引のパターン抽出(Sci Rep 2018)
- テンポラル・ネットワークにおける友好関係の検定(Backboning)

金融ネットワークとは？

- ノード：　金融機関（銀行、投資銀行、証券会社、保険会社、etc）
  金融資産（株式、証券、貸出債権、etc）

- エッジ：　資金貸借、株式の持ち合い
  資産の保有関係



Boss el al. 2004 Quant. Finan.

金融ネットワーク研究の大分類

実証的

１．データ解析

２．実データを用いたストレス・テスト

３．数理モデリング（ネットワーク論、マクロモデル）

４．ゲーム論的なアプローチ

規範的

Review article on financial networks


Journal of Computational Social Science

Network models of financial systemic risk: a review

Authors | Authors and affiliations

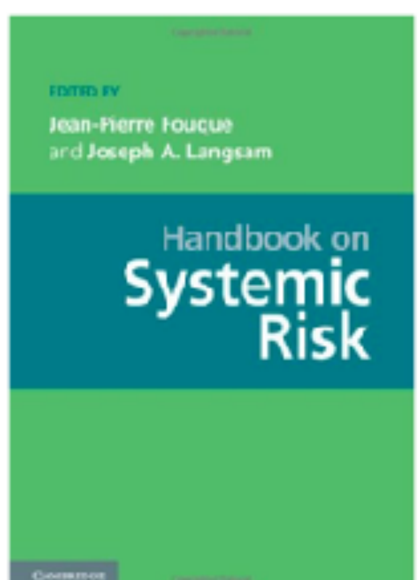
Fabio Caccioli, Paolo Barucca, Teruyoshi Kobayashi

Open Access | Survey Article

Abstract

The global financial system can be represented as a large complex network in which banks, hedge funds and other financial institutions are interconnected to each other through visible and invisible financial linkages. Recently, a lot of attention has been paid to the understanding

1. Fabio Caccioli, Paolo Barucca, Teruyoshi Kobayashi
   "Network models of financial systemic risk: a review"
   Journal of Computational Social Science, 2018



2. Handbook on Systemic Risk (2013)

なぜ金融ネットワークが重要か

- 破たんの連鎖現象を描写するモデル
- 伝播のしくみはネットワーク構造に依存（メゾスコピック）
- 法則性が十分（ほとんど？）解明されていない複雑系のひとつ

なぜ金融ネットワークが重要か　２

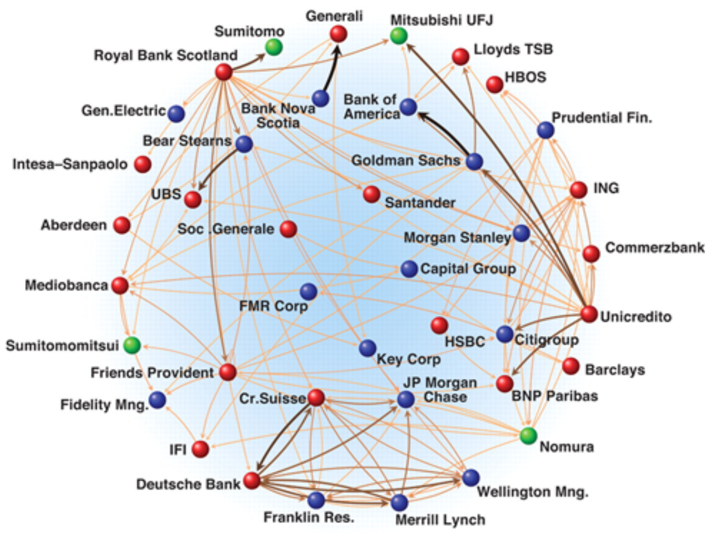
- 三洋証券の破綻、2008のリーマンショックは金融市場に甚大な影響

- マクロ経済の不景気も長期化

- ネットワーク論的観点からの政策対応はほぼ手付かず

→　社会的意義が大きい

金融ネットワークの特徴



“Economic networks: The new challenges”
Frank Schweitzer et al. Science, 2009.

1. 各ノードが先天的に同一ではない

例:

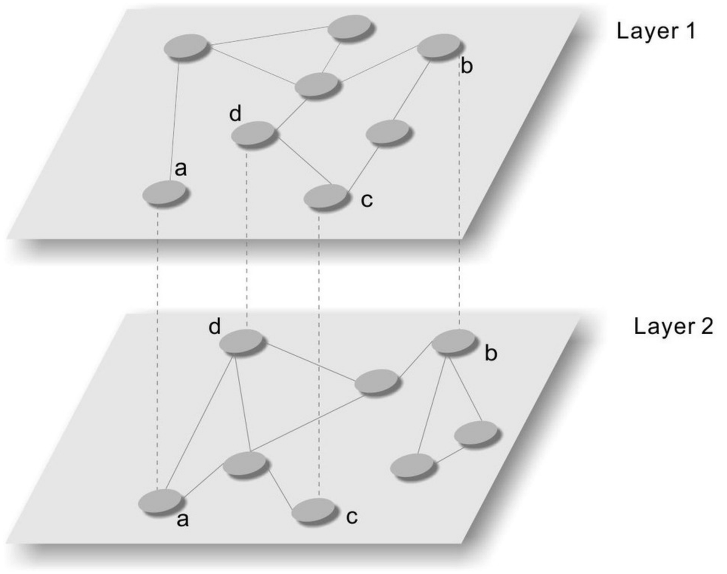
- 各金融機関の自己資本比率(i.e.,破綻確率)が異なる
- 保有資産のリスクが異なる
- 各金融機関の保有資産に相関がある

金融ネットワークの特徴

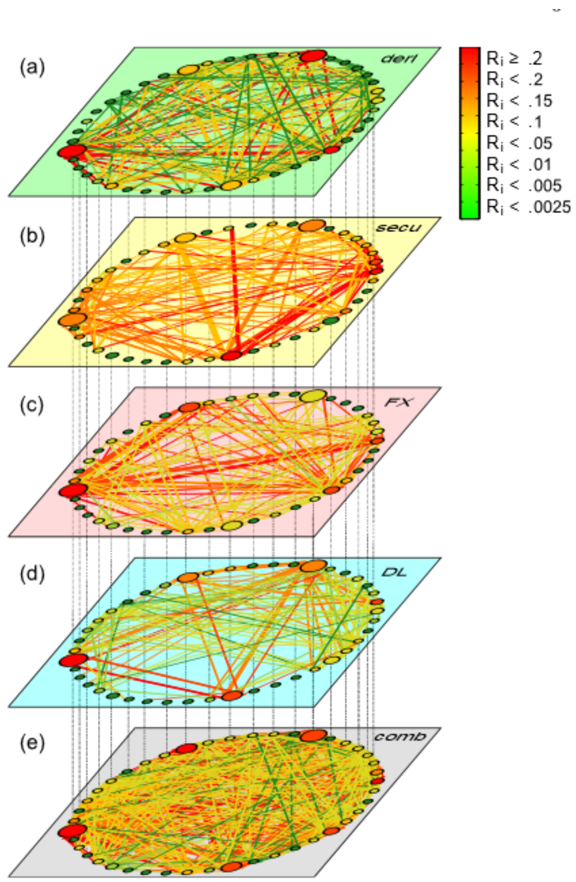
2. エッジの意味が多様 (多層構造)

例:

- 借入(貸出)資金に返済順位
- 満期の違い
- 担保の有無
- 資産のリターン相関



例：メキシコの銀行間ネットワーク



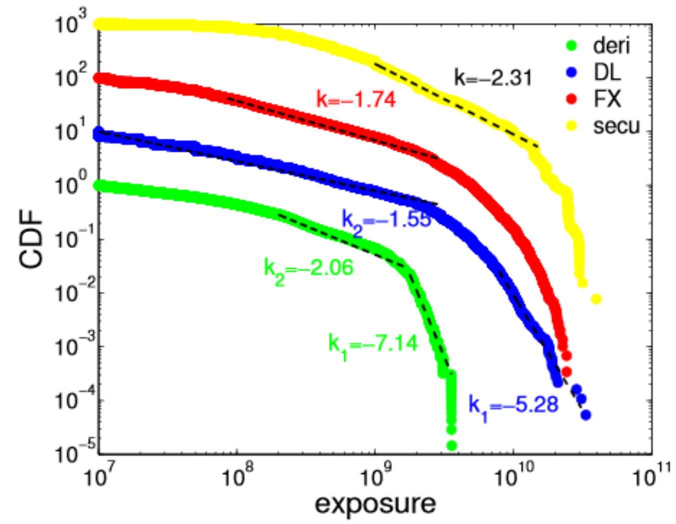


FIG. 1. Banking multi-layer network of Mexico on Sept 30 2013. (a) network of exposures from derivatives, (b) securities cross-holdings, (c) foreign exchange exposures, (d) deposits and loans and (e) combined banking network $L_{ij}^{\mathrm{comb}}(t)$. Nodes (banks) are colored according to their systemic impact R_i^{α} in the respective layer (see section III B): from systemically important banks (red) to systemically safe (green). Node-size represents banks' total assets. Link-width is the exposure size between banks, link-color is taken from the counterparty.

Poledna et al.
(2015, J.Finan.Stability)

金融ネットワークの特徴

3. 大域的情報の影響を受ける

例：

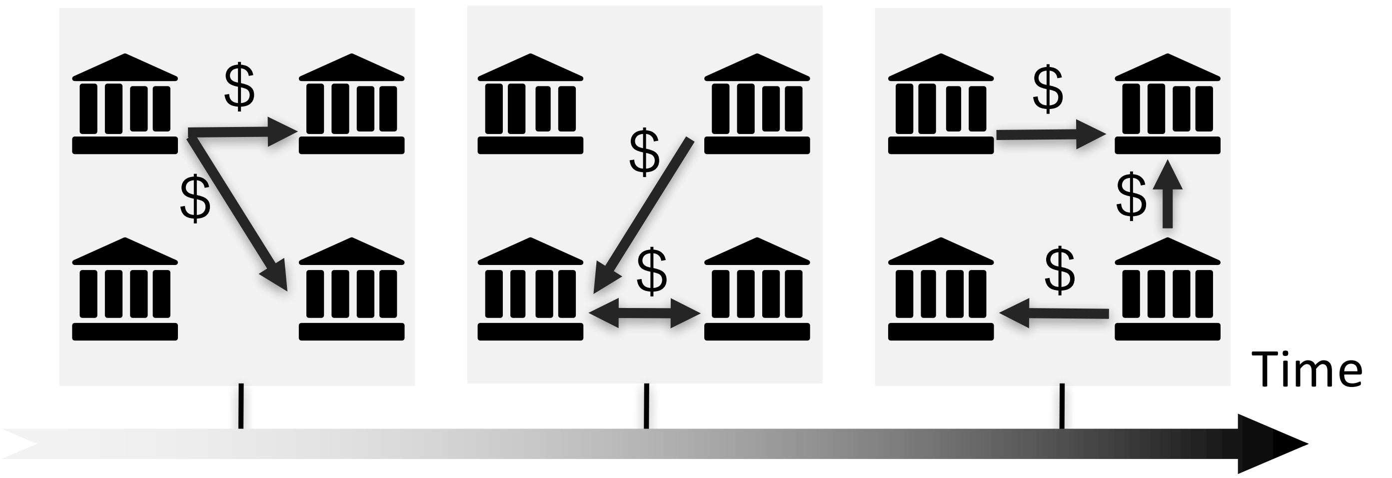
- 株価や債券価格
- GDPやインフレ

金融ネットワークの特徴

4. テンポラル性

例

- 銀行間の翌日物取引
- 頻繁な資産ポートフォリオの変化



具体的な課題

- 破綻の連鎖現象メカニズムの解明
- Too big to fail 問題（システム上重要な金融機関の選択）
- 「投売り」、「取り付け」などの同期現象メカニズム解明へのネットワーク論的アプローチ
- 数理工学的手法を用いた規制の最適化（規制の内容、対象先）

連鎖破綻のモデル

Watts (2002)の情報カスケードモデル

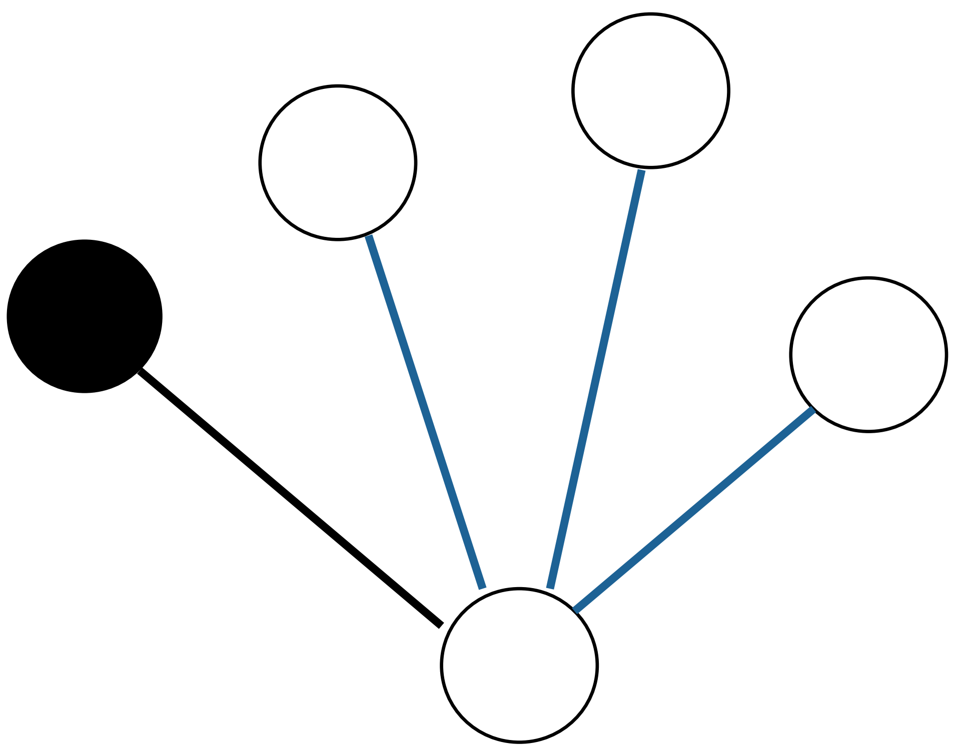
1. 初期はすべてのノードが inactive

2. active な隣接ノードの割合が ϕ より大きいと自身もactiveに

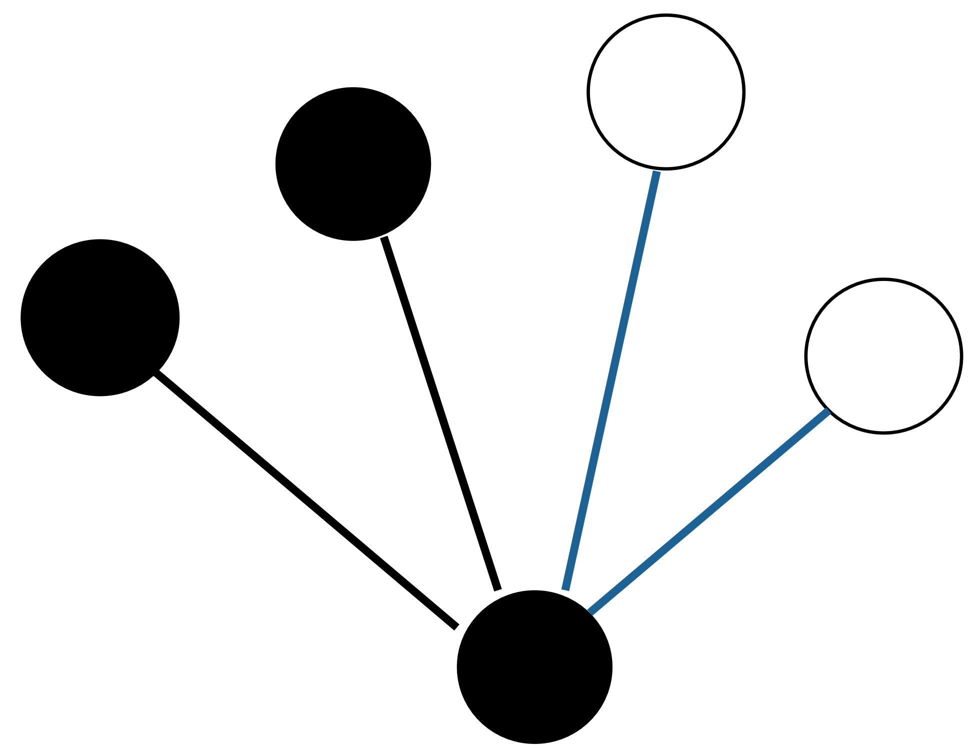
$$\mathrm{i.e.,}\ \frac{m}{k} > \phi$$

k : 次数
m : activeな 隣接ノードの数

例： $\phi = 1/3$ のケース



$1/4 < \phi$



$2/4 > \phi$

- Recursion equation for the cascade size ρ:

カスケードのサイズ：$\rho = \underbrace{\rho_0}_{\text{シードの割合}} + (1-\rho_0)\sum_{k=1}^{\infty} \underbrace{p_k}_{\text{次数分布}} \sum_{m=0}^{k} \binom{k}{m} q^m (1-q)^{k-m} F\left(\frac{m}{k}\right),$

$$q = \rho_0 + (1-\rho_0)\sum_{k=1}^{\infty} \frac{k}{z} p_k \sum_{m=0}^{k-1} \binom{k-1}{m} q^m (1-q)^{k-1-m} F\left(\frac{m}{k}\right),$$

Gleeson and Cahalane, PRE (2007)
Caccioli et al., JCSS (2018)

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一階微分係数が 1より大 (at $\rho_0 = 0$)

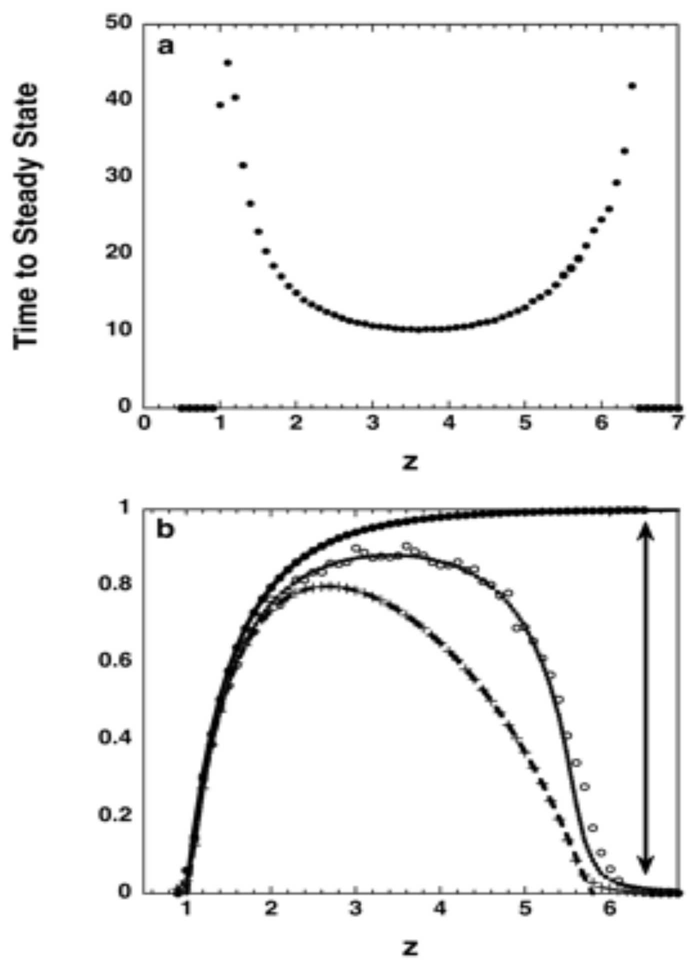
- First-order cascade condition:

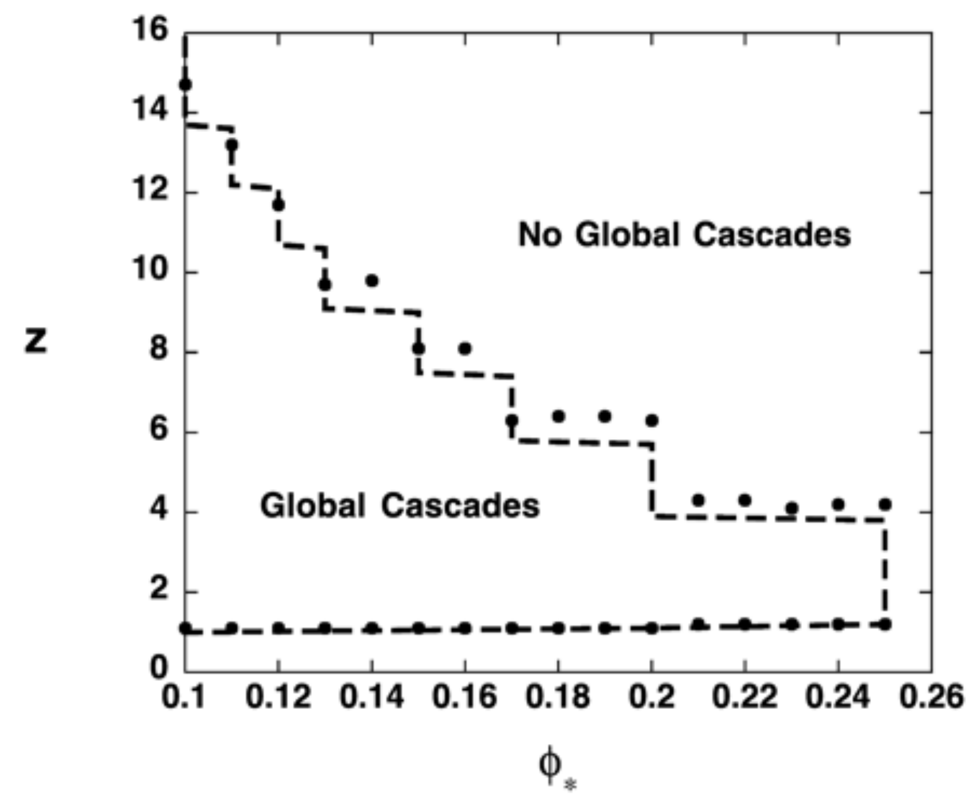
$$(1-\rho_0)\sum_{k=1}^{\infty} \frac{k(k-1)}{z} p_k \left[F\left(\frac{1}{k}\right) - F(0)\right] > 1.$$

$\boxed{\text{これが満たされると、無限小のノードが破綻するだけでグローバルなカスケードが発生する}}$

Gleeson and Cahalane, PRE (2007)
Caccioli et al., JCSS (2018)

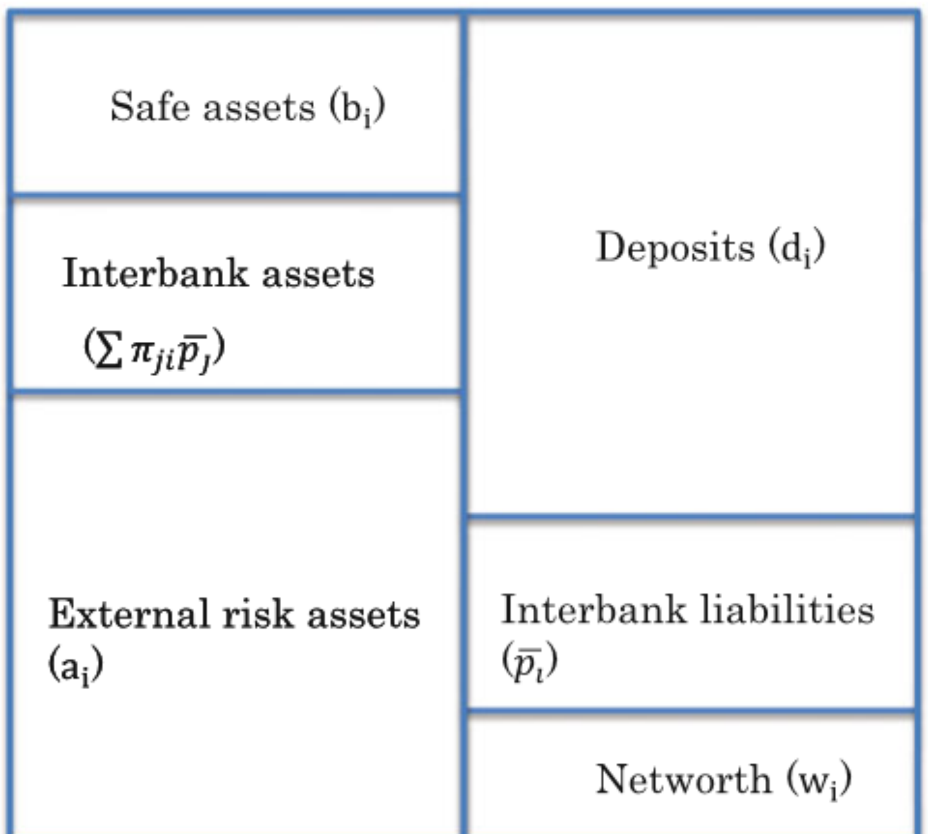
波及モデルの例：Watts (2002, PNAS)





単純(標準的)な連鎖破綻モデル

貸出先
k



$1-\theta$

θ:自己資本比率

自己資本比率 : θ

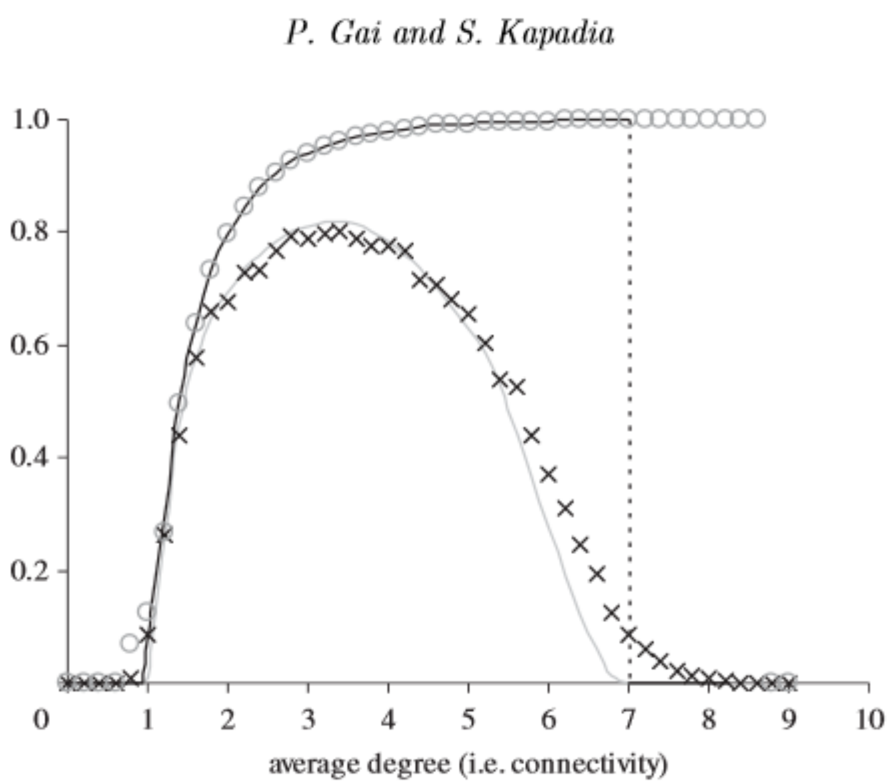
資産の損失 $> \theta \Rightarrow$ 破綻

Threshold model と同じ構造
A node is activated if $\frac{m}{k} > R$
(e.g., Watts, 2002 PNAS)

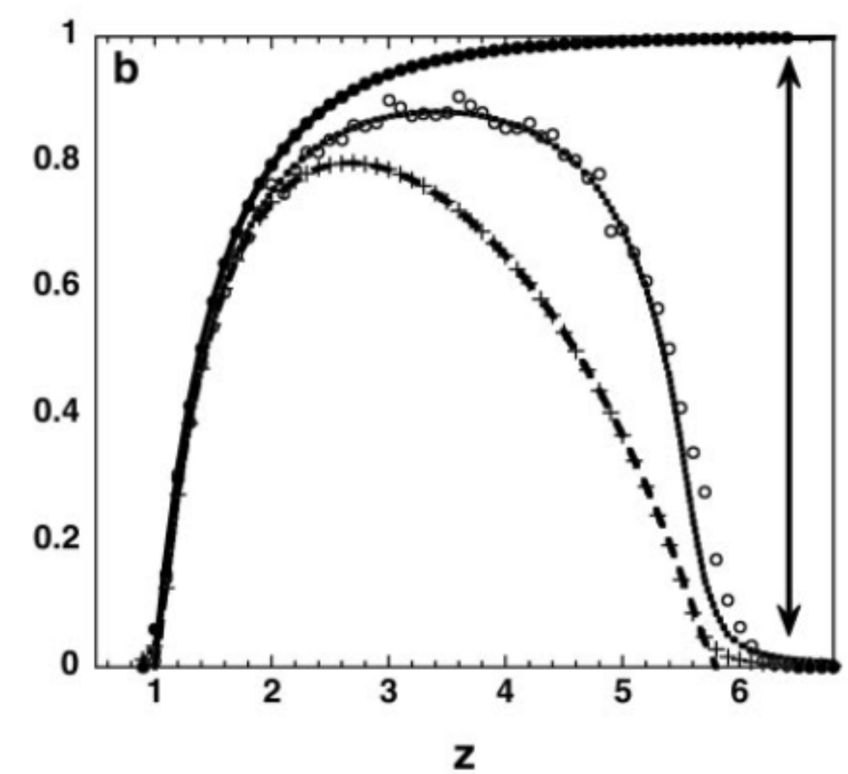
Gai and Kapadia, Proc. Roy. Soc. A (2010)

単純(標準的)な連鎖破綻モデル

Gai and Kapadia (2010, Proc. Roy.Soc. A)



Watts (2002, PNAS)



拡張版：多層型情報カスケードモデル

1. ノードは複数のレイヤーに属す

2. active になる閾値(ϕ)はレイヤーごとに異なる

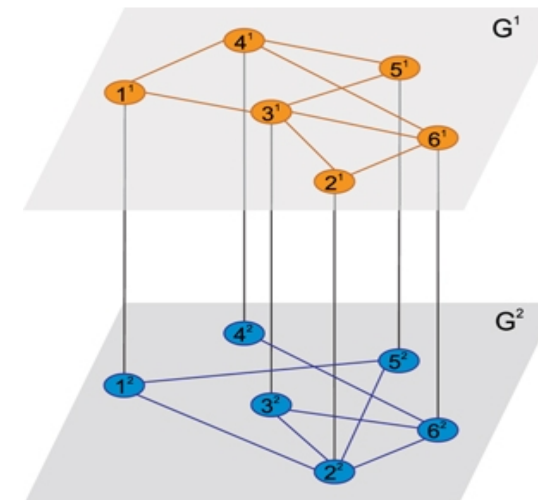


Figure 1 from Discrete-time distributed consensus on multiplex networks
I Trpevski et al 2014 New J. Phys. 16 113063 doi:10.1088/1367-2630/16/11/113063

$$\mathrm{e.g.,}\quad \frac{m_1 + m_2}{k_1 + k_2} > \phi_1 \Rightarrow$$ レイヤー１で active

$$\frac{m_1 + m_2}{k_1 + k_2} > \phi_2 \Rightarrow$$ レイヤー2 でも active

$$\phi_1 < \phi_2$$

Cascades in multiplex financial networks with debts of different seniority

債務の優先劣後構造：a multiplex financial network

- 標準的な連鎖破綻モデルは
  Watts (2002,PNAS)を基礎としている
  c.f. Gai and Kapadia (2010, Proc.Roy.Soc.A)

- 実際には返済には順位があり、
  優先権のある債権者から順に返済される

- 返済順位の存在が破綻メカニズムを
  どう変えるか？

Featured in Physics

Cascades in multiplex financial networks with debts of different seniority

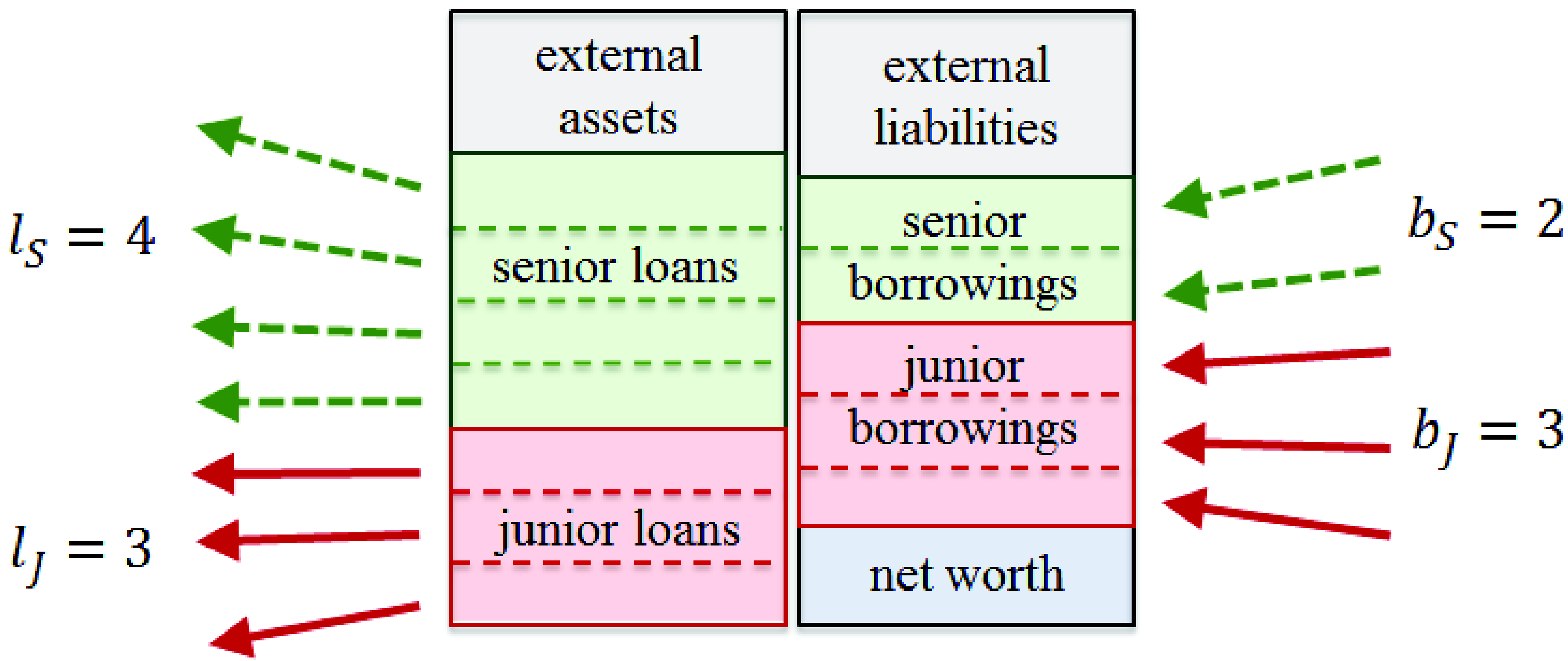
Charles D. Brummitt and Teruyoshi Kobayashi

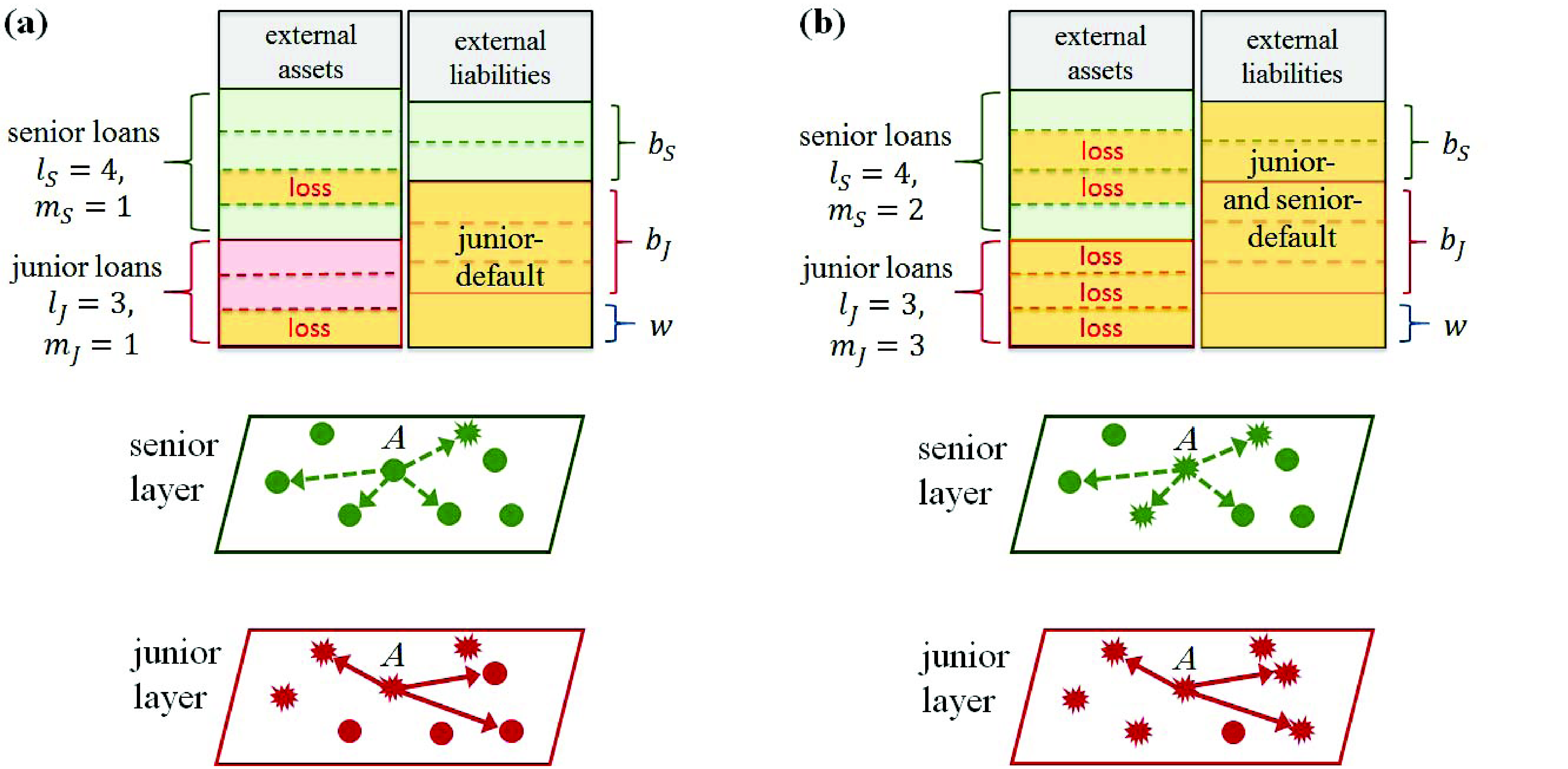
Phys. Rev. E **91**, 062813 (2015) – Published 24 June 2015

A model of a banking network predicts the balance of high- and low-priority debts that ensures financial stability.

Show Abstract +

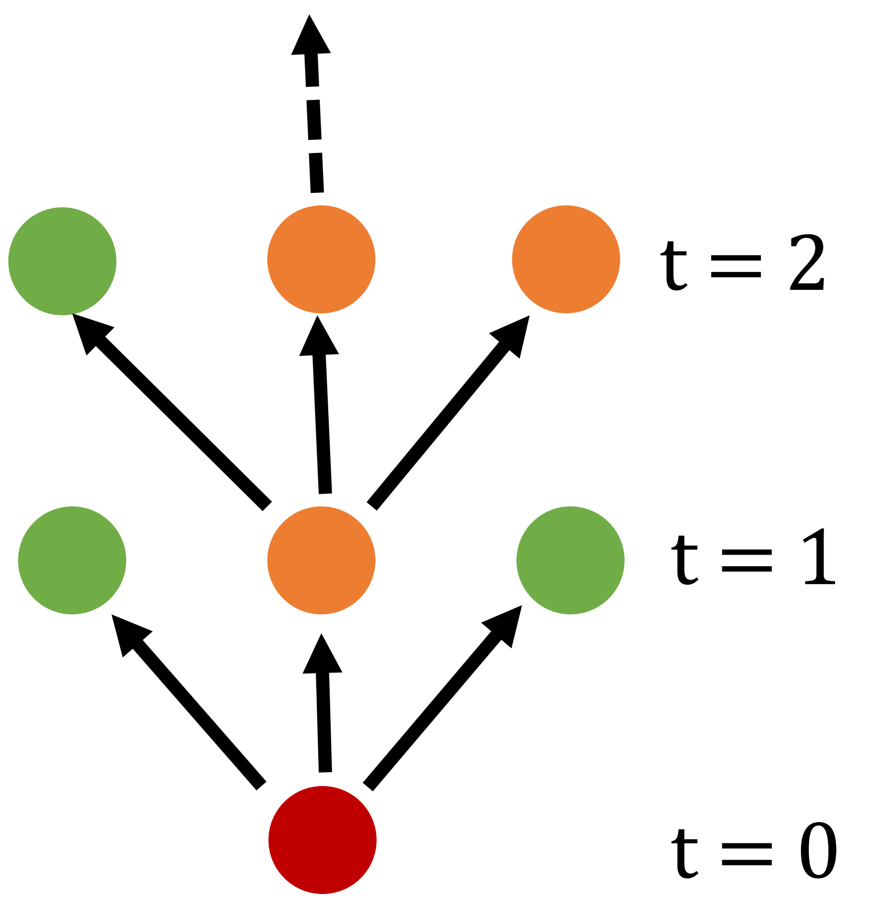
Balance sheet (2-layer model: senior and junior)




(a)
external assets
external liabilities
senior loans
$l_S = 4$,
$m_S = 1$
loss
junior loans
$l_J = 3$,
$m_J = 1$
loss
junior-default
b_S
b_J
w
senior layer
A
junior layer
A
(b)
external assets
external liabilities
senior loans
$l_S = 4$,
$m_S = 2$
loss
loss
junior loans
$l_J = 3$,
$m_J = 3$
loss
loss
loss
junior- and senior-default
b_S
b_J
w
senior layer
A
junior layer
A

Tree-like approximation for the cascade size
(Gleeson et al. 2007, 2008, PRE)

$$\begin{aligned}
\phi_{t+1}^{\alpha} &= g^{(\alpha)}(\phi_t^J, \phi_t^S) \\
&\equiv \phi_0^{\alpha} + (1 - \phi_0^{\alpha}) \sum_{l_J + l_S \geqslant 1} \sum_{b_J} p_{l_J}^{J,\mathrm{loan}} p_{l_S}^{S,\mathrm{loan}} p_{b_J}^{J,\mathrm{borrow}} \\
&\quad \times \sum_{m_J=0}^{l_J} \sum_{m_S=0}^{l_S} B_{m_J}^{l_J}(\phi_t^J) B_{m_S}^{l_S}(\phi_t^S) F_{\alpha}(\vec{l}, \vec{b}, \vec{m}), \qquad (3)
\end{aligned}$$



Tree-like approximation for the cascade size
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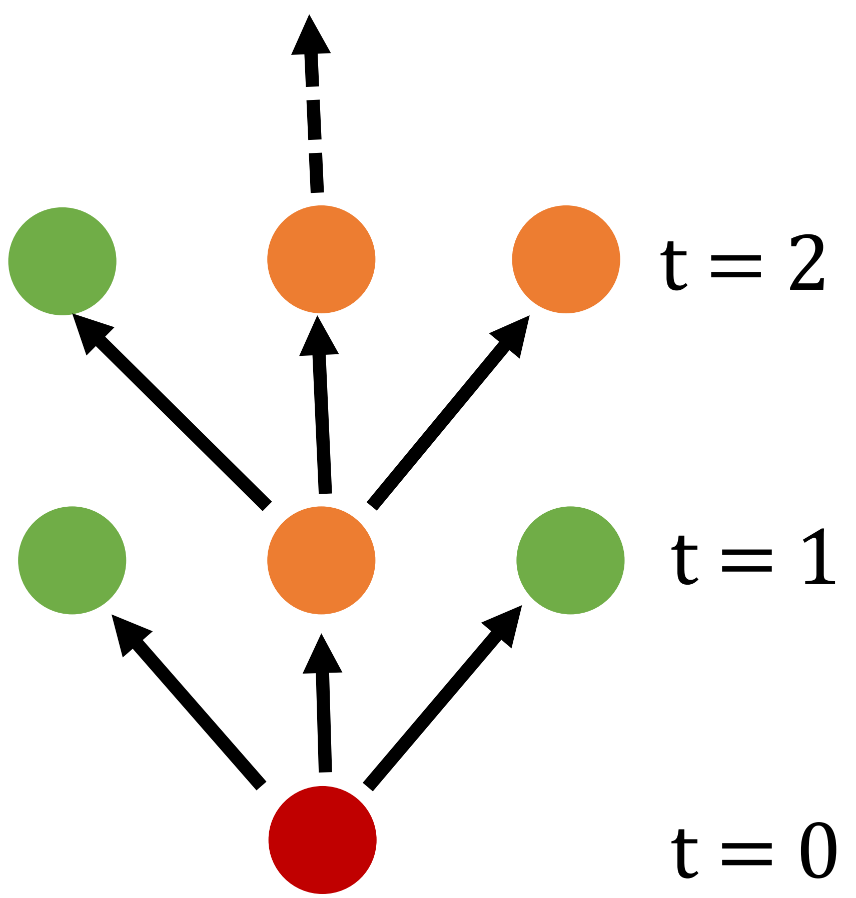
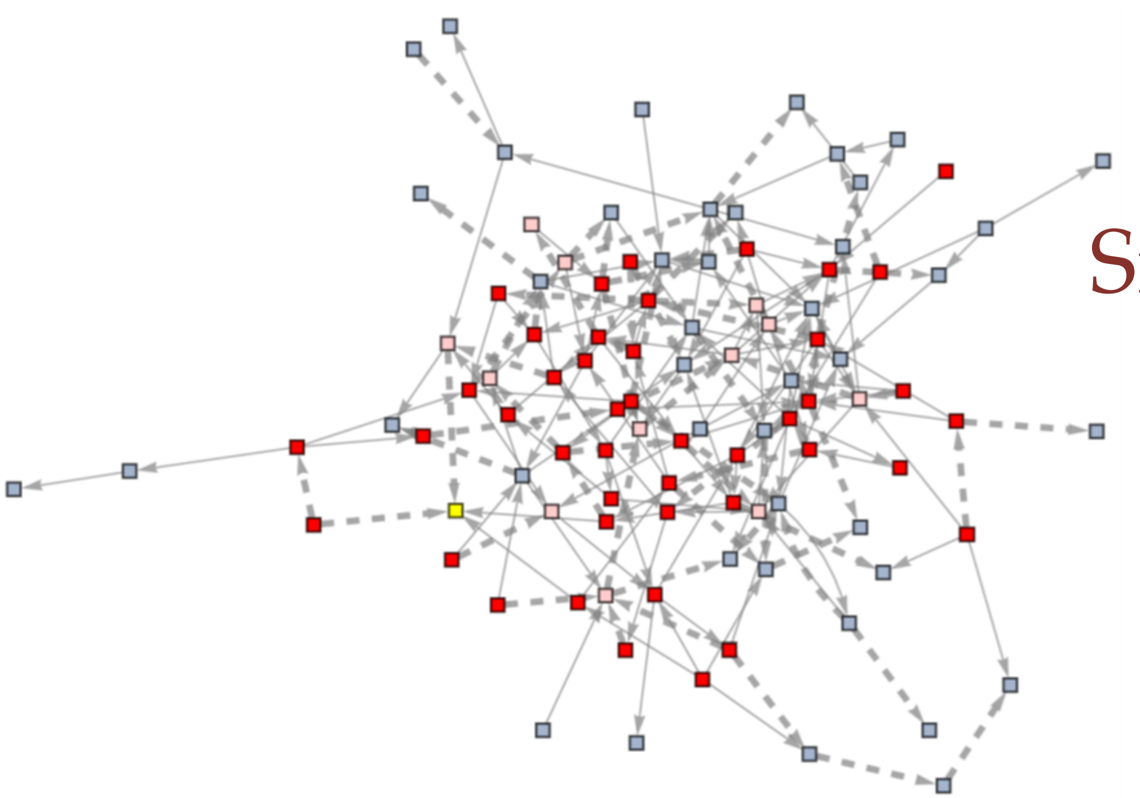
$$\phi_{t+1}^{\alpha} = g^{(\alpha)}(\phi_t^J, \phi_t^S)$$
$$\equiv \phi_0^{\alpha} + (1-\phi_0^{\alpha}) \sum_{l_J + l_S \geqslant 1} \sum_{b_J} p_{l_J}^{J,\text{loan}} p_{l_S}^{S,\text{loan}} p_{b_J}^{J,\text{borrow}}$$
$$\times \sum_{m_J=0}^{l_J} \sum_{m_S=0}^{l_S} B_{m_J}^{l_J}(\phi_t^J) B_{m_S}^{l_S}(\phi_t^S) F_{\alpha}(\vec{l},\vec{b},\vec{m}), \qquad (3)$$

$$\mathrm{tr}\mathcal{J}^M = \sum_{i=1}^{M} \prod_{k=1}^{i-1} p_0^{k,\text{borrow}} \sum_{\vec{l}:\sum_{\alpha=1}^{M} l_\alpha < 1/R_1} \prod_{s=1}^{M} p_{l_s}^{s,\text{loan}} l_i .$$

Cascade condition: $\mathrm{tr}\mathcal{J}^M > 1$

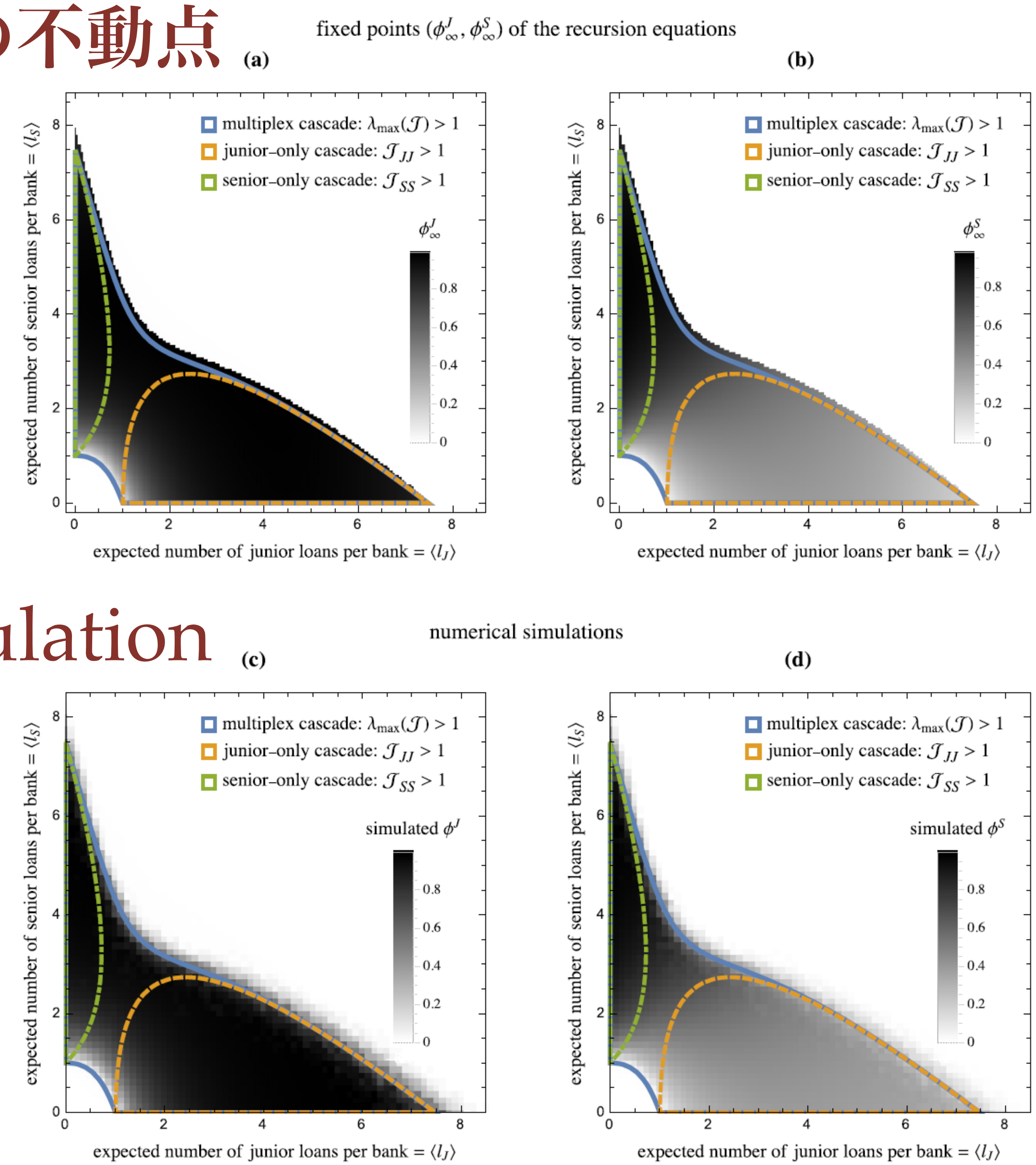


Simulation

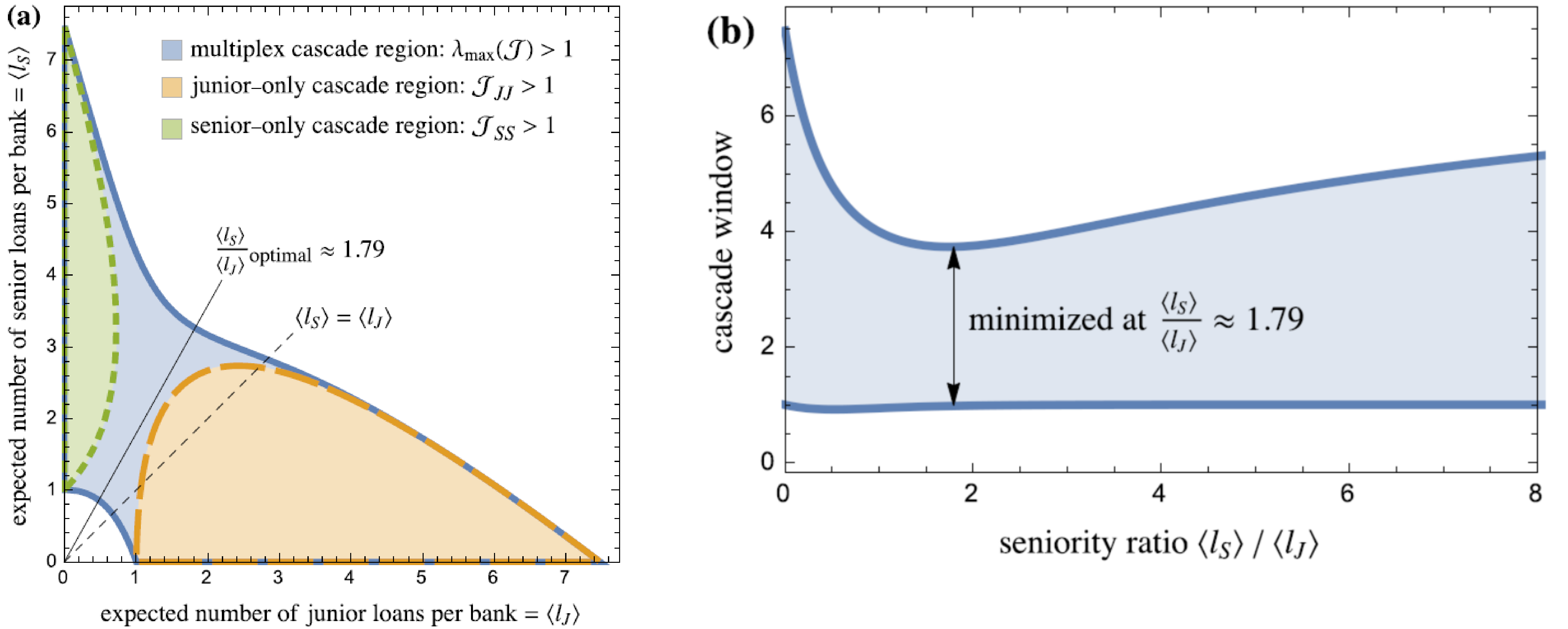


再帰式の不動点

Simulation



Cascade region and the optimal seniority ratio



まとめ

・債務の優先劣後構造がある場合の銀行間ネットワークをmultiplex model で表現できた

・任意のレイヤー数に対応する解析的なcascade condition を 導出した

・システミック・リスクが最小になる seniority ratio の存在を示した
　⇒ senior debts/ junior debts > 0.5

Identification of relationship lending in the interbank market

Question

- 信用不安が起こると、銀行間取引に何が起こるか
- 借手、貸手の取引相手選択の自由度に変化？
- “Substitutability of trading partners” を捉えたい!

Question

- ★ Measuring the substitutability of trading partners
- ✓ Number of trades?
- ✓ Frequency of trades?
- ✓ Volume of trades?

Question

- ★ Measuring the substitutability of trading partners
- ✓ Number of trades?
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Null model: ランダムに取引相手を選ぶ

Question

- ⋆ Measuring the substitutability of trading partners
- ✓ Number of trades?
- ✓ Frequency of trades?
- ✓ Volume of trades?

Null model: ランダムに取引相手を選ぶ

→ **Data**: 実際の取引回数はランダムで説明できるか？

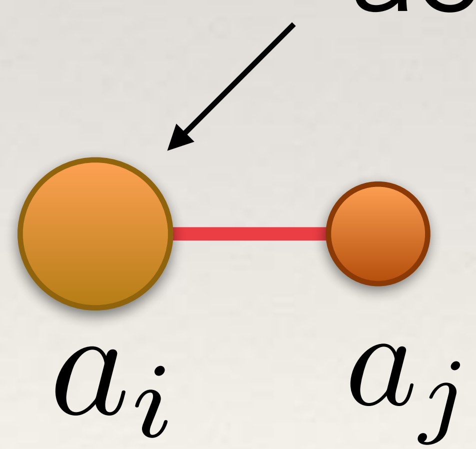
Null model

- Daily matching probability (undirected)

$$u(a_i, a_j) = a_i a_j$$

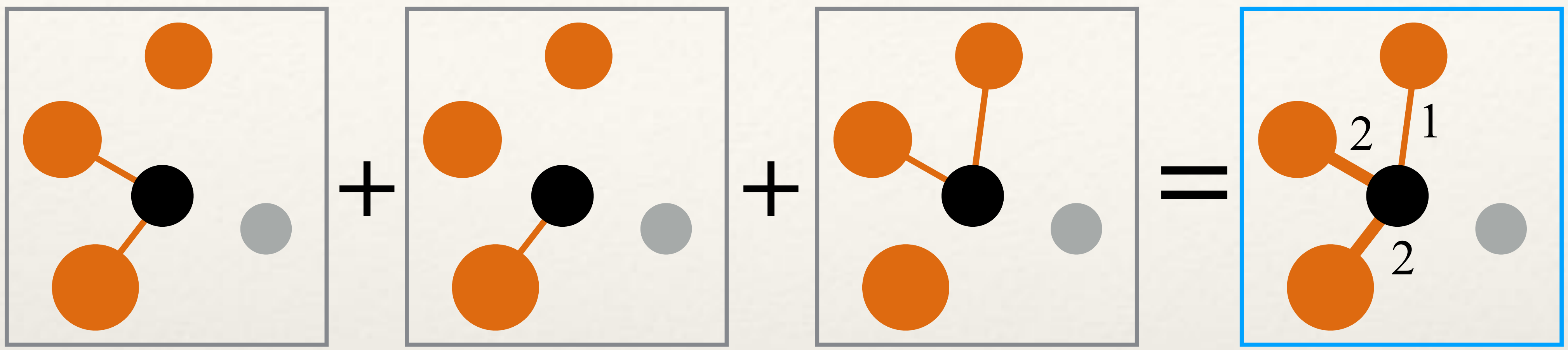
"Fitness model"

"activity" $\propto$ # contacts

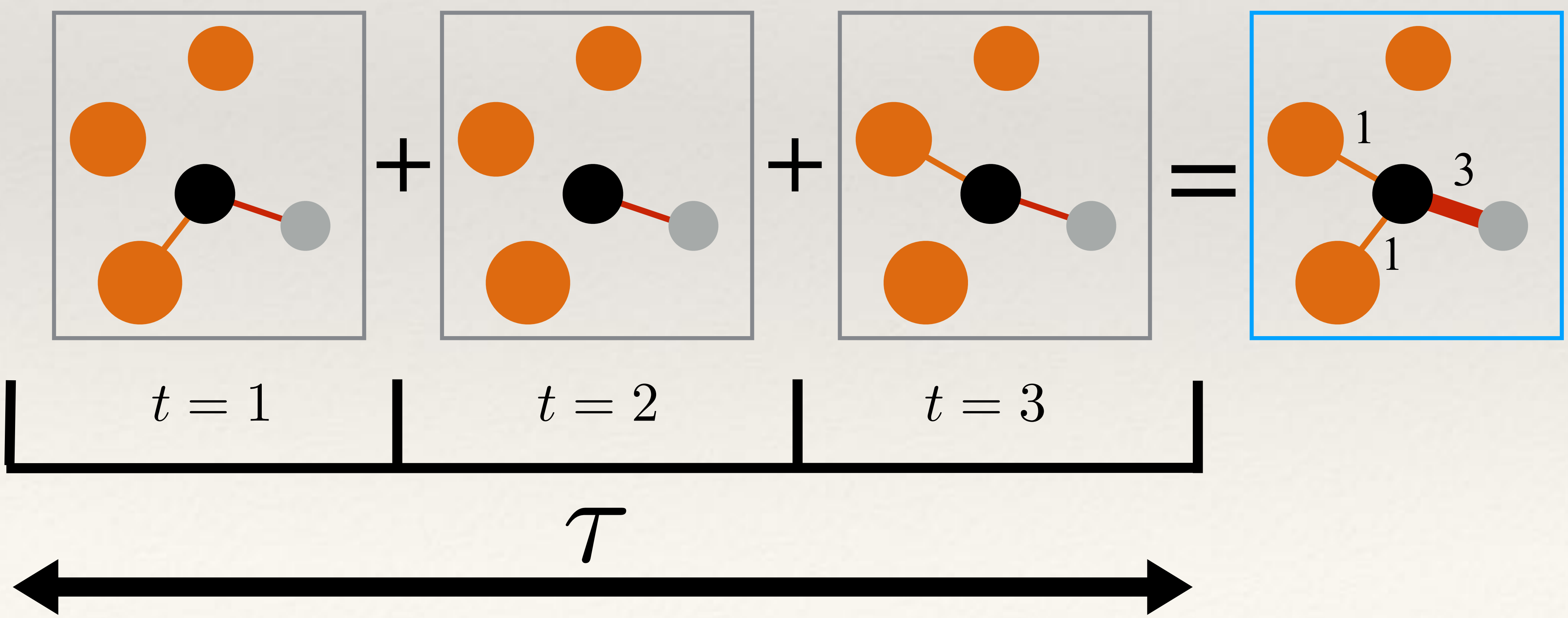


Significant ties in temporal networks

If random (= null hypothesis):



If there is a strong partnership:



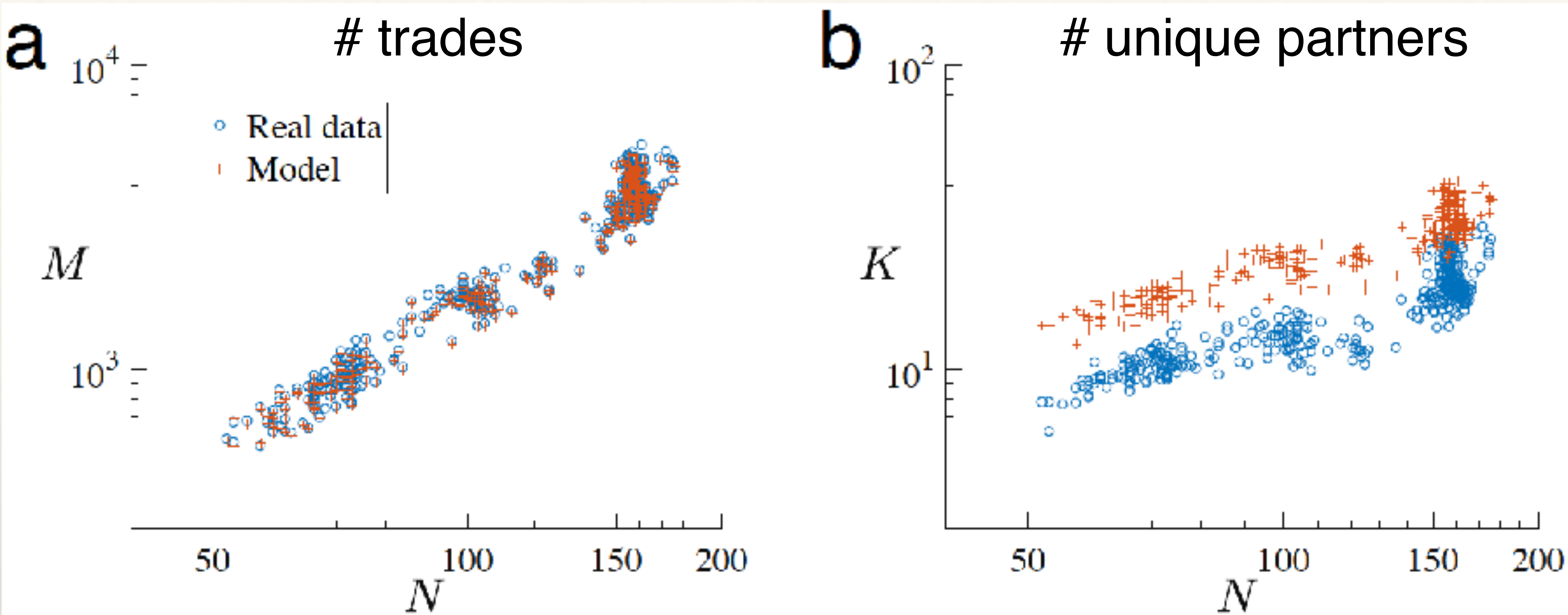
Under random matching, # bilateral transactions should follow a binomial distribution:

$$p(\{m_{ij}\}|\vec{a}) = \prod_{i,j:i\neq j} \binom{\tau}{m_{ij}} u(a_i, a_j)^{m_{ij}} (1 - u(a_i, a_j))^{\tau - m_{ij}},$$

Maximum-likelihood estimator:

$$F_i(\vec{a}^*) \equiv \sum_{j:j\neq i} \frac{m_{ij} - \tau(a_i^* a_j^*)}{1 - (a_i^* a_j^*)} = 0, \ \forall\, i = 1, \ldots, N,$$

Performance of ML



trades: real ≈ model
unique partners: real < model

→ preferential relationship?

Identification of significant ties

- **Edge-based test**

Under the null, m_{ij} should follow a binomial distribution:

$$g(m_{ij} \,|\, a_i^*, a_j^*) = \binom{\tau}{m_{ij}} u(a_i^*, a_j^*)^{m_{ij}} (1 - u(a_i^*, a_j^*))^{\tau - m_{ij}}, \ \forall\, i, j = 1, \ldots, N\,.$$

➡ $m_{ij} > m_{ij}^{\mathrm{C}}$ indicates the presence of a ***significant tie***.

- **Node-based test**

Under the null, aggregate degree K_i should follow a Poisson binomial distribution:

$$f(K_i \mid \overrightarrow{a}^*) \approx \frac{{\lambda_i^*}^{K_i} e^{-\lambda_i^*}}{K_i!}$$

➡ $K_i < K_i^{\mathrm{C}}$ indicates node i depends on significant ties (i.e., *relationship-dependent*).

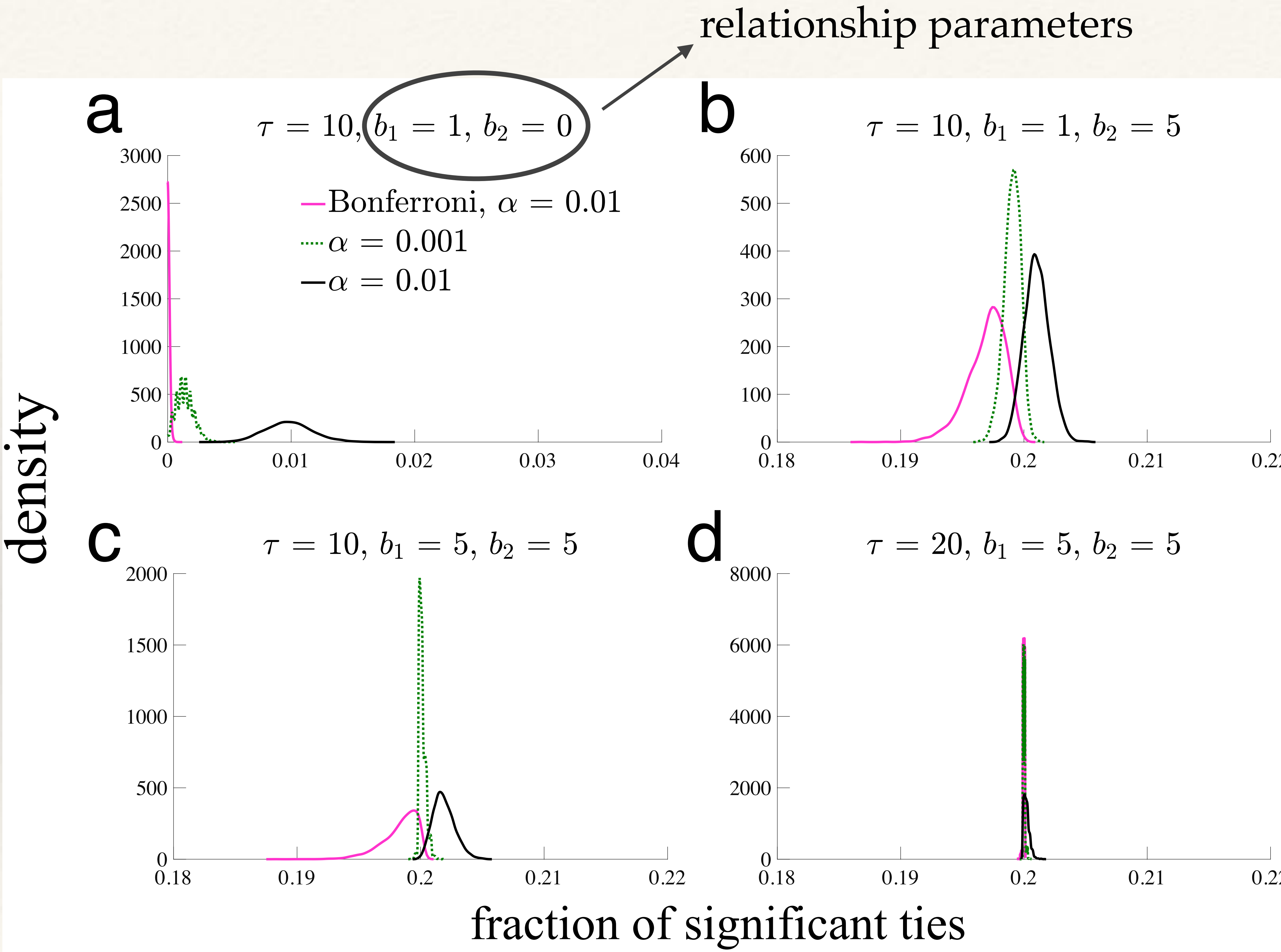
Experiments on synthetic networks

- **Introduce "relationship lending"**

1. Create random temporal networks
2. Assign a fraction of pairs as relationship pairs
3. Decreasing hazard prob for terminating a relationship:

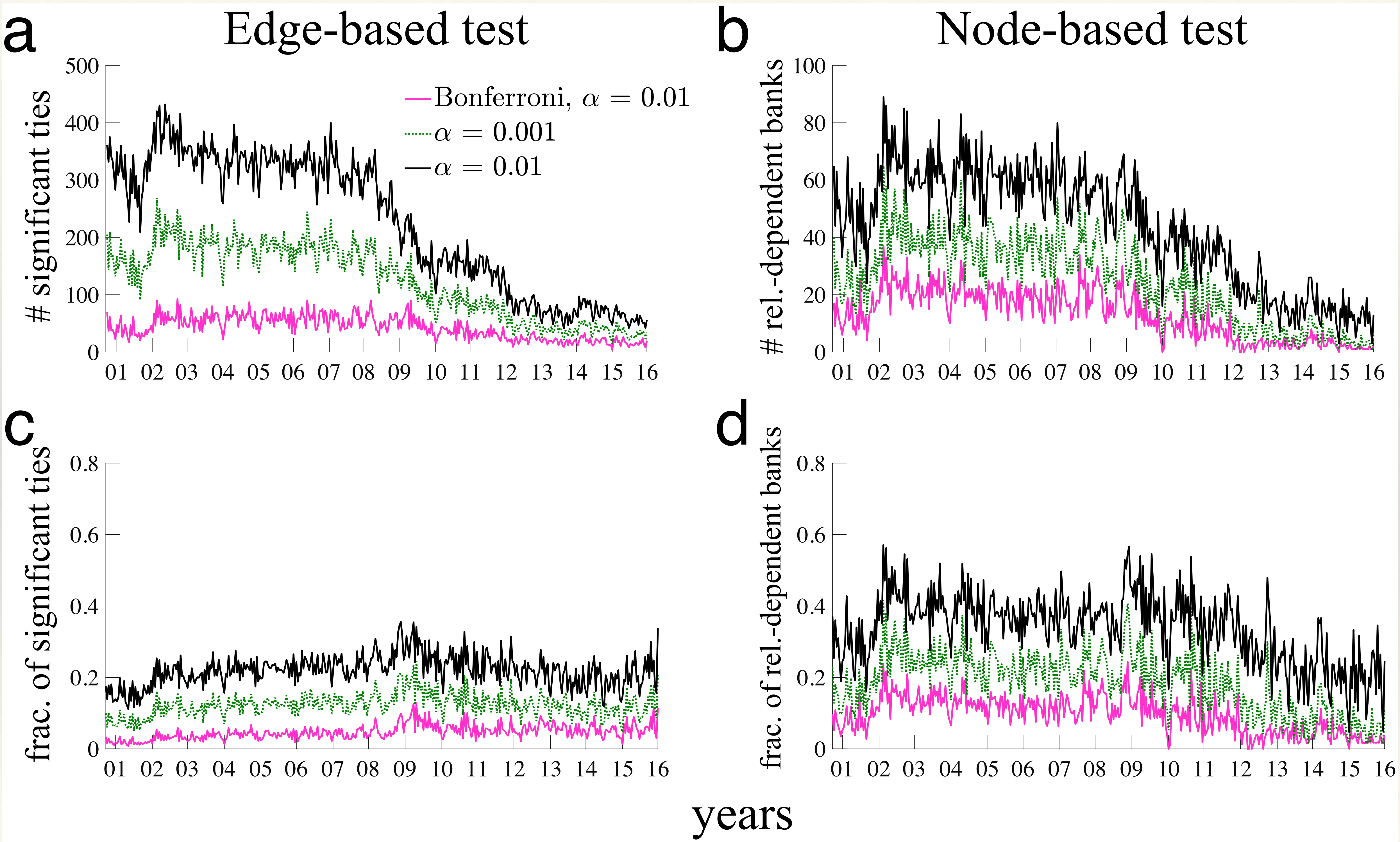
$$p_{ij}^{\mathrm{norel}}(t) = \frac{b_0}{b_1 + b_2 D_{ij}(t-1)},$$

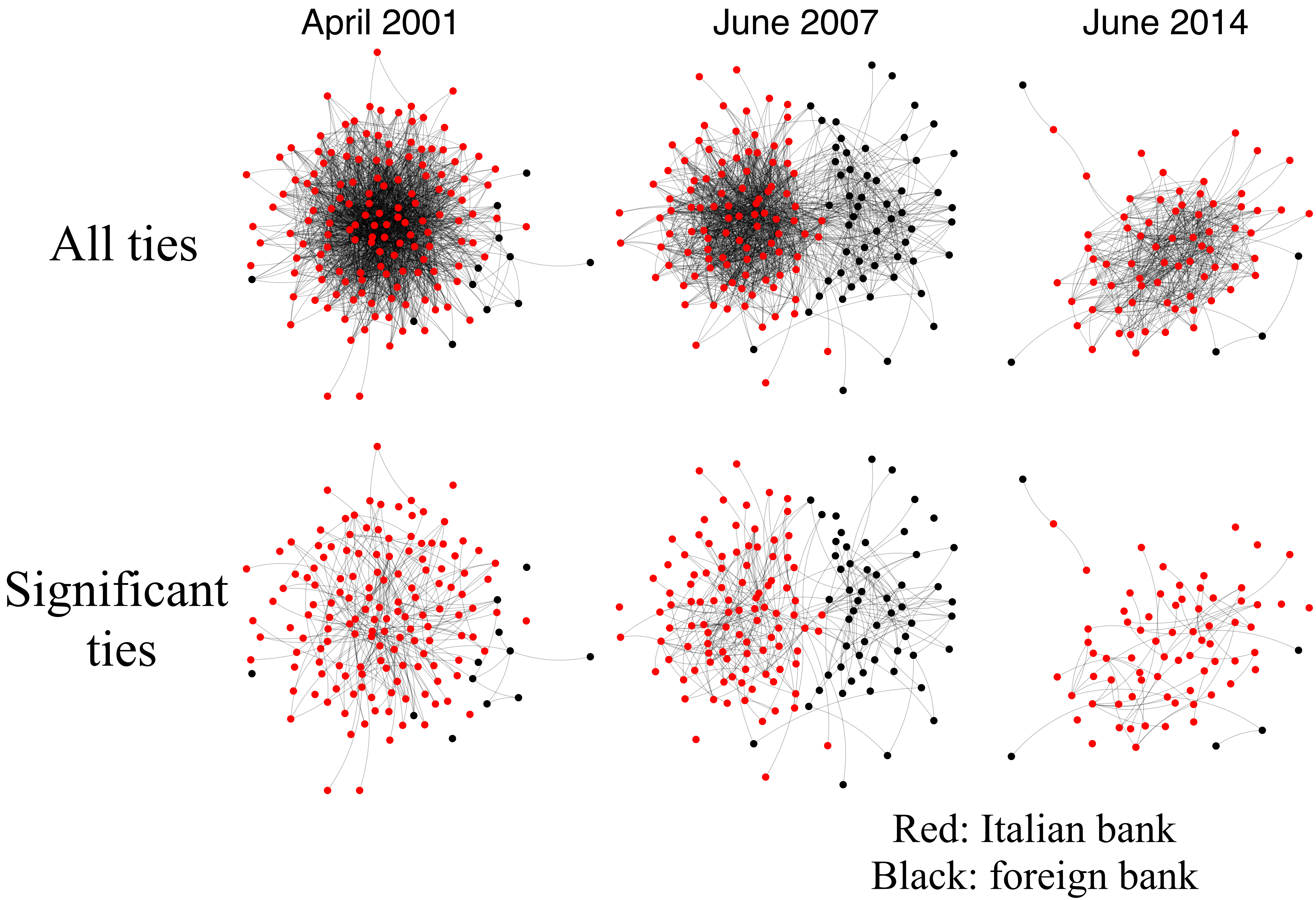
Results: synthetic network



Ground truth: 20%

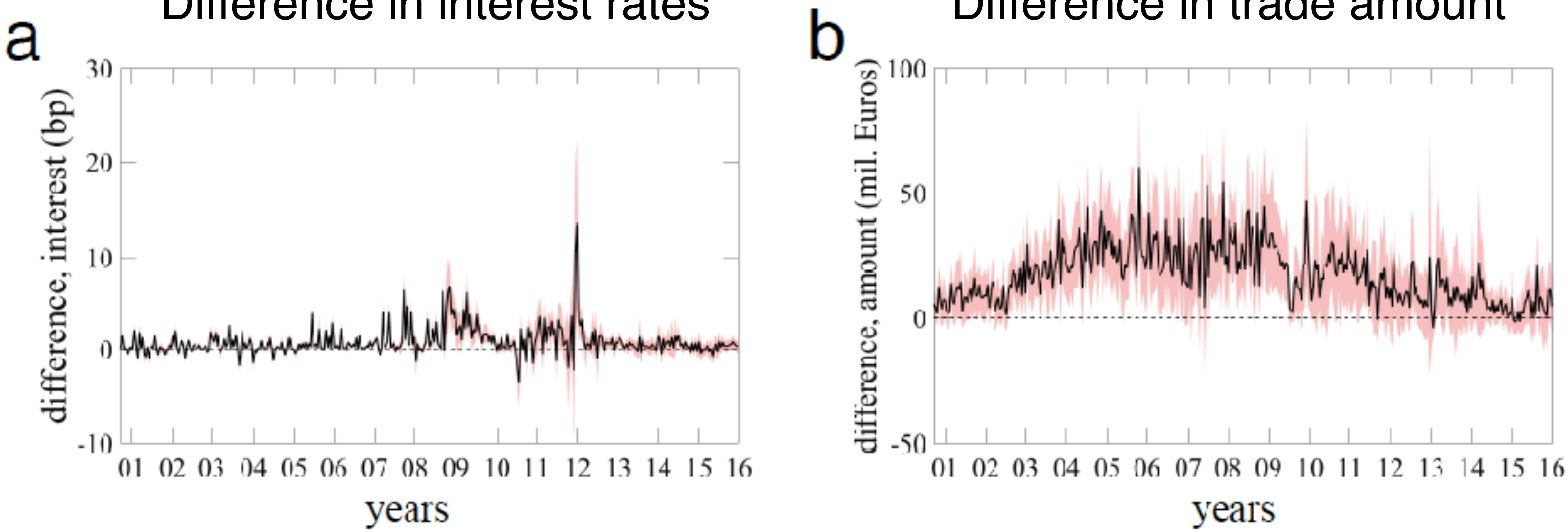
Results: empirical network




April 2001
June 2007
June 2014
All ties
Significant ties
Red: Italian bank
Black: foreign bank

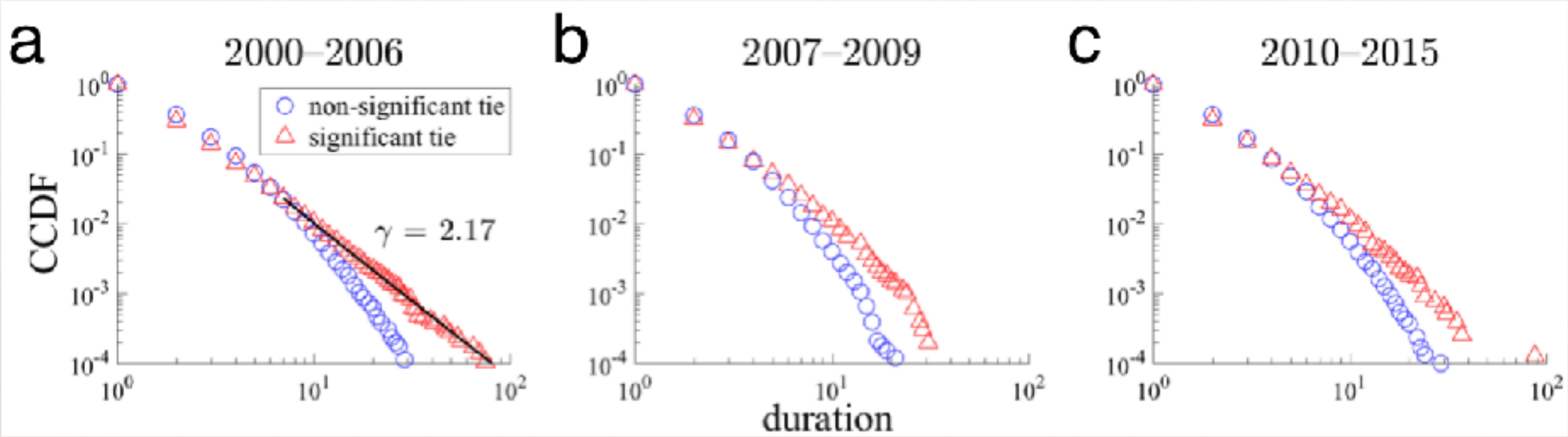
Difference in interest rates

Difference in trade amount

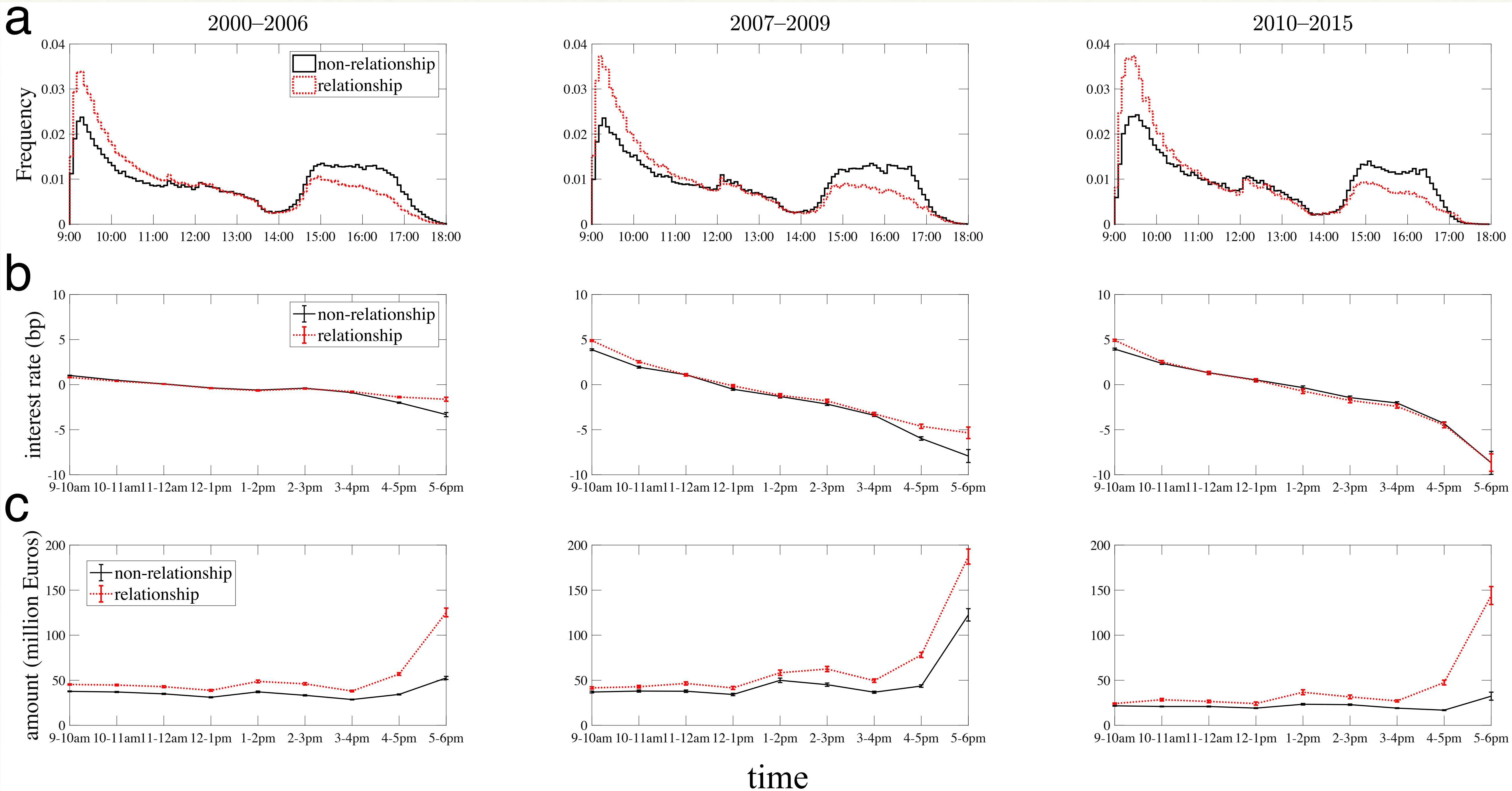


(relationship - non-relationship)

Duration of a significant tie



Intraday pattern



Conclusion

1. Fitness model works well as a null model (i.e., random matching).

2. Significant ties and relationship-dependent nodes are statistically identified by testing the null hypothesis.

3. Banks connected by significant ties trade in a different manner from the others.

Paper:

"Identifying relationship lending in the interbank market: A network approach"

***Journal of Banking & Finance*, in press.**

T. Kobayashi and Taro Takaguchi (LINE Corp),

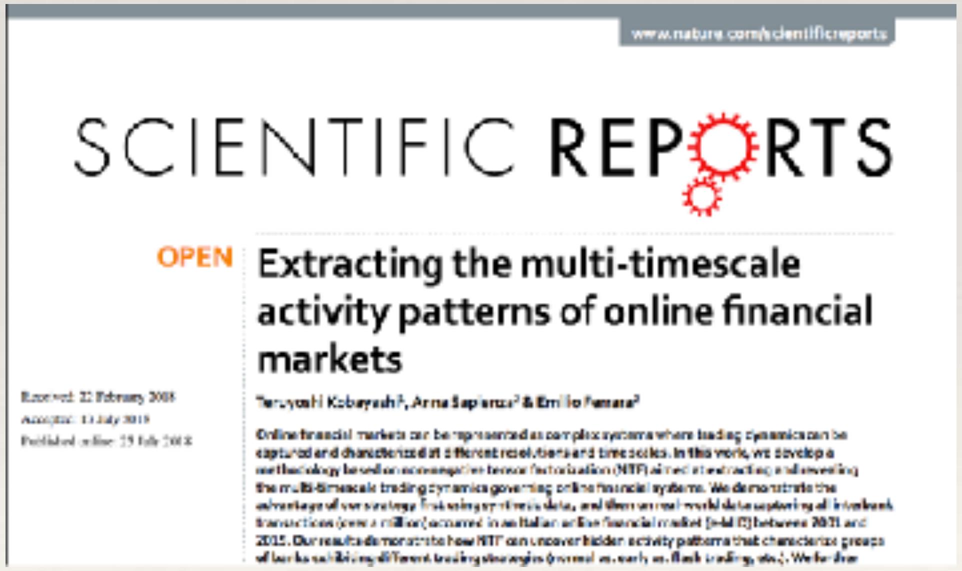
Extracting the multi-timescale activity patterns of online financial markets

Teruyoshi Kobayashi

Anna Sapienza, USC

Emilio Ferrara, USC

Scientific Reports, 2018



www.nature.com/scientificreports

SCIENTIFIC REPORTS

OPEN

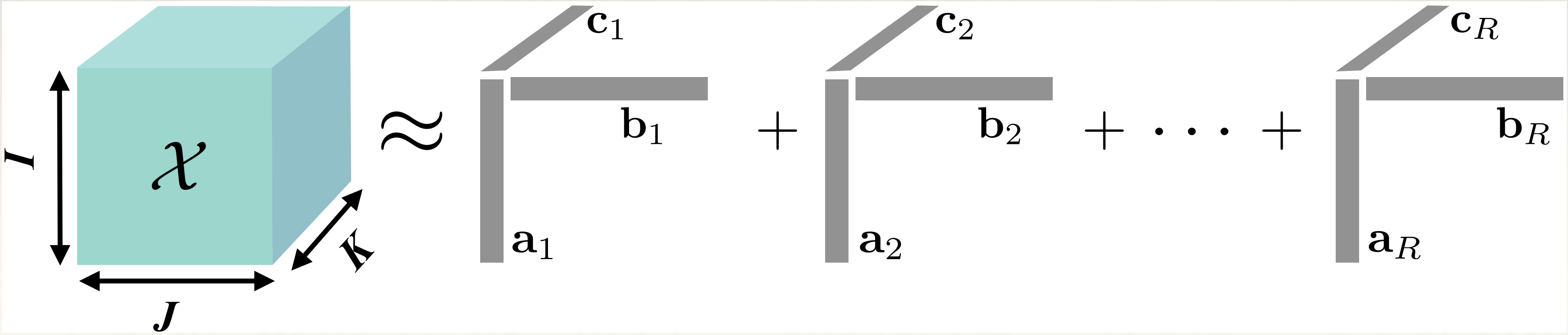
Extracting the multi-timescale activity patterns of online financial markets

Non-negative tensor factorization

- Extracting patterns from a high-dimensional data
- Representing a tensor by the sum of outer products

$$\mathcal{X} \approx \sum_{r=1}^{R} \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r, \quad \leftarrow \text{R 個の外積の和}$$

$$x_{ijk} \approx \sum_{r=1}^{R} a_{ir} b_{jr} c_{kr},$$



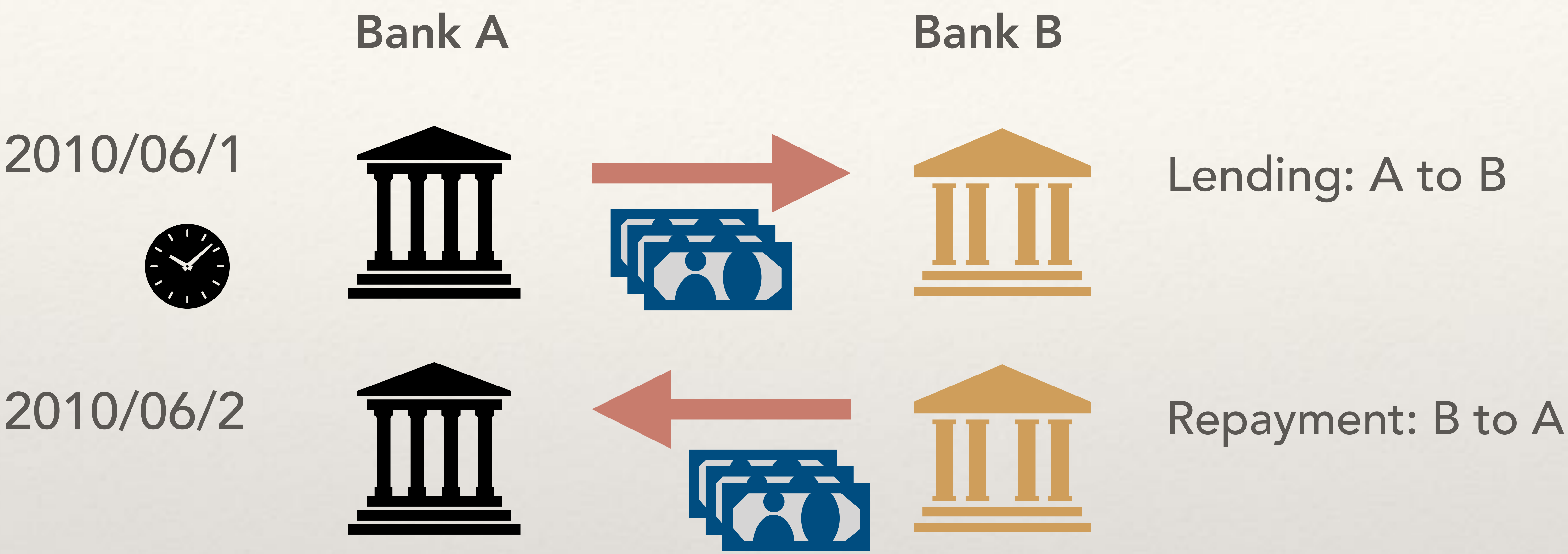
Non-negative tensor factorization

Previous applications:

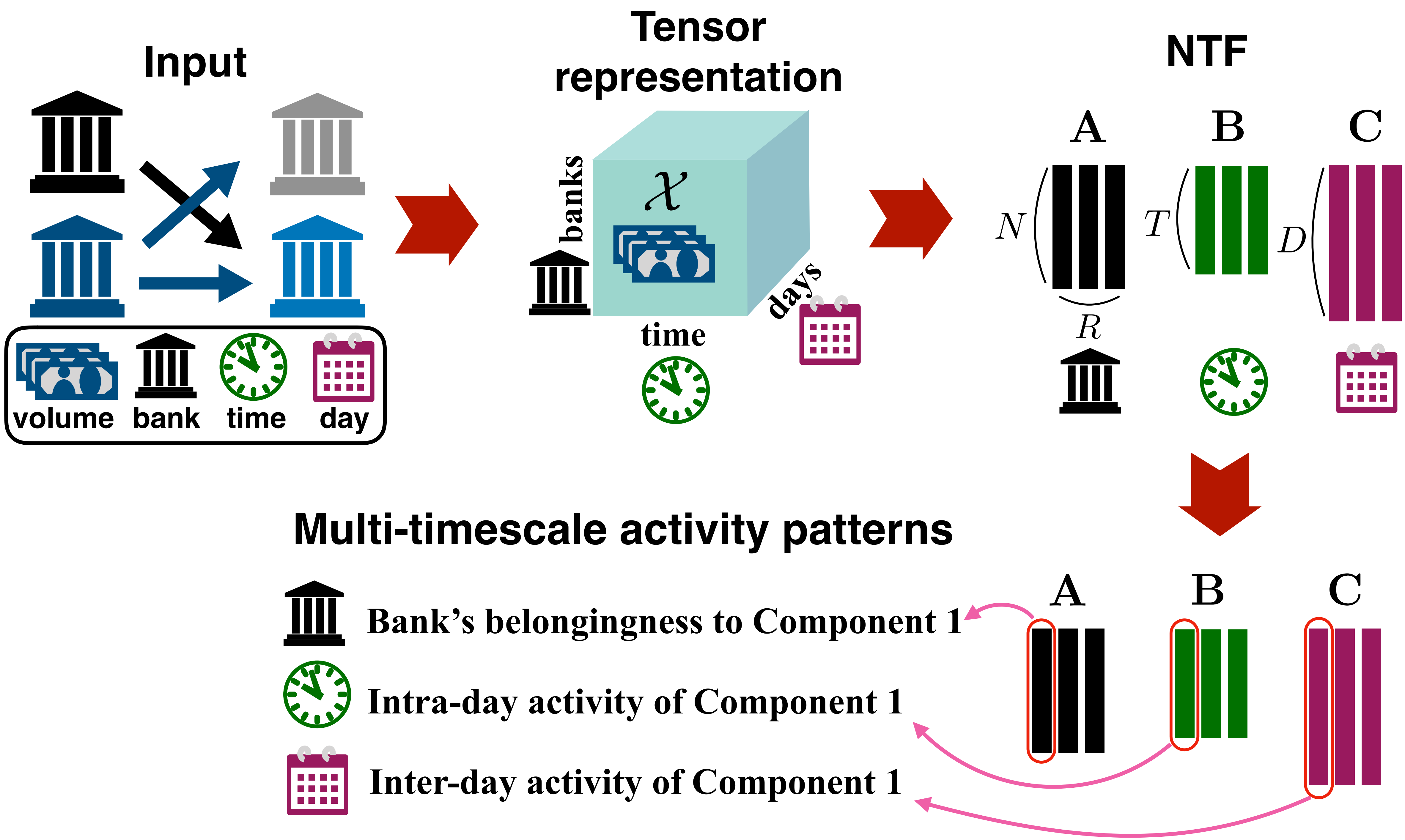
- Interaction patterns in online games (Sapienza et al, *Informaiton*, 2018), Twitter (Panisson, et al, WWW 2014)

- Detection of temporal community structure (Gauvin et al, 2014, PLOS ONE)

Aim of the work



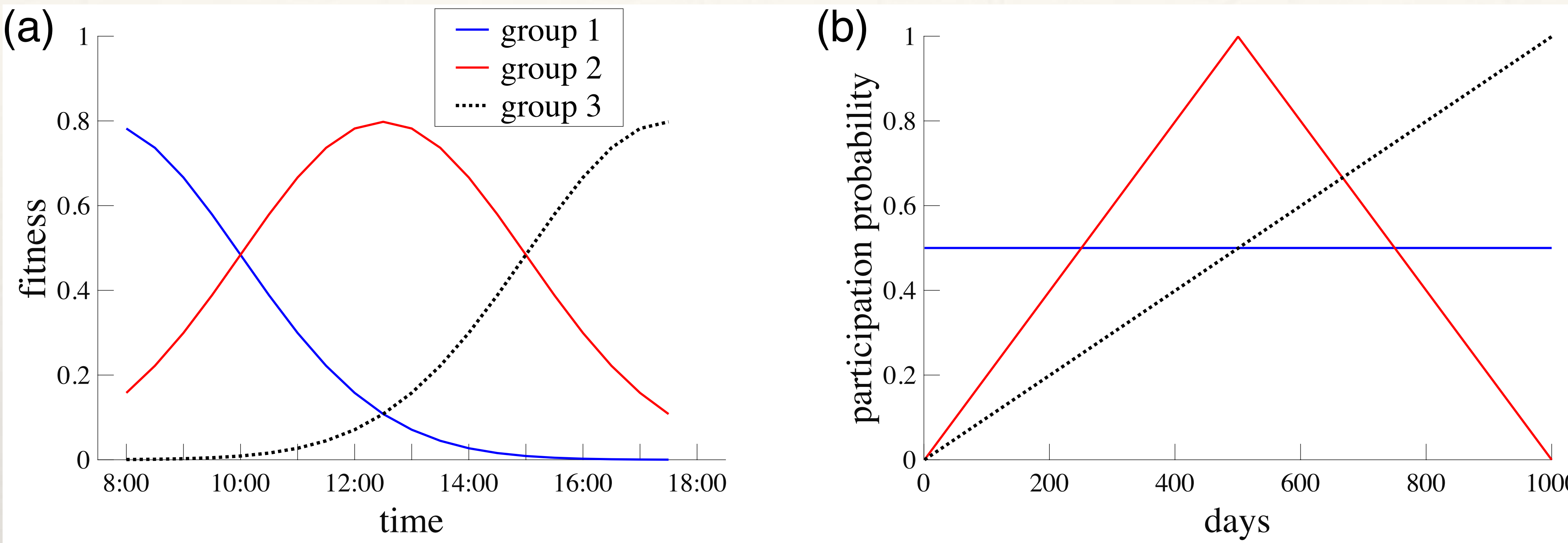
★<u>**Extract intra- and inter-day trading patterns!**</u>


Input
volume
bank
time
day
Tensor representation
$\mathcal{X}$
banks
time
days
NTF
A
B
C
N
T
D
R
Multi-timescale activity patterns
Bank's belongingness to Component 1
Intra-day activity of Component 1
Inter-day activity of Component 1
A
B
C

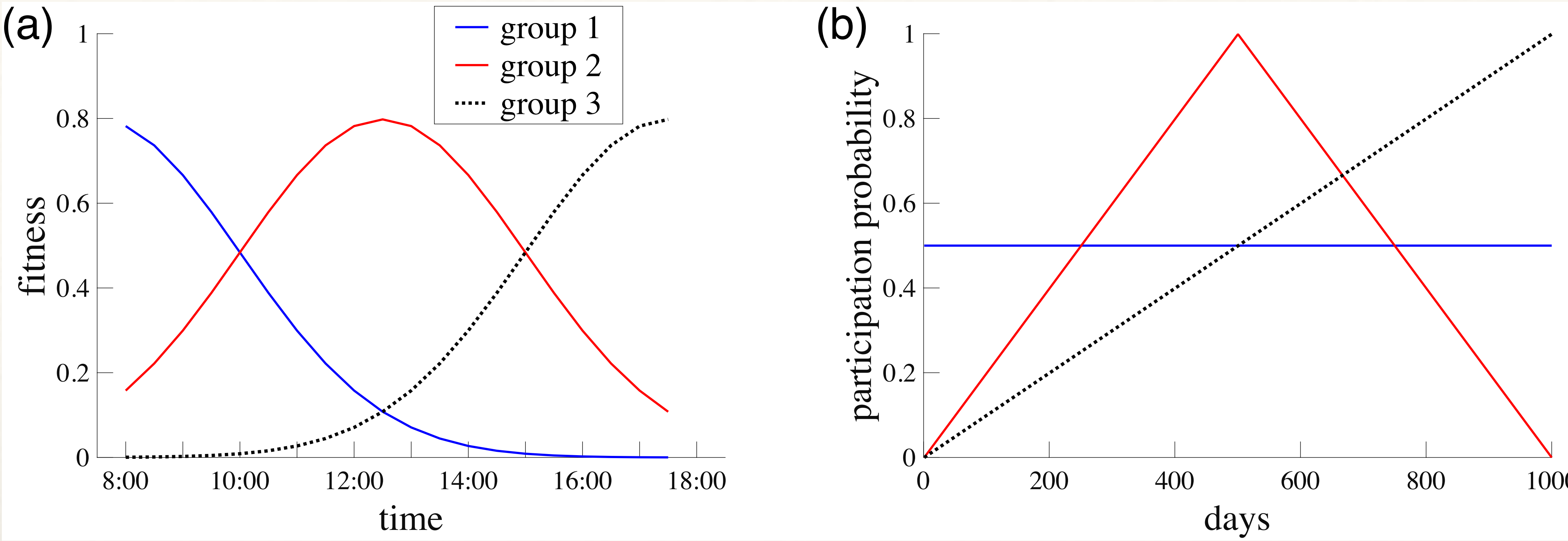
Synthetic network

Patterns embedded in synthetic temporal networks

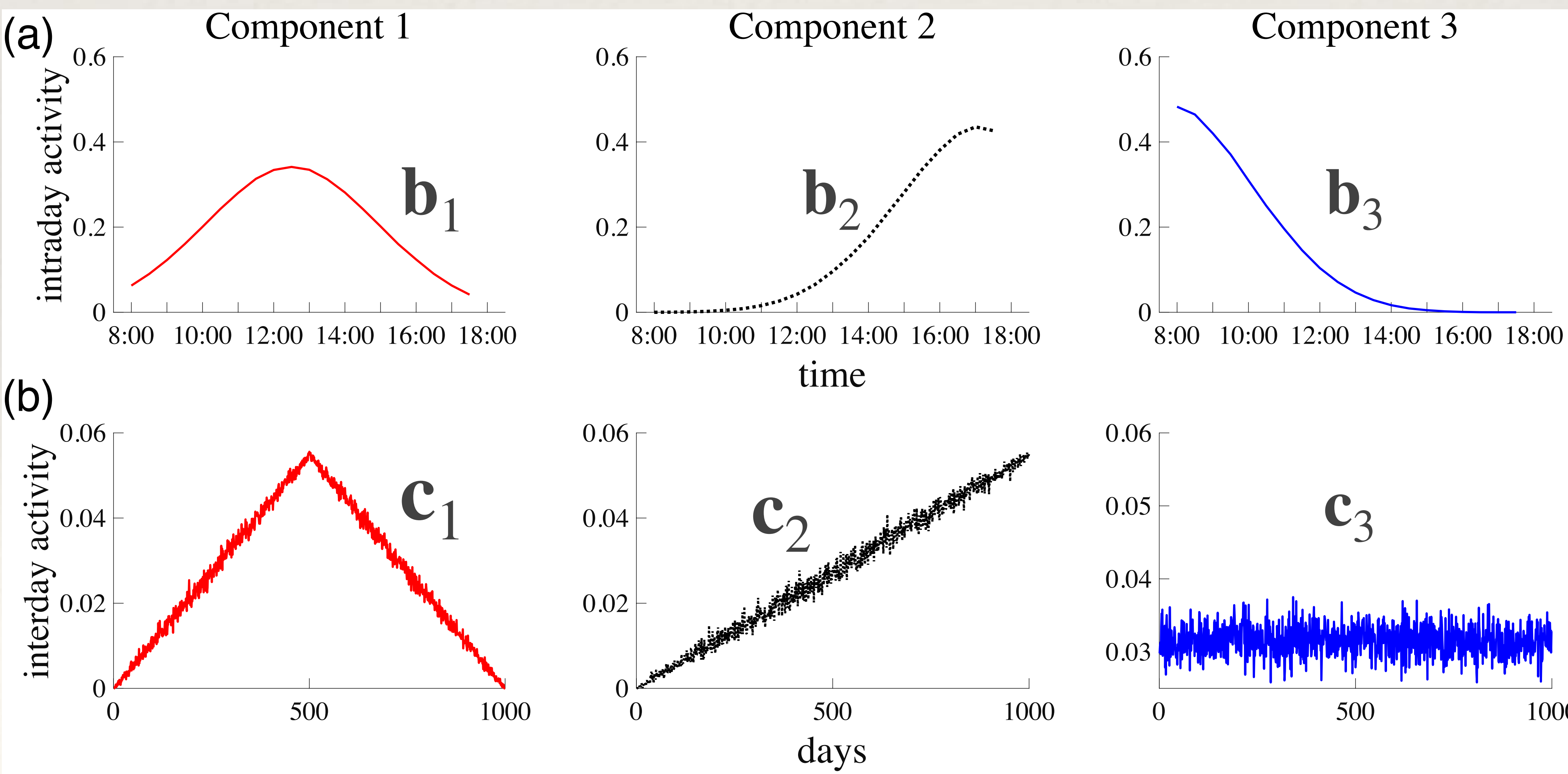
Fitness model: $p_{ij,t} = a_{i,t}a_{j,t}$



Patterns embedded in synthetic temporal networks



Patterns revealed by NTF



Determination of #components, *R*

If #component R is <u>large,</u>

- Overidentification problem

If #component R is <u>small,</u>

- Tensor decomposition will be less accurate

Determination of #components

PARAFAC or CP model:

$$x_{ijk} = \sum_{n=1}^{R} \sum_{m=1}^{R} \sum_{p=1}^{R} \lambda_{nmp} a_{in} b_{jm} c_{kp}, \qquad \lambda_{nmp} = \delta_{nm} \delta_{mp} \delta_{np}$$

Tucker3 model:

$$x_{ijk} = \sum_{n}^{R_n} \sum_{m}^{R_m} \sum_{p}^{R_p} g_{nmp} a_{in} b_{jm} c_{kp}$$

Determination of #components

PARAFAC or CP model:

$$x_{ijk} = \sum_{n=1}^{R} \sum_{m=1}^{R} \sum_{p=1}^{R} \lambda_{nmp} a_{in} b_{jm} c_{kp}, \qquad \lambda_{nmp} = \delta_{nm}\delta_{mp}\delta_{np}$$

コンポーネント間の相関許さない

Tucker3 model:

コンポーネント間の相関許す

$$x_{ijk} = \sum_{n}^{R_n} \sum_{m}^{R_m} \sum_{p}^{R_p} g_{nmp} a_{in} b_{jm} c_{kp}$$

Determination of #components

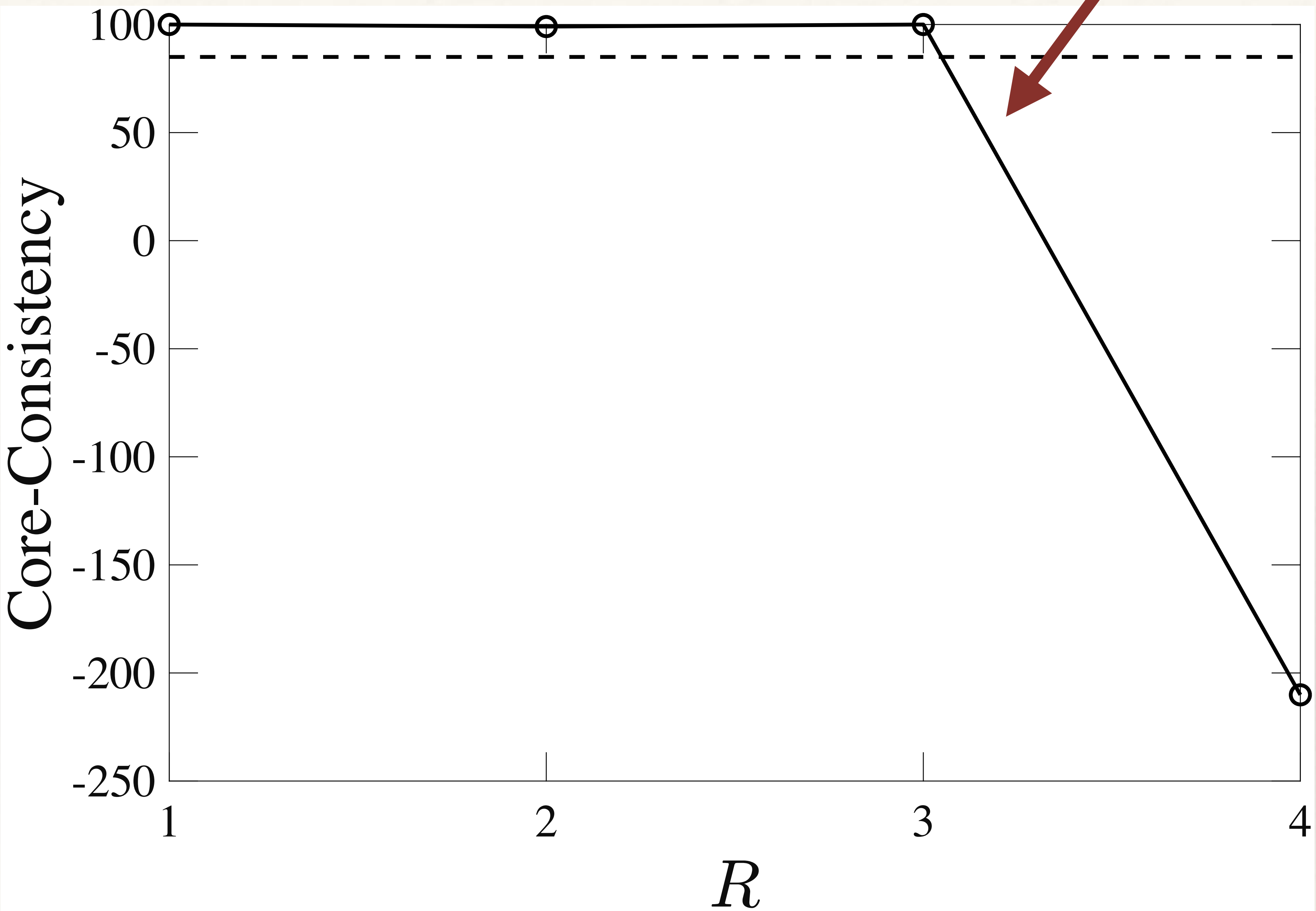
Core-consistency:

$$\mathrm{CC} = 100 \times \left(1 - \frac{\sum_{n=1}^{R} \sum_{m=1}^{R} \sum_{p=1}^{R} \left(g_{nmp} - \lambda_{nmp}\right)^2}{R} \right),$$

・100に近いほどPARAFACモデルが相対的に良い
・でもRが小さいと分解の精度が低い

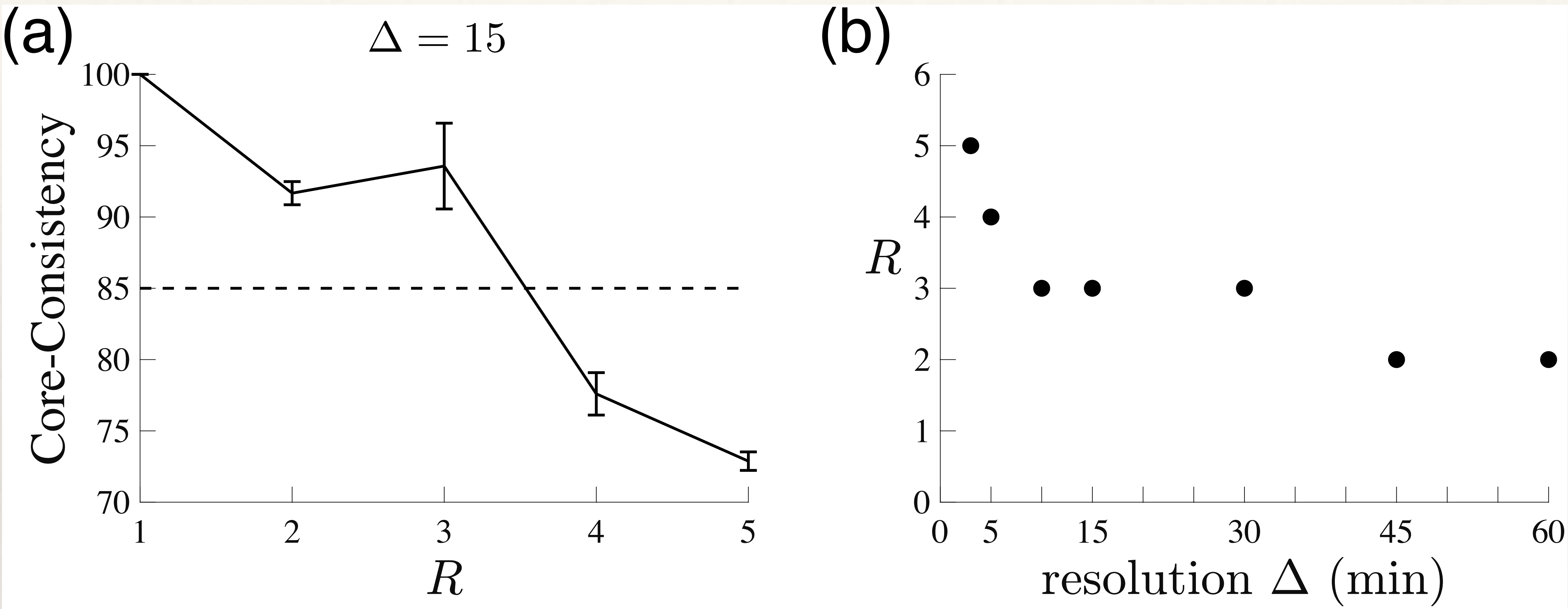
- Synthetic network

過剰にコンポーネントが
あると急激に低下する



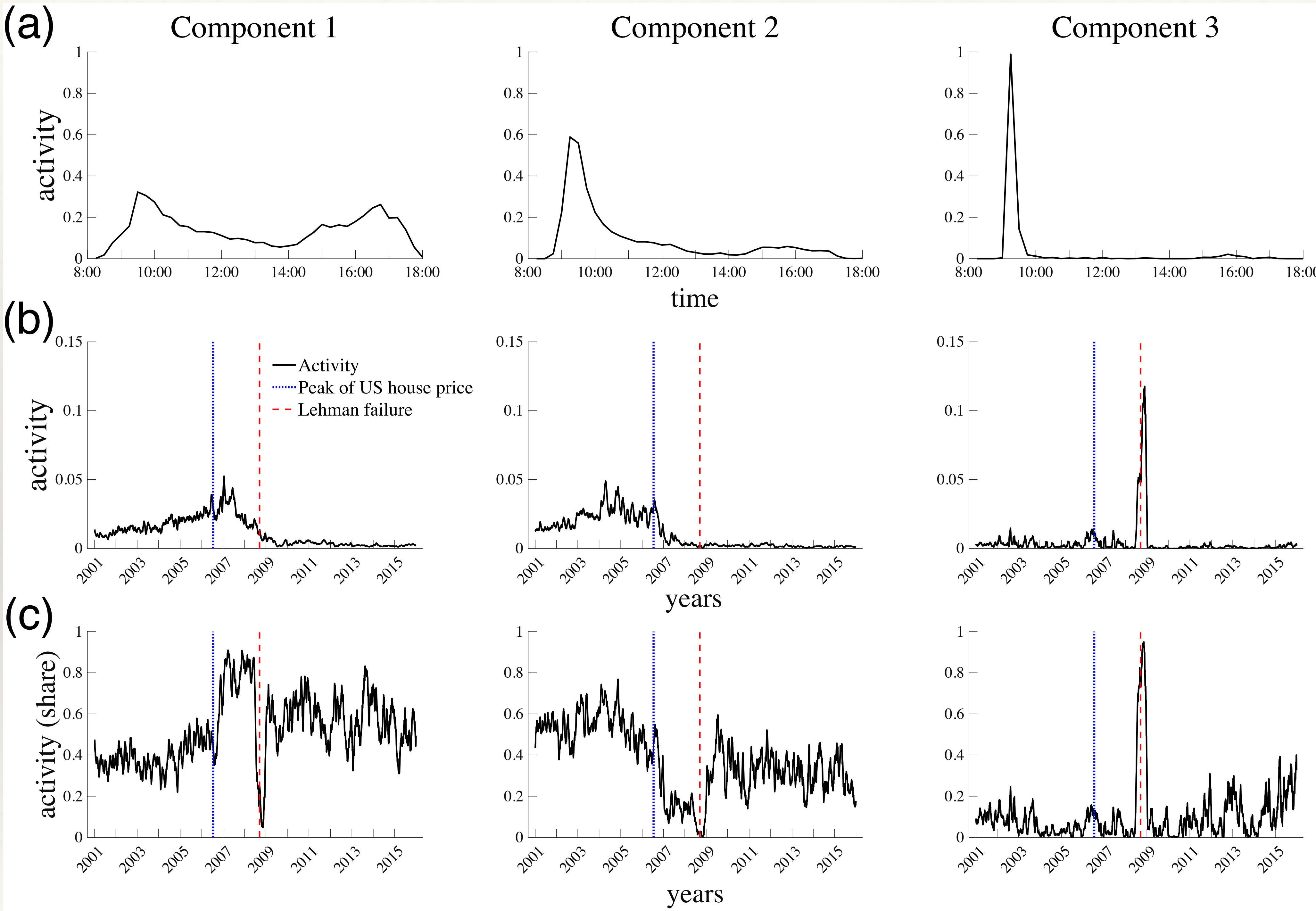
Interbank network

- Interbank network: core-consistency

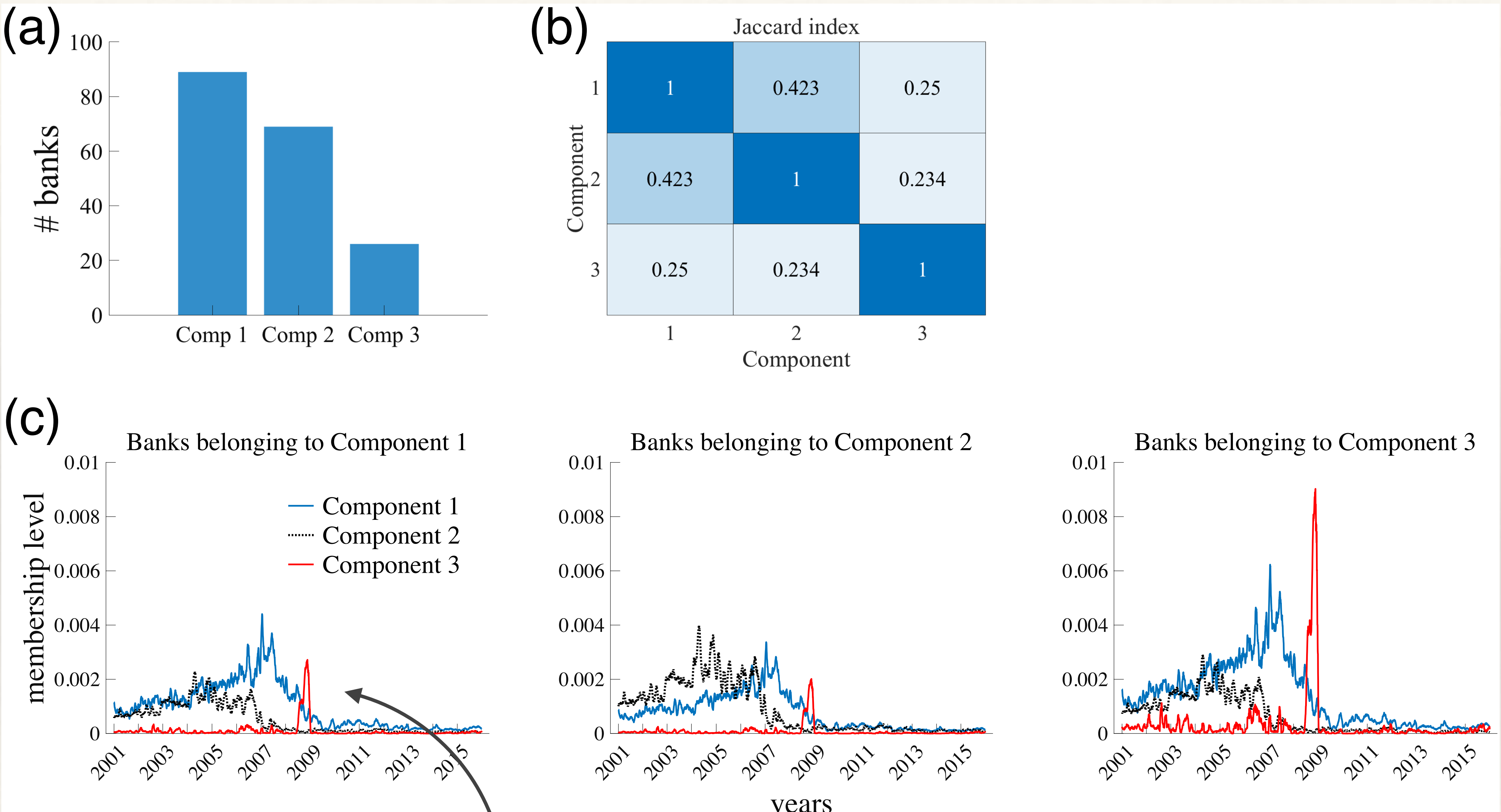


→ R = 3 を選択

• Interbank network: activity pattern



- Interbank network: banks' belongingness to components



	1	2	3
1	1	0.423	0.25
2	0.423	1	0.234
3	0.25	0.234	1

Row average of $\mathbf{a}_r \mathbf{c}_r^\top$

- Interbank network: characterization of components