Some properties of density functions on maxima of solutions to one-dimensional stochastic differential equations

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Abstract

In the talk, we shall deal with the following one-dimensional stochastic differential equation (SDE),

$$X_t = x_0 + \int_0^t b(s, X_s)ds + \int_0^t \sigma(s, X_s)dW_s, t \in [0, \infty)$$

where $b, \sigma : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ are measurable functions and $\{W_t, t \in [0, \infty)\}$ denotes a one-dimensional standard Brownian motion defined on a probability space $(\Omega, \mathcal{F}, P)$. We will consider discrete time maximum and continuous time maximum which are defined by $M_T^n := \max\{X_{t_1}, \cdots, X_{t_n}\}$ and $M_T := \max_{0 \leq t \leq T} X_t$, respectively, where the time interval $[0, T]$ and the time partition $\Delta_n : 0 < t_1 < \cdots < t_n = T$, $n \geq 2$ are fixed.

The main goal of the talk is to show some properties of the probability density functions of M_T^n and M_T denoted by $p_{M_T^n}$ and p_{M_T}, respectively. In particular, lower and upper bounds for $p_{M_T^n}$, an upper bound for p_{M_T} and a convergence of $p_{M_T^n}$ to p_{M_T} as $n \to \infty$ will be shown by means of integration by parts (IBP) formulas. Here, we say that an IBP formula for the random variables F and G holds if there exists an integrable random variable $H(F; G)$ such that

$$E^P[\varphi'(F)G] = E^P[\varphi(F)H(F; G)]$$

holds for any $\varphi \in C_b^1(\mathbb{R}; \mathbb{R})$. The random variables M_T^n and M_T are used to compute the price and risks of Lookback and Barrier options in mathematical finance and IBP formulas are applied to calculate the risks of options called the greeks. If time permits, other properties of $p_{M_T^n}$ and p_{M_T} shall be mentioned in the talk.