Two-agent discrete choice model with random coefficient utility functions for structural analysis on household labor supply*

PRELIMINARY AND INCOMPLETE

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Abstract

This paper discusses a bargaining model on discrete choices of individual household based on two-agent qualitative choice model. The two-agent qualitative choice model describes discrete choices made through bargaining interactions between two agents. This paper presents a bargaining model of discrete choices on labor force participation of wife and husband of a household.

This paper focuses on employee job opportunity, where employees' hours of work tends to be assigned by the employers. While hours of work is restricted, the choice each of the agents makes will not be continuous but discrete, i.e., a binary choice model of whether each of the agents works or not applies. This model explicitly demonstrates utility maximizing behavior of two interacting agents under such discrete constraint imposed on hours of work, describing both the labor force participating behavior of wife and that of husband endogenously.

As structural equations, an income-leisure preference function of wife and that of husband are introduced in this paper. These functions have random coefficients, which represent taste differences among wives as well as among husbands in population, so that the model gives probabilistic distributions for the outcomes of discrete choices made by husband as well as that made by wife on his or her labor supply.

This paper empirically utilized observations of labor force participating decisions made by Japanese households that consist of only one couple of wife and husband with a child or children under fifteen years old.

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1 Introduction

In the literature of path breaking analyses on qualitative economic decisions, McFadden (1973; 1981) as well as Hausman and Wise (1978) focuses on describing decisions of individual agents. Most of the models following these contributions, such as Dubin and McFadden (1984), describe discrete choices based on single-agent qualitative choice model. On the other hand, this paper discusses a bargaining model on discrete choices of individual household based on two-agent qualitative choice model. The two-agent qualitative choice model describes discrete choices made through bargaining interactions between two agents.

This paper presents a bargaining model of discrete choices on labor force participation of husband and wife of households. The model gives probabilistic distributions for the outcomes of discrete choices made by husband as well as that made by wife on his/her labor supply. According to the probabilistic distributions, the model describes binary choice behavior of whether the agents accept the employee job opportunity or not.

Contrary to self-employed workers, employees' hours of work h tends to be assigned by employers as $h = \overline{h}$, where $\overline{h}$ denotes the assigned hours. In the case where hours of work is restricted to $\overline{h}$, the choices the agents make will not be continuous but discrete, i.e., binary choice of whether each of the agents works or not applies.

This model explicitly demonstrates utility maximizing behavior of two interacting agents under such discrete constraint imposed on hours of work. As structural equations, an income-leisure preference function of wife and that of husband are introduced in this paper. These functions have random coefficients, which represent taste differences among wives and husbands in population. While indirect utility functions are implicitly introduced in the literature of empirical studies on qualitative economic decisions, direct income-leisure utility functions, as well as income-leisure restrictions are explicitly introduced as structural equations in this paper, and structural parameters of the utility functions are estimated.

Let μ_w denote wife's labor supply probability to her employee job opportunity, and let μ_h denote husband's labor supply probability to his employee job opportunity. Based on the estimated structural parameters, conditional simulation on μ_w as well as that on μ_h are performed in this paper, given observed wage rates as well as observed assigned hours of work.

In the literature of quantitative analyses on labor supply, the cross sectional analysis by Douglas (1934) gave a significant evidence that the observed job participation ratios of females are negatively correlated to the observed household income levels[1] . This finding implies that labor force participation decision made by household members may not be independent, so that the same finding suggests that interacting decision making behavior between household members need to be introduced in theory explicitly.

As for labor force participation behavior of females, Mincer (1962) described patterns of female labor force participation in a long run by introducing a lifetime hypothesis, and presented females' working hour allocation in their lifetime. More than that, Heckman (1974) demonstrated that introducing the concept of "reservation wage" gave a way to probabilistic approach to binomial working decisions made by married women.

In the literature of household decision making behavior with bargaining interactions between household members, Bjorn and Vuong (1984) presented a theory explicitly describing bargaining interactions between wife and husband in a household. The authors formulate their models of data generating process with a class of bivariate dummy endogenous variable models consisting of two equations, each of which describes whether wife works ($Y_w = 1$) or not ($Y_w = 0$) or husband works ($Y_h = 1$) or not ($Y_h = 0$). Because the author "assume that the difference in utility that the husband derives from working versus not working, when the wife works, differs only by a constant β_h from the utility he derives from working versus not working when the wife does not work. A similar simplifying assumption is made for the wife"[2] , their model has no problem of non-unique solution.

Bresnahan and Reiss (1990; 1991) pointed out the problem of "nonunique equilibrium solution"[3] which does not fulfill the Nash pure-strategy equilibrium conditions while the authors defined payoffs for two players in the simultaneous-move games. Bresnahan and Reiss (1990; 1991) precluded this problem in two different ways. Bresnahan and Reiss (1990) adopted sequential-move games, and Bresnahan and Reiss (1991) assumed a restriction on the payoffs. In this paper, simultaneous-move pure-strategy games is assumed and the author assumes constraints on the payoffs the least[4] , so that the problem of non-unique equilibrium solution arises[5] .

Instead of describing interactions between household members explicitly, Obi (1969a, 1969b, 1979) limited his analyses to the households where both wives and husbands face employee opportunities only. The author introduced an an-

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[1] See Douglas (1934) pp.279-294.

[2] Bjorn and Vuong (1984), page 7.

[3] Bresnahan and Reiss (1991), page 66. Bresnahan and Reiss (1990) also discussed this problem on page 536 through 537.

[4] Minimum a priori restrictions which keep the logical consistency of income-leisure preference fields is made on payoffs in this paper. See discussions on page 10 in Section 2.2 as well as discussions in Section 3.6, especially discussions on the Assumption 2 on page 25, for more details.

[5] Section 2.4 discusses this problem more in detail.

alytical notion of "principal earner" and "non-principal earners" of a household. "Principal earner" is defined as the household member whose wage rate is the highest among the wages of household members in each specific household. Given the observed wage difference between male's labor market and female's labor market, husbands are assigned as principal earners, and wives are assigned as non-principal earners in most cases. Obi described the non-principal earners' labor supply probability to their employee and/or self-employee job opportunities, given the principal earners' job participation, and thus given the principal earners' income level.

The model in this paper assumes that the players of two-person simultaneous-move game behave based on the pure strategy. In an empirical study, this paper utilizes observations of labor supply decisions made by Japanese households that consist only one couple of wife and husband only with a child/children under fifteen years old[6] . This paper explicitly describes labor force participating decisions of household through interactions between wife and husband in a household, thus both wife's work decision and husband's work decision are described endogenously.

In Section 2, I propose a two-agent discrete choice model of household on labor force participating decisions to employee job opportunities. Based on the model in Section 2, Section 3 gives a stochastic model of household labor supply. This model introduces random coefficients[7] to income-leisure preference functions representing taste difference among agents in population. Estimation of structural parameters and simulation is presented in Section 4. Concluding remarks are given in Section 5.

2 A two-agent discrete choice model of household on labor supply

This section presents a model of household labor supply of husband and wife for employee job opportunities. The household described here is supposed to consist of only one couple of husband and wife with their child/children under 15 years[8] of age, if any. In this sense, this model is a special case of a two-agent discrete choice model of household on labor supply.

Household members, not limited to husbands and wives, generally have choices among self-employed job opportunities as well as among employee job opportunities. The model in this paper exclusively focuses on the decision mak-

[6] Children under fifteen years old are prohibited to work by law in Japan, and thus the agents that make work decisions are limited to wife and husband in such a household.

[7] The model assumes that both the coefficient set of the wife and that of the husband at the ith household in population never changes over time and that the coefficient sets of the wives or the husbands are randomly distributed among agents, i.e., among wives or among husbands in population.

[8] See footnote 6.

ing behavior concerning employee job opportunities only[9] . Contrary to self-employed workers, employees' hours of work h tends to be assigned by the employers as $h = \overline{h}$, where $\overline{h}$ denotes the assigned hours. In this case, the decision making of labor supply has characteristics of discrete choice, because what each agent can choose is, not how long he or she works, but whether he or she works.

2.1 The income-leisure preference function and the constraints for its maximization

Non-labor income (in real term) which the ith household gains during a unit period is denoted as I_A^i. The wage rate and the assigned hours of work of the employee job opportunity, which the husband of the ith household faces, is denoted by w_h^i and $\overline{h}_h^i$ respectively. Analogously, the wage rate and the assigned hours of work, which the wife of the ith household faces, is denoted by w_w^i and $\overline{h}_w^i$ respectively. The wage rates, w_h^i and w_w^i, as well as the non-labor income I_A^i, are measured in real term. The total income of the ith household in real term is denoted by X^i. X^i is the sum of the non-labor income, I_A^i, and the income actually earned by the husband plus the income actually earned by the wife, where both of the husband and the wife belong to the same ith household.

Let the leisure of husband and that of wife be denoted by Λ_h^i and Λ_w^i respectively. The range of Λ_h^i and Λ_w^i should be $0 \leq \Lambda_h^i \leq T$ and $0 \leq \Lambda_w^i \leq T$ respectively, where T is the agent's maximum amount of consumable leisure during a unit period.

The total income, X^i is subject to expenditure by the wife as well as by the husband of the ith household. On the contrary, the husband's leisure, Λ_h^i, is not subject to consumption by his wife, nor the wife's leisure, Λ_w^i, is not subject to consumption by her husband. In short, it is reasonably assumed that each member of the ith household can exclusively consume his own or her own leisure on his/her own. Based on this reasoning, the following assumption is introduced in this paper.

> **Assumption 1**: Each of the husband and the wife of the ith household has the following utility indicator function, ω_h and ω_w respectively.

$$\omega_h^i = \omega_h(X^i, \Lambda_h^i \,|\, \mathbf{\Gamma}_h^i) \quad (1)$$
$$\omega_w^i = \omega_w(X^i, \Lambda_w^i \,|\, \mathbf{\Gamma}_w^i) \quad (2)$$

[9] See Miyauchi (1992) for a two-agent discrete/continuous choice model of household on labor supply for employee and/or self-employed job opportunities.

, where $\mathbf{\Gamma}_h^i$ and $\mathbf{\Gamma}_w^i$ are parameter vectors of the utility function, each for the ith household's husband and the wife respectively [10] . Note that the utility indicator functions ω_h^i and ω_w^i share the total income X_i (endogenous variable). Note also that wife's leisure Λ_w^i (endogenous variable) enters only her utility indicator function ω_w^i but not husband's utility indicator function ω_h^i. Similarly, husband's leisure Λ_h^i (endogenous variable) enters only his utility indicator function ω_h^i but not wife's utility indicator function ω_w^i.

Each of the husband and the wife maximizes his or her own utility indicator function subject to the constraints of;

$$X^i = I_A^i + w_h^i h_h^i + w_w^i h_w^i\,, \qquad \begin{cases} (h_h^i = 0 \text{ or } h_h^i = \overline{h}_h^i) \\ (h_w^i = 0 \text{ or } h_w^i = \overline{h}_w^i) \end{cases} \tag{3}$$

$$\Lambda_h^i = T - h_h^i\,, \qquad (h_h^i = 0 \text{ or } h_h^i = \overline{h}_h^i) \tag{4}$$

$$\Lambda_w^i = T - h_w^i\,, \qquad (h_w^i = 0 \text{ or } h_w^i = \overline{h}_w^i) \tag{5}$$

, where each of the endogenous variables h_h^i and h_w^i denotes hours of work of husband and that of wife which is subject to the husband's choice or to the wife's choice respectively, where both of the husband and the wife belong to the same ith household.

The husband of the ith household maximizes the utility indicator function (1) subject to the constraints (3) and (4). Similarly, the wife of the ith household maximizes the utility indicator function (2) subject to the constraints (3) and (5). Inserting these restrictions (3) through (5) into the utility indicator function (1) and (2) yields

$$\omega_h^i = \omega_h(I_A^i + w_h^i h_h^i + w_w^i h_w^i,\, T - h_h^i \,|\, \mathbf{\Gamma}_h^i) \tag{6}$$

$$\omega_w^i = \omega_w(I_A^i + w_h^i h_h^i + w_w^i h_w^i,\, T - h_w^i \,|\, \mathbf{\Gamma}_w^i) \tag{7}$$

$$\text{, where} \begin{cases} h_h = 0 & \text{or} \quad h_h = \overline{h}_h \\ h_w = 0 & \text{or} \quad h_w = \overline{h}_w \end{cases} .$$

Note that both h_h^i and h_w^i enter the each of the husband's utility function (6) and the wife's utility function (7). For this reason, the attainable maximum utility of each husband and wife is not independently determined, but affected by the labor supply choice of the spouse (or the partner) belonging to the same household.

2.2 Payoffs of labor supply choice in terms of preference

As shown in equations (6) and (7), each of the utility level of husband and that of wife, ω_h and ω_w respectively depends on the combination of (h_h, h_w). The

[10] Although the case $\dfrac{\partial \omega_h}{\partial \Lambda_w} \neq 0$ or $\dfrac{\partial \omega_w}{\partial \Lambda_h} \neq 0$ cannot be excluded a priori, a functional form such that these values are constantly zero is assumed for simplicity in this paper.

combination of (h_h, h_w) is exhaustible within the following four cases according to the labor supply choice made by husband and that of wife.

(i)	$(h_h, h_w) = (0, 0)$	neither works,
(ii)	$(h_h, h_w) = (\overline{h}_h, 0)$	only husband works,
(iii)	$(h_h, h_w) = (0, \overline{h}_w)$	only wife works,
(iv)	$(h_h, h_w) = (\overline{h}_h, \overline{h}_w)$	both work,

where the superscript i is suppressed for simplicity.

Let the above (i) through (iv) be noted as the "combination of labor supply choice." The husband's preference order related to the "combination of labor supply choice" (i) through (iv) is determined by the inequalities among the set of values of ω_h's, which are obtained by inserting the each value of (h_h, h_w) in (i) through (iv) into the right hand side of the husband's utility function (6). In other words, the preference order is determined by the shapes of the husband's indifference curves. The wife's preference order is also determined similarly by inserting the each value of (h_h, h_w) in (i) through (iv) into her utility function (7), which is equivalent to the statement that the order is dependent on the shapes of the wife's indifference curves.

Income-leisure restrictions, (3) through (5), are graphically depicted in Figure 1. The preference field depicted on the right half of Figure 1 is the husband's, and that of wife is depicted on the left half. The vertical axis depicts the total income X, which commonly enters the right hand sides of the husband's utility function (1) and that of the wife's utility function (2). The horizontal axis depicts the leisure of husband Λ_h or that of wife Λ_w, according to whose field it belongs to. Note that Λ_h and Λ_w are depicted separately in Figure 1, because Λ_h enters exclusively into the husband's utility function (1), and so does Λ_w into the wife's utility function (2). The length of $\overline{\text{or}}$, and that of $\overline{\text{o'r'}}$ shows the husband's and the wife's maximum amount of consumable leisure during a unit period, T, respectively.

The set of the wage rate, w_h, and the assigned hours of work, $\overline{h}_h$, of husband's employee job opportunity is depicted by the portion of the line $\overline{\text{ab}}$ or by that of the line $\overline{\text{cd}}$, depending on whether his wife works or not. The portion of the line $\overline{\text{ab}}$ and that of the line $\overline{\text{cd}}$ are depicted so that $\tan\alpha = w_h$ holds.

On the other hand, the set of the wage rate, w_w, and the assigned hours of work, $\overline{h}_w$, of wife's employee job opportunity is depicted by the portion of the line $\overline{\text{a'b'}}$ or by that of the line $\overline{\text{c'd'}}$, depending on whether her husband works or not. The portion of the line $\overline{\text{a'b'}}$ and that of the line $\overline{\text{c'd'}}$ are depicted so that $\tan\beta = w_w$ holds.

Let us see the Figure 1 closely for each case of the "combination of labor supply choice" (i) through (iv). (i) In case neither works $[(h_h, h_w) = (0, 0)]$, the husband is located at point a, and the wife is at point a'. Let the intersection point with the Λ_h axis and a vertical line passing point b and d be denoted by H. The length of the portion of the line $\overline{\text{rH}}$ is the husband's assigned hours of work. Similarly, let the intersection point with the Λ_w axis and a vertical

Wife's income-leisure preference field

$\overline{h}_w$:Wife's assigned hours of work

w_w:Wife's wage rate in real term

$\tan\beta = w_w$

Husband's income-leisure preference field

$\overline{h}_h$:Husband's assigned hours of work

w_h:Husband's wage rate in real term

$\tan\alpha = w_h$

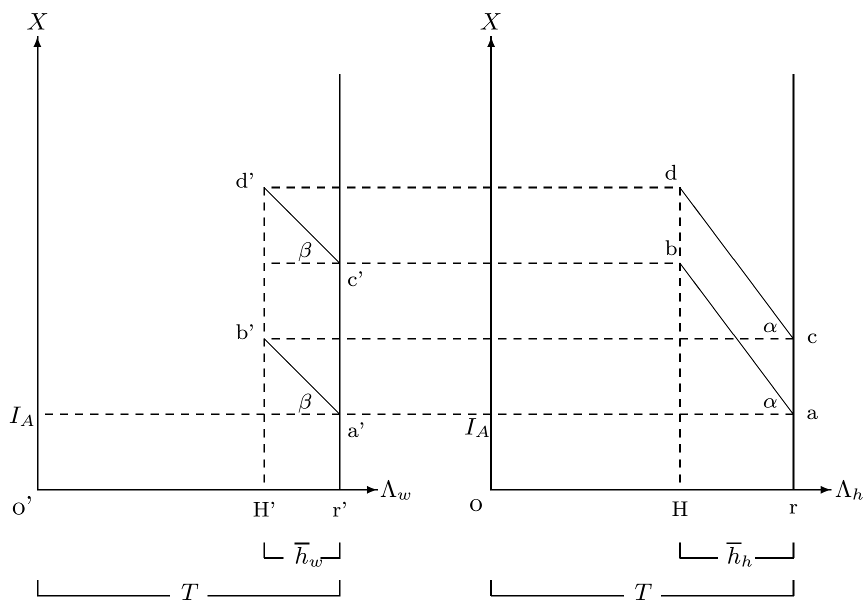

Figure 1: Husband's and wife's income-leisure preference field and their income-leisure constraints

line passing point b' and d' be denoted by H'. The length of the portion of the line $\overline{\text{r'H'}}$ is the wife's assigned hours of work. (ii) In case only husband works $[(h_h, h_w) = (\overline{h}_h, 0)]$, the husband is located at point b, and the wife is at point c'. (iii) In case only wife works $[(h_h, h_w) = (0, \overline{h}_w)]$, the husband is at point c, and the wife at point b'. Lastly (iv) in case both work $[(h_h, h_w) = (\overline{h}_h, \overline{h}_w)]$, the husband is at point d, and the wife at point d'.

Because the total income X, which is common to the husband and the wife, enters both the husband's utility function (1) and the wife's utility function (2) simultaneously, the ordinate of the husband's position and that of the wife's position in their preference field are always the same to each other.

Given the income-leisure constraint equations, (3) and (4), the husband faces one of the two alternative choice sets {a, b} or {c, d}, depending upon the wife's decision of whether she works. If the wife works, the husband faces the choice set of {c, d}, otherwise {a, b}. The values of utility index can be assigned to these points a, b, c, and d by inserting the coordinates of these points to the right hand side of the formula (6), and those values are regarded as payoffs of the husband. Similarly, given the income-leisure constraint equations, (3) and (5), the wife faces one of the two alternative choice sets {a', b'} or {c', d'}, depending upon the husband's decision of whether he works. If the husband works, the wife faces the choice set of {c', d'}, otherwise {a', b'}. The values of utility index at these points a', b', c', and d' are given by the formula (7), and those values are regarded as payoffs of the wife.

The husband's payoffs vary depending upon the shapes of his indifference curves, and so do the wife's payoffs. How many types of payoffs possibly exist as the shapes of indifference curves vary? The permutation of husband's set of four points a, b, c, and d or that of wife's set of a', b', c', and d' is 4! = 24 in all respectively. However, only 6 types of payoffs out of 24 proved to be plausible under the conditions of the positivity of marginal utility and the convexity of indifference curves[11] .

Let the values of husband's preference index at the points a, b, c, and d be denoted by ω_h^{a}, ω_h^{b}, ω_h^{c}, and ω_h^{d} respectively. Similarly let the values of wife's preference index at the points of a', b', c', and d' be denoted by $\omega_w^{\text{a'}}$, $\omega_w^{\text{b'}}$, $\omega_w^{\text{c'}}$, and $\omega_w^{\text{d'}}$ respectively. Thus the payoffs of husband and these of wife are given as in Table 1 and Table 2 respectively.

Table 1: Husband's payoffs

wife / husband	$h_w = 0$	$h_w = \overline{h}_w$
$h_h = 0$	ω_h^{a}	ω_h^{c}
$h_h = \overline{h}_h$	ω_h^{b}	ω_h^{d}

Table 2: Wife's payoffs

wife / husband	$h_w = 0$	$h_w = \overline{h}_w$
$h_h = 0$	$\omega_w^{\text{a'}}$	$\omega_w^{\text{b'}}$
$h_h = \overline{h}_h$	$\omega_w^{\text{c'}}$	$\omega_w^{\text{d'}}$

Let the husband's payoff matrix of ω_h's in Table 1 be denoted by π_h^k $(k = 1, 2, \cdots, 6)$. Similarly let the wife's payoff matrix of ω_w's in Table 2 be denoted

[11] See Miyauchi(1991).

by π_w^ℓ $(\ell = 1, 2, \cdots, 6)$. The superscript k of π_h^k indicates the plausible kth type of husband's payoff matrix, and the superscript ℓ of π_w^ℓ indicates the plausible ℓth type of wife's payoff matrix.

The 6 types of husband's payoff matrix are listed in the left column of Table 3, while the 6 types of wife's payoff matrix are in the right column[12] . Since the utility index is ordinal, the payoffs in the matrices π_h^k and π_w^ℓ are indicated with the ordinal numbers 1, 2, 3, and 4, where 1 is the least preferred and 4 is the most preferred.

Table 3: Six types of payoff matrix

$\pi_h^1 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$	$\pi_w^1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
$\pi_h^2 = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$	$\pi_w^2 = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$
$\pi_h^3 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$	$\pi_w^3 = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$
$\pi_h^4 = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$	$\pi_w^4 = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$
$\pi_h^5 = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$	$\pi_w^5 = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$
$\pi_h^6 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$	$\pi_w^6 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

The payoff matrices π_h^k $(k = 1, 2, \cdots, 6)$ and π_w^ℓ $(\ell = 1, 2, \cdots, 6)$ shown in Table 3 are relevant and exhaustible for all the plausible cases under the basic assumption of utility function. In other words, the payoff matrices in Table 3 cover the whole plausible cases. Note that the wife's payoff matrix π_w^6 should be excluded from the plausible set of payoff matrix if and only if $w_h \overline{h}_h \geq w_w \overline{h}_w$ holds, and the husband's payoff matrix π_h^6 should be excluded from the plausible set of payoff matrix if and only if $w_h \overline{h}_h \leq w_w \overline{h}_w$ hols[13] .

2.3 Given income of husband and that of wife

Let I_h^0 denote husband's given income, that is the amount of income available to husband even if he does not work. I_h^0 is the sum of non-labor income of the household, I_A, and the income earned by his wife, $w_w h_w$, as

$$I_h^0 = I_A + w_w h_w \tag{8}$$

[12] See Miyauchi(1991) for more precise discussions on the relation between the types of payoff matrix and the shapes of indifference curves.
[13] See Miyauchi(1991) for more details.

where $h_w = 0$ in case his wife does not work, and where $h_w = \overline{h}_w$ in case she works[14] .

Similarly, let I_w^0 denote wife's given income, that is the sum of non-labor income of the household, I_A, and the income earned by her husband, $w_h h_h$, as

$$I_w^0 = I_A + w_h h_h \tag{9}$$

where $h_h = 0$ in case her husband does not work, and where $h_h = \overline{h}_h$ in case he works.

2.4 The Nash equilibrium solution of the two-person simultaneous-move game

The model in this paper assumes that the players of two-person simultaneous-move game behave based on the pure strategy.

Combining the husband's payoff matrices, π_h^k $(k = 1, 2, \cdots, 6)$, and the wife's payoff matrices, π_w^ℓ $(\ell = 1, 2, \cdots, 6)$, payoff table, $\Pi^{k-\ell}$ $(k, \ell = 1, 2, \cdots, 6)$, is obtained. For example, combining the husband's payoff matrix, π^1, and the wife's payoff matrix, π^3, yields a payoff table Π^{1-3}.

$$\Pi^{1-3} = \begin{bmatrix} (1,1) & (3,2) \\ (2,4) & (4,3) \end{bmatrix}$$

Some payoff tables have the Nash equilibrium solution, and others not. Among the payoff tables that have the Nash equilibrium solution, some tables have the unique Nash equilibrium solution and others have non-unique equilibrium solution. In the left half of Table 4-1 through Table 4-3, the Nash equilibrium solution corresponding to each payoff table is shown by $\circ$ or $\odot$ if the Nash equilibrium solution exists.

Neither the payoff table Π^{2-3} nor the payoff table Π^{3-2} fulfills the condition of the Nash equilibrium solution[15] .

The payoff table Π^{3-3} has two Nash equilibrium solutions, which means the outcome of the labor force participating decisions are either of the following two cases[16] .

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[14] Although $X > 0$ must hold for all relevant households, in an emprical study there might exist some cases where $I_h^0 < 0$ because both $I_A^i < 0$ and $|\, w_w h_w \,| < |\, I_A \,|$, which is the out of the scope of the model in this paper.

[15] If we observe the outcomes of labor force decisions are so frequently changed over time at some households, we might be able to understand such households have the payoff table Π^{2-3} or the payoff table Π^{3-2}. Since the data set utilized in this paper is cross sectional, we have no way of observing such households. Section 3.6 in this paper consistently argues that the Assumption 2 on page 25 precludes both the payoff table Π^{2-3} and the payoff table Π^{3-2} from occurring.

[16] If the model cannot preclude the payoff tables Π^{3-3} from occurring, there arises a problem of identification. If this is the case, we have no way of finding a unique set of parameters of a model of data generating process.

- Only the husband works.
- Only the wife works.

All other payoff tables fulfill the condition of the unique Nash equilibrium solution.

2.4.1 Cooperative solution of the two-person game

Let the husband's and wife's utility indicator corresponding to a unique Nash equilibrium solution be denoted by ω_h^* and ω_w^* respectively. Moreover, let the husband's and wife's utility indicator, that they obtain when they change their choice on labor supply corresponding to the Nash equilibrium solution simultaneously, be denoted by ω_h' and ω_w' respectively. Cooperative solution can exist if and only if the two inequalities

$$\omega_h^* < \omega_h' \tag{10}$$

$$\omega_w^* < \omega_w' \tag{11}$$

hold simultaneously. For the payoff tables, Π^{2-4}, Π^{4-2}, and Π^{4-4}, there exists cooperative solution, as shown in the right half of the Table 4-1 through Table 4-3. (Actually, cooperative solution exists only for the payoff tables in Table 4-1.)

Table 4-1: Solution of the game (case 1)

$\pi_h^k - \pi_w^\ell$	the Nash equilibrium solution				cooperative solution			
	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$
1–1				$\odot$				
1–2				$\odot$				
1–3		$\odot$						
1–4		$\odot$						
1–5		$\odot$						
2–1				$\odot$				
2–2				$\odot$				
2–3	no equilibrium solution							
2–4	$\circ$							$\odot$
2–5	$\odot$							
3–1			$\odot$					
3–2	no equilibrium solution							
3–3		$\circ$	$\circ$					
3–4		$\odot$						
3–5		$\odot$						
4–1			$\odot$					
4–2	$\circ$							$\odot$
4–3			$\odot$					
4–4	$\circ$							$\odot$
4–5	$\odot$							
5–1			$\odot$					
5–2	$\odot$							
5–3			$\odot$					
5–4	$\odot$							
5–5	$\odot$							

Table 4-2: Solution of the game (case 2)

valid only for the case $w_h\overline{h}_h > w_w\overline{h}_w$

$\pi_h^k - \pi_w^\ell$	the Nash equilibrium solution				cooperative solution			
	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$	$h_h = 0$ $h_w = 0$	$h_h = \overline{h}_h$ $h_w = 0$	$h_h = 0$ $h_w = \overline{h}_w$	$h_h = \overline{h}_h$ $h_w = \overline{h}_w$
6–1				$\odot$				
6–2				$\odot$				
6–3		$\odot$						
6–4		$\odot$						
6–5		$\odot$						

Table 4-3: Solution of the game (case 3)

valid only for the case $w_h \overline{h}_h < w_w \overline{h}_w$

$\pi_h^k - \pi_w^\ell$	the Nash equilibrium solution				cooperative solution			
	$h_h = 0$, $h_w = 0$	$h_h = \overline{h}_h$, $h_w = 0$	$h_h = 0$, $h_w = \overline{h}_w$	$h_h = \overline{h}_h$, $h_w = \overline{h}_w$	$h_h = 0$, $h_w = 0$	$h_h = \overline{h}_h$, $h_w = 0$	$h_h = 0$, $h_w = \overline{h}_w$	$h_h = \overline{h}_h$, $h_w = \overline{h}_w$
1–6				$\odot$				
2–6				$\odot$				
3–6			$\odot$					
4–6			$\odot$					
5–6			$\odot$					

3 A stochastic model of household labor supply of husband and wife

A stochastic model of household labor supply, which describes the joint probability of dichotomous choice of wife and that of husband of household in population, is constructed based on the discussions presented in Section 2.

3.1 Threshold income of labor supply (TILS)

In this section, a concept of "Threshold Income of Labor Supply (TILS)" is defined. TILS is the threshold point on either positive region of wife's given income I_w^0 or that of husband's given income I_h^0, which cuts off the region into two non-overlapping regions[17] .

Let I_w^* denote the wife's TILS. The wife's TILS, I_w^*, is the level of wife's given income, I_w^0, which makes it indifferent for her whether she works or not. The precise definition of the wife's TILS, I_w^* follows. Given the wife's wage rate, w_w, and also given the wife's assigned hours of work, $\overline{h}_w$, the wife's TILS, I_w^*, is defined as the income level such that the equation

$$\omega_w(I_w^*,\, T \,|\, \mathbf{\Gamma}_w^i) = \omega_w(I_w^* + w_w\,\overline{h}_w,\, T - \overline{h}_w \,|\, \mathbf{\Gamma}_w^i) \tag{12}$$

holds, where the function, ω_w, is given by the formula (2). Solving the equation (12) with respect to I_w^* yields the formula

$$I_w^* = I_w^*(w_w,\, \overline{h}_w \,|\, \mathbf{\Gamma}_w^i) \tag{13}$$

indicating that I_w^* is a function of w_w, $\overline{h}_w$, and parameter vector, $\mathbf{\Gamma}_w^i$.

[17] Due to the convexity of indifference curves, both $H_w^* > 0$ and $H_h^* > 0$ must hold.

Let us define the two regions of wife's given income, I_w^0, located at the both sides of wife's TILS as

$$\begin{aligned}\mathcal{I}_w^L &\equiv \{I_w^0 \,|\, I_w^0 < I_w^*\} \\ \mathcal{I}_w^U &\equiv \{I_w^0 \,|\, I_w^* < I_w^0\}.\end{aligned}$$

Note that it is not self-evident that the wife works while $I_w^0 \in \mathcal{I}_w^L$ holds. Whether she works or not while $I_w^0 \in \mathcal{I}_w^L$ holds depends on the shapes of her income-leisure indifference curves[18] .

Analogously, let I_h^* denote the husband's TILS. The husband's TILS, I_h^*, is the level of husband's given income, I_h^0, which makes it indifferent for him whether he works or not. The precise definition of the husband's TILS, I_h^* follows. The husband's TILS, I_h^*, is defined as the income level such that the equation

$$\omega_h(I_h^*,\, T|\,\boldsymbol{\Gamma}_h^i) = \omega_h(I_h^* + w_h\,\overline{h}_h,\, T - \overline{h}_h|\,\boldsymbol{\Gamma}_h^i) \tag{14}$$

holds, given the husband's wage rate, w_h, as well as given the husband's assigned hours of work, $\overline{h}_h$. The function, ω_h, is given by the formula (1). Solving the equation (14) with respect to I_h^* yields

$$I_h^* = I_h^*(w_h,\, \overline{h}_h \,|\, \boldsymbol{\Gamma}_h^i) \tag{15}$$

indicating that I_h^* is a function of w_h, $\overline{h}_h$, and parameter vector, $\boldsymbol{\Gamma}_h^i$.

Let us define the two regions of husband's given income, I_h^0, located at the both sides of husband's TILS as

$$\begin{aligned}\mathcal{I}_h^L &\equiv \{I_h^0 \,|\, I_h^0 < I_h^*\} \\ \mathcal{I}_h^U &\equiv \{I_h^0 \,|\, I_h^* < I_h^0\}.\end{aligned}$$

Note again that it is not self-evident that the husband works while $I_h^0 \in \mathcal{I}_h^L$ holds. Section 3.3 presents more detailed discussions.

A brief explanation on the relation between labor supply decision and TILS follows. Figure 2 describes the relation between husband's decision on labor supply and his TILS, given the wage rate, w_h, and also given the assigned hours of work, $\overline{h}_h$, of his employee opportunity. Suppose his given income level is the amount of the length $\overline{\text{TA}}$. In this case, he will accept the employee opportunity, because his utility indicator at point B, where he is if works, is higher than his utility indicator at point A, where he is if does not work. Next, suppose his given income increases up to the amount of the length $\overline{\text{TE}}$. Contrarily in this case, he will reject the employee opportunity, because his utility indicator at point E, where he is if does not work, is higher than his utility indicator at point F, where he is if he works. Finally, suppose his given income, I_h^0, is exactly the amount of the length $\overline{\text{TC}}$. He will be at point C if he does not work, and he will be at point D if he does work. In this final case, whether he works or

[18] See Section 3.3 for more detailed discussions.

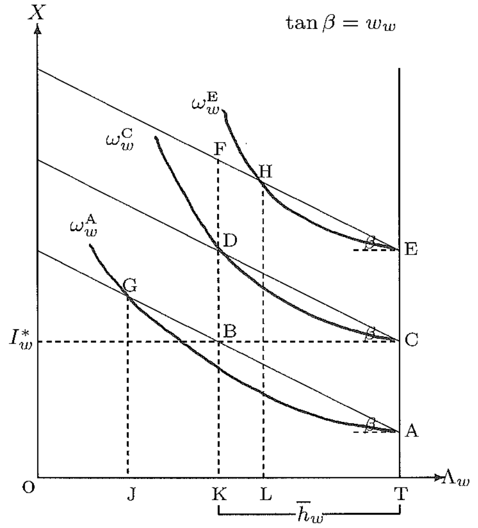


Figure 2: Wife's TILS and MHLS where $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$ holds.

not is exactly indifferent to him, because both point C and D are on the same indifference curve, ω_h^{C}. The given income level that makes it indifferent whether the husband works or not is the husband's TILS, I_h^*, such as the amount of the length $\overline{\mathrm{TC}}$ in Figure 2. The wife's TILS can be explained analogously.

3.2 Maximum hours of labor supply

In this section, a concept of "Maximum Hours of Labor Supply (MHLS)" is defined.

Given the wage rate, w_h, and also given the given income, I_h^0, husband's MHLS is defined as the hours of work, h_h, such that the equation

$$\omega_h(I_h^0,\, T\,|\,\mathbf{\Gamma}_h^i) = \omega_h(I_h^0 + w_h\, h_h,\, T - h_h\,|\,\mathbf{\Gamma}_h^i) \tag{16}$$

holds, where the function, ω_h, is given by the formula (1). Note that h_h is a variable and not necessarily equal to $\overline{h}_h$. Solving the equation (16) with respect to h_h yields the formula

$$h_h^x = h_h^x(I_h^0,\, w_h\,|\,\mathbf{\Gamma}_h^i) \tag{17}$$

where the husband's MHLS is denoted by h_h^x. The formula (17) indicates that the husband's MHLS is the function of husband's given income, I_h^0, wage rate, w_h, and parameter vector, $\mathbf{\Gamma}_h^i$, of his utility function.

Analogously, wife's MHLS is defined as the hours of work, h_w, such that

$$\omega_w(I_w^0, T|\mathbf{\Gamma}_w^i) = \omega_w(I_w^0 + w_w\, h_w, T - h_w|\mathbf{\Gamma}_w^i) \tag{18}$$

holds, given the wage rate, w_w, as well as given the given income, I_w^0. The function, ω_w, is given by the formula (2). Note that h_w is also a variable and not necessarily equal to $\overline{h}_w$. Solving the equation (18) with respect to h_w yields

$$h_w^x = h_w^x(I_w^0, w_w|\mathbf{\Gamma}_w^i) \tag{19}$$

where the wife's MHLS is denoted by h_w^x. The formula (19) indicates that the wife's MHLS is the function of wife's given income, I_w^0, wage rate, w_w, and parameter vector, $\mathbf{\Gamma}_w^i$, of her utility function.

Husband's MHSL can be shown in Figure 2. Points L, K, J, are the feet of perpendiculars to Λ_h axis originating points H, D, G, respectively. Suppose husband's given income, I_h^0, is the amount of the length $\overline{\text{TA}}$. In this case, the husband will accept the employee opportunity of wage rate, w_h, and assigned hours of work, $\overline{h}_h$. Given his given income, $I_h^0 = \overline{\text{TA}}$, and also given the wage rate, w_h, he will accept any employee opportunity as long as the assigned hours of work is at most the amount of length $\overline{\text{TJ}}$. He will reject the employee opportunity if the assigned hours of work exceeds the length $\overline{\text{TJ}}$. The husband's MHLS is the assigned hours of work that makes it indifferent whether he works or not. The wife's MHLS can be shown analogously.

Note that the husband's MHLS, h_h^x, decreases as his given income, I_h^0, increases, i.e., $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ holds in Figure 2. On the contrary, husband's indifference curves depicted in Figure 3 make his MHLS increase as his given income increases, so that $\dfrac{\partial h_h^x}{\partial I_h^0} > 0$ holds in Figure 3.

It should be noted that whether the sign of $\dfrac{\partial h_h^x}{\partial I_h^0}$ is positive or negative depends on the characteristics of his indifference curves. Moreover, note that the sign of $\dfrac{\partial h_h^x}{\partial I_h^0}$ determines the corresponding relationship between his decision of labor supply and the regions of his given income, I_h^0, separated by his TILS, I_h^*, i.e., $\mathcal{I}_h^L$ and $\mathcal{I}_h^U$, both of which were defined in the previous section.

Similar statements apply to the case of wife.

The next section precisely discusses the corresponding relationship between her or his decision of labor supply and the two sets of her given income $\mathcal{I}_w^L$ and $\mathcal{I}_w^U$ or the two sets of his given income $\mathcal{I}_h^L$ and $\mathcal{I}_h^U$.

3.3 Relation between TILS and decision on labor supply

Let the relation between husband's TILS and his decision be described first. Given the husband's given income, I_h^0, let the husband's utility index be denoted

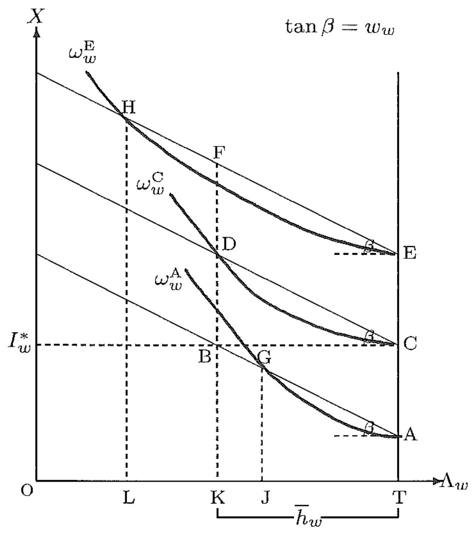


Figure 3: Wife's TILS and MHLS where $\dfrac{\partial h_w^x}{\partial I_w^0} > 0$ holds.

by ω_h^0 in case he does not accept his employee job opportunity of w_h and $\overline{h}_h$, and let it be denoted by ω_h^1 in case he does accept the same employee opportunity. The values of ω_h^0 and ω_h^1 can be given by the formula

$$\begin{aligned} \omega_h^0 &= \omega_h(I_h^0, T|\mathbf{\Gamma}_h^i) \\ \omega_h^1 &= \omega_h(I_h^0 + w_h\overline{h}_h, T - \overline{h}_h|\mathbf{\Gamma}_h^i) \end{aligned}$$

where ω_h is the husband's utility function (1). The relation between two inequalities, i.e., the inequality between the region of his given income I_h^0 and his TILS, I_h^*, and the inequality between ω_h^0 and ω_h^1 is

$$\left.\begin{array}{l} I_h^0 \in \mathcal{I}_h^L \rightleftharpoons \omega_h^0 < \omega_h^1 \\ I_h^0 \in \mathcal{I}_h^U \rightleftharpoons \omega_h^0 > \omega_h^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_h^x}{\partial I_h^0} < 0 \tag{20}$$

$$\left.\begin{array}{l} I_h^0 \in \mathcal{I}_h^L \rightleftharpoons \omega_h^0 > \omega_h^1 \\ I_h^0 \in \mathcal{I}_h^U \rightleftharpoons \omega_h^0 < \omega_h^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_h^x}{\partial I_h^0} > 0 \tag{21}$$

For the case of wife, let the wife's utility index be denoted by ω_w^0 in case she does not accept her employee job opportunity of w_w and $\overline{h}_w$, and let it be denoted by ω_w^1 in case she does accept the same employee opportunity, given her given income, I_w^0. The values of ω_w^0 and ω_w^1 can be given by the formula

$$\omega_w^0 = \omega_w(I_w^0, T|\mathbf{\Gamma}_w^i)$$

$$\omega_w^1 \quad = \quad \omega_w(I_w^0 + w_w \overline{h}_w, T - \overline{h}_w | \mathbf{\Gamma}_w^i)$$

where ω_w is the wife's utility function (2). The relation between two inequalities, i.e., the inequality between the region of her given income I_h^0 and her TILS, I_w^*, and the inequality between ω_w^0 and ω_w^1 is

$$\left.\begin{array}{l} I_w^0 \in \mathcal{I}_w^L \rightleftharpoons \omega_w^0 < \omega_w^1 \\ I_w^0 \in \mathcal{I}_w^U \rightleftharpoons \omega_w^0 > \omega_w^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{22}$$

$$\left.\begin{array}{l} I_w^0 \in \mathcal{I}_w^L \rightleftharpoons \omega_w^0 > \omega_w^1 \\ I_w^0 \in \mathcal{I}_w^U \rightleftharpoons \omega_w^0 < \omega_w^1 \end{array}\right\} \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{23}$$

The wife's indifference curves depicted in Figure 2 correspond to the case $\frac{\partial h_w^x}{\partial I_w^0} < 0$. In Figure 2, the wife's TILS, I_w^*, is shown by the length $\overline{\text{TC}}$. Note that as long as her given income, I_w^0, is equal to her TILS, I_w^*, her MHLS, h_w^x, is equalized to the assigned hours of work, $\overline{h}_w$, of her employee opportunity ($I_w^0 = I_w^* \rightleftharpoons h_w^x = \overline{h}_w$). Suppose her given income, I_w^0, falls below her TILS, I_w^* , i.e., $I_w^0 \in \mathcal{I}_w^L$, to the level such as the length $\overline{\text{TA}}$. As long as $\frac{\partial h_w^x}{\partial I_w^0} < 0$ holds as depicted in Figure 2, $h_w^x > \overline{h}_w$ holds for the region of $I_w^0 < I_w^*$. This means that as long as $\frac{\partial h_w^x}{\partial I_w^0} < 0$ holds as in Figure 2, the point G on the line AB is necessarily located at the left side of the point B in Figure 2, because $h_w^x > \overline{h}_w$ holds and because the amount of h_w^x is, by definition, the distance between the line TE and point G, which is the intersection of line AB and the indifference curve passing on point A, ω_w^{A}. Considering the convexity of the indifference curves to the origin, it is concluded that utility index on point B is necessarily higher than that on point A, because both point A and G is on the same indifference curve and because point B is located between A and G on the line AG. Thus, the relation

$$I_w^0 \in \mathcal{I}_w^L \rightharpoonup \omega_w^0 < \omega_w^1, \qquad \text{if } \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{24}$$

must hold.

Next, on the contrary, suppose wife's given income, I_w^0, rises above her TILS, I_w^* , where $I_w^0 \in \mathcal{I}_w^U$, to the level such as the length $\overline{\text{TE}}$. In this case $h_w^x < \overline{h}_w$ holds for the region of $I_w^0 \in \mathcal{I}_w^U$, as long as $\frac{\partial h_w^x}{\partial I_w^0} < 0$ holds as depicted in Figure 2. By the similar reasoning above, it is concluded that point H on the line EF is necessarily located at the right side of point F and that the utility index on point F is necessarily higher than that on point E. Thus, the relation

$$I_w^0 \in \mathcal{I}_w^U \rightharpoonup \omega_w^0 > \omega_w^1, \qquad \text{if } \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{25}$$

must hold.

Combining the formulae (24) and (25) yields the relation (22).

In the similar manners as described above, it is shown that the relation (23) holds under the condition $\dfrac{\partial h_w^x}{\partial I_w^0} > 0$, using Figure 3.

The relations (22) and (23) show that each of these formulae of inequalities between the given income and the TILS can be assigned to on of the payoff matrices.

Analogously, the relations (20) and (21) are derived for husband.

3.4 Introducing a random coefficient into utility function

The subsequent Section 3.9 introduces both husband's utility function with random coefficient(s) and wife's utility function with random coefficient(s). The model assumes that the coefficient sets of the wives and those of the husbands are randomly distributed among agents, i.e., among wives as well as among husbands in population but that both the coefficient set of the wife and that of the husband at the ith household in population never change over time.

For sake of simplicity, the model assumes that husbands in population share the same parameters of their utility function except the intercept (constant term) parameter of his marginal utility of leisure, which means that the intercept parameter distributes among households in population. Similarly, it is also assumed that wives in population also share the same parameters of their utility function except the intercept parameter of her marginal utility of leisure. This section deduces the distribution of husband's or wife's Threshold Income of Labor Supply (TILS).

It is assumed that each of the husband's and wife's parameter vectors of their utility function,

$$\begin{aligned} {}^{\mathrm{t}}\boldsymbol{\Gamma}_h &= (\gamma_{h1}, \gamma_{h2}, \cdots, \gamma_{hm}) \\ {}^{\mathrm{t}}\boldsymbol{\Gamma}_w &= (\gamma_{w1}, \gamma_{w2}, \cdots, \gamma_{wm}) \end{aligned}$$

contains random coefficients, γ_{h4} and γ_{w4}, respectively. γ_{h4} is the intercept parameter of husband's marginal utility of leisure, and γ_{w4} is the intercept parameter of wife's marginal utility of leisure. The superscript i of the parameter vector is suppressed for simplicity.

The random coefficients, γ_{h4} and γ_{w4}, are also assumed to follow the joint probability density function

$$f(\gamma_{h4}, \gamma_{w4} \,|\, \boldsymbol{\zeta}) \tag{26}$$

where $\boldsymbol{\zeta}$ is the parameter vector of the probability density function f.

The distribution of husband's TILS, I_h^*, and that of wife's TILS, I_w^*, are obtained as follows. Solving the formulae (15) and (13) of husband's and wife's

TILS, with respect to γ_{h4} and γ_{w4} respectively, yields

$$\gamma_{h4} = \gamma_{h4}(I_h^*, w_h, \overline{h}_h | \widetilde{\mathbf{\Gamma}}_h) \tag{27}$$

$$\gamma_{w4} = \gamma_{w4}(I_w^*, w_w, \overline{h}_w | \widetilde{\mathbf{\Gamma}}_w) \tag{28}$$

where $\widetilde{\mathbf{\Gamma}}_h$ and $\widetilde{\mathbf{\Gamma}}_w$ are the parameter vectors of husband's and wife's including the common parameters among households in population,

$$^{\mathrm{t}}\widetilde{\mathbf{\Gamma}}_h = (\gamma_{h1}, \gamma_{h2}, \gamma_{h3}, \gamma_{h5}, \cdots, \gamma_{hm})$$

$$^{\mathrm{t}}\widetilde{\mathbf{\Gamma}}_w = (\gamma_{w1}, \gamma_{w2}, \gamma_{w3}, \gamma_{w5}, \cdots, \gamma_{wm})$$

excluding the random coefficients γ_{h4} and γ_{w4}.

Inserting the formula (27) and (28) into (26) yields

$$f[\,\gamma_{h4}(I_h^*,\, w_h,\, \overline{h}_h \,|\, \widetilde{\mathbf{\Gamma}}_h),\, \gamma_{w4}(I_w^*,\, w_w,\, \overline{h}_w \,|\, \widetilde{\mathbf{\Gamma}}_w) \,|\, \boldsymbol{\zeta}\,] \tag{29}$$

The joint probability density function of I_h^* and I_w^* is obtained as a product of the formula (29) and the Jacobean

$$J \equiv \begin{vmatrix} \frac{\partial \gamma_{h4}}{\partial I_h^*} & \frac{\partial \gamma_{h4}}{\partial I_w^*} \\ \frac{\partial \gamma_{w4}}{\partial I_h^*} & \frac{\partial \gamma_{w4}}{\partial I_w^*} \end{vmatrix}$$

For the case where each of wife's utility function and that of husband's utility function is in the form of the formulae (46) and (42) respectively in Section 3.9, the Jacobean reduces into the form

$$J = \left| \frac{\partial \gamma_{h4}}{\partial I_h^*} \cdot \frac{\partial \gamma_{w4}}{\partial I_w^*} \right| \tag{30}$$

because $\dfrac{\partial \gamma_{h4}}{\partial I_w^*} = \dfrac{\partial \gamma_{w4}}{\partial I_h^*} = 0$ holds. Multiplying the formula (29) by the Jacobean J (30) yields the joint distribution density function of I_h^* and I_w^*, $g(I_h^*, I_w^*)$, as

$$\begin{aligned} g(I_h^*, I_w^*) &= f[\,\gamma_{h4}(I_h^*, w_h, \overline{h}_h \,|\, \widetilde{\mathbf{\Gamma}}_h) \\ &\qquad ,\gamma_{w4}(I_w^*, w_w, \overline{h}_w \,|\, \widetilde{\mathbf{\Gamma}}_w) \,|\, \boldsymbol{\zeta}\,] \cdot \left| \frac{\partial \gamma_{h4}}{\partial I_h^*} \cdot \frac{\partial \gamma_{w4}}{\partial I_w^*} \right| \end{aligned} \tag{31}$$

The formula (31) shows that the shape of the joint distribution density function of I_h^* and I_w^*, $g(I_h^*, I_w^*)$, depends upon the variables w_h, $\overline{h}_h$, w_w, $\overline{h}_w$ and the parameter vectors, $\widetilde{\mathbf{\Gamma}}_h$ and $\widetilde{\mathbf{\Gamma}}_w$, as well as the parameter vector, $\boldsymbol{\zeta}$, of the joint distribution density function f.

Let the probability density function of each husband's TILS, I_h^*, and wife's TILS, I_w^*, be denoted by $g_h(I_h^*)$ and $g_w(I_w^*)$ respectively. The probability density function $g_h(I_h^*)$ and $g_w(I_w^*)$ are the marginal distributions of the joint distribution, $g(I_h^*, I_w^*)$, given by the formula (31).

3.5 The sign of $\frac{\partial h_w^x}{\partial I_w^0}$ and observation on labor supply

Let the fitted value of wife's labor supply probability be denoted by $\widehat{\mu}_w$. Given the values of w_h, $\overline{h}_h$, w_w, $\overline{h}_w$ and the parameter vectors, $\widetilde{\mathbf{\Gamma}}_h$, $\widetilde{\mathbf{\Gamma}}_w$ and $\boldsymbol{\zeta}$ in the formula (31), the wife's labor supply probability $\widehat{\mu}_w$ can be calculated by using the probability density function of wife's TILS, $g_w(I_w^*)$. The probability, $\widehat{\mu}_w$, in the population where the wife's given income is I_w^0 is given by the formula

$$\widehat{\mu}_w = \int_{I_w^0=I_A+I_h^j}^{\infty} g_w(I_w^*)dI_w^* \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{32}$$

or

$$\widehat{\mu}_w = \int_{-\infty}^{I_w^0=I_A+I_h^j} g_w(I_w^*)dI_w^* \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{33}$$

depending upon the sign of $\frac{\partial h_w^x}{\partial I_w^0}$. The formula (32) is justified by the relation (22), and the formula (33) is justified by the relation (23). Differentiating $\widehat{\mu}_w$ in respect to wife's given income, I_w^0, yields

$$\frac{\partial \widehat{\mu}_i}{\partial I_w^0} = -g_w(I_w^0) < 0 \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} < 0 \tag{34}$$

or

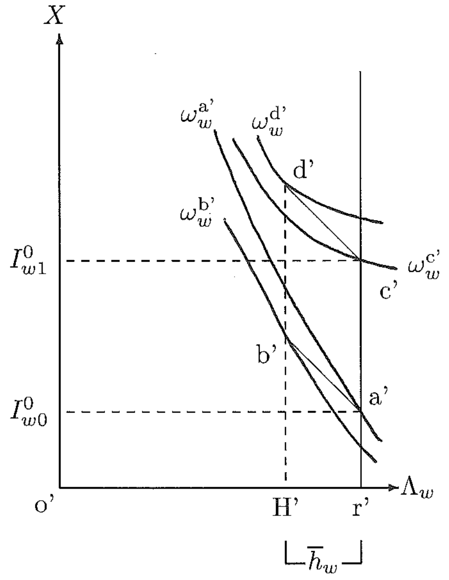
$$\frac{\partial \widehat{\mu}_i}{\partial I_w^0} = g_w(I_w^0) > 0 \quad \text{if and only if} \quad \frac{\partial h_w^x}{\partial I_w^0} > 0 \tag{35}$$

again depending upon the sign of $\frac{\partial h_w^x}{\partial I_w^0}$.

Taking the observations of Douglas(1934) into account, the sign of $\frac{\partial h_w^x}{\partial I_w^0}$ is necessarily negative because it is observed that household income levels and wife's labor supply ratios are negatively correlated. Thus the equality in formula (35) is inconsistent with observed characteristics of wife's labor supply ratio. This concludes that the inequality $\frac{\partial h_w^x}{\partial I_w^0} < 0$ must hold as for the wife's MHLS, h_w^x.

3.6 Consistency between the sign of $\frac{\partial h_h^x}{\partial I_h^0}$, $\frac{\partial h_w^x}{\partial I_w^0}$ and the payoff matrices

The condition, $\frac{\partial h_w^x}{\partial I_w^0} < 0$, which is clarified in Subsection 3.5, is consistent to the wife's payoff matrices listed in Table 3 except π_w^2. This means that the



$$\pi_w^2 = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Figure 4: Wife's indifference curves yielding her payoff matrix π_w^2

whole set of wife's indifference curves that generate her payoff matrix, π_w^2, is not consistent with the condition, $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$.

Indifference curves that generate wife's payoff matrix, π_w^2, is depicted in Figure 4. The wife's given income when her husband does not work is denoted by I_{w0}^0 in Figure 4. $I_{w0}^0 = I_A$ holds where I_A is the unearned income of the household. Next, in Figure 4, the wife's given income when her husband does work is denoted by I_{w1}^0, where $I_{w0}^0 = I_A + w_h \overline{h}_h$ holds.

Let the region of wife's TILS, I_w^* of the wife's , be examined if the wife's payoff matrix is π_w^2. Note that wife's indifference curves, which are consistent with her payoff matrix, π_w^2, is depicted in Figure 4.

Let wife's utility index at point a' in Figure 4 be denoted by $\omega_w^{\text{a'}}$, and similarly, let the wife's utility indexes at point b', c', and d' be denoted by $\omega_w^{\text{b'}}$, $\omega_w^{\text{c'}}$, and $\omega_w^{\text{d'}}$ respectively. The relation between the group of ω_w^0, ω_w^1 and the group of $\omega_w^{\text{a'}}$, $\omega_w^{\text{b'}}$, $\omega_w^{\text{c'}}$, $\omega_w^{\text{d'}}$ is

$$\left.\begin{array}{l}\omega_w^0 = \omega_w^{\text{a'}},\\ \omega_w^1 = \omega_w^{\text{b'}}\end{array}\right\} \quad \text{if} \quad I_w^0 = I_{w0}^0$$
$$\left.\begin{array}{l}\omega_w^0 = \omega_w^{\text{c'}},\\ \omega_w^1 = \omega_w^{\text{d'}}\end{array}\right\} \quad \text{if} \quad I_w^0 = I_{w1}^0$$

depending whether her husband works or not.

Now suppose wife's indifference curves fulfill the condition $\dfrac{\partial h_w^x}{\partial I_w^0} < 0$. Let us examine the inequality between $\omega_w^{\text{a'}}$ and $\omega_w^{\text{b'}}$, and the inequality between $\omega_w^{\text{c'}}$ and $\omega_w^{\text{d'}}$, according to the relation (22).

1. If $I_w^* < I_{w0}^0 < I_{w1}^0$ hods,
   then $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} > \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_w^* < I_{w0}^0$ does not apply because the inequality $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ holds in Figure 4.

2. If $I_{w0}^0 < I_w^* < I_{w1}^0$ holds,
   then $\omega_w^{\text{a'}} < \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} > \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_{w0}^0 < I_w^* < I_{w1}^0$ does not apply because $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ holds in Figure 4.

3. If $I_{w0}^0 < I_{w1}^0 < I_w^*$ holds,
   then $\omega_w^{\text{a'}} < \omega_w^{\text{b'}}$ and $\omega_w^{\text{c'}} < \omega_w^{\text{d'}}$ must follow.
   Nevertheless, $I_{w1}^0 < I_w^*$ does not apply because $\omega_w^{\text{a'}} > \omega_w^{\text{b'}}$ holds in Figure 4.

Following the reasoning 1 through 3 above, the region of wife's TILS, I_w^*, does not exist if her payoff matrix, π_w^2, is assumed. This concludes wife's payoff matrix, π_w^2, is inconsistent with the condition $\frac{\partial h_w^x}{\partial I_w^0} < 0$. Thus, the wife's payoff matrix, π_w^2, is excluded from the further discussion.

Two payoff tables, Π^{2-3} and Π^{3-2}, are the two cases where no equilibrium solution exist. (See Table 4-1.) Setting the consistency condition $\frac{\partial h_w^x}{\partial I_w^0} < 0$ can preclude the payoff table Π^{3-2}, which gives no equilibrium solution, because wife's payoff matrix, π_w^2, is no longer relevant.

Following the similar reasoning above, it can be concluded that husband's payoff matrix, π_h^2, is inconsistent with the condition $\frac{\partial h_h^x}{\partial I_h^0} < 0$, because the region of husband's TILS, I_h^*, does not exist if his payoff matrix, π_h^2, is assumed.

As for the wife's MHLS, h_w^x, the condition, $\frac{\partial h_w^x}{\partial I_w^0} < 0$, is required for the consistency with observations. Along with this condition, if the condition, $\frac{\partial h_h^x}{\partial I_h^0} < 0$, is assumed for the husband's MHLS, h_h^x, we can exclude the payoff table, Π^{2-3}, where neither Nash equilibrium solution nor cooperative solution exists, because the husband's payoff matrix, π_h^2, becomes no longer relevant. Thus, combining the assumption $\frac{\partial h_h^x}{\partial I_h^0} < 0$ with the consistency condition $\frac{\partial h_w^x}{\partial I_w^0} < 0$ precludes all the cases where no equilibrium solution exist[19] .

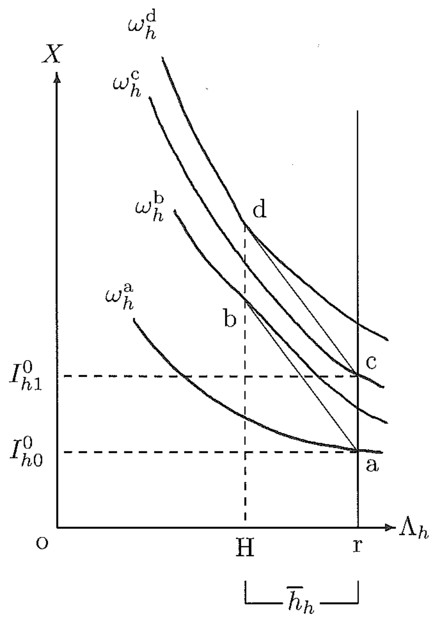
Assumption 2: $$\frac{\partial h_h^x}{\partial I_h^0} < 0$$

3.7 The relation between payoff matrix and the region of TILS

In this section, the relations between the payoff matrices, except π_h^2 and π_w^2, listed in Table 3 and the regions of "Threshold Income of Labor Supply (TILS)" is considered.

Firstly, let the region where husband's TILS, I_h^*, exists be considered when his payoff matrix is π_h^1. Husband's indifference curves that generate his payoff matrix π_h^1 is depicted in Figure 5. Let his given income when his wife does not work be denoted by I_{h0}^0. Now $I_{h0}^0 = I_A$ holds where I_A is the unearned income of the household. Next, let his given income when his wife does work be denoted by I_{h1}^0, where $I_{h0}^0 = I_A + w_w \overline{h}_w$ holds. I_{h0}^0 and I_{h1}^0 are depicted in Figure 5.

[19] Note that the inequality $\frac{\partial h_w^x}{\partial I_w^0} < 0$ is theoretically induced, as discussed above, from the evidence on the observed wife's employee job opportunity participation rate is negatively correlated with the household income as Douglas(1934) found.



$$\pi_h^1 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Figure 5: Husband's indifference curves yielding his payoff matrix π_h^1

Let husband's utility index at point a in Figure 5 be denoted by ω_h^{a}, and similarly, let the husband's utility indexes at point b, c, and d be denoted by ω_h^{b}, ω_h^{c}, and ω_h^{d} respectively. The relation between the group of ω_h^0, ω_h^1 and the group of ω_h^{a}, ω_h^{b}, ω_h^{c}, ω_h^{d} is

$$\left.\begin{array}{l}\omega_h^0 = \omega_h^{\mathrm{a}} \\ \omega_h^1 = \omega_h^{\mathrm{b}}\end{array}\right\} \quad \text{if} \quad I_h^0 = I_{h0}^0$$
$$\left.\begin{array}{l}\omega_h^0 = \omega_h^{\mathrm{c}} \\ \omega_h^1 = \omega_h^{\mathrm{d}}\end{array}\right\} \quad \text{if} \quad I_h^0 = I_{h1}^0$$

depending whether his wife works or not.

Let us examine the inequality between ω_h^{a} and ω_h^{b}, and the inequality between ω_h^{c} and ω_h^{d}. The relation (20) gives the relevant inequalities between these utility indexes because $\dfrac{\partial h_h^x}{\partial I_h^0} < 0$ is assumed.

1' $\quad I_h^* < I_{h0}^0 < I_{h1}^0 \rightleftharpoons \omega_h^{\mathrm{a}} > \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} > \omega_h^{\mathrm{d}}$ holds.
Nevertheless, $I_h^* < I_{h0}^0$ does not apply because the inequality $\omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ holds in Figure 5.

2' $\quad I_{h0}^0 < I_h^* < I_{h1}^0 \rightleftharpoons \omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} > \omega_h^{\mathrm{d}}$ holds.
Nevertheless, $I_{h0}^0 < I_h^* < I_{h1}^0$ does not apply because the inequality $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ holds in Figure 5.

3' $\quad I_{h0}^0 < I_{h1}^0 < I_h^* \rightleftharpoons \omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ holds.
The inequalities $\omega_h^{\mathrm{a}} < \omega_h^{\mathrm{b}}$ and $\omega_h^{\mathrm{c}} < \omega_h^{\mathrm{d}}$ comply with the preference curves shown in Figure 5.

The above reasoning 1', 2', and 3' concludes that the region of husband's TILS, $I_h^* = I_h^*(w_h, \overline{h}_h \,|\, \mathbf{\Gamma}_h)$, is $I_{h1}^0 < I_h^*$.

The plausible regions of husband's TILS for the rest of husband's payoff matrices, π_h^3, π_h^4, π_h^5, and π_h^6 are obtained following the similar reasoning above. The plausible regions of wife's TILS for wife's payoff matrices, π_w^1, π_w^3, π_w^4, π_w^5, and π_w^6 are also obtained similarly. The plausible regions of husband's and wife's TILS are shown in Table 5-1 and 5-2 respectively.

Table 5-1: Regions of husband's TILS corresponding to his payoff matrices

Husband's payoff matrix k's index of π_h^k	Region of husband's TILS, I_h^*
1	$I_{h1}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \mathbf{\Gamma}_h)$
3	$I_{h0}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \mathbf{\Gamma}_h) \quad < I_{h1}^0$
4	$I_{h2}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \mathbf{\Gamma}_h) \quad < I_{h0}^0$
5	$I_h^*(w_h, \overline{h}_h \vert \mathbf{\Gamma}_h) \quad < I_{h2}^0$
6	$I_{h1}^0 < \quad I_h^*(w_h, \overline{h}_h \vert \mathbf{\Gamma}_h)$

Note that $I_{h0}^0 = I_A, \quad I_{h1}^0 = I_A + w_w\overline{h}_w$.

See formula (38) for the definition of I_{h2}^0.

Table 5-2: Regions of wife's TILS corresponding to her payoff matrices

Wife's payoff matrix ℓ's index of π_w^ℓ	Region of wife's TILS, I_w^*
1	$I_{w1}^0 < \quad I_w^*(w_w, \overline{h}_w \vert \mathbf{\Gamma}_w)$
3	$I_{w0}^0 < \quad I_w^*(w_w, \overline{h}_w \vert \mathbf{\Gamma}_w) \quad < I_{w1}^0$
4	$I_{w2}^0 < \quad I_w^*(w_w, \overline{h}_w \vert \mathbf{\Gamma}_w) \quad < I_{w0}^0$
5	$I_w^*(w_w, \overline{h}_w \vert \mathbf{\Gamma}_w) \quad < I_{w2}^0$
6	$I_{w1}^0 < \quad I_w^*(w_w, \overline{h}_w \vert \mathbf{\Gamma}_w)$

Note that $I_{w0}^0 = I_A, \quad I_{w1}^0 = I_A + w_h\overline{h}_h$.

See formula (40) for the definition of I_{w2}^0.

In Table 5-1, a variable I_{h2}^0 bounds the region of husband's TILS, which corresponds to his payoff matrix, π_h^5. Similarly in Table 5-2, a variable I_{w2}^0 bounds the region of wife's TILS, which corresponds to her payoff matrix, π_w^5. Some description should be made on these variables, I_{h2}^0 and I_{w2}^0.

Let the variable, I_{h2}^0, be described first. Husband's indifference curves yielding his payoff matrix, π_h^5, is depicted in Figure 6. The coordinates of husband's available leisure, Λ_h, and his available income, X, are indicated by points a, b, c, and d, given the wage rate, w_h, and the assigned hours of work, $\overline{h}_h$, of his employee opportunity. Let an auxiliary line passing point a and d be drawn in Figure 6. The gradient of the line ad to the horizontal axis, θ, is given by the formula

$$\tan\theta = \frac{w_h\overline{h}_h + w_w\overline{h}_w}{\overline{h}_h}$$

Suppose husband's wage rate raises up to

$$W_h \equiv \frac{w_h\overline{h}_h + w_w\overline{h}_w}{\overline{h}_h}$$

and his assigned hours of work unchanged as $\overline{h}_h$. The point d is where husband would be located if he accepts this employee opportunity of W_h and $\overline{h}_h$, in case his wife does not work. Since his utility index at point a is higher than his

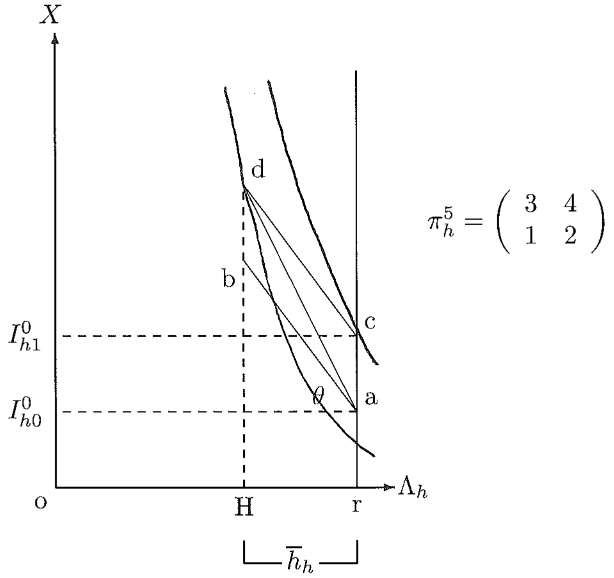


Figure 6: Husband's indifference curves yielding his payoff matrix π^5_w

utility index at d, he does not accept the employee opportunity of W_h and $\overline{h}_h$. Let $\Re_h$ denote the subset consisting of $\mathbf{\Gamma}_h$'s such that inequality $\omega_a > \omega_d$ holds. For the $\mathbf{\Gamma}_h$'s in the subset $\Re_h$,

$$I_h^*(W_h, \overline{h}_h | \mathbf{\Gamma}_h) < I_{h0}^0, \quad \forall \mathbf{\Gamma}_h \in \Re_h \tag{36}$$

holds according to the relation (20). Note that the argument W_h, instead of w_h, enters the husband's TILS in formula (36).

For the $\mathbf{\Gamma}_h$'s in the subset $\Re_h$, what will be the region of husbands' TILS, I_h^*, when the wage rate again reduces to w_h, with the assigned hours of work being constant as $\overline{h}_h$? Suppose the parameter vector, ${}^t\mathbf{\Gamma}_h = (\gamma_{h1}, \gamma_{h2}, \cdots, \gamma_{hm})$, of husband's utility function has constant elements over the population except γ_{h4}, which is randomly distributed in the population. Solving the equation

$$I_h^*(W_h,\, \overline{h}_h \,|\, \mathbf{\Gamma}_h) = I_{h0}^0$$

in respect to γ_{h4} yields the formula

$$\gamma_{h4} = \gamma_{h4}(I_{h0}^0, W_h, \overline{h}_h \,|\, \widetilde{\mathbf{\Gamma}}_h) \tag{37}$$

Inserting the formula (37) into the element γ_{h4} in the parameter vector, $\mathbf{\Gamma}_h$, of the formula (15) yields

$$\begin{aligned} I_{h2}^0 &= I_h^*\big[w_h, \overline{h}_h \,|\, \big(\gamma_{h1}, \gamma_{h2}, \gamma_{h3}, \gamma_{h4}(I_{h0}^0, W_h, \overline{h}_h \,|\, \widetilde{\mathbf{\Gamma}}_h), \cdots, \gamma_{hm}\big)\big] \\ &= I_{h2}^0(I_{h0}^0, w_h, \overline{h}_h, w_w, \overline{h}_w \,|\, \widetilde{\mathbf{\Gamma}}_h) \end{aligned} \tag{38}$$

Using the formula (38), the region of husbands' TILS, whose payoff matrix are π_h^5, can be shown as

$$I_h^*(w_h,\, \overline{h}_h \,|\, \mathbf{\Gamma}_h) < I_{h2}^0, \quad \forall \mathbf{\Gamma}_h \in \Re_h \tag{39}$$

For the variable, I_{w2}^0, bounding the region of wife's TILS, which corresponds to her payoff matrix, π_w^5 in Table 5-2, a formula

$$I_{w2}^0 = I_{w2}^0(I_{w0}^0, w_h, \overline{h}_h, w_w, \overline{h}_w \,|\, \widetilde{\mathbf{\Gamma}}_w) \tag{40}$$

can be obtained. Using the formula (40), the region of wives' TILS, whose payoff matrix are π_w^5, can be shown as

$$I_w^*(w_w, \overline{h}_w \,|\, \mathbf{\Gamma}_w) < I_{w2}^0 \tag{41}$$

Regions of TILS corresponding to payoff matrices can be shown graphically in figure 7 and 8 [20] .

[20] For the inequalities between I_{h2}^0 and I_{h0}^0, and for the inequalities between I_{w2}^0 and I_{w0}^0, it can be proved that $\dfrac{\partial h_h^x}{\partial I_h^0} < 0 \rightharpoonup I_{h2}^0 < I_{h0}^0$ and $\dfrac{\partial h_w^x}{\partial I_w^0} < 0 \rightharpoonup I_{w2}^0 < I_{w0}^0$ holds. See Miyauchi(1991) for precise discussion.

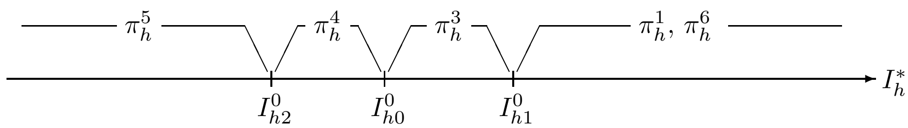


Figure 7: Correspondence between husband's payoff matrixes and the region of his TILS, I_h^*

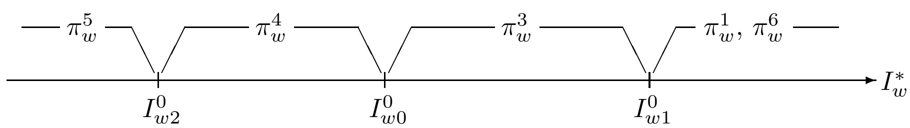


Figure 8: Correspondence between wife's payoff matrixes and the region of her TILS, I_w^*

3.8 Joint distribution of TILS and decision on household labor supply

In Subsection 3.7, husband's and wife's TILS, I_h^* and I_w^* respectively, are partitioned into regions corresponding to each of their own payoff matrices. Employing this result, the two dimensional plane of TILS, (I_h^*, I_w^*), can be mapped to the household's Nash equilibrium solutions and cooperative solutions clarified in Section 2.

Combining the I_h^* axis in Figure 7 and the I_w^* axis in right angle in Figure 8 yields two-dimensional coordinate system of (I_h^*, I_w^*), which is shown in Figure 9. Horizontal axis is of I_h^* and vertical axis is of I_w^*. As shown in Figure 9, the (I_h^*, I_w^*) plane is partitioned into regions by orthogonal lines ii, jj, kk passing $I_{h1}^0, I_{h0}^0, I_{h2}^0$ on the I_h^* axis respectively, and also by orthogonal lines $\ell\ell$, mm, nn passing $I_{w1}^0, I_{w0}^0, I_{w2}^0$ on the I_w^* axis. Note that any region on (I_h^*, I_w^*) plane partitioned by lines ii, jj, kk, $\ell\ell$, mm, nn can be assigned one of the payoff tables listed in Table 3. By showing the superscript of payoff tables, Figure 9 assigns the corresponding payoff table to each region on (I_h^*, I_w^*) plane.

Figure 10 assigns the corresponding Nash equilibrium solutions or cooperative solutions on husband's and wife's labor supply to the region on (I_h^*, I_w^*) plane, based on Figure 9 and Table 4-1 through 4-3. The region in Figure 10 corresponding to the payoff table Π^{3-3} (i.e., the region indicated by $(3-3)$ in

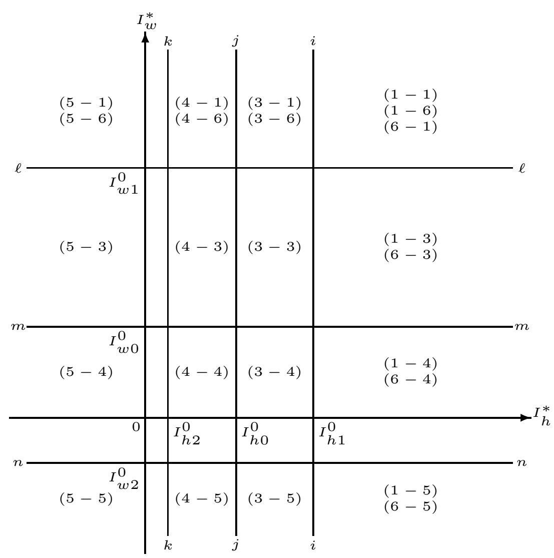


Figure 9: Correspondence between the payoff tables and the region of (I_h^*, I_w^*)

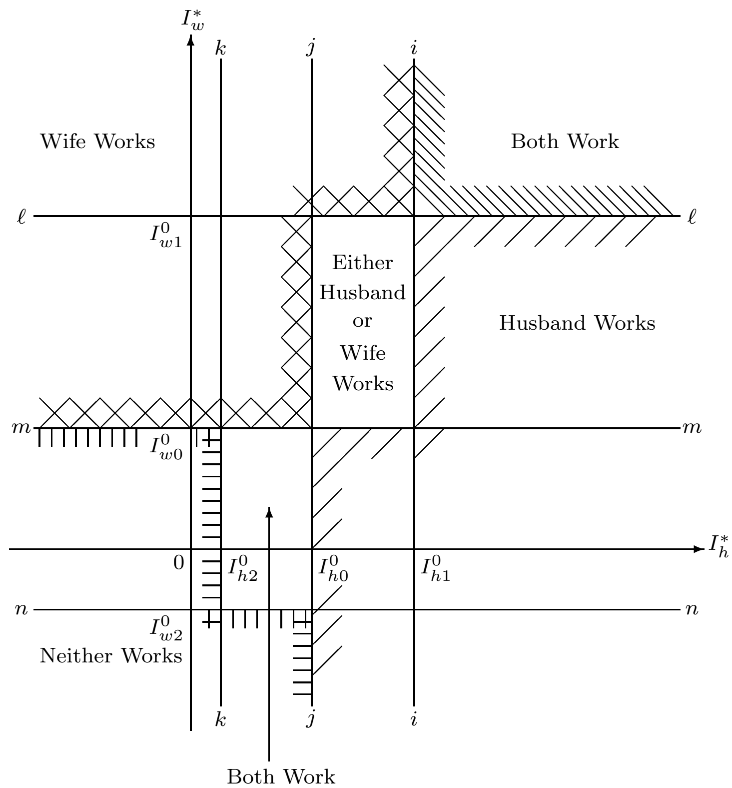


Figure 10: Mapping the region of (I_h^*, I_w^*) to household decisions

Figure 9) will be of special interest. Note that the households in population where the coordinate of $(I_h^*,\ I_w^*)$ belongs to this region yield non unique Nash equilibrium solution, so that the theory can only suggest either husband works or wife works.

Integrating the probability density function $g(I_h^*, I_w^*)$ in each region shown in Figure 10 yields probabilities of both work, husband works, wife works, and neither works.

3.9 Utility function in quadratic form

Utility functions of husband is specified in quadratic forms as

$$\omega_h = \frac{1}{2}\gamma_{h1}X^2 + \gamma_{h2}X + \gamma_{h3}X\Lambda_h + \gamma_{h4}\Lambda_h + \frac{1}{2}\gamma_{h5}\Lambda_h^2 \tag{42}$$

where γ_{hj} $(j = 1, \cdots, 5)$ are the parameters of the utility function. γ_{h4} is defined as $\gamma_{h4} \equiv \gamma_{h4}^0 + \overline{\gamma_{h4}} \cdot u_h$, where stochastic variable u_h, is assumed to follow logarithmic normal distribution.

$$\log_e u_h \sim N(m_h, \sigma_h^2) \tag{43}$$

γ_{h4} is the intersection of marginal utility of husband's leisure, which is assumed to be a random coefficient distributed in population. Other parameters, γ_{h1}, γ_{h2}, γ_{h3}, γ_{h4}^0, $\overline{\gamma}_{h4}$, and γ_{h5} are assumed to be common over the population. γ_{h1} is normalized as $\gamma_{h1} \equiv -1$.

Let u_h^* denote the stochastic variable following the standard normal distribution. Standardizing the stochastic variable $\log_e u_h$ in formula (43) yields

$$\frac{\log_e u_h - m_h}{\sigma_h} = u_h^* \rightharpoonup u_h = \exp(m_h) \cdot \exp(\sigma_h \cdot u_h^*)$$

where a constraint, $m_h = -\frac{1}{2}\sigma_h^2$, is imposed on m_h, so that $E(u_h) = 1$ follows. Thus a formula

$$u_h = \exp(-\frac{1}{2}\sigma_h^2) \cdot \exp(\sigma_h \cdot u_h^*) \tag{44}$$

is obtained.

Given the formula (42) and (44), the formula of husband's TILS is

$$I_h^* = H_0^h + H_2^h \cdot \exp(\sigma_h \cdot u_h^*) \tag{45}$$

where

$$H_0^h \equiv \frac{\gamma_{h4}^0 - \gamma_{h2}w_h - \gamma_{h3}w_h(T - \overline{h}_h) + \gamma_{h5}(T - \frac{1}{2}\overline{h}_h) - \frac{1}{2}\gamma_{h1}w_h^2\overline{h}_h}{\gamma_{h1}w_h - \gamma_{h3}}$$

$$H_2^h \equiv \frac{\overline{\gamma}_{h4}}{\gamma_{h1}w_h - \gamma_{h3}} \cdot \exp(-\frac{1}{2}\sigma_h^2)$$

Similarly, utility functions of wife is specified as quadratic forms as

$$\omega_w = \frac{1}{2}\gamma_{w1}X^2 + \gamma_{w2}X + \gamma_{w3}X\Lambda_w + \gamma_{w4}\Lambda_w + \frac{1}{2}\gamma_{w5}\Lambda_w^2 \tag{46}$$

where γ_{wj} $(j = 1, \cdots, 5)$ are parameters of the utility function. γ_{w4} is defined as $\gamma_{w4} \equiv \gamma_{w4}^0 + \overline{\gamma}_{w4} \cdot u_w$, where stochastic variable u_w, is assumed to follow logarithmic normal distribution.

$$\log_e u_w \sim N(m_w, \sigma_w^2) \tag{47}$$

where a constraint, $m_w = -\frac{1}{2}\sigma_w^2$, is imposed on m_w, so that $E(u_w) = 1$ follows. γ_{w4} is the intersection of marginal utility of wife's leisure, which is assumed to be a random coefficient distributed in population. Other parameters, γ_{w1}, γ_{w2}, γ_{w3}, γ_{w4}^0, $\overline{\gamma}_{w4}$, and γ_{w5} are assumed to be common over the population. γ_{w1} is normalized as $\gamma_{w1} \equiv -1$.

Given the formula (46) and (47), the formula of wife's TILS

$$I_w^* = H_0^w + H_2^w \cdot \exp(\sigma_w \cdot u_w^*) \tag{48}$$

is obtained where

$$H_0^w \equiv \frac{\gamma_{w4}^0 - \gamma_{w2} w_w - \gamma_{w3} w_w (T - \overline{h}_w) + \gamma_{w5}(T - \frac{1}{2}\overline{h}_w) - \frac{1}{2}\gamma_{w1} w_w^2 \overline{h}_w}{\gamma_{w1} w_w - \gamma_{w3}}$$

$$H_2^w \equiv \frac{\overline{\gamma}_{w4}}{\gamma_{w1} w_w - \gamma_{w3}} \cdot \exp(-\frac{1}{2}\sigma_w^2)$$

Let ρ denote the correlation coefficient of two dimensional normal distribution of u_h^* and u_w^*.

3.10 A priori restrictions on structural parameters

A priori restrictions imposed on structural parameters are as follows [21] .

a priori restrictions on parameters of utility function:

1. $\frac{\partial h_h^x}{\partial I_h^0} < 0$, $\frac{\partial h_w^x}{\partial I_w^0} < 0$
2. $\frac{\partial \omega_h}{\partial X} > 0$, $\frac{\partial \omega_w}{\partial X} > 0$
3. $\frac{\partial \omega_h}{\partial \Lambda_h} > 0$, $\frac{\partial \omega_w}{\partial \Lambda_w} > 0$
4. Indifference curves are convex to the origin.
5. $H_0^h > I_h^{0\,max}$ and $H_0^w > I_w^{0\,max}$ must hold, where $I_h^{0\,max}$ and $I_w^{0\,max}$ are the maximum values of observed given income of husband and wife respectively.
6. $H_2^h < 0$, $H_2^w < 0$

a priori restrictions on distribution parameters of u_h and u_w:

a $\sigma_h > 0$

b $\sigma_w > 0$

c $|\rho| < 1$

[21] See Miyauchi(1991) for detailed discussions on a priori restrictions on structural parameters.

4 Estimation of structural parameters

Applying Japanese data to the model discussed in Section 3, the structural parameters were estimated. The estimation was performed by minimizing the χ^2 for observed cases of household labor supply and these simulated values. Optimal parameter set was searched within a parameter space that is consistent with a priori restrictions discussed in Subsection 3.10.

Observations on household labor supply probability are obtained by "Household Labor Status Survey" for the years of 1971, 1974, 1977, 1979, and 1982. Observations on wage rate and assigned hours of work are obtained by "Wage Census" for the corresponding years. Because appropriate observations on unearned income, I_A, was not available, the value of I_A was assumed to be zero for the first attempt. These observations are stratified by husband's and wife's age classes.

The estimates of the structural parameters are as follows.

$$
\begin{array}{llll}
\gamma_{h2} & = 6540.59 & \gamma_{w2} & = 1150.25 \\
\gamma_{h3} & = 520.02 & \gamma_{w3} & = -21.36 \\
\overline{\gamma}_{h4} & = 816058.3 & \overline{\gamma}_{w4} & = 269220.5 \\
\gamma_{h5} & = -32724.1 & \gamma_{w5} & = -962.0 \\
\gamma^0_{h4} & = 79727.4 & \gamma^0_{w4} & = 26956.1 \\
\sigma_h & = 2.534 & \sigma_w & = 1.0474 \\
\rho & = 0.602 & &
\end{array}
$$

A comparison between observed and simulated labor supply probabilities is presented graphically in Figure 11. Although the observed and the simulated probabilities are obtained for each stratum of husband's and wife's age classes, let the graphical comparison be presented on the basis of aggregating these strata for the sake of briefness.

5 Concluding remarks

1. The magnitude in probabilities of husband's and wife's labor supply is significantly different by these patterns of household decision, i.e., both work, husband works, wife works, and neither works. The model presented in this paper simulates well the difference in the magnitude of these probabilities.

2. The model simulates well the time trend observed in these probabilities, although systematic biases in simulated probabilities are persistent in each year.

3. The simulated probability in the region corresponding to the payoff table, Π_h^{3-3}, where the model fails to give a unique equilibrium solution on house-

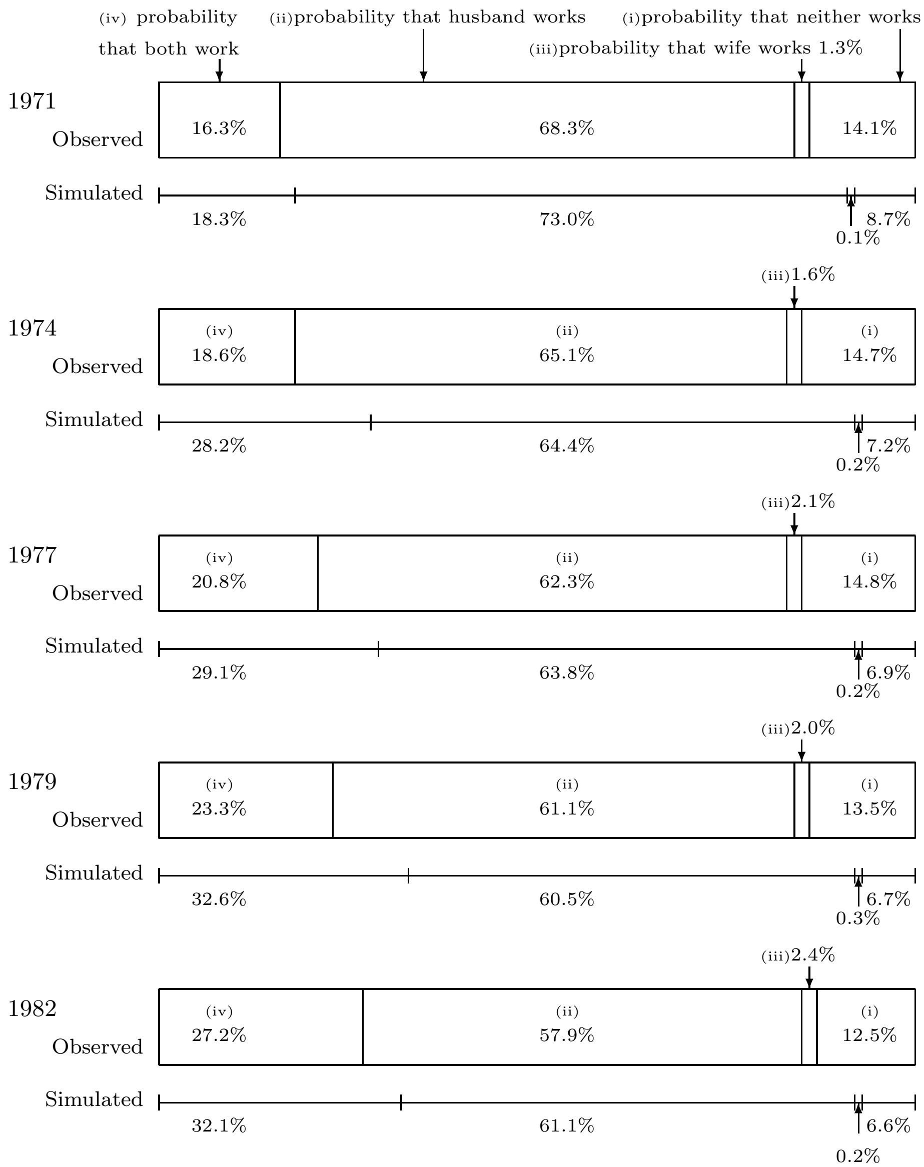


Figure 11: Observed and simulated probabilities of household decision on labor supply

hold labor supply, is less than 0.001, although this fact never precludes the problem of identification.

4. The model gives systematic biases in simulated probabilities in each year. The model should be modified so that the systematic biases can be resolved.

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