Existence of Equilibria for Atomless Economies with Bads

Tomoki Inoue[*]

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Abstract

We consider an exchange economy where there are a continuum of consumers and some commodities are bads. Bads are commodities that cause disutility to consumers. Under any price vector with positive for goods and negative for bads, consumers can increase the consumption of both goods and bads while satisfying the budget constraint and, therefore, their budget sets are not bounded. Hara [1] exemplified that an equilibrium fails to exist. In his economy, a small portion of consumers wants to consume quite a large amount of bads and the resulting demand function is not integrable. The integrability of demand function represents that every consumer is negligible relative to the whole economy. Thus, it is one of the conditions for an equilibrium.

We prove the existence of an equilibrium by making assumptions on the asymptotic cone of consumers' upper contour sets. The asymptotic cone of the upper contour set of consumption vector x is the set of all directions that as desirable as x. As Werner [2] proved, the asymptotic cone of the upper contour set characterizes the set of price vectors under which the demand set is nonempty. We assume that;

(i) For every consumer, his upper contour sets have the same asymptotic cone.

[*]School of Political Science and Economics, Meiji University, 1-1 Kanda-Surugadai, Chiyoda-ku, Tokyo 101-8301, Japan; tomoki.inoue@gmail.com.

(ii) There exists $\delta > 0$ such that, for every direction z, if consumers whose asymptotic cones contain z are of positive measure, then the measure is greater than δ.

Assumption (i) is met if consumers have concave or quasi-linear utility functions. Even if consumers' preference relations are different, their asymptotic cones of the upper contour set can be equal. Furthermore, if some consumers with positive measure have large asymptotic cones, assumption (ii) is likely to be satisfied. Thus, assumption (ii) is not so strong.

The union of all consumers' asymptotic cones is called the *total desirabilty cone* that consists of desirablity directions for some consumers. By the definition of the asymptotic cone, the distance from every consumer's upper contour set to its asymptotic cone converges to zero. Since the total desirability cone contains every consumer's asymptotic cone, the distance from his upper contour set to the total desirablity cone converges to zero. We assume that;

(iii) The rate of convergence of the distance from consumers' upper contour sets to the total desirabilty cone is similar.

We prove that an equilibrium exists under assumptions (i)-(iii).

Keywords: Bads; Existence of equilibrium; Atomless economy

JEL classification: C62, D41

References

[1] Hara, C., 2005. Existence of equilibria in economies with bads. Econometrica 73, 647-658.

[2] Werner, J., 1987. Arbitrage and the existence of competitive equilibrium. Econometrica 55, 1403-1418.