Equilibrium Yield Curve, Phillips Correlation, and Monetary Policy

Mitsuru Katagiri
International Monetary Fund

October 24, 2017
@Keio University

Disclaimer

The views expressed here are those of the author and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

Motivation

- Yield curves are upward sloping on average (=positive term premiums) in most advanced economies
- In a standard consumption based asset pricing model, term premiums are usually *negative* and *small*
  $\Rightarrow$ "**bond premium puzzle**" (Backus et al., 1989)

Motivation

- Yield curves are upward sloping on average (=positive term premiums) in most advanced economies
- In a standard consumption based asset pricing model, term premiums are usually *negative* and *small*
  $\Rightarrow$ "**bond premium puzzle**" (Backus et al., 1989)

Questions:

- Can positive term premiums be rationalized by consumers' optimization under the observed income/inflation?

Motivation

- Yield curves are upward sloping on average (=positive term premiums) in most advanced economies
- In a standard consumption based asset pricing model, term premiums are usually *negative* and *small*
  $\Rightarrow$ "**bond premium puzzle**" (Backus et al., 1989)

Questions:

- Can positive term premiums be rationalized by consumers' optimization under the observed income/inflation?
- How does the monetary policy influence term premiums?
  - ⋆ In particular, what does the model predict about yield curves under permanently low interest rates with the ZLB?

This Paper

- Analyze the equilibrium yield curve in a model with optimal savings as a buffer stock (Deaton, 1991)
  - ⋆ The exogenous income/inflation process is estimated by data
  - ⋆ Nominal interest rates are set by a monetary policy rule
    $\Rightarrow$ Changes in a monetary policy behavior can be analyzed

This Paper

- Analyze the equilibrium yield curve in a model with optimal savings as a buffer stock (Deaton, 1991)
  - ⋆ The exogenous income/inflation process is estimated by data
  - ⋆ Nominal interest rates are set by a monetary policy rule
    $\Rightarrow$ Changes in a monetary policy behavior can be analyzed

- Conduct a counterfactual simulation for a permanently low interest rate environment with the ZLB (low-for-long)

Previous Literature on Term Structure

Euler equation with exogenous consumption and inflation

- **Approach #1**: compute equilibrium yield curves by the Euler equation under *exogenous* inflation and consumption

  Backus et al. (1989), Boudoukh (1993), Wachter (2006), Piazzesi and Schneider (2007), Bansal and Shaliastovich (2013), Branger et al. (2016)

- **Main takeaway**: To rationalize positive term premiums, $corr(\Delta c_t, \pi_t) < 0$ is necessary. Why?

Previous Literature on Term Structure

Why is $corr(\Delta c_t, \pi_t) < 0$ necessary?

- Let us think about 2-period bond price, $Q_{2,t}$

$$\begin{aligned} Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1}M_{t,t+1}\right) \quad \text{where } M_{t,t+1}\text{: Nominal SDF} \\ &= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) \end{aligned}$$

Previous Literature on Term Structure

Why is $corr(\Delta c_t, \pi_t) < 0$ necessary?

- Let us think about 2-period bond price, $Q_{2,t}$

$$
\begin{aligned}
Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1} M_{t,t+1}\right) \quad \text{where } M_{t,t+1}\text{: Nominal SDF} \\
&= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1})
\end{aligned}
$$

- Term premiums > 0 iff $cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) < 0$

Previous Literature on Term Structure

Why is $corr(\Delta c_t, \pi_t) < 0$ necessary?

- Let us think about 2-period bond price, $Q_{2,t}$

$$
\begin{aligned}
Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1} M_{t,t+1}\right) \quad \text{where } M_{t,t+1}\text{: Nominal SDF} \\
&= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1})
\end{aligned}
$$

- Term premiums > 0 iff $cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) < 0$
- Autocorrelation for Δc_t and π_t is positive in data
  $\Rightarrow$ *Negative* and *small* term premiums in a standard model

Previous Literature on Term Structure

Why is $corr(\Delta c_t, \pi_t) < 0$ necessary?

- Let us think about 2-period bond price, $Q_{2,t}$

$$
\begin{aligned}
Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1}M_{t,t+1}\right) \quad \text{where } M_{t,t+1}\text{: Nominal SDF} \\
&= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1})
\end{aligned}
$$

- Term premiums > 0 iff $cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) < 0$
- Autocorrelation for Δc_t and π_t is positive in data
  $\Rightarrow$ *Negative* and *small* term premiums in a standard model
- $corr(\Delta c_t, \pi_t) < 0$ is necessary for positive term premium
- **Intuition**: An inflation hike is a bad news for both nominal bond prices and future consumption growth

Previous Literature on Term Structure

Empirical observations and challenges

- $corr(\Delta c_t, \pi_t) < 0$ is empirically observed in most economies
  - ⋆ $corr(\Delta c_t, \pi_t) = -0.19$ in the US from 1959Q2 to 2017Q1
- Euler eq. based on EZ preference accounts for the US yield curve under $corr(\Delta c_t, \pi_t) < 0$ (Piazzesi and Schneider, 2007)

Previous Literature on Term Structure

Empirical observations and challenges

- $corr(\Delta c_t, \pi_t) < 0$ is empirically observed in most economies
  - $corr(\Delta c_t, \pi_t) = -0.19$ in the US from 1959Q2 to 2017Q1
- Euler eq. based on EZ preference accounts for the US yield curve under $corr(\Delta c_t, \pi_t) < 0$ (Piazzesi and Schneider, 2007)

Challenges for Approach #1:

1. Changes in a monetary policy behavior (e.g., the zero lower bound) are difficult to analyze
2. Consistency with macroeconomic variables is out of scope

Previous Literature on Term Structure

A general equilibrium model with endogenous consumption (and inflation)

- **Approach #2**: analyzes equilibrium yield curves in a general equilibrium with *endogenous* consumption (and inflation)

  Campbell et al. (2012), Rudebusch and Swanson (2008, 2012), van Binsgergen et al. (2012), Hsu et al. (2015), Gourio and Ngo (2016), Nakata and Tanaka (2016), Gallmeyer et al. (2007, 2017)

- Nominal interest rates are not endogenous variables but set by a policy reaction function to inflation,

$$R_t = \phi_\pi \pi_t \quad \text{where} \quad \phi_\pi > 1$$

- Again, $corr(\Delta c_t, \pi_t) < 0$ is a necessary feature for positive term premiums. Why?

Previous Literature on Term Structure

A general equilibrium model with endogenous consumption (and inflation)

- Let us think about 2-period bond price, $Q_{2,t}$, again. The covariance term is:

$$\begin{aligned} cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) &= cov(1/R_{t+1}, M_{t,t+1}) \\ &\approx cov(1/\pi_{t+1}, 1/\Delta c_{t+1}) \end{aligned}$$

- The covariance term is negative iff $corr(\Delta c_t, \pi_t) < 0$

Previous Literature on Term Structure

A general equilibrium model with endogenous consumption (and inflation)

- Let us think about 2-period bond price, $Q_{2,t}$, again. The covariance term is:

$$\begin{aligned} cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) &= cov(1/R_{t+1}, M_{t,t+1}) \\ &\approx cov(1/\pi_{t+1}, 1/\Delta c_{t+1}) \end{aligned}$$

- The covariance term is negative iff $corr(\Delta c_t, \pi_t) < 0$

Challenges for Approach #2:

1. Inflation determinacy in the low-for-long faces some difficulty

Previous Literature on Term Structure

A general equilibrium model with endogenous consumption (and inflation)

- Let us think about 2-period bond price, $Q_{2,t}$, again. The covariance term is:

$$\begin{aligned} cov(\mathbb{E}_{t+1}(M_{t+1,t+2}), M_{t,t+1}) &= cov(1/R_{t+1}, M_{t,t+1}) \\ &\approx cov(1/\pi_{t+1}, 1/\Delta c_{t+1}) \end{aligned}$$

- The covariance term is negative iff $corr(\Delta c_t, \pi_t) < 0$

Challenges for Approach #2:

1. Inflation determinacy in the low-for-long faces some difficulty
2. It is hard to reconcile $corr(\Delta c_t, \pi_t) < 0$ with macroeconomic behaviors including the Phillips correlation
   - ⋆ Does $corr(\Delta c_t, \pi_t) < 0$ mean that real economic activity and inflation should be *negatively* correlated?

Preview of Main Results

- The model can account for upward sloping yield curves even under the Phillips correlation
  - ⋆ A difference between a stationary and non-stationary part of income and their relationship with inflation is a key
  - ⋆ Real term premiums explain a large part of term premiums
- Low-for-long would entail flatter yield curves by reducing real term premiums around the zero lower bound
  - ⋆ Bank profits from the maturity transformation could face difficulty in the low-for-long (GFSR, 2017)

Empirical Preliminaries

Yield Curve

	Average Level			Standard Deviation		
	O/N	5Y	10Y	O/N	5Y	10Y
U.S.	5.13	5.87	6.20	3.63	3.04	2.82
(1959Q2-2017Q1)	(0.00)	(0.74)	(1.07)	(1.00)	(0.83)	(0.77)
U.K.	6.18	7.32	7.65	4.00	3.82	3.62
(1957Q2-2017Q1)	(0.00)	(0.51)	(0.84)	(1.00)	(0.96)	(0.91)
Germany	3.16	5.06	5.57	2.40	2.77	2.50
(1975Q2-2017Q1)	(0.00)	(1.90)	(2.41)	(1.00)	(1.15)	(1.04)
Japan	2.98	3.43	3.83	3.30	3.08	2.81
(1975Q2-2017Q1)	(0.00)	(0.45)	(0.85)	(1.00)	(0.94)	(0.85)

- Average yield curves are upward sloping for all countries
- Volatility is a bit smaller for longer yields except for Germany

Phillips Correlation

Dependent variable: Inf_t

	U.S. (59Q2-17Q1)		U.K. (57Q2-17Q1)		Germany (75Q2-17Q1)		Japan (75Q2-17Q1)	
Inf_{t-1}	0.83^{**}	0.83^{**}	0.58^{**}	0.57^{**}	0.52^{**}	0.49^{**}	0.85^{**}	0.85^{**}
gap_t	0.03	–	0.03	–	0.05	–	0.04	–
gap_{t-1}	–	0.05^{**}	–	0.13^{**}	–	0.15^{**}	–	0.04

Note: Variables with ** and * are statistically significant at 1% and 5% level.

- Phillips correlation is statistically significant for lagged income gap (i.e., gap_{t-1}) for all countries except for Japan
  - Income gap is defined as a deviation of household disposable income from the HP-filter trend
  - Inflation is Q-on-Q changes in the PCE deflatpr

Model

Overview

- A representative agent model with buffer-stock savings pioneered by Deaton (1991) and Carroll (1992)
- One-period nominal bonds are only choice for savings
- Income/inflation follow an exogenous process with correlation
- Nominal interest rates are set by a monetary policy rule

Budget Constraint

- The representative household's budget constraint:

$$P_t c_t + \frac{B_t}{R_t} + \sum_{n>1} Q_{n,t} B_{n,t} + \Phi\left(\frac{B_t}{R_t}\right) = P_t Y_t + B_{t-1} + \sum_{n>1} Q_{n-1,t} B_{n,t-1}$$

P_t: price level, Y_t: real income, c_t: consumption, R_t: nominal interest rate, B_t: nominal one-period bond, $B_{n,t}$: n-period nominal bonds, $Q_{n,t}$: n-period bond prices, $\Phi(\cdot)$: costs for bond holdings

- Assume a tiny cost for bonds to avoid divergence satisfying

$$\Phi'\left(\frac{B_t}{R_t}\right) > 0 \text{ and } \Phi''\left(\frac{B_t}{R_t}\right) > 0$$

Income and inflation

- Household real income (Y_t) consists of the non-stationary part (y_t^*) and the stationary part (y_t):

$$\log(Y_t) = \log(y_t^*) + \log(y_t)$$

where $g_t \equiv y_t^*/y_{t-1}^*$ and y_t are stationary

Income and inflation

- Household real income (Y_t) consists of the non-stationary part (y_t^*) and the stationary part (y_t):

$$\log(Y_t) = \log(y_t^*) + \log(y_t)$$

where $g_t \equiv y_t^*/y_{t-1}^*$ and y_t are stationary

- Similarly, inflation ($\Pi_t \equiv P_t/P_{t-1}$) consists of the trend (π_t^*) and the cycle (π_t) as in Cogley and Sbordone (2008):

$$\log(\Pi_t) = \log(\pi_t^*) + \log(\pi_t)$$

where $\xi_t \equiv \pi_t^*/\pi_{t-1}^*$ and π_t are stationary

Detrending

- The economy is detrended by P_t, π_t^* and y_t^*:

$$\textbf{Bond}: b_t = B_t/(P_t y_t^* \pi_t^*) \;\text{ and }\; b_{n,t} = B_{n,t}/(P_t y_t^* {\pi_t^*}^n)$$

$$\textbf{Price}: \tilde{R}_t = R_t/\pi_t^* \;\text{ and }\; \tilde{Q}_{n,t} = {\pi_t^*}^n Q_{n,t}$$

$$\textbf{Consumption}: \tilde{c}_t = c_t/y_t^*$$

Detrending

- The economy is detrended by P_t, π_t^* and y_t^*:

$$\textbf{Bond} : b_t = B_t/(P_t y_t^* \pi_t^*) \;\text{ and }\; b_{n,t} = B_{n,t}/(P_t y_t^* {\pi_t^*}^n)$$

$$\textbf{Price} : \tilde{R}_t = R_t/\pi_t^* \;\text{ and }\; \tilde{Q}_{n,t} = {\pi_t^*}^n Q_{n,t}$$

$$\textbf{Consumption} : \tilde{c}_t = c_t/y_t^*$$

- The budget constraint after detrending:

$$\tilde{c}_t + \frac{b_t}{\tilde{R}_t} + \sum_{n>1} \tilde{Q}_{n,t} b_{n,t} + \Phi\left(\frac{b_t}{\tilde{R}_t}\right) = 1 + \frac{b_{t-1}}{g_t \pi_t \xi_t} + \frac{\sum_{n>1} \tilde{Q}_{n-1,t} b_{n,t-1}}{g_t \pi_t {\xi_t}^n}$$

Monetary Policy and Bond Holding Cost

- Nominal interest rates, R_t, are set by a policy rule

$$\begin{aligned} R_t &= R_{t-1}^{\phi_r}\left[\pi_t^* g^* \left(\frac{\Pi_t}{\pi_t^*}\right)^{\phi_\pi}\left(\frac{Y_t}{y_t^*}\right)^{\phi_y}\right]^{1-\phi_r} \\ \Rightarrow \tilde{R}_t &= \left(\frac{\tilde{R}_{t-1}}{\xi_t}\right)^{\phi_r}\left[g^* \pi_t^{\phi_\pi} y_t^{\phi_y}\right]^{1-\phi_r} \end{aligned}$$

- A cost for bond holdings is assumed to be quadratic:

$$\Phi\left(\frac{b_t}{\tilde{R}_t}\right) \equiv \frac{\phi_b}{2}\left(\frac{b_t}{\tilde{R}_t} - \frac{b^*}{R^*}\right)^2 \tilde{R}_t$$

Optimization problem

- The household maximizes the value function based on the Epstein-Zin-Weil preference:

$$V_t = \left\{ c_t^{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1-\sigma}{1-\alpha}} \right\}^{\frac{1}{1-\sigma}}$$

where σ: inverse of IES, α: CRRA coefficient

Optimization problem

- The household maximizes the value function based on the Epstein-Zin-Weil preference:

$$V_t = \left\{ c_t^{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1-\sigma}{1-\alpha}} \right\}^{\frac{1}{1-\sigma}}$$

where σ: inverse of IES, α: CRRA coefficient

- The supply of $b_{n,t}$ is assumed to be zero in equilibrium

Optimization problem

- The household maximizes the value function based on the Epstein-Zin-Weil preference:

$$V_t = \left\{ c_t^{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\alpha} \right]^{\frac{1-\sigma}{1-\alpha}} \right\}^{\frac{1}{1-\sigma}}$$

  where σ: inverse of IES, α: CRRA coefficient

- The supply of $b_{n,t}$ is assumed to be zero in equilibrium

- The model consists of two endogenous, $(b_{t-1}, \tilde{R}_{t-1})$, and four exogenous state variables, (g_t, ξ_t, y_t, π_t)

Euler Equation

- The equilibrium is characterized by the Euler equations

$$R_t E_t[M_{t,t+1}] = 1$$

and

$$E_t[Q_{n-1,t+1} M_{t,t+1}] = Q_{n,t}, \ \forall n > 1$$

- $M_{t,t+1}$ is the nominal SDF from period t to $t+1$:

$$M_{t,t+1} = \frac{\beta}{\pi_{t+1}} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \left[\frac{V_{t+1}}{\mathbb{E}_t \left(V_{t+1}^{1-\alpha} \right)^{\frac{1}{1-\alpha}}} \right]^{\sigma-\alpha}$$

Yield Curve and Term Premiums

- The yield for each maturity, $R_{n,t}$:

$$R_{n,t} = \left(\frac{1}{Q_{n,t}}\right)^{\frac{1}{n}}$$

- Bond prices and yields for risk-neutral agents, $\hat{Q}_{n,t}$ and $\hat{R}_{n,t}$:

$$\frac{1}{R_t}\mathbb{E}_t[\hat{Q}_{n-1,t+1}] = \hat{Q}_{n,t} \quad \text{and} \quad \hat{R}_{n,t} = \left(\frac{1}{\hat{Q}_{n,t}}\right)^{\frac{1}{n}}, \forall n > 1$$

- Term premiums, $\psi_{n,t}$, are defined as:

$$\psi_{n,t} = R_{n,t} - \hat{R}_{n,t}$$

Quantitative Analysis

Calibration

- Most parameter values are calibrated to standard values
  - ⋆ b^* is the average asset-income ratio in the U.S.
  - ⋆ Examines several values for α and the monetary policy rule

Parameters	Values
Discount rate, β	0.9985
Inverse of IES, σ	1.0
Risk averseness, α	100.0
Cost for bond holdings, ϕ_b	0.001
Steady-state savings, b^*	4.8
Monetary policy rule:	
Response to inflation, ϕ_π	1.50
Response to growth, ϕ_y	0.25
Interest rate smoothing, ϕ_r	0.80

Income and inflation processes

- **Income:** $g_t \equiv y_t^*/y_{t-1}^*$ and y_t follow

$$\log(g_t) = \rho_g \log(g_{t-1}) + \varepsilon_{g,t}$$
$$\log(y_t) = \rho_y \log(y_{t-1}) + \varepsilon_{y,t}$$

where $cov(\varepsilon_{g,t}, \varepsilon_{y,t}) = 0$

Income and inflation processes

- **Income:** $g_t \equiv y_t^* / y_{t-1}^*$ and y_t follow

$$\begin{aligned} \log(g_t) &= \rho_g \log(g_{t-1}) + \varepsilon_{g,t} \\ \log(y_t) &= \rho_y \log(y_{t-1}) + \varepsilon_{y,t} \end{aligned}$$

where $cov(\varepsilon_{g,t}, \varepsilon_{y,t}) = 0$

- **Inflation:** $\xi_t \equiv \pi_t^* / \pi_{t-1}^*$ and π_t follow

$$\begin{aligned} \log(\xi_t) &= \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t} \\ \log(\pi_t) &= \rho_\pi \log(\pi_{t-1}) + \kappa \log(y_{t-1}) + \varepsilon_{\pi,t} \end{aligned}$$

Income and inflation processes

- **Income:** $g_t \equiv y_t^*/y_{t-1}^*$ and y_t follow

$$\begin{aligned}\log(g_t) &= \rho_g \log(g_{t-1}) + \varepsilon_{g,t}\\ \log(y_t) &= \rho_y \log(y_{t-1}) + \varepsilon_{y,t}\end{aligned}$$

  where $cov(\varepsilon_{g,t}, \varepsilon_{y,t}) = 0$

- **Inflation:** $\xi_t \equiv \pi_t^*/\pi_{t-1}^*$ and π_t follow

$$\begin{aligned}\log(\xi_t) &= \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}\\ \log(\pi_t) &= \rho_\pi \log(\pi_{t-1}) + \kappa \log(y_{t-1}) + \varepsilon_{\pi,t}\end{aligned}$$

- Phillips correlation is captured by two channels:
  1. The lagged income gap, y_{t-1}, can influence π_t as in data
  2. $\varepsilon_{\pi,t}$ can be correlated with income shocks: $cov(\varepsilon_{\pi,t}, \varepsilon_{g,t}) \neq 0$ and $cov(\varepsilon_{\pi,t}, \varepsilon_{y,t}) \neq 0$

Estimation of Income and inflation processes

- The parameters are estimated by a Bayesian method using $\Delta \log(\Pi)$ and $\Delta \log(Y_t)$ as observable variables
  - Data: Personal disposable income and PCE deflator
  - Sample periods: 1957Q2 - 2017Q1 for UK, 1959Q2 - 2017Q1 for US, and 1975Q2 - 2017Q1 for Germany and Japan

Estimation of Income and inflation processes

- The parameters are estimated by a Bayesian method using $\Delta \log(\Pi)$ and $\Delta \log(Y_t)$ as observable variables
  - ⋆ Data: Personal disposable income and PCE deflator
  - ⋆ Sample periods: 1957Q2 - 2017Q1 for UK, 1959Q2 - 2017Q1 for US, and 1975Q2 - 2017Q1 for Germany and Japan

- For identification of trend and cycle, assume that:
  1. Ratio of trend and cycle volatility for inflation, σ_ξ / σ_π, is 1.0%
  2. There is a tight prior distribution for ρ_y and ρ_ξ

Estimation of Income and inflation processes

- The parameters are estimated by a Bayesian method using $\Delta \log(\Pi)$ and $\Delta \log(Y_t)$ as observable variables
  - ⋆ Data: Personal disposable income and PCE deflator
  - ⋆ Sample periods: 1957Q2 - 2017Q1 for UK, 1959Q2 - 2017Q1 for US, and 1975Q2 - 2017Q1 for Germany and Japan
- For identification of trend and cycle, assume that:
  1. Ratio of trend and cycle volatility for inflation, σ_ξ / σ_π, is 1.0%
  2. There is a tight prior distribution for ρ_y and ρ_ξ
- 10 parameters to be estimated:

$$(\rho_g, \rho_y, \sigma_g, \sigma_y, \rho_\xi, \rho_\pi, \sigma_\pi, \kappa, \sigma_{\pi,g}, \sigma_{\pi,y})$$

Estimation Result

Name	Prior	Posterior			
		US	UK	Germany	Japan
ρ_g	Beta	0.54	0.62	0.33	0.71
	(0.50,0.25)	[0.32,0.75]	[0.39,0.86]	[0.01,0.62]	[0.59,0.83]
ρ_y	Beta	0.76	0.72	0.77	0.77
	(0.75,0.05)	[0.68,0.84]	[0.64,0.82]	[0.69,0.85]	[0.69,0.85]
ρ_ξ	Beta	0.97	0.96	0.94	0.98
	(0.95,0.03)	[0.95,0.99]	[0.94,0.99]	[0.89,0.98]	[0.96,1.00]
ρ_π	Beta	0.53	0.22	0.23	0.58
	(0.50,0.25)	[0.35,0.70]	[0.11,0.34]	[0.08,0.39]	[0.42,0.75]
κ	Uniform	**0.13**	**0.14**	**0.22**	**-0.05**
	(-1,1)	[-0.04,0.32]	[0.04,0.23]	[0.10,0.33]	[-0.37,0.24]

- The lagged income gap has positive effects on inflation (i.e., $\kappa > 0$) except for Japan

Estimation Result (Cont'd)

Name	Prior	Posterior			
		US	UK	Germany	Japan
σ_g	Inv.Gamma	0.24	0.20	0.27	0.09
	(0.3,Inf)	[0.10,0.40]	[0.06,0.33]	[0.08,0.47]	[0.06,0.11]
σ_y	Inv.Gamma	0.26	1.89	0.39	0.09
	(0.4,Inf)	[0.15,0.37]	[1.51,2.28]	[0.19,0.57]	[0.07,0.11]
σ_π	Inv.Gamma	0.11	0.99	0.17	0.09
	(0.3,Inf)	[0.09,0.13]	[0.82,1.15]	[0.13,0.21]	[0.07,0.11]
$\sigma_{\pi,g}$	Uniform	**-0.37**	**-0.48**	**-0.57**	**-0.28**
	(-1,1)	[-0.57,-0.07]	[-0.86,-0.11]	[-0.92,-0.24]	[-0.67,0.12]
$\sigma_{\pi,y}$	Uniform	**0.13**	**0.01**	**0.10**	**0.19**
	(-1,1)	[-0.16,0.40]	[-0.16,0.16]	[-0.22,0.46]	[-0.06,0.46]

- Non-stationary and stationary part of income is *negatively* and *positively* correlated with inflation ($\sigma_{\pi,g} < 0$ and $\sigma_{\pi,y} > 0$)

Phillips Correlation for Estimated Income Gap

Dependent variable: Inf_t

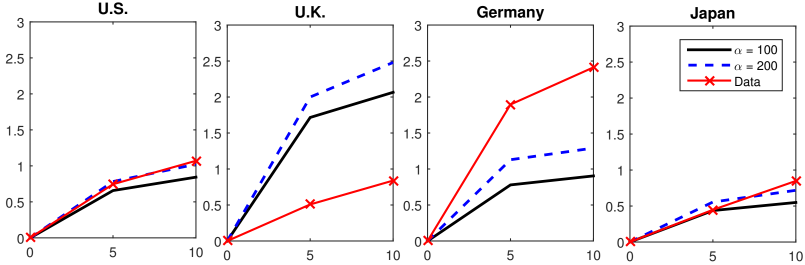
	U.S. (59Q2-17Q1)		U.K. (57Q2-17Q1)		Germany (75Q2-17Q1)		Japan (75Q2-17Q1)	
Inf_{t-1}	0.79^{**}	0.78^{**}	0.58^{**}	0.58^{**}	0.53^{**}	0.55^{**}	0.82^{**}	0.87^{**}
gap_t	0.15^{**}	–	-0.12	–	-0.09	–	0.17^{*}	–
gap_{t-1}	–	0.17^{**}	–	0.03	–	0.17^{*}	–	-0.13

Note: Variables with ** and * are statistically significant at 1% and 5% level.

- Phillips correlation is observed for the estimated income gaps (i.e., y_t and y_{t-1}) except for UK

Result: Equilibrium Yield Curve

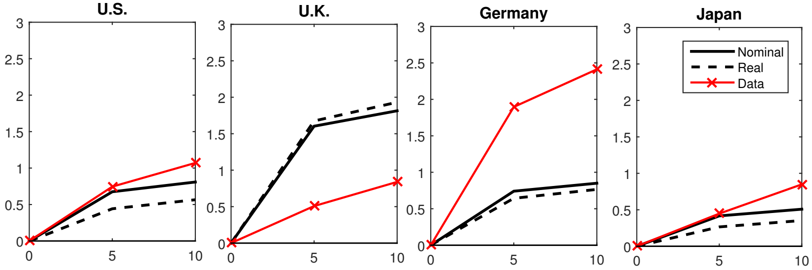
Different level of risk averseness



- The model can replicate upward sloping yield curves even under the Phillips correlation!
  - ⋆ The equilibrium yield curve is computed by putting estimated g_t, y_t, ξ_t and π_t into the model for each country
  - ⋆ High risk averseness entails steeper equilibrium yield curves

Result: Equilibrium Yield Curve

Nominal and real yield curve



- Real term premiums explain a large part of term premiums
  - ⋆ It is in contrast with the previous literature, but in line with empirical works (e.g., Abrahams et. al., 2016)

Mechanism behind the Equilibrium Yield Curve

- Difference between stationary and non-stationary income is a key to understanding the equilibrium yield curve

Mechanism behind the Equilibrium Yield Curve

- Difference between stationary and non-stationary income is a key to understanding the equilibrium yield curve
  - ⋆ The stationary income shock entails the Phillips correlation because $\sigma_{y,\pi} > 0$ and $\kappa > 0$

Mechanism behind the Equilibrium Yield Curve

- Difference between stationary and non-stationary income is a key to understanding the equilibrium yield curve
  - The stationary income shock entails the Phillips correlation because $\sigma_{y,\pi} > 0$ and $\kappa > 0$
  - But, under the PIH, consumption growth is more influenced by non-stationary income than stationary one
    $\Rightarrow \sigma_{g,\pi} < 0$ induces $corr(\Delta c_t, \pi_t) < 0$ in the model

Mechanism behind the Equilibrium Yield Curve

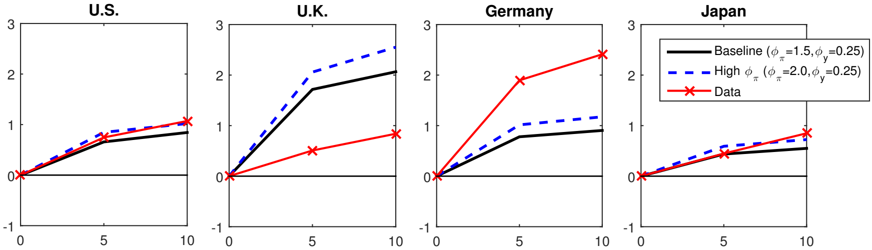
- Difference between stationary and non-stationary income is a key to understanding the equilibrium yield curve
  - ⋆ The stationary income shock entails the Phillips correlation because $\sigma_{y,\pi} > 0$ and $\kappa > 0$
  - ⋆ But, under the PIH, consumption growth is more influenced by non-stationary income than stationary one
    $\Rightarrow \sigma_{g,\pi} < 0$ induces $corr(\Delta c_t, \pi_t) < 0$ in the model
  - ⋆ Non-stationarity of supply shock is long well-known in the VAR literature (e.g., Blanchard and Quah, 1989)

Mechanism behind the Equilibrium Yield Curve

- Difference between stationary and non-stationary income is a key to understanding the equilibrium yield curve
  - ⋆ The stationary income shock entails the Phillips correlation because $\sigma_{y,\pi} > 0$ and $\kappa > 0$
  - ⋆ But, under the PIH, consumption growth is more influenced by non-stationary income than stationary one
    $\Rightarrow \sigma_{g,\pi} < 0$ induces $corr(\Delta c_t, \pi_t) < 0$ in the model
  - ⋆ Non-stationarity of supply shock is long well-known in the VAR literature (e.g., Blanchard and Quah, 1989)
- Positive real term premiums are caused by $\phi_\pi > 1$ (i.e., the Taylor principle)
  $\Rightarrow$ Real long-term bond is also a poor hedge against inflation!

Monetary Policy Rule and Equilibrium Yield Curve

Simulation results under different parameter values



- Higher values of ϕ_π lead to steeper yield curves. Why?

- Let us think about 2-period bond price, $Q_{2,t}$

$$\begin{aligned} Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1} M_{t,t+1}\right) \\ &= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(1/R_{t+1}, M_{t,t+1}) \end{aligned}$$

- $cov(1/R_{t+1}, M_{t,t+1}) \downarrow \Rightarrow$ Term premiums $\uparrow$

- Let us think about 2-period bond price, $Q_{2,t}$

$$
\begin{aligned}
Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1}M_{t,t+1}\right) \\
&= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(1/R_{t+1}, M_{t,t+1})
\end{aligned}
$$

- $cov(1/R_{t+1}, M_{t,t+1}) \downarrow \Rightarrow$ Term premiums $\uparrow$

- Comparative statics for ϕ_π:
  - ⋆ High $\phi_\pi \Rightarrow cov(1/R_{t+1}, \pi_{t+1}) \downarrow \Rightarrow cov(1/R_{t+1}, \Delta c_{t+1}) \uparrow$

- Let us think about 2-period bond price, $Q_{2,t}$

$$\begin{aligned} Q_{2,t} &= \mathbb{E}_t\left(Q_{1,t+1}M_{t,t+1}\right) \\ &= \mathbb{E}_t(Q_{1,t+1})/R_t + cov(1/R_{t+1}, M_{t,t+1}) \end{aligned}$$

- $cov(1/R_{t+1}, M_{t,t+1}) \downarrow \Rightarrow$ Term premiums $\uparrow$

- Comparative statics for ϕ_π:
  - ⋆ High $\phi_\pi \Rightarrow cov(1/R_{t+1}, \pi_{t+1}) \downarrow \Rightarrow cov(1/R_{t+1}, \Delta c_{t+1}) \uparrow$

- Differences in term structure across countries are possibly explained by differences in monetary policy rules

Volatility Curve and Trend Inflation

Relative volatility for long-term interest rates

	Data		Baseline		Fixed π_t^*	
	5Y	10Y	5Y	10Y	5Y	10Y
U.S.	0.83	0.77	0.73	0.71	0.39	0.20
U.K.	0.96	0.91	0.86	0.90	0.30	0.15
Germany	1.15	1.04	0.59	0.54	0.33	0.17
Japan	0.94	0.85	0.74	0.59	0.34	0.17

- Relative volatility of long-term interest rates fit the data well
- Volatility induced by the trend inflation is a key
  - ⋆ With fixed trend inflation, volatility of long-term interest rates are much smaller than data (Fuhrer, 1996)

Macroeconomic Moments for Consumption Growth

		U.S.	U.K.	Germany	Japan
$std(\Delta c_t)$	Model	1.15	1.90	1.09	1.15
	Data	0.67	1.07	0.93	0.86
$corr(\Delta c_t, \Delta y_t)$	Model	0.16	-0.09	0.00	0.17
	Data	0.53	0.30	0.57	0.21
$corr(\Delta c_t, \pi_t)$	Model	-0.25	-0.52	-0.49	-0.21
	Data	-0.19	-0.22	-0.12	0.11
$corr(\Delta c_{t-1}, c_t)$	Model	0.19	-0.01	0.00	0.38
	Data	0.33	-0.04	-0.25	0.20

1. $std(\Delta c_t)$ is too large ("excess smoothness" in data)
2. $corr(\Delta c_t, \Delta y_t)$ is too small ("excess sensitivity" in data)

Rule-of-Thumb Consumers

- The excess smoothness and sensitivity are commonly observed in the PIH model (e.g., Deaton and Campbell, 1989)

Rule-of-Thumb Consumers

- The excess smoothness and sensitivity are commonly observed in the PIH model (e.g., Deaton and Campbell, 1989)
- Campbell and Mankiw (1990) point out the existence of the Rule-of-thumb (ROT) consumers

Rule-of-Thumb Consumers

- The excess smoothness and sensitivity are commonly observed in the PIH model (e.g., Deaton and Campbell, 1989)
- Campbell and Mankiw (1990) point out the existence of the Rule-of-thumb (ROT) consumers
- With ROT consumers, consumption growth is redefined as:

$$\begin{aligned} \Delta c_t &= \lambda \Delta c_t^{ROT} + (1-\lambda)\Delta c_t^{PIH} \\ &= \lambda \Delta y_t + (1-\lambda)\Delta c_t^{PIH} \end{aligned}$$

- Set $\lambda = 0.25$ as in a previous literature

Macroeconomic Moments with ROT Consumer

		U.S.	U.K.	Germany	Japan
$std(\Delta c_t)$	Baseline	1.15	1.90	1.09	1.15
	With ROT	0.92	1.45	0.85	0.89
	Data	0.67	1.07	0.93	0.86
$corr(\Delta c_t, \Delta y_t)$	Baseline	0.16	-0.09	0.00	0.17
	With ROT	0.36	0.18	0.25	0.30
	Data	0.53	0.30	0.57	0.21
$corr(\Delta c_t, \pi_t)$	Baseline	−0.25	−0.52	−0.49	−0.21
	With ROT	−0.28	−0.55	−0.56	−0.19
	Data	−0.19	−0.22	−0.12	0.11
$corr(\Delta c_{t-1}, c_t)$	Baseline	0.19	−0.01	0.00	0.38
	With ROT	0.24	0.05	0.11	0.40
	Data	0.33	−0.04	−0.25	0.20

- With ROT consumers, $std(\Delta c_t)$ and $corr(\Delta c_t, \Delta y_t)$ fit better

The low-for-long Economy

- The economy with permanently low growth and low inflation is a real risk for advanced economies (c.f., secular stagnation)

The low-for-long Economy

- The economy with permanently low growth and low inflation is a real risk for advanced economies (c.f., secular stagnation)
- What does the model predict about the term structure in the low-for-long environment?
  - ⋆ It is important issue for financial stability because the maturity transformation is a key for bank profitability (GFSR, 2017)

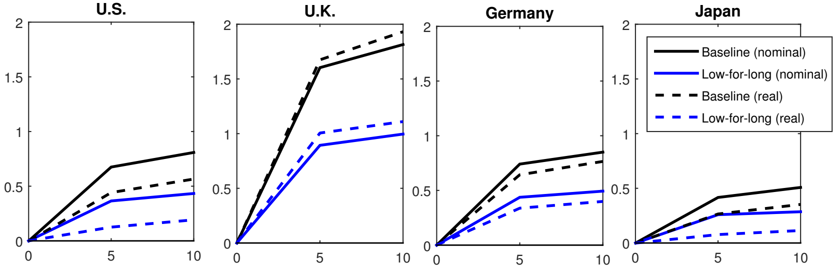
The low-for-long Economy

- The economy with permanently low growth and low inflation is a real risk for advanced economies (c.f., secular stagnation)
- What does the model predict about the term structure in the low-for-long environment?
  - ⋆ It is important issue for financial stability because the maturity transformation is a key for bank profitability (GFSR, 2017)
- Conduct counterfactual simulations for the low-for-long by:
  1. Setting $\log(g^*) = 0$ and $\log(\pi_t^*) = 0$
  2. Introduce the zero lower bound of nominal interest rate

$$R_t = \max\left\{1.0, R_{t-1}^{\phi_r}\left[\pi_t^* g^* \left(\frac{\Pi_t}{\pi_t^*}\right)^{\phi_\pi}\left(\frac{Y_t}{y_t^*}\right)^{\phi_y}\right]^{1-\phi_r}\right\}$$

Equilibrium Yield Curve under the Low-for-Long

Nominal and real yield curves



- The low-for-long would entail a flatter yield curve in addition to lower level of interest rates
- Real term premiums decline a lot while inflation premiums are almost unchanged. What is the intuition?

Equilibrium Yield Curve under the Low-for-Long

Logic behind the flattening under the Low-for-Long

- Responses of real interest rates to inflation would be changed under the Low-for-Long environment due to the ZLB
  - ⋆ **Above the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \uparrow$ by Taylor principle
  - ⋆ **Around the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \downarrow$ due to the ZLB

Equilibrium Yield Curve under the Low-for-Long

Logic behind the flattening under the Low-for-Long

- Responses of real interest rates to inflation would be changed under the Low-for-Long environment due to the ZLB
  - ⋆ **Above the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \uparrow$ by Taylor principle
  - ⋆ **Around the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \downarrow$ due to the ZLB
- Hence, the positive correlation between real SDF and real interest rates would be weaken
- **Intuition**: Inflation decreases income/consumption growth but increases real bond prices under the low-for-long
  $\Rightarrow$ A long-term bond is insurance rather than a risky asset!

Equilibrium Yield Curve under the Low-for-Long

Logic behind the flattening under the Low-for-Long

- Responses of real interest rates to inflation would be changed under the Low-for-Long environment due to the ZLB
  - ⋆ **Above the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \uparrow$ by Taylor principle
  - ⋆ **Around the ZLB**: $\pi_t \uparrow \Rightarrow R_t/\pi_{t+1} \downarrow$ due to the ZLB
- Hence, the positive correlation between real SDF and real interest rates would be weaken
- **Intuition**: Inflation decreases income/consumption growth but increases real bond prices under the low-for-long
  $\Rightarrow$ A long-term bond is insurance rather than a risky asset!
- Bank profitability could face difficulty in the low-for-long

Concluding Remarks

- The model can replicate an upward sloping yield curve even under the Phillips correlation
  - ⋆ Differences between stationary and non-stationary income is a key to understanding the relationship
- The low-for-long would entail a flatter yield curve due to a decline in real term premiums around the ZLB
  - ⋆ Bank profitability could face difficulty in the low-for-long

Concluding Remarks

- The model can replicate an upward sloping yield curve even under the Phillips correlation
  - ⋆ Differences between stationary and non-stationary income is a key to understanding the relationship
- The low-for-long would entail a flatter yield curve due to a decline in real term premiums around the ZLB
  - ⋆ Bank profitability could face difficulty in the low-for-long
- Future work:
  - ⋆ How do we rationalize the dynamic relation between income growth and inflation in a general equilibrium framework?
  - ⋆ What determines the differences in term structure of interest rates across countries?