Strategic Experimentation with Random Serial Dictatorship*

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First Draft: April 21, 2017; Current Draft: June 16, 2017

Abstract

We consider one-sided matching problems in which agents can endogenously acquire information about objects to evaluate their values more precisely. In such situations, whether agents acquire information or not crucially depends on their beliefs about their choice set, i.e., the set of objects each agent can obtain by changing his report. In this paper, we fix the assignment rule to the random serial dictatorship, and study the efficiency of each disclosure policy of choice sets. With a stylized environment where there is only one object that has ex ante unobservable private-value component, we demonstrate that the full disclosure policy, which always discloses each agent's choice set, is typically inefficient, because it fails to internalize the positive externality of information acquisition. Then, we illustrate that obscuring the information about the best available fixed-valued objects, we can induce more efficient information acquisition. We also show that in the worst case, the loss of the full disclosure policy relative to the optimal one is large.

*First Presentation: October 27, 2016. Shunya Noda has been supported by the Funai Overseas Scholarship and the E. K. Potter fellowship. I gratefully acknowledge that Fuhito Kojima gave me invaluable guidance. I am also grateful to Itai Ashlagi, Alex Bloedel, Gabriel Carroll, Juan Camilo Castillo, Weixin Chen, Laura Doval, Atsushi Iwasaki, Michihiro Kandori, Paul Milgrom, Michael Ostrovsky, Alvin Roth, Julia Salmi and all the seminar participants in Stanford for helpful comments. All remaining errors are my own.

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1 Introduction

The random serial dictatorship mechanism (RSD) is used in many real-world problems because it satisfies a lot of nice properties — RSD is strategy-proof, simple, fair, transparent and easy to understand and implement. Practically, there are various ways to implement RSD. Some clearing houses disclose neither information regarding priority orders nor choices made by earlier dictators (sometimes, such clearing houses draw the lotteries after receiving agents' preference reports) while the others implement RSD in a sequential form, i.e., fully disclose the *choice set* (the set of available objects) to the current dictator and ask him to pick a favorite one.

The difference in disclosure policies does not make difference in resultant matchings if preferences are exogenously endowed. Because RSD is strategy-proof, regardless of their beliefs about the choice sets, agents report truthful preference order. However, if agents endogenously acquire information about their preferences, an agent may want to acquire more information on his taste given a particular choice set, while he would take something without further investigation when his choice set is something else. Furthermore, disclosure policies crucially affect agents' beliefs about their choice sets, and therefore, different disclosure policies typically generate very different decision makings for information acquisition. In this study, we present the optimal choice set disclosure policy that induces efficient information acquisition.

The following are two illustrative examples for different disclosure policies implemented in real-world problems:

Example 1 (Sequential Disclosure)**.** If an allocation problem is not too large, RSD is typically implemented in a *sequential form*. For example, the cubicle (office space) assignment mechanism for Ph.D. students in Stanford Economics Department employs (a variant of) RSD in a sequential form. The seniority is used as the primary key of the order of priorities, and the random numbers are used to sort students within each year-in-program. The cubicles are allocated to second to sixth-year students. Third to sixth-year students, who are already assigned some cubicles, need to relinquish their cubicles to participate in the mechanism; thus, once a student decides to participate, he does not have the property right for his current cubicle. The detailed process is as follows:

1. The administrator draws the lottery to assign choice ordering. Then, she announces all the realized lottery numbers to the public.

2. [Four days later] Students who are currently assigned a cubicle notify the administrators of their intention to relinquish their cubicle and enter the draw. If incumbents (third to sixth-year students) want to change their cubicle, they need to relinquish the property right for their cubicles at this moment.

3. [Three days later] Third to sixth-year students (wanting to change their cubicles) are assembled, and sequentially choose the cubicles by pointing their favorite cubicles from the set of remaining ones. All the choices are immediately disclosed to the public.

4. [Two days later] Second-year students are assembled, and sequentially choose their cubicles.

The crucial facts are (i) students have enough time to check the surroundings of some cubicles after observing their endowed priority (including the realization of the random lottery number), and (ii) the sequential picking process is divided to two stages so that students of low priority (second-year students) can acquire information contingent on the choices of earlier dictators (third to sixth-year students). Hence, each student can flexibly change which information to acquire contingent on his belief about the set of cubicles available for him.

Example 2 (No Disclosure)**.** If an allocation problem is large, it is difficult to assemble participants to implement RSD in a sequential form, and therefore, RSD is run through some computer systems. In this case, participants are physically separated, and cannot directly observe the choices of the others. Hence, if the clearing house chooses to not disclose the information about the lottery numbers and/or earlier dictators' choices, each participant has much less information about his choice set than Example 1 when he submits his preference sheet to the clearing house.

For example, in the graduate student housing assignment of Stanford university, while the students are aware of their priority from their characteristics (e.g. incoming Ph.D.

students have higher priority than already enrolled master's students), they are not informed their lottery numbers beforehand (the realized lottery numbers are disclosed after the assignment is finalized). Therefore, students have to write their preference sheet without observing sufficient information for specifying their options available in their turn, and they cannot acquire information for residences contingent on the set of room-types available for them.

Example 3 (Lottery Number Disclosure)**.** Some clearing houses, including those of the housing assignment mechanism of Rutgers University,[1] Fairfield University,[2] Lehigh university,[3] and Washington University in St. Louis,[4] explicitly declare that they disclose the lottery numbers of the applicants at the beginning of the assignment process. While applicants cannot observe the actual available room-types, they can infer the available room-types in their turns from the lottery numbers, and can make contingent information acquisition.

While only a few clearing houses stipulate how the information about the remaining objects and the lottery numbers are disclosed, it is not difficult to find more anecdotal evidences with which the clearing houses employs various different disclosure policies. However, the performance of the different disclosure policies has not been well-examined in the literature.

Mathematically, choice set disclosure policies for RSD can be represented as signal structures. Contingent on the current dictator's priority and the set of remaining objects, the planner sends a signal to the current dictator. Observing the signal, the current dictator forms a posterior belief about his choice set, and decides how he acquires information. After observing additional information (if any), the current dictator's preference order, i.e., the order of expected payoffs from the objects conditional on the acquired information, is finalized. Since RSD is strategy-proof, he truthfully reports his preference list to the planner, and the planner mechanically assigns the best remaining object to him.

[1] http://ruoncampus.rutgers.edu/lottery/lottery-application-process/ ; last visited on 05/10/17.

[2] https://www.fairfield.edu/media/fairfielduniversitywebsite/documents/student/res_lotterybooklet12-13.pdf ; last visited on 05/10/17

[3] http://financeadmin.lehigh.edu/content/housing-selection-process-rising-3rd-4th-5th-year-undergraduate-students ; last visited on 05/10/17

[4] http://reslife.wustl.edu/applying-for-housing/housing-selection-subsite/rounds/ ; last visited on 05/10/17

Then, the set of remaining objects is updated, and it becomes the next dictator's turn. In this environment, the planner controls the disclosure policy to maximize the social welfare.

Surprisingly, even with independent and private values (i.e., there are no hidden states that commonly influence multiple agents' preferences) the full disclosure policy does not always maximize the ex ante expected payoffs — the no disclosure policy sometimes outperforms the full disclosure policy, and typically there exists an optimal disclosure policy which outperforms both of these two. This is because information acquisition has positive externalities: if an agent investigates an uncertain-valued object and takes it only when its private value for him is large, he can sometimes preserve an ex ante more valuable object for the next mover. Therefore, if an early dictator investigates more, a late dictator is better off. The full disclosure policy fails to internalize this externality, and therefore, inefficient. Accordingly, the welfare-maximizing planner wants to encourage more information acquisition than the level achieved by full disclosure.

How can we induce more information acquisition when monetary transfers are not allowed? Here, we can use the technique of Bayesian persuasion (introduced by Kamenica and Gentzkow (2011)) to improve the social welfare. As we have argued, the beliefs about choice sets are crucial for agents' decision making for information acquisition. More specifically, *whether or not an agent investigates the uncertain-valued object crucially depends on the value of the outside option*, which he can take when the investigation reveals that the uncertain-valued object is unattractive for him. By controlling this information, the planner can effectively convince agents to make more investigations.

The optimal disclosure policy sometimes largely outperforms the full disclosure policy. We show that the difference between the social welfare achieved by optimal disclosure and full disclosure is asymptotically equal to the maximal loss from discarding the uncertain-valued object. In this sense, the gain from using an optimal disclosure policy, instead of full disclosure, is unbounded.

The remainder of the paper is organized as follows. Section 2 surveys the literature. Section 3 describes the model. Section 4 characterizes (i) the equilibrium investigation strategy under full disclosure, (ii) the first-best investigation strategy, and (iii) the optimal disclosure policy. Section 5 compares the performance of these three, and derives the worst-case welfare difference. Section 6 discusses the generality of our qualitative results. Section 7 concludes.

2 Related Literature

A number of papers have investigated pros and cons of the (non-random) serial dictatorship mechanism (SD) and RSD. Abdulkadiroğlu and Sonmez (1998) prove that the matching generated by RSD coincides the core from random endowments. Svensson (1999) characterizes that SD is a unique mechanism that is group strategy-proof and neutral. While Bogomolnaia and Moulin (2001) show that RSD is not always ex ante ordinally efficient, Che and Kojima (2010) show that the inefficiency of RSD disappears in some large markets; thus, it is approximately ordinally efficient in such markets. Li (2016) shows that if RSD is implemented in a sequential form, it satisfies a stronger notion of strategy-proofness, which is named obvious strategy-proofness. Later, Pycia and Troyan (2016) characterized that RSD (implemented in the sequential-form) is a unique mechanism that is obviously strategy-proof, efficient, and symmetric. Overall, the performance of RSD is highly appreciated, and therefore, widely used in practice.

There are also a large amount of preceding papers on designing information structures. Kamenica and Gentzkow (2011) study the sender-receiver game and characterize the optimal signal structure, which effectively persuade the receiver to take the action demanded by the sender. This technique is called Bayesian persuasion. The choice set disclosure problem, which we study in this paper, is a special case of this situation as the sender (the planner) wants to control the receiver's activity (the agents' information acquisition) through disclosure of the information of the choice sets, in order to maximize the sender's expected payoff (the social welfare).

Gershkov and Szentes (2009), Kremer, Mansour, and Perry (2014), Che and Hörner (2015), Doval and Ely (2016), and Glazer, Kremer, and Perry (2016) study the design of the recommendation system that enhances social learning. Similar to our work, in their model, the planner control the agents' beliefs through disclosure of the information, which the planner can manage (e.g. the order of moves and the results of experimentation by earlier agents), to induce more information acquisition. It is worth noting that they study incentives in information acquisition for the common state, while we assume that the states are idiosyncratic. If the model contains a hidden state whose realization affects the payoffs of all the agents, information acquisition naturally benefits the other agents. On the other hand, we show that even with private values (i.e., there is no common states), if each object has a limited capacity, information acquisition also has positive externality effects, and a similar technique can improve the social welfare in the one-sided matching problems. In addition, to our knowledge, this is the first study that investigates the structure of strategic experimentation in one-sided matching problems.

Bade (2015) studies endogenous information acquisition in house allocation problems, and proves that (non-random) SD together with the full disclosure policy is a unique mechanism that satisfies ordinal efficiency and group strategy-proofness. Importantly, unlike our work, Bade (2015) evaluates mechanisms by ordinal efficiency without requiring mechanisms to be fair (hence, the order of dictatorship does not have to be randomized). Fairness makes a crucial difference between our analysis; since more information always benefits the current mover, disclosing full information to the current mover always leads to ordinal efficiency, even when it hurts the successive movers. Indeed, we verify that under some circumstances, disclosing full information to earlier dictators reduces the expected payoffs of successors. Furthermore, when the priority order must be randomized, agents' ex ante expected payoffs are sometimes improved by increasing late dictators' payoffs at the expense of early dictators' payoffs. Accordingly, the full disclosure policy is inefficient if (i) we need to randomize the priority order to achieve a fair assignment, and (ii) agents' payoffs are evaluated ex ante, i.e., before drawing the lotteries.

In terms of the search technology, this paper benefits from the literature of the box-opening problem, originated by Weitzman (1979). In particular, in our model, the problem that the agent faces under the full disclosure policy can be reduced to Doval (2016)'s one-box problem. These papers study an optimal investigation strategy of single agent who searches the value of objects and then makes a decision for which one to choose. We extend this box-opening problem to a multi-agent setting, and discover the positive externality of information acquisition.

Finally, a number of papers study the incentives in information acquisition when monetary transfers are allowed. See, for example, Bergemann and Välimäki (2002), Hatfield, Kojima, and Kominers (2015), and Matsushima and Noda (2016). The literature shows that as long as we assume private values and agents cannot acquire information over the preferences of the others, the VCG mechanism can induce the first-best efficient information acquisition.[5] In contrast, we study the way to improve the social welfare without monetary transfers, and find that we can achieve it by controlling agents' information about choice sets.

3 Model

Consider a one-sided matching problem of assigning indivisible objects to agents who can consume at most one object each. There are finitely many ex ante symmetric agents $i \in \mathcal{I} \triangleq \{1, 2, \cdots, K+1\}$, and finitely many different objects $k \in \mathcal{K} \triangleq \{0, 1, \cdots, K\}$. Since $|\mathcal{I}| = |\mathcal{K}| = K + 1$, we can focus on one-to-one perfect matchings. Each agent receives von Neumann–Morgenstern utility from his assignment — if agent i's object is $k \in \mathcal{K}$, he receives v^i_k. We call object 0 the *box* because each agent i does not know its precise value ex ante. The value of object 0, which is denoted by v^i_0, follows a cumulative distribution function F, i.i.d. across agents (hence, we are assuming *private values*, rather than common values). We assume $\int_{-\infty}^{\infty} |v^i_0| dF(v^i_0) < \infty$. The values of objects $1, 2, \cdots, K$, which we refer *alternatives*, are deterministic, and satisfy $v^i_k = v_k$ for $k \in \mathcal{K} \backslash \{0\}$. Without loss of generality, we label them $v_1 \geq v_2 \geq \cdots \geq v_K$.[6]

The assignment rule is fixed to RSD. Since agents are ex ante symmetric, without loss of generality, we can label agent i as the i-th dictator of the serial dictatorship. On the other hand, we control information of *choice sets*, i.e., the set of remaining objects, $H \subset \mathcal{K}$, in each agent's turn. More formally, at the beginning, the planner commits to a *disclosure policy* (S, π), where S is a finite realization space and $\pi : \mathcal{I} \times (2^{\mathcal{K}}) \to \Delta(S)$ is the signal distributions conditional on the agent's priority i and his choice set $H \subset \mathcal{K}$.

[5] Kleinberg, Waggoner, and Weyl (2016) show that if information acquisition is instantaneous, i.e., each agent can immediately acquire information at any moment, Dutch auctions may induce more information acquisition.

[6] Compared with the literature on the one-sided matching problems, our environment is very stylized. On the other hand, to study information design for strategic experimentation and social learning, we usually consider stylized environments to obtain clear-cut results. For example, agents only have binary choices of actions in the model of Kremer et al. (2014) and Che and Hörner (2015). We will discuss how we can generalize the insights from our stylized environment in Subsection 6.1.

Note that the planner can conceal each agent's priority; thus, each agent is not aware of his priority i, unless the planner decides to disclose it.

After observing a realization of the signal $s \in S$ but before making the preference report, each agent can decide whether or not to *investigate* the box. Therefore, whether an agent investigates the box can depends on his *posterior belief*, which is generated by the signal realization $s \in S$. If agent i chooses to investigate, he will pay the investigation cost, which is normalized to 1, and observes the realization of v_0^i. If agent i does not investigate, object 0 is worth $\mathbb{E}_F[v_0^i] \triangleq \mu$ to him on average. Since RSD is strategy-proof, after evaluating object 0, agents report their preferences truthfully, and take the most preferred object in H.

Controlling the disclosure policy (S, π), he planner wants to achieve *ex ante ordinal efficiency*, which is a basic efficiency criterion for random mechanisms, defined by Bogomolnaia and Moulin (2001). Since agents are assumed ex ante symmetric and the priority order is determined uniformly randomly, a disclosure policy is ex ante ordinally efficient if and only if it maximizes the average payoff of agents. Hence, the aim of the central planner is equivalent to maximization of the average payoff of agents.

Under every disclosure policy, all of the alternatives ($\mathcal{K} \setminus \{0\} = \{1, 2, \cdots, K\}$) are consumed, and the payoffs from them are the same across disclosure policies. Hence, the only variable part of the average expected payoff is the expected valuation of the box and the investigation cost for it. Therefore, an optimal disclosure policy maximizes the net payoff from the box, i.e.,

$$\begin{aligned}&\text{(the equilibrium expected valuation of the box given the disclosure policy)}\\ -&\text{(the equilibrium expected investigation cost given the disclosure policy)},\end{aligned}$$

which we define as the *social welfare*. We mathematically describe the form of it later.

4 Characterization

4.1 Full Disclosure

We say that (S, π) is the *full disclosure policy* if $S = 2^{\mathcal{K}}$ and $\pi(i, H)(H) = 1$ for every $i \in \mathcal{I}$ and $H \in 2^{\mathcal{K}}$. If the planner takes the full disclosure policy, each agent can obtain all the information about his choice set; thus, if agent i's choice set is H, he believes that his choice set is H with probability 1. If we implement RSD in a sequential form and endow each dictator enough time for investigating the box (if any) in each dictator's turn, the signal sent to the agent is equivalent to the full disclosure policy.

If $0 \notin H$, i.e., object 0 is not available, clearly the agent does not investigate it because he cannot choose it even if he finds it attractive. Similarly, if $H \setminus \{0\} = \emptyset$, i.e., only object 0 is available, clearly the agent does not investigate either because he cannot change his assignment from the box even if he finds the box unattractive. Hence, for these two cases, the box is never investigated.

If both object 0 and at least one alternative is available, each agent needs to decide whether or not investigate the box. In this case, his decision problem reduces to the single-agent one-box search problem, which is well-studied by Doval (2016). Define $k \triangleq \min_{l \in H \setminus \{0\}} l$. Since younger alternative is better, $v_k = \max_{l \in H \setminus \{0\}} v_l$. Since the agent always takes either (i) the box (object 0) or (ii) the best alternative (object k), the value of v_k provides us sufficient information for the agent's decision making.

If agent i investigates the box, he would choose object 0 if and only if $v_0^i > v_k$. On the other hand, if he chooses to not investigate, he would choose either object 0 or k without investigation, and gets an expected payoff of $\max\{\mu, v_k\}$. Hence, it is optimal for agent i to investigate the box if and only if

$$\int_{v_k}^{\infty} v_0^i dF(v_0^i) + F(v_k)v_k - 1 \geq \max\{\mu, v_k\}. \tag{1}$$

Defining

$$C(v_k) \triangleq \int_{v_k}^{\infty} v_0^i dF(v_0^i) + F(v_k)v_k - 1 - \max\{\mu, v_k\},$$

(1) is equivalently rewritten as $C(v_k) \geq 0$. Since

$$C'(v) = \begin{cases} F(v) \geq 0 & \text{if } v \leq \mu \\ -(1 - F(v)) \leq 0 & \text{otherwise,} \end{cases}$$

C is maximized at $v = \mu$. Furthermore, whenever F is not degenerate, $0 < F(\mu) < 1$, and therefore, μ is the unique maximizer of C. Hence, when $C(\mu) > 0$, (1) is satisfied if and only if $v_k \in [\beta, \rho]$, where β and ρ are two solutions of

$$C(v) = 0$$

with $\rho > \beta$.

If $C(\mu) < 0$, investigation and too costly; thus, no agents investigates object 0 in all the cases. To make the problem nontrivial, from now, we always assume that $C(\mu) > 0$.

Given $C(\mu) > 0$, clearly, $\rho > \mu > \beta$ holds. We call ρ the *reservation value* and β the *backup value*. Intuitively, if $v_k > \rho$, the gain from investigation is small because there is a sufficiently attractive available alternative. Hence, he wants to take object k without spending the investigation cost. On the contrary, if $v_k < \beta$, the alternative is too unattractive and it is not likely that v_0^i is much smaller than v_k. Hence, the agent wants to take the box without spending the investigation cost. Accordingly, agent i wants to investigate the box if and only if the value of the best alternative locates in the middle.

Proposition 1 (Doval (2016))**.** *For $H \in 2^{\mathcal{K}}$ such that $H \setminus \{0\} \neq \emptyset$, define $k(H) \triangleq \min_{l \in H \setminus \{0\}} l$. Under the full disclosure policy, the following investigation strategy $\sigma : 2^{\mathcal{K}} \setminus \{\emptyset\} \to \{0, 1\}$ maximizes the agent's expected payoff:*

$$\sigma(H) \triangleq \begin{cases} 1 & \textit{if } 0 \in H, H \setminus \{0\} \neq \emptyset, \textit{ and } v_{k(H)} \in [\beta, \rho] \\ 0 & \textit{otherwise.} \end{cases}$$

What is the social welfare achieved by the full disclosure policy? If $\{k \in \mathcal{K} : \beta \leq v_k \leq$

$\rho\} = \emptyset$, it is equal to μ because no agents investigate the box. Otherwise, let

$$r \triangleq \arg \min_{k \in \mathcal{K} \setminus \{0\}} \{k \in \mathcal{K} \setminus \{0\} : \beta \leq v_k \leq \rho\},$$
$$b \triangleq \arg \max_{k \in \mathcal{K} \setminus \{0\}} \{k \in \mathcal{K} \setminus \{0\} : \beta \leq v_k \leq \rho\}$$

Then, $v_{r-1} > \rho \geq v_r \geq \cdots \geq v_b \geq \beta > v_{b+1}$. Hence, each agent investigates the box if and only if $k \in \{r, r+1, \cdots, b\}$. Accordingly, the social welfare from the full disclosure policy is

$$\begin{aligned} W^{Full} =& \left[\int_{v_r}^{\infty} v_0^i dF(v_0^i) - 1\right] + F(v_r)\left[\int_{v_{r+1}}^{\infty} v_0^i dF(v_0^i) - 1\right] \\ &+ \cdots + F(v_r) \cdot \cdots \cdot F(v_{b-1})\left[\int_{v_b}^{\infty} v_0^i dF(v_0^i) - 1\right] \\ &+ F(v_r) \cdot \cdots \cdot F(v_{b-1})F(v_b) \cdot \mu \\ =& \sum_{k=r}^{b}\left[\prod_{l=r}^{k-1} F(v_l)\right]\left[\int_{v_k}^{\infty} v_0^i dF(v_0^i) - 1\right] + \left[\prod_{k=r}^{b} F(v_k)\right]\mu \\ =& \mu + \sum_{k=r}^{b}\left[\prod_{l=r}^{k-1} F(v_l)\right]\left[\int_{v_k}^{\infty} v_0^i dF(v_0^i) - 1 - \mu + F(v_k)\mu\right]. \end{aligned}$$

Define

$$G(v_k) \triangleq \int_{v_k}^{\infty} v_0^i dF(v_0^i) - 1 - \mu + F(v_k)\mu.$$

Then,

$$W^{Full} = \mu + \sum_{k=r}^{b}\left[\prod_{l=r}^{k-1} F(v_l)\right] G(v_k).$$

Hence, the increment of the social welfare from voluntary investigation is a weighted sum of $G(v_k)$. Next proposition says that this is positive as long as $k \in \{r, r+1, \cdots, b\}$, i.e., agents voluntarily investigate the box under full disclosure.

Proposition 2. For all $v_k \in \mathbb{R}$,

$$G(v_k) \geq C(v_k) \tag{2}$$

The equality holds if and only if $v_k = \mu$.

Proofs are relegated to the appendix.

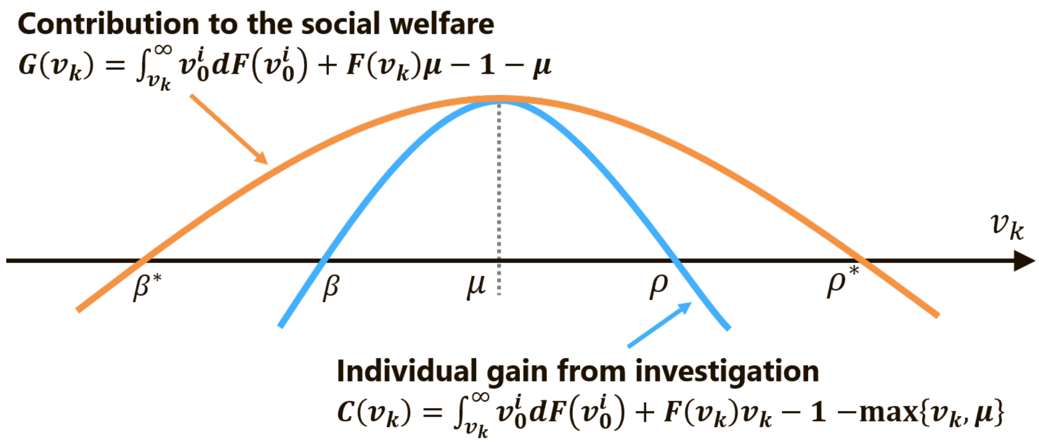

Figure 1: Proposition 2 shows that whenever an agent wants to investigate under the full disclosure policy (i.e., the blue line is above zero), its contribution to the social welfare is also positive (i.e., the orange line is above zero).

Proposition 2 indicates that if an agent is "nearly indifferent" for whether or not acquire information, there is a significant welfare gain from investigation. When $v_k \approx \rho$, the alternative is ex ante more valuable than the box. Then, if an agent investigates the box and finds that it has a good taste to him, the best alternative, which is commonly attractive for all the agents, is preserved for the next agent. Similarly, when $v_k \approx \beta$, the box is ex ante more valuable than the alternative. Then, if an agent investigates the box and finds that it has a bad taste to him, then the box, which is better than the alternative on average, is preserved. In both cases, investigation increases the probability that an ex ante more valuable object is preserved for the next agent, and later agents will receive a significant welfare gain from the current agent's investigation. Here, investigation has positive externality.

Proposition 2 also indicates the following two facts. First, voluntary investigation induced by the full disclosure policy is never socially wasteful, but it always increases the net value of object 0. Second, typically, the full disclosure policy provides a too weak incentive for investigation. If we can induce more investigation, we can sometimes improve the social welfare.

4.2 First-Best Investigation Strategy

Next, assuming that the planner can enforce each agent to investigate or not, we will characterize the *first-best investigation strategy.* Formally, a first-best investigation strategy maximizes the following social welfare function:

$$\begin{aligned} W(p) &\triangleq \sum_{l=1}^{K}\left[\prod_{m=1}^{l-1}(1-p_m(1-F(v_m)))\right]p_l\left[\int_{v_l}^{\infty} v_0^i dF(v_0^i)-1\right] \\ &\quad + \prod_{m=1}^{K}(1-p_m(1-F(v_m)))\,\mu \\ &= \mu + \sum_{l=1}^{K}\left[\prod_{m=1}^{l-1}(1-p_m(1-F(v_m)))\right]p_l G(v_l) \end{aligned}$$

where $p=(p_1,\cdots,p_K)$, subject to $p_l \in [0,1]$ for $l=1,2,\cdots,K$. Here, p_k denotes the probability that the planner enforces the agent to investigate the box conditional on the event that (i) object 0 is available, i.e., $0 \in H$, and (ii) there exists at least one available alternative, and the best alternative is object k, i.e., $k=\min_{l\in H\setminus\{0\}} l$. Whenever either (i) or (ii) is not satisfied, investigation is simply wasteful, and therefore, the planner never wants to make agents to investigate the box. The maximizer of W, p^{FB} is called the *first-best investigation strategy.* Indeed, if the planner can observe whether each agent investigates or not (thus, she can enforce agents to take a socially optimal investigation strategy) while she cannot observe the realized value of the box, the first-best investigation strategy is the one the welfare-maximizing planner would take.

p^{FB} can be constructed by dynamic programming. It is convenient to define the *continuation social welfare*:

$$W_k(p_{k:K}) \triangleq \sum_{l=k}^{K}\left[\prod_{m=k}^{l-1}(1-p_m(1-F(v_m)))\right]p_l G(v_l)$$

where $p_{k:l}$ denotes $(p_k,p_{k+1},\cdots,p_l)$. Then, $W_0(p)+\mu=W(p)$ and the first-best investigation strategy p^{FB} maximizes W_k for every $k\in\mathcal{K}\setminus\{0\}$. We define $W_k^{FB}\triangleq W_k(p_{k:K}^{FB})$.

We construct p_k^{FB} and W_k^{FB} backward. $W_{K+1}^{FB}=0$ because object 0 is no longer investigated if no alternatives are available. Given W_{k+1}^{FB}, p_k^{FB} solves

$$W_k^{FB} = \max_{p_k \in [0,1]} \left\{ p_k G(v_k) + (1 - p_k(1 - F(v_k)) W_{k+1}^{FB} \right\}.$$

Taking the derivative, we obtain that $p_k^{FB} = 1$ is optimal if

$$\frac{1}{1 - F(v_k)} G(v_k) \geq W_{k+1}^{FB}. \tag{3}$$

In this manner, we can construct the first-best investigation strategy p^{FB}.

Although we need to use dynamic programming to obtain p^{FB}, p^{FB} can partially be characterized by some threshold values. Define the *social reservation value* ρ^* and *social backup value* β^* by two solutions of

$$G(v) = 0 \tag{4}$$

with $\rho^* > \beta^*$. Since we assume $C(\mu) > 0$, and $G(v) \geq C(v)$ holds by Proposition 2, (4) actually has two solutions. Note that

$$G'(v) = (\mu - v) \cdot f(v),$$

which implies that G is increasing for $v < \mu$, and decreasing for $v > \mu$. Therefore, $G(v_k) \geq 0$ if and only if $v_k \in [\beta^*, \rho^*]$. The following inequalities are also immediate from Proposition 2:

$$\rho^* > \rho > \mu > \beta > \beta^*.$$

This relationship is also illustrated in Figure 1.

We can obtain the partial threshold-value characterization of the first-best investigation strategy with β^*, ρ, and ρ^*. Specifically, p^{FB} satisfies the following properties.

Proposition 3.

1. *$v_k < \beta^*$ implies $p_k^{FB} = 0$.*

2. *$v_k > \rho^*$ implies $p_k^{FB} = 0$.*

3. $v_k \in (\beta^*, \rho)$ *implies* $p_k^{FB} = 1$.

Importantly, the first-best strategy investigates more often than the equilibrium strategy under the full disclosure policy. The first-best strategy always investigates when the value of the best alternative is in (β, ρ), i.e., when an agent is willing to investigate with full information. In addition, the first-best strategy (i) always investigates the box when the value of the best alternative is in (β^*, β), and (ii) sometimes investigates the box when the value of the best alternative is in (ρ, ρ^*).

Proposition 3 does not tell us the value of p_k when $v_k \in (\rho, \rho^*)$. Indeed, whether the box is investigated with object k by the first-best strategy depends not only on v_k but also the sequence of $(v_{k+1}, v_{k+2}, \cdots, v_K)$. This is why we need to use dynamic programming to obtain the first-best strategy. The following observations are crucial for understanding this feature.

1. The planner cannot prevent the agent from taking a good alternative even when she can enforce the agent to investigate the box. Therefore, if $v_k < \rho^*$ is large, the benefit of investigation is small, because the agent would not take the box when $v_0^i \in [\rho, v_k]$ while the planner wants the agent to take it.

2. Earlier investigation (with large v_k) crowds out the probability of the successive investigation. If an investigation with the alternative k reveals that $v_0^i > v_k$, investigations with alternatives $k+1, k+2, \cdots$ are not made, while they might be more welfare-improving. Hence, the first-best strategy recommends investigation with $\rho < v_k < \rho^*$ if and only if the continuation social welfare from $k+1$ (which is determined by $(v_{k+1}, v_{k+2}, \cdots, v_K)$) is relatively small.

Examples 4 and 5 show that if $v_k \in (\rho, \rho^*)$, actually we cannot identify the value of p_k, without observing $(v_{k+1}, \cdots, v_K)$.

Example 4. Assume that $\rho < v_1 < \rho^*$ and $K = 1$.[7] In this case, if $p_1 = 0$, the social welfare is μ. On the other hand, if $p_1 = 1$, the social welfare is

$$\int_{v_1}^{\infty} v_0^i dF(v_0^i) + F(v_1)\mu - 1 = G(v_1) + \mu.$$

[7] Alternatively, we can assume K is large but $v_k < \beta^*$ for $k = 2, 3, \cdots, K$ to obtain the same result.

Hence, the planner demands investigation if and only if $G(v_1) > 0$, or equivalently, $v_1 \in (\beta^*, \rho^*)$, which is actually satisfied by assumption. In this case, $p_1^{FB} = 1$.

Before showing the next example, we introduce a fundamental property of the reservation value, ρ.

Fact 1. *For $\{v \in \mathbb{R} : F(v) < 1\}$, $\frac{1}{1-F(v)}G(v)$ increasing for $v < \rho$, and decreasing for $v > \rho$. Hence, it is uniquely maximized at $v = \rho$. Furthermore,*

$$\frac{1}{1-F(\rho)}G(\rho) = \rho - \mu.$$

Fact 1 implies that if ω is drawn from F and a decision maker can choose whether to (i) make the current sample, ω, as his payoff, or (ii) discard the current sample and draw another sample from F, the decision maker's expected payoff is maximized when he sets ρ as the threshold (i.e., if $\omega > \rho$, he takes it, and otherwise, he picks another sample). This is why ρ is named the "reservation" value. This also implies that in our setting, the social welfare gets close to the supremum if there are approximately infinitely many alternatives with value $v_k = \rho$. Note that the resultant social welfare in this case is

$$\mu + \frac{1}{1-F(\rho)}G(\rho) = \rho.$$

Example 5. Again, assume that $\rho < v_1 < \rho^*$, but now assume that for a large K, $v_2 = v_3 = \cdots = v_K = \rho - \epsilon$ where $\epsilon > 0$ is a small number. $p_1^{FB} = 1$ is optimal only if

$$\frac{1}{1-F(v_1)}G(v_1) \geq W_2^{FB}. \tag{5}$$

On the other hand, since $v_2 = v_3 = \cdots = v_K = \rho - \epsilon$,

$$W_2^{FB} = \sum_{k=2}^{K} [F(\rho - \epsilon)]^{k-2} G(\rho - \epsilon).$$

Hence, taking $\epsilon > 0$ small and $K \in \mathbb{Z}_{++}$ large, we can make W_2^{FB} arbitrarily close to

$$\frac{1}{1-F(\rho)}G(\rho) = \rho - \mu.$$

Therefore, for all $v_1 > \rho$, there exist $\epsilon > 0$ and $K \in \mathbb{Z}_{++}$ such that (5) is not satisfied. In such a case, $p_1^{FB} = 0$.

4.3 Optimal Disclosure

In this subsection, we maximize the optimal disclosure policy, which induces the investigation strategy that maximizes the social welfare. By Proposition 1 of Kamenica and Gentzkow (2011), we can restrict our attention to straightforward signals, whose realization space is equal to the action space, i.e., $S = \{0, 1\}$. In other words, without loss of generality, we can focus on disclosure policies that recommend "investigation" ($s = 1$) or "not" ($s = 0$). The optimal disclosure policy p^{Opt} solves

$$
\begin{aligned}
&\max_{p} \sum_{l=1}^{K} \left[\prod_{m=1}^{l-1} (1 - p_m(1 - F(v_m))) \right] p_l G(v_l) \\
&\text{s.t. } p_k \in [0,1] \quad \text{for } k = 1, 2, \cdots, K \\
&\qquad \sum_{l=1}^{K} \left[\prod_{m=1}^{l-1} (1 - p_m(1 - F(v_m))) \right] p_l C(v_l) \geq 0. \qquad (6)
\end{aligned}
$$

Here, (6) is the incentive compatibility (IC) constraint for each agent to obey the planner's recommendation of "investigate," i.e., the condition that agents actually want to make investigation when he observes $s = 1$. Given a disclosure policy p and the agent is recommended to "investigate" (receives $s = 1$), by Bayes' rule, his posterior belief about the best available alternative is object l is

$$
\frac{\prod_{m=1}^{l-1} (1 - p_m(1 - F(v_m)))\, p_l}{\sum_{h=1}^{K} \left[\prod_{m=1}^{h-1} (1 - p_m(1 - F(v_m))) \right] p_h}. \qquad (7)
$$

Since the denominator of (7) is positive and common across all the terms in (6), it can be crossed out. When the best alternative is l, his net gain from investigation is $C(v_l)$. Accordingly, if LHS of (6), which represents the expected net payoff from the investigation conditional on the planner send him $s = 1$, is non-negative, the agent would obey the recommendation by the planner. Note that by Proposition 4 of Kamenica and Gentzkow (2011), we can ignore the obedience constraint for $s = 0$.

Clearly, for $k = r, r+1, \cdots, b$, we should take $p_k^{Opt} = 1$ because the incentives of the planner and the agents coincide. Letting $p_k^{Opt} = 1$ for these k, not only the value of the objective function is improved, but also the IC constraint is relaxed. However, for $v_k \in (\beta^*, \beta)$ and $v_k \in (\rho, \rho^*)$, increasing p_k tighten the constraint. Hence, if the IC constraint is binding, i.e., the first-best investigation strategy p^{FB} does not satisfy the IC constraint, we need to select when to send the recommendation for "investigation" for $v_k \in (\beta^*, \beta)$ and $v_k \in (\rho, \rho^*)$.

First, we show that the optimal disclosure policy takes the form of "an interval."

Lemma 1.

1. *If $v_m > v_n > \rho$ and $p_m^{Opt} > 0$, then $p_n^{Opt} = 1$.*

2. *If $\beta > v_m > v_n$ and $p_n^{Opt} > 0$, then $p_m^{Opt} = 1$.*

Intuitively, as we can see in Figure 1, optimal disclosure has an interval structure because if v_k is closer to $[\beta, \rho]$, the agent feels less painful to make investigation with this alternative while the social gain from investigation is larger. Accordingly, investigation with v_k close to $[\beta, \rho]$ is both cheap and valuable; thus, always prioritized. Accordingly, the optimal disclosure policy has an interval structure.

Next, we need to choose whether to increase p_k for the upper side or the lower side. More formally, suppose that we currently have $p_k = 1$ for $k = u, u+1, \cdots, d$, and $p_k = 0$ for the others. Furthermore, the IC constraint is not binding, and $\rho^* > v_{u-1}$ and $v_{d+1} > \beta^*$, i.e., we may be able to improve the social welfare by recommending more investigation. Given Lemma 1, we have two choices — increasing p_{u-1} or increasing p_{d+1}. We cannot always recommend to investigate with both alternatives because the IC constraint may get binding. Which one should we choose first?

Proposition 4 provides a criterion for this choice.

Proposition 4. *Define*

$$r^* \triangleq \min\left\{k \in \mathcal{K} \setminus \{0\} : p_k^{FB} = 1\right\}$$
$$b^* \triangleq \max\{k \in \mathcal{K} \setminus \{0\} : p_k^{FB} = 1\}.$$

For $r^* \leq u < r$ and $b > d \geq b^*$, define

$$U_{u:d} \triangleq -\frac{G(v_u) + (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)G(v_k)}{C(v_u) + (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k)},$$

$$D_d \triangleq -\frac{G(v_d)}{C(v_d)}.$$

Suppose that $p^{Opt}_{u+1} = \cdots = p^{Opt}_{d-1} = 1$. Then, p^{Opt} satisfies the following five conditions:

1. *$p^{Opt}_u > 0$ and $p^{Opt}_d = 0$ imply $U_{u:d} \geq D_d$.*

2. *$p^{Opt}_u = 0$ and $p^{Opt}_d > 0$ imply $U_{u:d} \leq D_d$.*

3. *$p^{Opt}_u < 1$ and $p^{Opt}_d = 1$ imply $U_{u:d} \leq D_d$.*

4. *$p^{Opt}_u = 1$ and $p^{Opt}_d < 1$ imply $U_{u:d} \geq D_d$.*

5. *$p^{Opt}_u \in (0, 1)$, $p^{Opt}_d \in (0, 1)$ imply $U_{u:d} = D_d$.*

While $U_{u:d} - D_d$ is not equal to the total derivative of the objective function when we increase p_u and decrease p_d, its sign always coincides. Accordingly, checking the sign of $U_{u:d} - D_d$, we can obtain a local optimality condition.

Using this as the criterion, we can run a greedy algorithm for obtaining the optimal disclosure policy. The disclosure policy generated by the following greedy algorithm always satisfies the local optimality condition. Furthermore, since $U_{u:d}$ and D_d are monotone (see Lemmas 2 and 3 in the appendix for the detail), we can show that there exists an essentially unique disclosure policy that satisfies the necessary condition of Lemma 4 together with the IC constraint. Therefore, the disclosure policy generated by the greedy algorithm is globally optimal.

Proposition 5. *The optimal disclosure policy p^{Opt} can be constructed by the following greedy algorithm.*

1. *Initialize $u = r$, $d = b$ and set $p_r = p_{r+1} = \cdots = p_b = 1$, and $p_k = 0$ for the others.*

2. *If $U_{u-1:d+1} \geq D_{d+1}$, increase p_{u-1}. Otherwise, increase p_{d+1}. If the IC constraint binds, stop. If $p_{u-1} = 1$ or $p_{d+1} = 1$ is achieved before the IC constraint binds and $p = p^{FB}$ holds, stop. Otherwise, iterate Step 2.*

3. Return p as p^{Opt}.

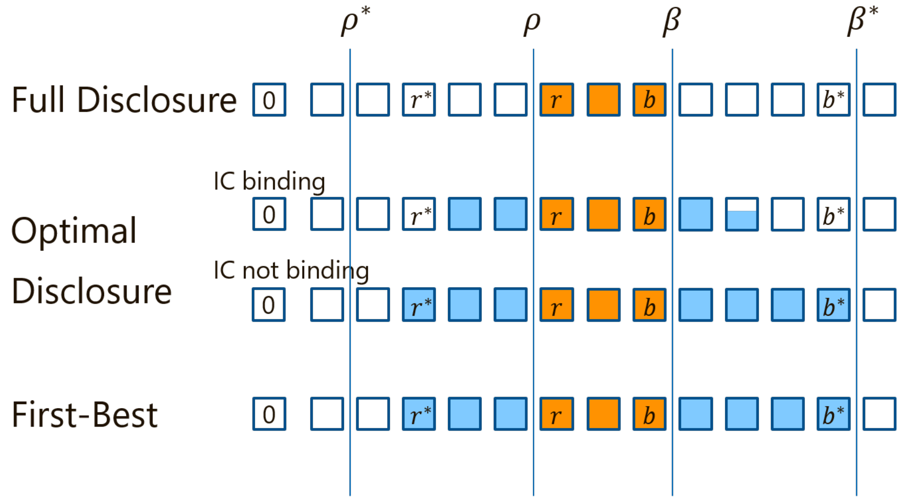

Figure 2: Comparison of full disclosure, optimal disclosure, and the first-best strategy.

5 Welfare Evaluation

5.1 Fixed F

We will evaluate the maximum difference in the performance between (i) the full disclosure policy and the first-best investigation strategy, and (ii) the full disclosure policy and the optimal disclosure policy. In this subsection, we fix F (the distribution of v_0^i), and only change the sequence of the values of the alternatives $(v_1, v_2, \cdots, v_K)$. Because F is fixed, μ, β, and ρ are also fixed.

First, we compare the performance of the full disclosure policy and the first-best strategy. Since Proposition 2 indicates that agents' voluntary investigation under the full disclosure policy always improves the social welfare, the worst case of the full disclosure is "no alternatives induce investigation." Indeed, no investigation takes place when for all $k \in \mathcal{K} \setminus \{0\}$, $v_k \notin [\beta, \rho]$, and then, the social welfare achieved under full disclosure is μ.

Conversely, as we have shown in Fact 1, if there are a large number of alternatives whose value is close to ρ, the social welfare from the first-best strategy would get close to the supremum of the achievable social welfare, ρ.[8] Furthermore, if v_k's are slightly above

[8]Recall that the supremum is ρ itself, as shown in Fact 1.

ρ, i.e., $v_k = \rho + \epsilon$ with $\epsilon > 0$, the existence of such alternatives would not change the investigation under full disclosure. Accordingly, the difference in the welfare between full disclosure and the first-best can be arbitrary close to $\rho - \mu$.

Next, we compare the performance of the full disclosure and the optimal disclosure policy. In order to provide an incentive for investigating the box, we need to add an object that satisfies $v_k \in (\beta, \rho)$ (otherwise, investigation cannot be optimal). To minimize the social welfare from full disclosure, we must choose v_k that minimizes

$$\begin{aligned} &\int_{v_k}^{\infty} v_0^i dF(v_0^i) + F(v_k)\mu - 1 \\ =&\mu + G(v_k) \end{aligned}$$

Since G has an inverse U-shape, it is minimized either when v_k is close to β or close to ρ.

The following proposition summarizes the above observations.

Proposition 6. *Given F,*

1. *For all $(v_1, v_2, \cdots, v_K)$,*

$$W^{FB} - W^{Full} < \rho - \mu$$

   This bound is tight.

2. *There exists $(v_1, v_2, \cdots, v_K)$ such that*

$$W^{Opt} - W^{Full} < \rho - \mu - \min\{G(\rho), G(\beta)\}$$

   This bound is tight.

5.2 The Worst Case of Worst Cases

Next, we control F to evaluate the performance of disclosure policies further. We choose F satisfying $support(F) = [\underline{v}, \bar{v}]$ with some $\underline{v}, \bar{v}$, and derive the worst case difference as a function of $\bar{v} - \underline{v}$.

Again, first, we compare the performance of the full disclosure and the first-best strategy. As Doval (2016) shows, if F is a mean-preserving spread of F', $\rho(F) \geq \rho(F')$ holds. Since $\rho - \mu$ is increasing in ρ, to maximize the difference between full disclosure and the first-best strategy, we can focus on the most dispersed distributions, i.e., the case of binary distributions. Formally, we assume that F is parametrized by $q \in (0,1)$ such that $v_0^i = \bar{v}$ with probability q and $v_0^i = \underline{v}$ with probability $1-q$. Clearly, $F(\beta) = F(\rho) = 1-q$.

In the case of binary values, ρ and β are determined by

$$\begin{aligned}
&\int_\rho^\infty v_0^i dF(v_0^i) + F(\rho)\rho - 1 - \rho = 0 \\
\Leftrightarrow \quad & q\bar{v} + (1-q)\rho - 1 - \rho = 0, \\
&\int_\beta^\infty v_0^i dF(v_0^i) + F(\beta)\beta - 1 - \mu = 0 \\
\Leftrightarrow \quad & q\bar{v} - (1-q)\beta - 1 - (q\bar{v} + (1-q)\underline{v}) = 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\rho &= \bar{v} - \frac{1}{q}, \\
\beta &= \underline{v} + \frac{1}{1-q}.
\end{aligned}$$

Given this,

$$\rho - \mu = (1-q)(\bar{v} - \underline{v}) - \frac{1}{q}.$$

This is maximized by $q = 1/\sqrt{\bar{v} - \underline{v}}$. Hence, the worst case welfare difference between the full disclosure and the first-best is

$$(\bar{v} - \underline{v}) - 2\sqrt{\bar{v} - \underline{v}}. \tag{8}$$

Next, we will evaluate the worst-case difference between the full disclosure policy and the optimal disclosure policy. Unlike the comparison with the first-best, it is difficult to obtain a tight upper bound of the welfare difference, because the binary distribution may not maximize the difference. However, (i) (8) is also a loose upper bound of $W^{Opt} - W^{Full}$ because

$$W^{Opt} - W^{Full} \leq W^{FB} - W^{Full} < (\bar{v} - \underline{v}) - 2\sqrt{\bar{v} - \underline{v}},$$

and (ii) by considering the worst-case in the case of binary distributions, we can evaluate the tight upper bound from below.

It follows from

$$\begin{aligned} G(\rho) = G(\beta) &= q(1-q)(\bar{v} - \underline{v}) - 1 \\ &= q \cdot \left\{ (1-q)(\bar{v} - \underline{v}) - \frac{1}{q} \right\} \\ &= q \cdot (\rho - \mu) \end{aligned}$$

that

$$\begin{aligned} &\rho - \mu - \min\{G(\rho), G(\beta)\} \\ =&(1-q) \cdot (\rho - \mu). \end{aligned} \tag{9}$$

The maximizer of (9) does not have a tractable form. Here, we again use $q = 1/\sqrt{\bar{v} - \underline{v}}$, which is a maximizer of $\rho - \mu$, to evaluate the tight upper bound from below. While $q = 1/\sqrt{\bar{v} - \underline{v}}$ does not maximize (9), it is approximately optimal when $\bar{v} - \underline{v}$ is large because $q = 1/\sqrt{\bar{v} - \underline{v}}$ is small in such a case. Substituting $q = 1/\sqrt{\bar{v} - \underline{v}}$, we have

$$(\bar{v} - \underline{v}) - 3\sqrt{\bar{v} - \underline{v}} + 2.$$

Now, we verified that there exist F (which is a binary distribution) and $(v_1, v_2, \cdots, v_K)$ such that $W^{Opt} - W^{Full}$ is close to $(\bar{v} - \underline{v}) - 3\sqrt{\bar{v} - \underline{v}} + 2$.

Proposition 7. *There exists $\underline{v}, \bar{v}$ and F such that support$(F) = [\underline{v}, \bar{v}]$ that satisfy the following:*

1. *For all $(v_1, v_2, \cdots, v_K)$,*

$$W^{FB} - W^{Full} < (\bar{v} - \underline{v}) - 2\sqrt{\bar{v} - \underline{v}}.$$

This bound is tight.

2. *For all* $(v_1, v_2, \cdots, v_K)$,

$$W^{Opt} - W^{Full} < (\bar{v} - \underline{v}) - 2\sqrt{\bar{v} - \underline{v}}.$$

Furthermore, for all $\epsilon > 0$*, there exists* $(v_1, v_2, \cdots, v_K)$ *such that*

$$W^{Opt} - W^{Full} > (\bar{v} - \underline{v}) - 3\sqrt{\bar{v} - \underline{v}} + 2 - \epsilon.$$

Proposition 7 indicates that the loss of the full disclosure policy might be large. Since

$$\begin{aligned} \frac{(\bar{v} - \underline{v}) - 2\sqrt{\bar{v} - \underline{v}}}{\bar{v} - \underline{v}} &\to 1 \quad \text{as } (\bar{v} - \underline{v}) \to \infty, \\ \frac{(\bar{v} - \underline{v}) - 3\sqrt{\bar{v} - \underline{v}} + 2}{\bar{v} - \underline{v}} &\to 1 \quad \text{as } (\bar{v} - \underline{v}) \to \infty, \end{aligned}$$

when $\bar{v} - \underline{v}$ is large (i.e., relative to the investigation cost, which is normalized to 1), the loss of full disclosure is asymptotically equal to $\bar{v} - \underline{v}$. This is the maximum possible welfare loss. Even when we assign object 0 to an agent only when it has the worst realization, i.e., $v_0^i = \underline{v}$, the loss cannot be larger than $\bar{v} - \underline{v}$. In this sense, the loss of full disclosure is asymptotically "unbounded."

6 Discussion

6.1 Generalization

The model of this paper is stylized in the sense that we have made several restrictive assumptions:

1. There exists only one type of uncertain-valued object.

2. The unique uncertain-valued object only has capacity one.

3. The values of alternatives are common across agents.

It is difficult to relax the first assumption. First, if there are multiple uncertain-valued objects, we can consider a very large variety of search technologies. Thus, the model specification itself is already ambiguous. Second, even if we specify the search technology in a certain manner, deriving each agent's optimal strategy given disclosure policies is difficult. Doval (2016) studies a single-agent search problem in which the agent can sequentially acquire information (i.e., can acquire information about one object after observing the result of prior investigations), and finds that even with two uncertain-valued objects, characterization of the optimal strategy is difficult without a strong structure. If we obscure the information about choice sets, each agent's problem would become more complicated. Needless to say, characterizing an optimal disclosure policy is even more difficult. However, we conjecture that there must be a similar externality as what we have shown in this paper, and a similar technique can improve the welfare in more general environments.

The second and third assumptions are not difficult to relax. Regarding the second assumption, note that each agent only cares about whether or not the residual capacity is non-zero. Therefore, even if object 0 has multiple capacities, the only difference in each agent's problem is the formulation of the posterior beliefs after observing signals, which can be computed straightforwardly. Hence, the second assumption is just for simplicity.

The third assumption can also be relaxed significantly. In our model, due to the common preference over alternatives, investigation increases the probability that the better object is preserved for the next dictator, and this is the role of the assumption of common preferences over alternatives. Even if we do not assume the perfect correlation in preference over alternatives, as long as the current dictator's ex ante preferred object is *more likely* to be preferred by the later dictators, investigation has a similar positive externality. Hence, as long as agents' preferences over alternatives are positively correlated, relaxation of the third assumption would not change qualitative results.

6.2 Simultaneous Disclosure Policies and Large Market Approximation

The full disclosure policy and the optimal disclosure policy need a long running time because we need to allow sequential information acquisition. To implement these disclosure policies, the planner needs to endow enough time to each agent for acquiring information after observing signals. Hence, their running time increases linearly in the number of agents, K. Linear running time is not bad in the context of computational complexity. However, with large K, sequential disclosure policies may be intractable in this situation, because human beings cannot complete information acquisition within a millisecond.

Practically, some clearing houses want to reduce the running time, and in such cases, they may want to use *simultaneous disclosure policies*, which does not allow agents to move sequentially. When sequential moves are not allowed, the only thing the planner can disclose is each agent's realized lottery number. In that case, the maximum achievable social welfare is inevitably decreased, because we cannot eliminate wasteful information acquisition (i.e., investigate the box while either the box is not available or it is the only available object).[9]

However, if the market is large, i.e., there are many copies of objects (including the box) and agents, by the law of large numbers, one can accurately predict choice sets of a large fraction of agents from the lottery numbers. Given that choice sets are predictable, the planner can use an approximation of sequential disclosure policies by using predicted choice sets as substitutes of the actual choice sets. Hence, even when the planner must use a simultaneous disclosure policy, she can achieve an approximate optimum in a large market.

6.3 Performance of No Disclosure Policy

In real-world house allocation problems, no disclosure policies, which do not disclose any information about choice sets, are also commonly used. Mathematically, (S, π) is a no disclosure policy if $|S| = 1$. No disclosure policy is typically suboptimal. Since it obscures

[9]Partially sequential disclosure policies are also practically used — for example, in Example 1, agents are divided to two groups (third to sixth-year students and second-year students), and the second group can make information acquisition contingent on the choice of the first group.

not only the best available alternatives, but also the availability of the box, it always induces wasteful investigation with some probability. On the other hand, no disclosure policy sometimes outperforms the full disclosure policy, because just like the optimal disclosure policy, it sometimes induces some more valuable information acquisition than the full disclosure policy.

It is trivial that there exists an environment where full disclosure strictly outperforms no disclosure. We will show the converse, i.e., there exists an environment where no disclosure outperforms. Suppose that $v_0^i = 90$ with probability 0.1 and $v_0^i = -10$ with probability 0.9. Then, $\mu = 0$, $\rho = 80$ and $\beta = -80/9 = -8.88...$. Suppose also that $K = 2$, $v_1 = 81$ and $v_2 = 10$.

In this environment, under full disclosure, only the second dictator, who faces the choice between object 0 and 2, investigates, and the social welfare is

$$\frac{1}{10} \cdot 90 + \frac{9}{10} \cdot 0 - 1 = 9.$$

On the other hand, if all the three agents investigate, the social welfare is

$$\frac{19}{100} \cdot 90 + \frac{81}{100} \cdot 0 - 3 = 14.1,$$

because at least one of the first two dictators finds $v_0^i = 90$, the box is consumed by a good-fit agent. This is strictly larger than the social welfare from the full disclosure policy.

Indeed, under no disclosure, there exists an equilibrium in which all the agents investigate. Given that the other two agents investigate, if an agent does not investigate, his expected payoff is

$$\frac{1}{3} \cdot 81 + \frac{1}{3} \cdot \left(\frac{1}{10} \cdot 81 + \frac{9}{10} \cdot 10\right) + \frac{1}{3} \cdot \left(\frac{19}{100} \cdot 10 + \frac{81}{100} \cdot 0\right) = 33.33... .$$

On the other hand, if this agent investigates, his expected payoff is

$$
\begin{aligned}
&\frac{1}{10}\cdot\left\{\frac{1}{3}\cdot 90+\frac{1}{3}\cdot\left(\frac{1}{10}\cdot 81+\frac{9}{10}\cdot 90\right)+\frac{1}{3}\cdot\left(\frac{19}{100}\cdot 10+\frac{81}{100}\cdot 90\right)\right\}\\
&+\frac{1}{10}\cdot\left\{\frac{1}{3}\cdot 81+\frac{1}{3}\cdot\left(\frac{1}{10}\cdot 81+\frac{9}{10}\cdot 10\right)+\frac{1}{3}\cdot\left(\frac{19}{100}\cdot 10+\frac{81}{100}\cdot(-10)\right)\right\}\\
&-1\\
&=35.04,
\end{aligned}
$$

which is strictly larger than the payoff from no investigation. Hence, there exists an equilibrium in which all the three agents investigate.

6.4 Strategic Experimentation with Other Matching Mechanisms

If we extend the idea of "choice sets" as the set of objects an agent can obtain by changing his report, the information about choice sets affect agents' information acquisition not only with RSD, but also with more general mechanisms, e.g., the Boston mechanism (BOS), the deferred acceptance algorithm (DA, Gale and Shapley (1962)), the top trading cycle algorithm (TTC, Shapley and Scarf (1974)), and the probabilistic serial mechanism (PS, Bogomolnaia and Moulin (2001)). If we can nicely control the information about the other agents' choices (or preference reports) and induce more efficient information acquisition, we may also improve the social welfare in more general problems.

However, it is more difficult to control the information about choice sets with these mechanisms than with RSD. Importantly, under SD and RSD, the current dictator's choice set is solely determined by the preference reports of the *earlier* dictators. In other words, for all $j > i$, agent j's preference report does not change agent i's choice set. Accordingly, if the planner hears the preference reports sequentially, she can identify the current dictator's choice set before the current dictator acquires any information. This property is very peculiar to SD and RSD, and indeed, BOS, DA, TTC and PS do not satisfy this property. Therefore, it is more difficult to introduce a sophisticated disclosure policy to the mechanisms other than RSD.

The comparison with PS is particularly important because the aim of the RSD and

PS are the same — they are designed to resolve one-sided matching problems without property rights, but with fairness concerns. The literature shows that PS is ex ante more ordinally efficient than RSD (Bogomolnaia and Moulin (2001), Manea (2009)). While PS is not strategy-proof, with some assumptions, it becomes strategy-proof in a large market (Kojima and Manea (2010)). However, with RSD, it is easier than with PS to control the information about choice sets to induce more information acquisition. The comparison of the overall performance of RSD and PS is interesting, but beyond the scope of this paper. We leave it for future research.

7 Concluding Remarks

In this paper, we study the optimal choice set disclosure policy. When preferences are endogenous and the mechanism is fixed to RSD, investigation of a dictator increases the expected payoff of the successive dictators. As a result, the full disclosure policy is typically inefficient because it fails to internalize the positive externality of investigation. To induce more efficient information acquisition, we can use the technique of Bayesian persuasion. The gain from using the optimal disclosure policy is large in the worst case.

Practically, it is difficult to construct an optimal disclosure policy because agents face more complex search problems in the real world. However, still, we may be able to extend the insights from this paper to make some policy suggestions:

1. This paper articulates the importance of the "outside option" (the value of the best remaining alternative) for agents' investigation, and shows that by controlling the information about the value of the outside options, we can improve the social welfare. If there exists a situation where the control of the information about the outside option is easier, Bayesian persuasion might be easier to implement.

2. While constructing an optimal disclosure policy in general environments is difficult, marginal improvement of the social welfare should be easier. Indeed, the value for the marginal improvement seems large — as we can see in Figure 1, when an agent is "almost indifferent" between investigate or not (i.e., $v_k \approx \beta$ or $v_k \approx \rho$), if we can make the agent to investigate the box, the welfare gain is significantly large.

Both are interesting research questions, but we leave them for the future research.

Appendix

A Proofs

A.1 Proof of Proposition 2

$$
\begin{aligned}
& G(v_k) - C(v_k) \\
=& \left\{ \int_{v_k}^{\infty} v_0^i dF(v_0^i) - 1 - \mu + F(v_k)\mu \right\} - \left\{ \int_{v_k}^{\infty} v_0^i dF(v_0^i) - 1 - \max\{\mu, v_k\} + F(v_k)v_k \right\} \\
=& \max\{\mu, v_k\} - \mu + F(v_k) \cdot (\mu - v_k) \\
=& \max\left\{ F(v_k)(\mu - v_k), -(1 - F(v_k))(\mu - v_k) \right\} \\
\geq& 0.
\end{aligned}
$$

The inequality follows from the fact that the sign of $F(v_k)(\mu - v_k)$ and $-(1-F(v_k))(\mu - v_k)$ are different if they are not zero. Furthermore, the inequality is satisfied with equality if and only if $\mu = v_k$. ■

A.2 Proof of Proposition 3

1. $v_k < \beta^*$ implies $G(v_l) < 0$ for $l \geq k$. Accordingly,

$$
\begin{aligned}
W_{k+1}^{FB} &= \sum_{l=k+1}^{K} \left[\prod_{m=k+1}^{l-1} \left(1 - p_m^{FB}(1 - F(v_m))\right) \right] p_l^{FB} G(v_l) \\
&\leq 0.
\end{aligned}
$$

Clearly, $W_{k+1}^{FB} = 0$ is achievable by $p_{k+1}^{FB} = p_{k+2}^{FB} = \cdots = 0$. Therefore, for this case, (3) is equivalent to $G(v_k) \geq 0$. Hence, $p_k^{FB} = 0$ is optimal. ■

2. Towards a contradiction, suppose that there exists k such that $v_k > \rho^*$ but $p_k^{FB} > 0$. Among such k, take the largest one, i.e., $l = k+1, k+2, \cdots$, $v_l > \rho^*$ implies $p_l^{FB} = 0$. Then,

$$\begin{aligned} W_{k+1}^{FB} &= \sum_{l=k+1}^{K} \left[\prod_{m=k+1}^{l-1} \left(1 - p_m^{FB}(1 - F(v_m^{FB}))\right) \right] p_l^{FB} G(v_l) \\ &\geq 0. \end{aligned}$$

The inequality follows from (i) part 1 ($p_l^{FB} > 0$ only if $v_l \geq \beta^*$) and (ii) by the choice of k, for $l > k$, $p_l^{FB} > 0$ only if $v_l < \rho^*$. Then, $v_k > \rho^*$ implies $G(v_k) < 0$, and this implies

$$\frac{1}{1 - F(v_k)} G(v_k) < 0 \leq W_{k+1}^{FB}.$$

Hence, $p_k^{FB} > 0$ is not optimal. ■

3. Suppose that $v_k \in (\beta^*, \rho)$. First, we show that

$$W_l^{FB} \leq \frac{1}{1 - F(v_k)} G(v_k) \tag{10}$$

for $l = k+1, \cdots, K, K+1$.

We verify (10) by mathematical induction. Clearly,

$$\frac{1}{1 - F(v_k)} G(v_k) \geq 0 \triangleq W_{K+1}^{FB}$$

because $v_k \in [\beta^*, \rho^*]$ implies $G(v_k) \geq 0$, and $W_{K+1}^{FB} \triangleq 0$ always holds.

Given that (10) holds for $l = m+1$, then,

$$W_m^{FB} = \max\left\{W_{m+1}^{FB}, G(v_m) + F(v_m) W_{m+1}^{FB}\right\}.$$

If $W_m^{FB} = W_{m+1}^{FB}$, (10) for $l = m$ is immediate from the induction hypothesis. Suppose $W_m^{FB} = G(v_m) + F(v_m) W_{m+1}^{FB}$. Then,

$$\frac{1}{1 - F(v_m)} G(v_m) \geq W_{m+1}^{FB},$$

and therefore,

$$\begin{aligned}
W_m^{FB} &= G(v_m) + F(v_m)W_{m+1}^{FB} \\
&\leq G(v_m) + \frac{F(v_m)}{1-F(v_m)}G(v_m) \\
&= \frac{1}{1-F(v_m)}G(v_m).
\end{aligned}$$

Furthermore,

$$W_m^{FB} \leq \frac{1}{1-F(v_m)}G(v_m) \leq \frac{1}{1-F(v_k)}G(v_k)$$

is immediate from Fact 1. Hence, (10) holds for $l = k+1, \cdots, K, K+1$.

In particular, we have

$$W_{k+1}^{FB} \leq \frac{1}{1-F(v_k)}G(v_k).$$

Then, by (3), $p_k^{FB} = 1$.

A.3 Proof of Fact 1

First, we show that $\frac{1}{1-F(v)}G(v)$ is increasing for $v < \rho$. Using the fact that

$$\frac{d}{dv}\left\{\int_v^\infty (v_0^i - v)dF(v_0^i) - 1\right\} = -(1-F(v)) < 0 \quad \text{for } F(v) \neq 1 \tag{11}$$

and

$$\begin{aligned}
& C(\rho) = 0 \quad \text{and} \quad \rho \geq \mu \\
\Leftrightarrow \quad & \int_\rho^\infty v_0^i dF(v_0^i) - 1 - \rho + F(\rho)\rho = 0 && (12) \\
\Leftrightarrow \quad & \int_\rho^\infty (v_0^i - \rho)dF(v_0^i) - 1 = 0, && (13)
\end{aligned}$$

it follows from

$$
\begin{aligned}
&\frac{d}{dv}\left(\frac{1}{1-F(v)}\cdot G(v)\right)\\
=&\frac{f(v)}{(1-F(v))^2}G(v)+\frac{1}{1-F(v)}G'(v)\\
=&\frac{f(v)}{(1-F(v))^2}\cdot\left\{\int_v^\infty v_0^i dF(v_0^i)-1-\mu+F(v)\mu+(1-F(v))(\mu-v)\right\}\\
=&\frac{f(v)}{(1-F(v))^2}\cdot\left\{\int_v^\infty (v_0^i-v)dF(v_0^i)-1\right\}\\
>&\frac{f(v)}{(1-F(v))^2}\left[\int_\rho^\infty (v_0^i-\rho)dF(v_0^i)-1\right]\quad \text{for } v<\rho\\
=&0
\end{aligned}
$$

that $\frac{1}{1-F(v)}G(v)$ is actually increasing in for $v<\rho$ as desired.

Similarly,

$$
\begin{aligned}
&\frac{d}{dv}\left(\frac{1}{1-F(v)}\cdot G(v)\right)\\
=&\frac{f(v)}{(1-F(v))^2}\cdot\left\{\int_v^\infty (v_0^i-v)dF(v_0^i)-1\right\}\\
<&\frac{f(v)}{(1-F(v))^2}\left[\int_\rho^\infty (v_0^i-\rho)dF(v_0^i)-1\right]\quad \text{for } v>\rho\\
=&0
\end{aligned}
$$

implies that $\frac{1}{1-F(v)}G(v)$ is decreasing for $v>\rho$.

Finally,

$$
\begin{aligned}
\frac{1}{1-F(\rho)}G(\rho)&=\frac{1}{1-F(\rho)}\left[C(\rho)+(1-F(\rho))(\rho-\mu)\right]\\
&=\rho-\mu. \blacksquare
\end{aligned}
$$

A.4 Proof of Lemma 1

We only show part 1. The proof for part 2 is similar. Towards a contradiction, suppose that If $v_m>v_n>\rho$ and $p_m>0$, but $p_n<1$. If there exist such m and n, there exists k such that $v_k>v_{k+1}>\rho$, $p_k>0$ but $p_{k+1}<1$. Clearly, if $F(v_k)=1$, $p_k>0$ is not optimal (the agent never takes the box after any investigation); thus, $F(v_k) < 1$. We will show that decreasing p_k in compensation for increasing p_{k+1}, we can improve the value of the objective function, keeping the incentive constraint satisfied. Change p_k and p_{k+1} keeping

$$(1 - p_k + p_k F(v_k))\,(1 - p_{k+1} + p_{k+1} F(v_{k+1}))$$

constant. Then, by the implicit function theorem,

$$\frac{dp_{k+1}}{dp_k} = -\frac{(1 - F(v_k))\,(1 - p_{k+1} + p_{k+1} F(v_{k+1}))}{(1 - F(v_{k+1}))\,(1 - p_k + p_k F(v_k))}.$$

Hence, the total derivative of the objective function w.r.t. p_k is

$$\begin{aligned}
&\prod_{l=1}^{k-1} (1 - p_l + p_l F(v_l))\, G(v_k) \\
&\quad - \prod_{l=1}^{k-1} (1 - p_l + p_l F(v_l))\, (1 - F(v_k))\, p_{k+1} G(v_{k+1}) \\
&\quad + \prod_{l=1}^{k} (1 - p_l + p_l F(v_l))\, \frac{dp_{k+1}}{dp_k} G(v_{k+1}) \\
=&\prod_{l=1}^{k-1} (1 - p_l + p_l F(v_l))\, (1 - F(v_k)) \left\{ \frac{G(v_k)}{1 - F(v_k)} - \frac{G(v_{k+1})}{1 - F(v_{k+1})} \right\} \\
<&0
\end{aligned}$$

The last inequality follows from $v_k > v_{k+1} > \rho$ and Fact 1. Hence, decreasing p_k slightly in compensation for increasing p_{k+1}, the value of the objective function is improved.

Similarly, the total derivative of the right hand side of the constraint w.r.t. p_k is

$$\begin{aligned}
&\prod_{l=1}^{k-1}(1-p_l+p_lF(v_l))\,C(v_k)\\
&\quad-\prod_{l=1}^{k-1}(1-p_l+p_lF(v_l))\,(1-F(v_k))\,p_{k+1}C(v_{k+1})\\
&\quad\quad+\prod_{l=1}^{k}(1-p_l+p_lF(v_l))\,\frac{dp_{k+1}}{dp_k}C(v_{k+1})\\
=&\prod_{l=1}^{k-1}(1-p_l+p_lF(v_l))\,(1-F(v_k))\left\{\frac{C(v_k)}{1-F(v_k)}-\frac{C(v_{k+1})}{1-F(v_{k+1})}\right\}\\
<&0
\end{aligned}$$

The last inequality follows from the fact that $v_k > v_{k+1} > \rho$ and

$$\begin{aligned}
&\frac{d}{dv}\left\{\frac{C(v)}{1-F(v)}\right\}\\
=&\frac{d}{dv}\left\{\frac{1}{1-F(v)}\cdot\left[\int_v^\infty v_0^i dF(v_0^i)-1-(1-F(v))v\right]\right\}\\
=&\frac{f(v)}{(1-F(v))^2}\left\{\int_v^\infty (v_0^i-v)dF(v_0^i)-1\right\}-1\\
<&\frac{f(v)}{(1-F(v))^2}\left\{\int_\rho^\infty (v_0^i-\rho)dF(v_0^i)-1\right\}-1\\
=&-1
\end{aligned}$$

for $v > \rho$ and $F(v) \neq 1$. Hence, decreasing p_k slightly in compensation for increasing p_{k+1}, the incentive constraint is also satisfied. ■

A.5 Proof of Lemma 4

We only prove the part 1. The proofs for parts 2, 3, 4, and 5 are similar. For the optimality, the value of the social welfare should not be improved by such manipulation, i.e., the total derivative of the social welfare should be non-negative. By assumption,

$p^{Opt}_{u+1} = \cdots = p^{Opt}_{d-1} = 1$. Therefore, applying the implicit function theorem,

$$
\begin{aligned}
& dp_d/dp_u \\
&= - \frac{\left\{ \begin{array}{ll} & C(v_u) + (F(v_u)-1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k) \\ + & (F(v_u)-1)\prod_{l=u+1}^{d-1} F(v_l)\cdot p_d \cdot C(v_d) \end{array} \right\}}{(1-p_u+p_uF(v_u))\cdot \prod_{l=u+1}^{d-1} F(v_l)C(v_d)}
\end{aligned}
$$

The total derivative of the social welfare is

$$
\begin{aligned}
& G(v_u) + (F(v_u)-1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)G(v_k) \\
& + (F(v_u)-1)\prod_{l=u+1}^{d-1} F(v_l)p_dG(v_d) \\
& + (1-p_u+p_uF(v_u))\cdot \prod_{l=u+1}^{d-1} F(v_l)\cdot \frac{dp_d}{dp_u}\cdot G(v_d).
\end{aligned}
\tag{14}
$$

Since $\rho < v_u$ implies

$$
-\left\{ \begin{array}{ll} & C(v_u) + (F(v_u)-1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k) \\ + & (F(v_u)-1)\prod_{l=u+1}^{d-1} F(v_l)\cdot p_d \cdot C(v_d) \end{array} \right\} > 0,
$$

the sign of (14) is equal to the sign of

$$
-\frac{\left\{ \begin{array}{ll} & G(v_u) + (F(v_u)-1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)G(v_k) \\ + & (F(v_u)-1)\prod_{l=u+1}^{d} F(v_l)p_dG(v_d) \end{array} \right\}}{\left\{ \begin{array}{ll} & C(v_u) + (F(v_u)-1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k) \\ + & (F(v_u)-1)\prod_{l=u+1}^{d-1} F(v_l)\cdot p_dC(v_d) \end{array} \right\}} - D_d. \tag{15}
$$

Note that (15) coincides $U_{u:d} - D_d$ when $p_d = 0$. Finally, the sign of (15) is independent from the value of p_d, because

$$D_d = -\frac{G(v_d)}{C(v_d)},$$

and

$$(F(v_u) - 1) \prod_{l=u+1}^{d-1} F(v_l) p_d G(v_d) \cdot C(v_d) = (F(v_u) - 1) \prod_{l=u+1}^{d-1} F(v_l) \cdot p_d C(v_d) \cdot G(v_d).$$

Hence, we can substitute $p_d = 0$ to (15), in order to evaluate the sign of (14). Then, we obtain that the sign of total derivative of the objective function with respect to p_u is equal to the sign of $U_{u:d} - D_d$. Hence, $U_{u:d} - D_d \geq 0$ is necessary for optimality. ■

A.6 Proof of Proposition 5

First, we show the following to lemmas.

Lemma 2. *$U_{u:d} - D_d$ is increasing in u.*

Proof of Lemma 2 First, we show that the numerator of $U_{u:d}$ is increasing in u as follows:

$$\begin{aligned}
&G(v_u) + (F(v_u) - 1) \sum_{k=u+1}^{d-1} \prod_{l=u+1}^{k-1} F(v_l) G(v_k) \\
&- \left\{ G(v_{u+1}) + (F(v_{u+1}) - 1) \sum_{k=u+2}^{d-1} \prod_{l=u+2}^{k-1} F(v_l) G(v_k) \right\} \\
=& G(v_u) - G(v_{u+1}) \\
&- (1 - F(v_u)) \left\{ G(v_{u+1}) + (F(v_{u+1}) - 1) \sum_{k=u+2}^{d-1} \prod_{l=u+2}^{k-1} F(v_l) G(v_k) \right\} \\
\leq& 0.
\end{aligned}$$

The last inequality follows from $G(v_u) \leq G(v_{u+1})$ and $p_{u+1}^{FB} = 1$ implies

$$G(v_{u+1}) + (F(v_{u+1}) - 1) \sum_{k=u+2}^{d-1} \prod_{l=u+2}^{k-1} F(v_l) G(v_k) \geq 0.$$

Similarly, we can show that the denominator of $U_{u:d}$ is increasing in u. Recall that

$$U_{u:d} \triangleq -\frac{G(v_u) + (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)G(v_k)}{C(v_u) + (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k)}.$$

Since $G(v_u) + (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)G(v_k)$ is non-negative and increasing in u, and $-C(v_u) - (F(v_u) - 1)\sum_{k=u+1}^{d-1}\prod_{l=u+1}^{k-1} F(v_l)C(v_k)$ is positive and decreasing in u, $U_{u:d}$ is increasing in u. Accordingly, $U_{u:d} - D_d$ is increasing in u. ■

Lemma 3. $U_{u:d} - D_d > 0$ *implies* $U_{u:d+1} - D_{d+1} > 0$.

Proof of Lemma 3 Suppose that $U_{u:d} - D_d > 0$, or equivalently,

$$\begin{aligned} &C(v_d) \cdot \left\{ G(v_u) + (F(v_u) - 1) \sum_{k=u+1}^{d-1} \prod_{l=u+1}^{k-1} F(v_l)G(v_k) \right\} \\ <&G(v_d) \cdot \left\{ C(v_u) + (F(v_u) - 1) \sum_{k=u+1}^{d-1} \prod_{l=u+1}^{k-1} F(v_l)C(v_k) \right\}. \end{aligned}$$

Adding $(F(v_u) - 1)\prod_{l=u+1}^{d-1} F(v_l)G(v_d)C(v_d)$ to both sides, we have

$$\begin{aligned} &C(v_d) \cdot \left\{ G(v_u) + (F(v_u) - 1) \sum_{k=u+1}^{d} \prod_{l=u+1}^{k-1} F(v_l)G(v_k) \right\} \\ <&G(v_d) \cdot \left\{ C(v_u) + (F(v_u) - 1) \sum_{k=u+1}^{d} \prod_{l=u+1}^{k-1} F(v_l)C(v_k) \right\}. \end{aligned}$$

or equivalently, $U_{u:d+1} - D_d > 0$. Finally, it follows from $D_d \triangleq G(v_d)/(-C(v_d))$, $G(v_d) > 0$, $G(v_d) \geq G(v_{d+1})$, $-C(v_d) > 0$, and $-C(v_d) \leq -C(v_{d+1})$ that $D_d \geq D_{d+1}$. Therefore, $U_{u:d+1} - D_{d+1} > 0$, as desired. ■

Proof of Proposition 5 If the IC constraint is not binding, i.e., $p^{FB} = p^{Opt}$, the optimality of the greedy algorithm is trivial.

Towards a contradiction, suppose that there exists a disclosure policy p^{Opt} that achieves a strictly larger social welfare than the disclosure policy generated by the greedy algorithm, p^G. By Lemma 1, if there exists $k, l < r$ such that $p_k^{Opt}, p_l^{Opt} \in (0, 1)$, $v_k = v_l$. Accordingly, without loss of generality, we can assume that p^{Opt} has an interval structure, i.e., there exists $u^* \leq r$ and $d^* \geq b$ such that $p_k^{Opt} = 0$ for $k \leq u^* - 2$ and $k \geq d^* + 2$, $p_{u^*-1}^{Opt} \in [0, 1)$, $p_{u^*}^{Opt} = p_{u^*+1}^{Opt} = \cdots = p_{d^*}^{Opt}$, and $p_{d^*+1}^{Opt} \in [0, 1)$. On the other hand, by definition, p^G has at most one $k \in \mathcal{K} \setminus \{0\}$ such that $p_k \in (0, 1)$. Here, we prove for the case of $p_k^G = 0$ for $k \leq u^\dagger - 2$ and $k \geq d^\dagger + 1$, $p_{u^\dagger-1}^G \in [0, 1)$, and $p_{u^\dagger}^G = p_{u^\dagger+1}^G = \cdots = p_{d^\dagger}^G$. The proof for the case of $p_{u^\dagger}^G = 0$ and $p_{d^\dagger+1}^G \in [0, 1)$ is similar.

First, suppose that $u^* = u^\dagger$and $d^* = d^\dagger$. If $p_{d^*}^{Opt} = 0$, then $p^{Opt} = p^G$, which is a contradiction. If $p_{u^*-1}^G = 0$, to satisfy the IC constraint, $p_{u^*}^{Opt} = p_{d^*}^{Opt} = 0$ must be the case, and again we have $p^{Opt} = p^G$, contradiction. If $U_{u^*-1:d^*+1} - D_{d^*+1} = 0$, p^{Opt} and p^G achieves the same social welfare, which is also a contradiction. Suppose that $p_{d^*}^{Opt} > 0$, $p_{u^*-1}^G > 0$, and $U_{u^*-1:d^*+1} - D_{d^*+1} \neq 0$. Since $p_{d^*+1}^G = 0$, $U_{u^*-1:d^*+1} - D_{d^*+1} > 0$. Accordingly, p^{Opt} does not satisfy either part 2 or part 5 of Lemma 4, which contradicts the optimality of p^{Opt}.

Next, suppose $u^* < u^\dagger$. Since both p^{Opt} and p^G satisfy the IC constraint with equality, $d^* < d^\dagger$ must hold. It follows from $p_{u^\dagger-1}^G < 1$ and $p_{d^*+1}^G = 1$ that $U_{u^\dagger-1:d^*+1} - D_{d^*+1} \leq 0$. Since $U_{u:d}$ is increasing in u, if $u^* < u^\dagger - 1$, we have $U_{u^*:d^*+1} - D_{d^*+1} < 0$, which contradicts optimality of p^{Opt}. If $u^* = u^\dagger - 1$, we have $U_{u^*-1:d^*+1} - D_{d^*+1} < 0$ and $U_{u^*:d^*+1} - D_{d^*+1} \leq 0$, and therefore, we must have $p_{u^*-1}^{Opt} = 0$, $U_{u^*:d^*+1} - D_{d^*+1} = 0$ for the optimality of p^{Opt}. However, in this case, it follows from (i) $p_k^{Opt} = p_k^G = 1$ for $k = u^* + 1, \cdots, d^*$, (ii) $p_k^{Opt} = p_k^G = 0$ for $k \in \{1, \cdots, u^* - 1, d^* + 2, \cdots, K\}$ and $U_{u^*:d^*+1} - D_{d^*+1} = 0$ that p^{Opt} and p^G achieves the same social welfare, which is a contradiction.

When $u^* > u^\dagger$, we can derive a contradiction in a similar manner. Accordingly, the greedy algorithm finds an optimal disclosure policy. ■

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