

Why Are Exchange Rates So Smooth? A Household Finance Explanation

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Abstract

Empirical moments of asset prices and exchange rates imply that pricing kernels have to be almost perfectly correlated across countries. If they are not, real exchange rates are too smooth to be consistent with high Sharpe ratios in asset markets. However, the cross-country correlation of macro fundamentals is far from perfect. We reconcile these empirical facts in a two-country stochastic growth model with heterogeneous trading technologies for households and a home bias in consumption. In our model, only a small fraction of households actively participate in international risk sharing by frequently trading domestic and foreign equities. These active traders, who induce high cross-country correlation to the pricing kernels, are the marginal investors in foreign exchange markets. In a calibrated version of our model, we show that this mechanism can quantitatively account for the excess smoothness of exchange rates in the presence of highly volatile pricing kernels and weakly correlated macro fundamentals. We also provide evidence for this mechanism in the empirical part.

JEL codes: G15, G12, F31, F10.

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1 Introduction

A striking disconnect exists in international finance between the evidence gathered from asset prices and that gathered from quantities. When markets are complete, no arbitrage implies that the percentage rate of depreciation of the real exchange rate (RER) is given by the difference between the domestic and the foreign pricing kernels. We know from the data on RERs and asset prices that the volatility of RERs is much smaller than the volatility of the pricing kernels. Hence, the evidence from asset markets implies that the pricing kernels are highly correlated across countries. In fact, Brandt, Cochrane, and Santa-Clara (2006) conclude that the correlation of the pricing kernels across countries is close to 1. However, the quantity data paint a different picture. In standard representative agent models, the correlation of pricing kernels across countries is identical to that for aggregate consumption growth. Empirically, the correlation of aggregate consumption growth is below 50% for most industrialized country pairs. In this paper, we address this disconnect between prices and quantities in international finance in a two-country model with heterogeneous portfolios.¹

Household finance may hold the key to this disconnect. Standard macro-finance models assume that aggregate risk has been distributed across all households, but, in our model, most households do not bear their share of aggregate risk. We introduce heterogeneity in trading technologies in international equity and bond markets into an otherwise standard Lucas (1982) two-country model. In particular, we only allow a small fraction of international equity and bond market participants to optimally adjust their portfolios every period. These active investors, the only ones to respond elastically to variation in state prices, are marginal in foreign exchange (FX) markets and determine the dynamics of exchange rates.

The other critical feature in our model is moderate home bias in consumption. If agents have an extreme home bias, then there is little motive for international risk sharing, even for sophisticated investors, and RERs are too volatile. On the other hand, when agents have little home bias in consumption, frictionless trade yields too little volatility in RERs. In the intermediate case of a

¹There is an ongoing debate on whether incomplete market models help solve the puzzle caused by the disconnect between prices and quantities. When markets are incomplete, the percentage rate of depreciation of the RER may not be identical to the difference between the domestic and the foreign pricing kernels. Hence, the incomplete market model may help to resolve the puzzle as suggested by Favilukis, Garlappi, and Neamati (2015). Maurer and Tran (2016), who argue that a model embedded with risk entanglement, a refinement concept of incomplete market, can successfully explain these puzzles in the international finance. However, Lustig and Verdelhan (2015) show that market incompleteness cannot quantitatively resolve the puzzle without largely eliminating currency risk premia. Based on the empirical evidence of household finance, our view is that some households do not utilize the financial assets available to them and act as if in an incomplete market world.

moderate home bias, a calibrated version of our model can match RER volatility in the data.

Our model features global and country-specific aggregate risk, and household-specific idiosyncratic risk. In equilibrium, a large fraction of global aggregate risk is borne by the small pool of sophisticated investors who actively participate in both domestic and foreign equity and bond markets each period. These active domestic and foreign investors achieve a higher degree of risk sharing among themselves –across borders– than the average investors in these countries –within borders. Hence, the marginal investor’s consumption growth is highly correlated across countries, but the average investor’s is not. This mechanism can quantitatively account for the excess smoothness of the RER with pricing kernels that satisfy the Hansen-Jagannathan bounds.

Our approach is firmly grounded in the empirical evidence on household finance. The evidence suggests that most households do not purchase all assets available on the menu (see, e.g., Guiso and Sodini (2013) for an excellent survey of this literature). In fact, the composition of household asset holdings varies greatly across households, even in a financially developed country like the United States. Only 50% of U.S. households participate in the equity market, according to the 2010 Survey of Consumer Finance. The participation rate is even lower in other developed countries. Obviously, the nonparticipants bear no aggregate risk. Even among the equity market participants, many of them report equity shares that are significantly lower than the corresponding share of equities as a fraction of the market, i.e. all marketable securities. In addition, most of the equity market participants trade very infrequently and do not rebalance their portfolios often in response to changes in investment opportunities (see the evidence reported by Ameriks and Zeldes (2004), Brunnermeier and Nagel (2008), Calvet, Campbell, and Sodini (2009), and Alvarez, Guiso, and Lippi (2012)). In sum, the heterogeneity in observed portfolio choices data implies a highly uneven distribution of risk across investors and across time.

In the quantitative exercise, we parameterize our model to match moments of the world economy, in which the United States is the home country and the foreign country is the sum of France, Germany, Japan, and the United Kingdom. In our benchmark economy, we find that the international correlation of the pricing kernels exceeds 97%, while the international correlation of consumption growth is only 17%. Despite the high volatility of the pricing kernels as observed in the data, our model produces a 9.4% standard deviation of the RER, which is close to the data. Without heterogeneity in household types, the pricing kernels become much less volatile and weakly correlated, resulting in a violation of the Hansen-Jagannathan bounds implied by the pricing data.

Income and education are the two key predictors of equity market participation. To provide empirical support for the key mechanism in our model, we employ annual data on U.S. state-specific and Canada's province- or territory-specific income per capita as well as consumption per capita from 1997 to 2014, and income from equity investment as reported in individual income tax returns in 2012.² In our regression, the dependent variable is the state-province-specific correlation of consumption growth. The set of explanatory variables proxies the following: income level, investment income, the fraction of tax returns with investment income, and trade costs. We find that an increase in income level, an increase in investment income, and an increase in the fraction of tax returns with investment income significantly increase the consumption growth correlation. These results confirm that income level and equity investment have positive impacts on the consumption growth correlation, as in our model.

Most of the existing work on this puzzle modifies the preferences or the stochastic properties of endowment growth of an otherwise standard representative agent model, implicitly assuming that all risk sharing opportunities within a country have been exhausted. Colacito and Croce (2011) endow the representative investor with recursive preferences that impute a concern about long-run risk in consumption. When long-run risks are highly correlated across countries, the pricing kernels become highly correlated even though aggregate consumption growth is not³, while Farhi and Gabaix (2016) rely on correlated disaster risk. Finally, Stathopoulos (2016) analyzes a model in which the representative investor has preferences with external habit persistence, which induces high correlation of the pricing kernels, despite the low correlation of current consumption growth.

Prior work has explored market segmentation to understand exchange rates. Alvarez, Atkeson, and Kehoe (2002) develop a Baumol-Tobin model of nominal exchange rates in which money and securities markets are segmented. Recently, Gabaix and Maggiori (2015) consider a different form of market segmentation; only financial institutions are active in international bond and FX markets, while retail investors are not. Our model does not have a financial sector, but the sophisticated investors do clear FX markets and earn an equilibrium currency risk premium in return, much like the large financial institutions in Gabaix and Maggiori (2015).

Our paper introduces the Chien, Cole and Lustig (2011, 2012) heterogeneous trading technologies in a two-country Lucas (1982) model. Chien and Naknoi (2015) use a two-country Lucas (1982) model with limited participation that is similar to ours to study global imbalances. There

²We do not use Canada's household-specific data, because they span over a shorter period.

³Gavazzoni and Santacreu (2015) find that a calibrated model with the long-run risk that originates from international technology diffusion can quantitatively explain the RER volatility puzzle.

are two other recent research papers directly related to our work. These papers explore a segmented market explanation. Zhang (2015) offers an explanation of the puzzlingly high correlation of stock market returns and low correlation of fundamentals across countries, but her work does not address the exchange rate puzzle. Kim and Schiller (2015) focus on RER volatility in a segmented market model with trade frictions. Theirs is a standard limited participation model without heterogeneity among equity market participants. As a result, the residual aggregate risks in their calibrated model are not concentrated enough to reconcile the moments of asset prices and exchange rates.

Our main contribution to the literature is the integration of the micro evidence on household portfolio choices and trade frictions into a general equilibrium model to solve the exchange rate volatility puzzle. We show that distinguishing marginal FX investors from less sophisticated equity market investors is essential to solving the puzzle and enhances the performance of the model. The quantitative results indicate that some degree of trade frictions in goods market are also essential. Therefore, international trade in both goods market and asset markets matters to the exchange rate determination. Hence, our model offers an initial step to closing the gap between the asset market approach and the goods market approach of exchange rate determination. Moreover, our model does not require nonstandard preferences or aggregate risk specifications. Instead, the mechanism in our model relies on the skewness of the cross-sectional distribution of aggregate risks, which are strongly supported by the empirical evidence.

In addition, our study broadly contributes to the emerging literature that integrates international portfolio choices into international macroeconomics. Specifically, we demonstrate the importance of household portfolio heterogeneity in open economies, whereas the majority of open-economy macroeconomic models rely on a representative agent framework. Recent studies by Pavlova and Rigobon (2010) and Coeurdacier and Rey (2013) are prominent examples. Most international macroeconomic models assume either incomplete markets with only one asset or a complete market environment without portfolio heterogeneity. Although a complete menu of assets is traded in our model, we allow for heterogeneity in household trading technologies, as in the data.

The rest of our study is organized as follows. The next section describes our model. The quantitative results and counterfactual exercises are detailed in Section 3. Section 4 provides the empirical evidence. Finally, Section 5 concludes our study.

2 The Model

2.1 Environment

We consider an endowment economy with two countries, home and foreign. There are a large number of agents in each country with a unit measure. Each country is endowed with a nontraded good and an export good. For simplicity, we assume that home households consume the nontraded good and the foreign export good. Likewise, foreign households consume the nontraded good and the home export good.

Time is discrete, infinite, and indexed by $t \in [0, 1, 2, \dots]$. To have a stationary economy, we assume an identical average endowment growth rate for each country, while the actual growth rate may deviate from the average rate. More specifically, let $\ln m_t$ and $\ln m_t^*$ be the percentage deviations of endowment from trend growth. Then, the country-specific endowment, denoted by Y , in period t is

$$\begin{aligned}\ln Y_t &= t \ln \bar{g} + \ln m_t, \\ \ln Y_t^* &= t \ln \bar{g} + \ln m_t^*,\end{aligned}$$

where \bar{g} is the average growth rate of the endowments of both countries. The output growth dynamic is therefore governed by the evolution of m , which follows the following AR(1) process:

$$\begin{aligned}\ln m_{t+1} &= \rho \ln m_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2) \\ \ln m_{t+1}^* &= \rho \ln m_t^* + \varepsilon_{t+1}^*, \quad \varepsilon_{t+1}^* \sim N(0, \sigma_\varepsilon^{*2}).\end{aligned}$$

Define $z_t \equiv \{\varepsilon_t, \varepsilon_t^*\}$ as the aggregate shock in period t . Let z^t denote the history of aggregate shock up to period t . In each country, a constant fraction λ of the endowment is the nontraded good and the rest is the export good:

$$\begin{aligned}Y_n(z^t) &= \lambda Y(z^t) \text{ and } Y_x(z^t) = (1 - \lambda)Y(z^t) \\ Y_n^*(z^t) &= \lambda Y^*(z^t) \text{ and } Y_x^*(z^t) = (1 - \lambda)Y^*(z^t),\end{aligned}$$

where Y_n , Y_x , Y_n^* , and Y_x^* denote endowments of home nontraded, home export, foreign nontraded, and foreign export goods, respectively.

2.2 Aggregate Income

Aggregate income is the total value of exports and nontradable goods. Let the variables $q_n(z^t)$ and $q_x^*(z^t)$ denote the price of the home nontraded good in terms of the home consumption basket and the price of the home export good in terms of the foreign consumption basket, respectively. Then the total income at home, denoted $I(z^t)$, evaluated in terms of the home consumption basket is given by

$$I(z^t) = q_n(z^t)Y_n(z^t) + \frac{q_x^*(z^t)}{e_t(z^t)}Y_x(z^t),$$

where e_t denotes the RER, or the price of the home consumption basket relative to the foreign consumption basket. Similarly, the total income of the foreign country, denoted $I^*(z^t)$, in terms of the foreign consumption basket is

$$I^*(z^t) = q_n^*(z^t)Y_n^*(z^t) + q_x(z^t)e_t(z^t)Y_x^*(z^t),$$

where $q_n^*(z^t)$ and $q_x(z^t)$ denote the price of the foreign nontraded good in terms of the foreign consumption basket, and the price of the foreign export good in terms of the home consumption basket, respectively.

The total income in each country is divided into two parts: diversifiable income and nondiversifiable income. Claims to diversifiable income can be traded in financial markets while claims to nondiversifiable income cannot.⁴ We assume a constant share of nondiversifiable income, α , across countries and time. The nondiversifiable component is subject to idiosyncratic stochastic shocks. These shocks are i.i.d. across households and persistent over time in each country. We use η_t and η_t^* to denote the idiosyncratic shock in period t to the home and the foreign countries, respectively. Then, η^t and $\eta^{*,t}$ denote the history of idiosyncratic shocks to home and foreign households, respectively. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability that state (z^t, η^t) will be realized. These shock processes are assumed to be independent among z , η and η^* shocks.

⁴The non-diversifiable and diversifiable incomes here work exactly like labor income and capital income in the standard production economy, respectively.

2.3 Preferences

The household derives utility from consuming composites of goods,

$$\sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^t)} \frac{c(z^t, \eta^t)^{1-\gamma}}{1-\gamma} \pi(z^t, \eta^t),$$

where $\gamma > 0$, $0 < \beta < 1$. The parameter γ denotes the coefficient of relative risk aversion, β denotes the time discount factor, and $c(z^t, \eta^t)$ denotes the consumption basket. The home consumption basket is a Cobb-Douglas composite of the nontraded good c_n and the foreign export good c_x^* ,

$$c(z^t, \eta^t) = c_n(z^t, \eta^t)^\theta c_x^*(z^t, \eta^t)^{1-\theta}.$$

The parameter $\theta \in [0, 1]$ represents a home bias in consumption and governs the relative preferences over the nontraded and foreign export goods.

Similarly, the preferences for the foreign households are given by

$$\sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^{*,t})} \frac{c^*(z^t, \eta^{*,t})^{1-\gamma}}{1-\gamma} \pi(z^t, \eta^{*,t}),$$

where $\gamma > 0$, $0 < \beta < 1$. The foreign basket is similarly defined as

$$c^*(z^t, \eta^{*,t}) = c_n^*(z^t, \eta^{*,t})^\theta c_x(z^t, \eta^{*,t})^{1-\theta}.$$

2.4 Correlation of Consumption Growth

Given the preferences, the aggregate consumption basket becomes a composite of the home nontraded good and the foreign export good endowment:

$$C(z^t) = Y_n(z^t)^\theta Y_x^*(z^t)^{1-\theta}. \quad (1)$$

Likewise, foreign aggregate consumption is a composite of the foreign nontraded good and the home export good:

$$C^*(z^t) = Y_n^*(z^t)^\theta Y_x(z^t)^{1-\theta}. \quad (2)$$

As a result, the correlation of consumption growth is exogenously determined by the preferences parameter, θ , as well as the correlation of the endowment shock process. To see why, notice that the resource constraints in (1) and (2) together with the endowment imply that

$$\begin{aligned} \Delta \ln C(z^t) &= \theta \Delta \ln Y(z^t) + (1 - \theta) \Delta \ln Y^*(z^t) \\ \Delta \ln C^*(z^t) &= \theta \Delta \ln Y^*(z^t) + (1 - \theta) \Delta \ln Y(z^t), \end{aligned}$$

with an assumption of symmetric countries, $\sigma(\Delta \ln Y) = \sigma(\Delta \ln Y^*)$, where $\sigma(X)$ denotes the standard deviation of variable X . Let $\rho(X, X')$ denote the correlation between variables X and X' . Then, we can derive the correlation of consumption growth as

$$\rho(\Delta \ln C, \Delta \ln C^*) = \frac{2\theta(1 - \theta) + (\theta^2 + (1 - \theta)^2)\rho(\Delta \ln Y, \Delta \ln Y^*)}{(\theta^2 + (1 - \theta)^2) + 2\theta(1 - \theta)\rho(\Delta \ln Y, \Delta \ln Y^*)}. \quad (3)$$

The parameter θ governs the correlation of consumption growth in our model. If $\theta = 1$, then $\rho(\Delta \ln C, \Delta \ln C^*) = \rho(\Delta \ln Y, \Delta \ln Y^*)$. In this case, the preferences exhibit a complete home bias in consumption, and hence there is no goods trade between the two countries. We would like to think of this case as an approximation of having an extremely high trade friction, so that all goods trade is shut down and hence no international risk sharing. If $\theta = 0.5$, then $\rho(\Delta \ln C, \Delta \ln C^*) = 1$. The perfect correlation of consumption growth arises when there is no home bias in consumption. We interpret this case as an approximation of zero trade friction and countries reach full risk sharing.

2.5 Leverage and Asset Supply

In each country, three types of assets are available: state-contingent claims on aggregate shocks, risky equities, and risk-free bonds. Note that we assume that the idiosyncratic risk is uninsurable and hence there are no state contingent claims on idiosyncratic shocks. Both risky equities and risk-free bonds are claims to the diversifiable income. Equities represent a leveraged claim to

diversifiable income. The leverage ratio is constant over time and denoted by ϕ . Let $\bar{B}_t(z^t)$ denote the supply of a one-period risk-free bond in period t in the home country and $W_t(z^t)$ denote the price of a claim to the home country's total diversifiable income in period t . With a constant leverage ratio, the total supply of $\bar{B}_t(z^t)$ must be adjusted such that

$$\bar{B}_t(z^t) = \phi [W_t(z^t) - \bar{B}_t(z^t)].$$

By the previous equation, the aggregate diversifiable income can be decomposed into interest payments to bondholders and payouts to shareholders. The total payouts, including cash dividends and net repurchases, denoted $\bar{D}_t(z^t)$, are

$$\bar{D}_t(z^t) = (1 - \alpha)I(z^t) - R_{t,t-1}^f(z^{t-1})\bar{B}_{t-1}(z^{t-1}) + \bar{B}_t(z^t),$$

where $R_{t,t-1}^f(z^{t-1})$ denotes the home risk-free rate in period $t-1$. For simplicity, our model assumes that the supply of equity shares is constant. As a result, if a firm reissues or repurchases equity shares, it must be reflected by $\bar{D}_t(z^t)$ in our model. Similarly, the supply of foreign bonds is given by

$$\bar{B}_t^*(z^t) = \phi^* [W_t^*(z^t) - \bar{B}_t^*(z^t)],$$

while the payouts on foreign equity are given by

$$\bar{D}_t^*(z^t) = (1 - \alpha^*)I^*(z^t) - R_{t,t-1}^{*f}(z^{t-1})\bar{B}_{t-1}^*(z^{t-1}) + \bar{B}_t^*(z^t),$$

where $R_{t,t-1}^{*f}(z^{t-1})$ denotes the foreign risk-free rate in period $t-1$.

Finally, we denote the value of total home equity or a claim to total payouts on $\bar{D}_t(z^t)$ as $V_t(z^t)$. Likewise, we denote the value of total foreign equity or a claim to total payouts on $\bar{D}_t^*(z^t)$ as $V_t^*(z^t)$. The gross returns of home and foreign equities, or $R_{t,t-1}^d(z^t)$ and $R_{t,t-1}^{*d}(z^t)$, respectively,

are therefore given by

$$\begin{aligned} R_{t,t-1}^d(z^t) &= \frac{\bar{D}_t(z^t) + V_t(z^t)}{V_{t-1}(z^{t-1})} \\ R_{t,t-1}^{*d}(z^t) &= \frac{\bar{D}_t^*(z^t) + V_t^*(z^t)}{V_{t-1}^*(z^{t-1})}. \end{aligned}$$

2.6 Heterogeneity in Trading Technologies

There is significant portfolio heterogeneity not only across countries but also across investors within a country. To capture such heterogeneity, we implement the approach adopted by Chien, Cole, and Lustig (2011) and exogenously impose different restrictions on investor portfolio choices. These restrictions apply to the menu of assets that these investors can trade as well as the composition of household portfolios.

There are two classes of investors in terms of their asset trading technologies. The first class of investors faces no restrictions on portfolio choices and the menu of tradable assets. Specifically, these investors trade a complete set of contingent claims on the domestic and foreign endowments. We call these investors "Mertonian traders". They optimally adjust their portfolio choices in response to changes in the investment opportunity set. Hence, they are marginal traders and price exchange rate risk in our model.

The second class of investors faces restrictions on their portfolios and are called "non-Mertonian traders". Specifically, their portfolio composition is restricted to be constant over time. We assume two types of non-Mertonian traders: non-Mertonian equity investors, who can trade domestic equities and the domestic risk-free bonds, and nonparticipants, who invest in only the domestic risk-free bonds. Even though the portfolio composition of non-Mertonian traders is exogenously given, they can still optimally choose how much to save and consume.

Non-Mertonian equity investors deviate from the optimal portfolio choices in two dimensions. First, they cannot change the share of equities in their portfolios in response to changes in the market price of risk, which indicates missed market timing. Second, their portfolio share in equities might deviate from the optimal share on average.

We denote the fraction of different types of investors in the home country and the foreign country by μ_j and μ_j^* , where $j \in \{me, et, np\}$ represents Mertonian traders, non-Mertonian equity traders, and nonparticipants, respectively.

2.6.1 Mertonian Traders

We start by considering a version of our economy in which all trade occurs sequentially. Securities markets are segmented. Only the Mertonian traders have access to all securities markets. A home Mertonian trader who enters the period with net financial wealth $a_t(z^t, \eta^{t-1})$ in node (z^t, η^t) has accumulated domestic claims worth $a_{ht}(z^t, \eta^{t-1})$ and claims on foreign investments worth $a_{ft}(z^t, \eta^{t-1})$:⁵

$$a_t(z^t, \eta^{t-1}) = a_{ht}(z^t, \eta^{t-1}) + \frac{a_{ft}(z^t, \eta^{t-1})}{e_t(z^t)},$$

where a_{ht} denotes the payoff of state-contingent claims in the home country expressed in terms of the home consumption basket, and a_{ft} denotes the payoff on foreign state-contingent claims expressed in terms of the foreign consumption basket.

$Q(z^{t+1}|z^t)$ and $Q^*(z^{t+1}|z^t)$ are the state-contingent prices in the home country and the foreign country, expressed in units of the home and foreign consumption basket, respectively. At the end of the period, home Mertonian traders go to securities markets to buy domestic and foreign state-contingent claims $a_{h,t+1}(z^{t+1}, \eta^t)$ and $a_{f,t+1}(z^{t+1}, \eta^t)$, and they go to the goods market to purchase $c(z^t, \eta^t)$ units of the home consumption basket, subject to the following one-period budget constraint:

$$\begin{aligned} & \sum_{z_{t+1}} Q(z^{t+1}|z^t) a_{h,t+1}(z^{t+1}, \eta^t) + \sum_{z_{t+1}} Q^*(z^{t+1}|z^t) e_t(z^t) a_{f,t+1}(z^{t+1}, \eta^t) + c(z^t, \eta^t) \\ & \leq a_t(z^t, \eta^{t-1}) + \alpha I(z^t) \eta_t, \text{ for all } (z^t, \eta^t). \end{aligned}$$

Note that the numeraire is the home consumption basket. Investors can spend all of their nondiversifiable income and the accumulated wealth with which they entered the period.

Similarly, the net financial wealth $a_t^*(z^t, \eta^{*,t-1})$ of a foreign Mertonian trader at the start of the period consists of the net financial claims on the home endowment $a_{ht}^*(z^t, \eta^{*,t-1})$ and the claims on the foreign endowment $a_{ft}^*(z^t, \eta^{*,t-1})$:

$$a_t^*(z^t, \eta^{t-1}) = a_{ht}^*(z^t, \eta^{t-1}) e_t(z^t) + a_{ft}^*(z^t, \eta^{t-1}).$$

⁵The net financial wealth in node (z^t, η^t) does not depend on the realization of idiosyncratic shock, η_t , because of uninsurable idiosyncratic risks.

For the foreign households, their budget constraint is specified in terms of the foreign consumption basket. At the end of the period, the foreign Mertonian trader buys state-contingent claims $a_{h,t+1}^*(z^{t+1}, \eta^{*,t})$ and $a_{f,t+1}^*(z^{t+1}, \eta^{*,t})$ in financial markets and consumes $c^*(z^t, \eta^{*,t})$ units of the foreign consumption basket. The flow budget constraint is given by

$$\begin{aligned} & \sum_{z_{t+1}} Q(z^{t+1}|z^t) \frac{a_{h,t+1}^*(z^{t+1}, \eta^{*,t})}{e_t(z^t)} + \sum_{z_{t+1}} Q^*(z^{t+1}|z^t) a_{f,t+1}^*(z^{t+1}, \eta^{*,t}) + c^*(z^t, \eta^{*,t}) \\ & \leq a_t^*(z^t, \eta^{*,t-1}) + \alpha I^*(z^t) \eta_t^*, \text{ for all } (z^t, \eta^{*,t}). \end{aligned}$$

Note that all investors are subject to non-negative net wealth constraints, given by $a_t(z^t, \eta^{t-1}) \geq 0$ and $a_t^*(z^t, \eta^{*,t-1}) \geq 0$.

FX Arbitrageurs These Mertonian traders are FX arbitrageurs in our model. Only they enforce the no-arbitrage condition in the state-contingent claim market, which governs the evolution of exchange rates:

$$Q(z^{t+1}|z^t) = Q^*(z^{t+1}|z^t) \frac{e_{t+1}(z^{t+1})}{e_t(z^t)}.$$

Taking the logarithm of the no-arbitrage condition yields the following:

$$\ln \frac{e_{t+1}}{e_t} = \ln Q_{t+1} - \ln Q_{t+1}^*. \quad (4)$$

Hence, the percentage rate of depreciation of the RER is determined by the percentage difference between the home and foreign pricing kernels.

2.6.2 Non-Mertonian Traders

The non-Mertonian traders are restricted to fixed portfolio weights. Their total asset holdings at the beginning of period t , are given by their asset position at the end of the previous period, denoted by $\hat{a}_{t-1}(z^{t-1}, \eta^{t-1})$, multiplied by the gross portfolio return, $R_{t,t-1}^p(z^t)$, which depends on their fixed portfolio. These non-Mertonian traders face the following budget constraint for all (z^t, η^t) :

$$\hat{a}_t(z^t, \eta^t) + c(z^t, \eta^t) \leq R_{t,t-1}^p(z^t) \hat{a}_{t-1}(z^{t-1}, \eta^{t-1}) + \alpha I(z^t) \eta_t,$$

where all variables are expressed in units of the home consumption basket. The gross return on the fixed portfolio is given by

$$R_{t,t-1}^p(z^t) = \omega R_{t,t-1}^d(z^t) + (1 - \omega) R_{t,t-1}^f(z^{t-1}),$$

where ω denotes the fixed portfolio shares in domestic equities. In the case of nonparticipants, ω is zero.

The budget constraint of the non-Mertonian traders in the foreign country is given by

$$\hat{a}_t^*(z^t, \eta^{*,t}) + c^*(z^t, \eta^{*,t}) \leq R_{t,t-1}^{p*}(z^t) \hat{a}_{t-1}^*(z^{t-1}, \eta^{*,t-1}) + \alpha I^*(z^t) \eta_t^*,$$

for all $(z^t, \eta^{*,t})$. The gross return on the fixed portfolio is given by

$$R_{t,t-1}^{p*}(z^t) = \omega^* R_{t,t-1}^{d*}(z^t) + (1 - \omega^*) R_{t,t-1}^{f*}(z^{t-1}),$$

where ω^* denotes the fixed portfolio share in foreign equities. Finally, all traders are subject to nonnegative net wealth constraints, given by $\hat{a}_t(z^t, \eta^t) \geq 0$ and $\hat{a}_t^*(z^t, \eta^{*,t}) \geq 0$.

The details of the household problem and its associated Euler equations are described in Appendixes A.1 and A.2.

2.7 Competitive Equilibrium

A competitive equilibrium for this economy is defined in the standard way. It consists of allocations of consumption; allocations of state-contingent claim, bond, and equity choices; and a list of prices such that (i) given these prices, a household's asset and consumption choices maximize her expected utility subject to the budget constraints, the nonnegative net wealth constraints, and the constraints on portfolio choices; and (ii) all asset markets clear.

2.7.1 Pricing Kernel

We use a recursive multiplier method to solve for equilibrium allocations and prices.⁶ This approach has the advantage that we can express the household consumption share, which is consumption c relative to aggregate consumption C , in terms of a ratio of the household's recursive multiplier, ζ , to a single cross-sectional moment of the multiplier distribution, denoted h . To be precise,

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{-\frac{1}{\gamma}}}{h_t(z^t)}, \quad (5)$$

where $\zeta(z^t, \eta^t)$ is the recursive Lagrangian multiplier of the domestic household and $h_t(z^t)$ is defined as a $-1/\gamma$ moment of $\zeta(z^t, \eta^t)$ across traders (see to Appendix A.3 for details). Similarly, the same consumption sharing rule is applied to foreign traders:

$$\frac{c^*(z^t, \eta^t)}{C^*(z^t)} = \frac{\zeta^*(z^t, \eta^{*,t})^{-\frac{1}{\gamma}}}{h_t^*(z^t)}$$

As a result, the home stochastic discount factor is given by the standard Breeden-Lucas expression with a multiplicative adjustment:

$$Q_{t+1}(z^{t+1}|z^t) = \beta \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\gamma} \left(\frac{h_{t+1}(z^{t+1})}{h_t(z^t)} \right)^\gamma, \quad (6)$$

which can be interpreted as the intertemporal marginal rate of substitution (IMRS) of an unconstrained domestic Mertonian trader. The foreign stochastic discount factor of the foreign country is given by

$$Q_{t+1}^*(z^{t+1}|z^t) = \beta \left(\frac{C^*(z^{t+1})}{C^*(z^t)} \right)^{-\gamma} \left(\frac{h_{t+1}^*(z^{t+1})}{h_t^*(z^t)} \right)^\gamma, \quad (7)$$

which can be interpreted as the IMRS of an unconstrained foreign Mertonian trader. As a result, the percentage rate of depreciation of the RER is given by

$$\Delta \ln e_{t+1} = -\gamma(\Delta \ln C_{t+1} - \Delta \ln C_{t+1}^*) + \gamma(\Delta \ln h_{t+1} - \Delta \ln h_{t+1}^*).$$

⁶The use of cumulative multipliers in solving macro-economic equilibrium models was pioneered by Kehoe and Perri (2002), building on earlier work by Marimon and Marimon (1999). Chien, Cole, and Lustig (2011) extend the methodology to incomplete market environments.

2.7.2 Segmentation Mechanism

The key mechanism of our model works through the concentration of aggregate risk among a small pool of investors. As we show later, our calibrated model features a large pool of non-Mertonian traders whose exposure to aggregate risks is relatively low, given their low-risk and home-biased portfolios. In contrast, a relatively small set of investors equipped with better trading technologies —the so-called Mertonian traders —can accumulate a higher level of wealth and smooth consumption better. More importantly, their sophisticated trading technologies allow them to share the country-specific aggregate risk with foreign traders.

The heterogeneity of portfolio choices has a great impact on pricing. First, the concentration of aggregate risk among a small group of Mertonian traders generates a high market price of risk. Moreover, the equity home bias among non-Mertonian traders offers a good risk-sharing opportunity for Mertonian traders across countries. Given the fact that Mertonian traders are marginal investors who price risks, the pricing kernels could be highly correlated as a result of sharing country-specific risk among them, despite the limited sharing capacity at the aggregate level. In particular, the risk sharing at the aggregate level is restricted by a high degree of home bias in consumption.

3 Quantitative Results

We calibrate our model to evaluate the extent to which our model can account for the international correlation in pricing kernels, the volatility of the pricing kernel and the volatility of the RER seen in the data. Our benchmark model considers a symmetric two-country model in which both countries have identical preferences, portfolio restrictions, and shock processes. The benchmark model is calibrated to match several key features of data, including the data on trade in goods and assets. We then perform a number of counterfactual exercises to examine the effects of a home bias in consumption as well as heterogeneous portfolio choices on the dynamics behaviors of the RER and asset pricing.

In subsection 3.4, we demonstrate the effects of a home bias in consumption by varying the parameter θ . In addition, in subsection 3.5, we consider changes in the trader pool to highlight the role of equity market participation. The last subsection removes the heterogeneity of trading technologies – while keeping the home bias in consumption – to emphasize the importance of

different trading technologies.

3.1 Calibration

The home country in our model is set to the United States. The foreign country is an aggregation of four countries: France, Germany, Japan, and the United Kingdom. We collect annual data from International Financial Statistics from 1980 to 2012. The share of U.S. gross domestic product (GDP) in our hypothetical world economy is on average 52%, which is close to half. Thus, we assume an equal size for home and foreign economies. For simplicity and the demonstration of our mechanism, we set parameters such that the two economies are fully symmetric. Given that condition, all the parameters we calibrate applied to both countries.

According to our data, the trade to GDP in our hypothetical world is 0.32. Since there are only export and non-tradable goods in our model, we set the home bias parameter, θ , to one minus half of the trade to GDP ratio, which is 0.84. This calibration is based on that θ is also the share of the home goods in final consumption expenditure in our model. Given θ , the consumption growth process in each country is pinned down by the endowment process because of the preference assumption. The innovation terms in the output shock process, or ε and ε^* , are calibrated into a Markov process to match the following statistics: (1) The consumption growth correlation between two countries is 0.172 (2) The average consumption growth of each country is 2.13% with a standard deviation of 2.36% and (3) ρ is set to 0.95. The resulting correlation of home and foreign endowment growth is -0.21 .

We also consider a two-state first-order Markov chain for idiosyncratic shocks. The first state is low and the second state is high. Following Storesletten, Telmer, and Yaron (2004), we calibrate this shock process by two moments: the standard deviation of idiosyncratic shocks and the first-order autocorrelation of shocks, except that we eliminate the countercyclical variation in idiosyncratic risk. The Markov process for the log of the nondiversifiable income, or $\log \eta$, has a standard deviation of 0.71 and its autocorrelation is 0.89. The transition probability is denoted by

$$\pi(\eta' | \eta) = \begin{bmatrix} 0.9450 & 0.0550 \\ 0.0550 & 0.9450 \end{bmatrix}.$$

The two states of the idiosyncratic shocks, whose mean is normalized to 1, are $\eta_L = 0.3894$ and

$$\eta_H = 1.6106.$$

Following Mendoza, Quadrini, and Rios-Rull (2009), the fraction of nondiversifiable output is set to 88.75%. As shown in Section 2, equities in our model are simply leveraged claims to diversifiable income. Following Abel (1999) and Bansal and Yaron (2004), the leverage ratio parameter is set to 3.

The model operates at an annual frequency. We set the time discount factor $\beta = 0.95$ to deliver the low risk-free rate. The risk-aversion rate γ is set to 5.5 to help to produce a high risk premium in our benchmark calibration.

To match a high market price of risk, a small fraction of Mertonian traders must absorb a large amount of aggregate risk. We therefore set the fraction of Mertonian traders to 5% for both countries. Since 50% of U.S. investors do not hold stocks (according to the 2010 Survey of Consumer Finance data), we set 50% of investors as nonparticipants. The remaining investors are non-Mertonian equity investors, and they represent 45%. Their portfolio is assumed to be market portfolio. With the leverage is set to 3, the shares of home equities and the home risk-free bonds of a market portfolio are 25% and 75%, respectively.

3.2 Computation

The endowment processes of both countries share the same trend. Therefore, the ratio of aggregate consumption between these two countries is stationary. The RER is equal to the ratio of the marginal utility of consumption for unconstrained home and foreign Mertonian investors. Their intertemporal marginal utilities of consumption also decide the home and foreign pricing kernels. Therefore, the RERs as well as pricing kernels are stationary if there is a nonzero measure of non-binding Mertonian traders for both countries in every possible state. We will assume that this is the case.

By equations (6) and (7), the stationary Q and Q^* imply that the growth rates of h and h^* are also stationary despite the processes of h and h^* themselves are not. Similarly, the log difference between h and h^* has to be stationary as a result of stationary RERs. In other words, h and h^* have to share the same stochastic trend for a stationary RER.

To compute our model, we use summary statistics for the aggregate history as state variables, denoted by $z_k \in \mathcal{A}$. More specifically, the percentage deviation of endowment from the growth trend for both countries, $\ln m$ and $\ln m^*$, serve well as a summary statistics for the aggregate

history in our computation. Hence, for every possible states from z_k to z'_k , the pricing kernels are

$$\begin{aligned} Q(z'_k; z_k) &= \beta \left(\frac{C(z'_k)}{C(z_k)} \right)^{-\gamma} h_g(z'_k; z_k)^\gamma \\ Q^*(z'_k; z_k) &= \beta \left(\frac{C^*(z'_k)}{C^*(z_k)} \right)^{-\gamma} h_g^*(z'_k; z_k)^\gamma \end{aligned}$$

where h_g and h_g^* are the growth rate of h and h^* , respectively.

The RER at a grid point $z_k \in \mathcal{A}$ can be derived as follows:

$$e_t(z_k) = \frac{C(z_k)^{-\gamma} h(z_k)^\gamma}{C^*(z_k)^{-\gamma} h^*(z_k)^\gamma} = \left(\frac{m_t^*(z_k)}{m_t(z_k)} \right)^{-\gamma(1-2\theta)} b(z_k)^\gamma,$$

where $b(z_k)$ is the ratio of h and h^* conditional on the state z_k . (see Appendix A for details). Given the stationarity condition, the RER can be written only as a function of the summary statistics for aggregate history:

$$\ln e(z_k) = \gamma(2\theta - 1) \ln \left(\frac{m^*(z_k)}{m(z_k)} \right) + \gamma b(z_k) \quad (8)$$

In our computation, we record the growth rate of h and h^* . Given the stationarity of e_t , the growth rate of h at home and h^* abroad has to satisfy

$$\ln h_g(z'_k; z_k) - \ln h_g^*(z'_k; z_k) = b(z'_k) - b(z_k),$$

which results from equation (4).

Given $\{b(z_k), h_g(z'_k; z_k), h_g^*(z'_k; z_k)\}$, we can completely characterize an equilibrium of this economy, because we have the equilibrium prices, the RERs, and the allocations. See Appendix B for details.

3.3 Quantitative Results of the Benchmark Case

We report the model statistics for the benchmark case in Table I. Taking the standard deviation of equation (4) together with the assumption of symmetric countries, the standard deviation of RER

depreciation in our model is determined by the moments of pricing kernels as follows:

$$\sigma(\Delta \ln e_{t+1}) = \sigma(\ln Q) \sqrt{2(1 - \rho(\ln Q, \ln Q^*))}. \quad (9)$$

According to (9), the standard deviation of RER depreciation is increasing in the standard deviation of the pricing kernels and decreasing in the international correlation in the pricing kernels.

Our benchmark economy delivers high volatility and high correlation of pricing kernels, and these statistics are close to the observed statistics suggested by the pricing data. The standard deviation of the pricing kernels in the model is 0.42 and the correlation of the pricing kernels is 97.5%. According to (9), these statistics imply that the standard deviation of RER depreciation is 9.4%, whereas the observed volatility is 13%. Hence, our calibrated model is capable of producing reasonable volatility of the RER and the pricing kernels, despite the low international correlation in aggregate consumption.

Our success in matching the asset pricing moments relies on two mechanisms governing the two moments in (9). The first mechanism is the uneven distribution of aggregate risk across population. Most of the global aggregate risk is borne by Mertonian investors. The concentration of risk among a small set of investors leads to high volatility of the pricing kernels. The second mechanism works off the ability of the Mertonian investors in the two countries to share country-specific component of aggregate risks among themselves. Therefore, their consumption tends to synchronize, which produces highly correlated pricing kernels.

Also, as a group consumption and portfolio choices of the Mertonian investors are less restricted by the presence of nontraded consumption than those of the non-Mertonian investors because they represent only a small share of the population. However, if the international trade in goods is completely shut down, then there is no reasons for Mertonian investors to hold any external asset. We demonstrate in the next subsection that international trade is essential for the Mertonian investors to share risk across countries.

In Table I, last three rows shows the moments of consumption by investor group. The correlation of consumption and aggregate consumption for the non-Mertonian traders is almost perfect, 0.975, and the corresponding correlation for the Mertonian traders is much lower, 0.725. The correlation of consumption and income is higher for the non-Mertonian equity traders. We can compare this ranking with the estimates of income elasticity of demand in Hummels and Lee (2013). These authors use the U.S. consumer expenditure survey (CEX) data from 1994 to 2010 to estimate the

income elasticity of demand for export goods for each percentile of income distribution. They find that the investors' income elasticity of demand for export goods is decreasing in household income. In our model, the income elasticity of demand for export goods is identical to the income elasticity of demand for final consumption. Therefore, our ranking of the income elasticity of demand will be consistent with theirs if the Mertonian traders have higher income than the non-Mertonian equity traders. This is the case in our model, because the Mertonian traders accumulate a larger amount of high-return assets than the non-Mertonian equity traders.

The last row in Table I shows the standard deviation of the Mertonian traders' consumption relative to the standard deviation of the non-Mertonian equity traders consumption. This ratio in our model is 3.970, indicating that the Mertonian traders' consumption is more volatile. There are two reasons for this. First, the Mertonian traders share aggregate risk among themselves, while the non-Mertonian equity traders do not. Second, the idiosyncratic income shocks do not matter to volatility of aggregate consumption within the group, thanks to the assumption of the law of large numbers. The ranking of consumption volatility in our model is consistent with the evidence in Parker and Vissing-Jorgensen (2009). Using the CEX survey data from 1982 to 2004, they find that the top 5% of investors are estimated to be about 4.5 times more exposed to aggregate consumption shocks than those in the bottom 80%.

[Table 1 about here.]

3.4 Impacts of Home Bias in Consumption

In this subsection, we investigate the role of a home bias in consumption. Intuitively, without a home bias in consumption and without the nontraded good, the law of one price holds and the RER is constant. In this case, investors achieve full risk sharing through international goods markets regardless of frictions in financial markets. To the contrary, if there is no trade due to either a complete home bias or prohibitively large trade frictions, then there will be no incentives for investors to hold external assets. Intuitively, we interpret a larger consumption home bias as increasing international trade frictions.

To explore the impacts of a home bias in consumption on the RER volatility, we consider two exercises. First, we increase the share of the home goods in final consumption expenditure from 0.84 to 0.95, which significantly reduces the volume of trade in goods. The second column of Table II shows these results. There is virtually no change in the volatility of pricing kernel, while the

correlation of pricing kernels drops from 0.975 to 0.892.

A higher degree of home bias in consumption implies that consumers are less inclined to consume the foreign good and hence reduces the incentive to share the country-specific risk with foreign consumers. As a result, the international correlation in consumption growth falls from 16.9% to -10.9% . The fall in the correlation of consumption growth can also be understood by equation (3). As θ approaches unity, the correlation of consumption growth approaches to the correlation of endowment growth, which is -21% .

Evidently, a high degree of home bias in consumption significantly reduces the correlation in the pricing kernels, even though only a small fraction of investors are sharing the country-specific risk. Given equation (9), conditioning on unchanged volatility of the pricing kernels, the sharp fall in the correlation in the pricing kernels produces a sharp increase – as much as 20% – in the RER volatility. The magnitude is more than double that in the benchmark case. Such a positive impact of a home bias in consumption on the RER volatility is similar to the model by Warnock (2003), in which the RER is volatile as a result of nominal shocks.

The second exercise involves the removal of home bias in consumption by setting the share of the nontraded goods in consumption expenditure at 0.5. The third column of Table II confirms that without a home bias in consumption the RER is not volatile, although the pricing kernel is as volatile as in the benchmark case. In this case, the domestic pricing kernel is perfectly correlated with the foreign pricing kernel, as predicted by equation (3), and consequently international risk sharing is complete. Therefore, according to (9) the perfect correlation in pricing kernels produces zero standard deviation of the RER.

These two exercises demonstrate that the home bias in consumption is necessary for generating RER volatility, although a too-high degree of home bias in consumption can generate higher RER volatility than the data. In other words, international trade in goods is critical for international risk sharing. The introduction of frictions in both international trade and finance is novel compared with the existing models, that explain RER volatility as a result of international trade in assets without international trade in goods, such as Alvarez, Atkeson, and Kehoe (2002) and Colacito and Croce (2011). Moreover, our finding is supported by the recent empirical evidence of Fitzgerald (2012). She finds that trade costs impede risk sharing among developed countries, but financial frictions do not impede risk sharing among them. Her finding suggests that international trade in goods is necessary for international risk sharing, as in our model.

[Table 2 about here.]

3.5 Changes in Composition of Traders

In this subsection, we vary the composition of traders to examine the impacts of distribution of aggregate risk on the volatility of the RER, that of the pricing kernels, the international correlation of pricing kernels, and that of consumption growth.

First, we consider two special cases. The first one corresponds to the standard heterogeneous agents economy, such as Krusell and Smith (1998). The second one works like the standard segmented market model where all equity investors are marginal traders. These two model economies can be quantitatively evaluated under our framework by simply varying the composition of traders. For the first case, all the traders is set to Mertonian households. As for the segmented market case, a half of the population are non-participants according to the SCF data as in our benchmark case. The rest 50% of population are equity market participants, for whom we set all them as Mertonian traders, following the calibration strategy in the segmented market literature.

The second column of Table III reports the result of first case. The RER volatility, 13.1%, is happened to be a right number to match the data. However, the volatility of pricing kernel, only 0.131, is way too smooth that obviously violates the Hansen-Jagannathan bounds. The reason is that aggregate risk is equally distributed over the entire population, especially within a country. Since all investors respond to the change in investment opportunities in every period by optimally adjusting their portfolios and make no investment mistakes. Consequently, there is no residual risk to share across borders. For this reason, the correlation of the pricing kernels falls significantly from 0.975 to only 0.387.

What happens with the segmented market case? The third column of Table III reports the answers. Compared to the previous case, now a half of the population do not participate the equity market and create residual aggregate risks. However, there are still another half of population who can absorb the residual risks and hence the aggregate risks are not concentrated enough in order to produce a reasonable asset pricing results. The pricing kernel volatility rises to 0.189 from 0.119 of the first case while it is still far below from the prediction of the data by Hansen-Jagannathan bounds. The correlation of pricing kernel is 0.668, which is between the benchmark and the first case. Therefore, it is quantitatively important to distinguish the difference between marginal traders from the equity market participants as suggested by the empirical studies of household finance.

[Table 3 about here.]

Second, we vary equity market participation rate by changing the relative size of population between the nonparticipants and the non-Mertonian equity traders, while keeping the size of the Mertonian traders constant. In columns (2) and (3) of Table IV, we reduce the size of the nonparticipants from 50% to 40% and 30%, respectively. The results show that, as the equity market participation rate increases, country-specific risk becomes less concentrated among the Mertonian traders. Thus, the volatility of pricing kernel falls as the the equity market participation rate increases. A 10% increase in the equity market participation rate reduces the standard deviation of the pricing kernels by roughly 3% to 4%.

Since the equity portfolio held by the new equity traders is biased toward the domestic equity, their increasing participation forces the Mertonian traders to bear relatively higher residual risk from abroad. As a result, the Mertonian traders increase risk sharing across borders and, as a result, the correlation of the pricing kernels increases. A 10% increase in the equity market participation rate raises the correlation by 0.1% to 0.2%. Overall, according to equation (9), both the increase in the correlation of the pricing kernels and the decrease in their volatility reduce the RER volatility. Quantitatively, a 10% increase in the equity market participation rate reduces the standard deviation of the RER by roughly 1%.

Finally, we vary the composition of the equity market participants, holding the population size of nonparticipants unchanged. In columns 4-5 of Table IV, we increase the population size of the Mertonian traders from 5% to 10% and then to 20%, respectively. As the size of the Mertonian traders increases, aggregate risk becomes less concentrated among them. Thus, the volatility of the pricing kernels falls. As the size of the Mertonian traders increases from 5% to 10% and 10% to 20%, the standard deviation of the pricing kernels falls 7.9% and 8.2%, respectively.

In addition, an increase in the size of the Mertonian traders reduces the residual risk that is shared with their foreign counterparts; therefore the correlation of pricing kernels decreases. As we increase the size of the Mertonian traders in columns (4) and (5), the correlation of the pricing kernels falls from 97.5% to 95.7% and to 90.5%, respectively.

According to equation (9), the decrease in the correlation of the pricing kernels and the decrease in their volatility have competing effects on the RER volatility. Quantitatively, the decrease in the correlation of the pricing kernels dominates and the RER volatility increases as a result of an increase in the size of the Mertonian traders.

[Table 4 about here.]

4 Empirical Evidence

This section provides evidence that international correlation of consumption growth is positively affected by income level and participation in equity markets. In particular, we show that international correlation of consumption growth of high-income regions in the U.S. and Canada is higher than that of low-income regions, and the correlation is positively affected by investment in risky securities, after controlling for trade costs.

Let i denote a U.S. state, where $i = 1, \dots, 50$, and let j denote a Canada's province or territory where $j = 1, \dots, 13$. We estimate the following equation:

$$\rho(\Delta c_{it}, \Delta c_{jt}) = b_0 + b_1 \left((y_i + y_j) - \overline{(y_i + y_j)} \right) + b_2 I_i + b_3 I_j + b_4 \mathbf{T}_{ij} + v_{ij} \quad (10)$$

where $\rho(\Delta c_{it}, \Delta c_{jt})$ denotes the correlation of the first-difference of logarithm of per capita consumption of U.S. state i and that of Canada's province or territory j over all years, y_i denotes the average over time of logarithm of per capita income of U.S. state j , y_j denotes that of Canada's province or territory j , $\overline{(y_i + y_j)}$ is the cross-section average of the sum $y_i + y_j$ over all ij pairs, I_i is a measure of U.S. state-specific investment in equities, I_j is a measure of Canada's province- or territory-specific investment in equities, and \mathbf{T}_{ij} is a vector of their bilateral trade costs. The error term is denoted by v_{ij} .

We include trade costs in the estimating equation for two reasons. First, in theory international risk sharing requires international trade in both goods and assets, thus the presence of trade costs might explain low degree of risk sharing as documented by Fitzgerald (2012). Second, the empirical literature has found that trade costs influence bilateral trade flows in goods markets⁷ Our main coefficients of interests are b_1 , b_2 and b_3 . Our quantitative results in the previous section suggest the hypothesis that $b_1 > 0$, $b_2 > 0$ and $b_3 > 0$. For b_4 , we expect $b_4 < 0$, according to the trade literature.

Our dataset covers 50 U.S. states and 13 Canada's provinces and territories, hence the number of ij pairs is 650. The frequency of consumption and income data is annual, and the sample period is 1997-2014. The U.S. consumption data are from the Bureau of Economic Analysis, and Canada's consumption data are from the Statistics Canada. The consumption series for U.S. states is the real personal consumption expenditure, whereas that for Canada's provinces and territories is the

⁷See Anderson and van Wincoop (2004) for the literature review on trade costs.

per capita real final household expenditure. The income series for both U.S. states and Canada's provinces and territories are the real disposable household income per head, and they are from the OECD Database.

We construct two measures of region-specific investment in equities from records in the individual income tax returns from the U.S. Internal Revenue Service and individual income tax and the benefit returns from Canada's Revenue Agency for the tax year 2012.⁸ The first measure is income share of income from investment in equities. For U.S. states, we calculate it as the ratio of the sum of ordinary dividends amount and net capital gain (net loss) amount to the total income amount. For Canada's provinces or territories, we calculate it as the sum of taxable amount of dividends from taxable Canadian corporations, interest and other investment income, and taxable capital gains to the total income assessed. We include other investment income, because it may include non-taxable amount of dividends. However, other investment income is recorded as a bundle with interest income, thus we cannot exclude interest income from our measure of income share of investment income for Canada's provinces and territories.

The second measure of investment in equities is the share of tax returns reporting investment income from equities in total number of returns. For U.S. states, we calculate it as the ratio of the number of tax returns with ordinary dividends plus the number of tax returns with capital gains (net loss) to the total number of tax returns with total income. For Canada's provinces or territories, we calculate it as the ratio of the number of tax returns with investment income as the number one source of income to the total number of tax returns.

We consider two proxies of bilateral trade costs: the log of bilateral distance and the border dummy. The distance is the great circle distance from the most populated city in U.S. state i to that in Canada's province or territory j , and it is obtained from <http://www.gpsvisualizer.com/calculators>. The border dummy is 1 when U.S. state i shares border with Canada's province or territory j , and zero otherwise.

Descriptive statistics of our dataset are tabulated in Table V. The consumption growth correlation of U.S. states and Canada's provinces or territories is 0.37 on average, and varies from -0.24 to 0.79. As a comparison, the correlation growth correlation from the aggregate statistics is 0.65.⁹ A reason why the correlation of consumption growth from aggregate data is higher than the average from the regional data is that the average is a simple mean.

⁸The most available recent data are the tax years 2012 and 2013. We use data from the tax year 2012 in the benchmark estimation, and use data from the tax year in 2013 in the sensitivity analysis.

⁹The real consumption per capita series at the aggregate level for Canada and the U.S. is from the World Bank. The series are of constant prices.

The mean deviation of state-province-specific combined income ranges from -2% to 3%, whereas the average is 0% by construction. Its range reflects an asymmetry among the top half and the bottom half of the income distribution. Indeed, about 55% of the state-province pairs have combined income below the mean.

The share of equity investment income in total income for U.S. states varies from 4% to 18%, and its average is 8%. The corresponding share for Canada's provinces and territories varies from 1% to 11%, and its average is 6%. From these statistics, U.S. states appear to be more active in the equity market. The share of returns with investment income provides a similar comparison. To be precise, the share of returns with investment income for U.S. states is on average 35%, whereas the corresponding share for Canada's provinces and territories is on average only 3%. Moreover, the share of returns with investment income for U.S. states ranges from 20% to 49%, and that is well above the range for Canada's provinces and territories. Note that the corresponding share for Canada's provinces and territories varies from 1% to 7%.

As for the trade costs statistics, distance displays a much wider variation than the border dummy. In fact, only 15 state-province pairs out of 650 pairs share border.

[Table 5 about here.]

Table VI reports the estimation results from ordinary least squares. Regardless of whether we include trade costs measure, an increase in mean deviation of combined income, an increase in the share of investment income for both U.S. states and Canada's provinces or territories, and an increase in the share of returns reporting investment income as the number one source of income for Canada's provinces or territories are found to significantly increase the consumption growth correlation. However, the share of returns with investment income for U.S. states is found to have no significant impact on the consumption growth correlation.

Quantitatively, a 1% increase in mean deviation of combined income would increase the consumption growth correlation by 5 percentage points. A 1% increase in the share of investment income for U.S. states would increase the consumption growth correlation by 0.7 percentage points. A 1% increase in the share of investment income for Canada's provinces and territories would increase the consumption growth correlation by 0.8 percentage points. A 1% increase in the share of returns reporting investment income as the number one source of income for Canada's provinces or territories would increase the consumption growth correlation by 2 percentage points.

As anticipated, trade costs have negative impacts on the consumption growth correlation, and the negative impact is statistically significant. To be precise, a 1% reduction of distance increases

the consumption growth correlation by 3-4 basis points. However, whether the U.S. state shares border with the Canada's province or not does not have a significant impact on the consumption growth correlation.

[Table 6 about here.]

Next, as a sensitivity analysis we replace the 2012 data on investment income share and share of returns with investment income with the corresponding data in 2013. As reported in Table VII, the results are quite similar to those in Table VI. The quantitative impacts change very little.

[Table 7 about here.]

In sum, our empirical evidence substantiates our theory that the consumption growth correlation among high-income households and households investing in risky equities is higher than that among low-income households and households not investing in risky equities. The evidence supports our proposed story that households' differential ability to share aggregate risk in asset markets internationally offering an explanation for the empirical asset pricing pattern.

5 Conclusion

We use a general equilibrium model with asset trading restrictions and consumption home bias to demonstrate that RER volatility is related to frictions in both goods and financial markets. The asset trading restrictions imposed in our model are in line with the empirical evidence in the household finance literature. With a realistic assumption that most investors do not actively participate in the domestic and foreign equity markets, we reconcile highly correlated and volatile pricing kernels with low correlation in consumption growth.

The insight from our model is that the high cross-country correlation in the pricing kernels is not necessarily evidence of a high degree of international risk sharing. In particular, international risk sharing is aggressively undertaken by a small fraction of sophisticated investors facing no restrictions on asset trade. Despite the small size of fraction, these marginal investors are the arbitrageurs and their portfolio adjustment determines RER volatility. In fact, their portfolio adjustment still generates a positive while far from perfect correlation between the RER and relative consumption growth, as in equation (4). Hence, our model has not totally solved the Backus-Smith puzzle, which documents a zero or even a slightly negative correlation between the RER and relative consumption growth.

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Table I: Benchmark Results

	Benchmark	Data
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	> 0.40
$\sigma(\Delta \ln e)$	0.094	0.13
$\rho(\ln Q, \ln Q^*)$	0.975	> 0.95
$\rho(\Delta \ln C, \Delta \ln C^*)$	0.169	0.171
$\rho(\Delta \ln c, \Delta \ln C)_{Mertonian}$	0.725	Low
$\rho(\Delta \ln c, \Delta \ln C)_{Non-Mertonian}$	0.975	High
$\frac{\sigma(\Delta \ln c)_{Mertonian}}{\sigma(\Delta \ln c)_{Non-Mertonian}}$	3.970	4.5

Note: The simulation results are based on 18,000 agents for each type and 10,000 periods.

Table II: Results of Variation in Home Bias in Consumption

	Benchmark: $\theta = 0.84$	$\theta = 0.95$	$\theta = 0.5$
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.435	0.419
$\sigma(\Delta \ln e)$	0.094	0.202	0
$\rho(\ln Q, \ln Q^*)$	0.975	0.892	1
$\rho(\Delta \ln C, \Delta \ln C^*)$	0.169	-0.109	1

Note: The simulation results are based on 18,000 agents for each type and 10,000 periods.

Table III: Two Special Cases

	Benchmark	Case 1	Case 2
<i>Population size of investors</i>			
Mertonian	0.05	1	0.5
Non-Mertonian Equity	0.45	0	0
Nonparticipants	0.50	0	0.5
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.119	0.189
$\sigma(\Delta \ln e)$	0.094	0.131	0.154
$\rho(\ln Q, \ln Q^*)$	0.975	0.387	0.668

Note: The simulation results are based on 18,000 agents for each type and 10,000 periods.

Table IV: Results of Variations in the Trader Pool

	(1) Benchmark	(2)	(3)	(4)	(5)
<i>Population size of investors</i>					
Mertonian	0.05	0.05	0.05	0.10	0.20
Non-Mertonian Equity	0.45	0.55	0.65	0.40	0.30
Nonparticipants	0.50	0.40	0.30	0.50	0.50
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.396	0.360	0.344	0.262
$\sigma(\Delta \ln e)$	0.094	0.087	0.076	0.101	0.115
$\rho(\ln Q, \ln Q^*)$	0.975	0.976	0.978	0.957	0.905

Note: The simulation results are based on 18,000 agents for each type and 10,000 periods.

Table V: Descriptive statistics

Variables	Sample size	Mean	Std. Dev.	Min	Max
$\rho(\Delta c_{it}, \Delta c_{jt})$	650	0.37	0.21	-0.24	0.79
$(y_i + y_j) - (y_i + y_j)$	650	0.00	0.01	-0.02	0.03
I_i					
Income share of investment income	50	0.08	0.03	0.04	0.18
Share of returns with investment income	50	0.35	0.07	0.20	0.49
I_j					
Income share of investment income	13	0.06	0.03	0.01	0.11
Share of returns with investment income	13	0.03	0.02	0.01	0.07
Log of distance	650	7.25	0.61	4.29	8.67
Border dummy	650	0.02	0.15	0	1

Notes: i is index of U.S. states, and j is index of Canada's provinces and territories. c_{it} and c_{jt} are logarithm of per capita consumption in state i and province or territory j in year t . y_i and y_j are the average over time of logarithm of per capita income in state i and province or territory j .

Table VI: Benchmark Regression Result: Dependent variable is consumption growth correlation

	(1)	(2)	(3)	(4)	(5)	(6)
Mean deviation of combined income	4.708*** (0.857)	5.572*** (0.964)	4.992*** (0.840)	4.711*** (0.857)	5.572*** (0.964)	4.991*** (0.839)
Log of distance	-0.034** (0.014)	-0.038*** (0.015)	-0.043*** (0.014)	-0.032** (0.013)	-0.037*** (0.014)	-0.042*** (0.013)
Border dummy	-0.021 (0.057)	-0.020 (0.057)	-0.009 (0.057)			
US investment income share	0.741** (0.291)			0.735** (0.291)		
CA investment income share	0.827*** (0.285)			0.825*** (0.285)		
US investment returns share		-0.170 (0.142)			-0.171 (0.142)	
CA investment returns share		2.074*** (0.453)			2.066*** (0.452)	
Constant	0.509*** (0.113)	0.644*** (0.134)	0.685*** (0.104)	0.495*** (0.106)	0.631*** (0.129)	0.678*** (0.096)
Sample size	650	650	650	650	650	650
Adjusted R-squared	0.08	0.09	0.06	0.08	0.09	0.06
F-test	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Standard errors are in parentheses. ***, **, and * denotes statistical significance at 10%, 5% and 1%, respectively.

Table VII: Sensitivity Analysis: Data on investment income are from an alternative year

	(1)	(2)	(3)	(4)	(5)	(6)
Mean deviation of combined income	4.388*** (0.863)	5.650*** (0.957)	4.992*** (0.840)	4.389*** (0.863)	5.651*** (0.957)	4.991*** (0.839)
Log of distance	-0.034** (0.014)	-0.039*** (0.015)	-0.043*** (0.014)	-0.032** (0.013)	-0.037*** (0.014)	-0.042*** (0.013)
Border dummy	-0.020 (0.057)	-0.020 (0.057)	-0.009 (0.057)			
US investment income share	1.009*** (0.305)			1.005*** (0.304)		
CA investment income share	0.741*** (0.261)			0.738*** (0.261)		
US investment returns share		-0.188 (0.140)			-0.189 (0.140)	
CA investment returns share		1.937*** (0.432)			1.930*** (0.431)	
Constant	0.504*** (0.111)	0.658*** (0.134)	0.685*** (0.104)	0.490*** (0.104)	0.645*** (0.128)	0.678*** (0.096)
Sample size	650	650	650	650	650	650
Adjusted R-squared	0.08	0.09	0.06	0.08	0.09	0.06
F-test	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Standard errors are in parentheses. ***, **, and * denotes statistical significance at 10%, 5% and 1%, respectively.

A Time-Zero Trading Household Problem

A.1 Time-Zero Trading

We describe an equivalent version of this economy in which all households trade at time zero. The time-zero price of a claim that pays one unit of consumption in node z^t can be constructed recursively from the one-period-ahead Arrow prices:

$$P(z^t)\pi(z^t) = Q(z_t|z^{t-1})Q(z_{t-1}|z^{t-2})\dots Q(z_1|z^0)Q(z_0),$$

$$P^*(z^t)\pi(z^t) = Q^*(z_t|z^{t-1})Q^*(z_{t-1}|z^{t-2})\dots Q^*(z_1|z^0)Q^*(z_0).$$

The real exchange rate is the ratio of Arrow-Debreu prices in node z^t :

$$e_t(z^t) = \frac{P(z^t)}{P^*(z^t)}.$$

The net financial wealth position of any trader in the home country given the history can be stated as

$$-a_t(z^t, \eta^t) = \sum_{s \geq t} \sum_{(z^s, \eta^s) \succeq (z^t, \eta^t)} \tilde{P}(z^s, \eta^s) [\alpha I(z^s) \eta_s - c(z^s, \eta^s)],$$

where $\tilde{P}(z^t, \eta^t) = \pi(z^t, \eta^t)P(z^t)$. Similarly, the asset position of any foreign trader is

$$-a_t^*(z^t, \eta^{*,t}) = \sum_{s \geq t} \sum_{(z^s, \eta^{*,s}) \succeq (z^t, \eta^{*,t})} \tilde{P}(z^s, \eta^{*,s}) [\alpha I^*(z^t) \eta_s^* - c^*(z^s, \eta^{*,s})],$$

where $\tilde{P}^*(z^t, \eta^{*,t}) = \pi(z^t, \eta^{*,t})P^*(z^t)$. From the above equation, we are able to write the household problem in the form of time-zero trading fashion as shown in the next subsection.

A.2 Household Optimization Problem

Following Chien, Cole, and Lustig (2011), we state the household problem in this Arrow-Debreu economy.

A.2.1 Mertonian Traders

We start with the Mertonian traders' problem in the home country. There are two constraints. Let χ denote the multiplier on the present value budget constraint and $\varphi(z^t, \eta^t)$ denote the multiplier on debt constraints. The saddle-point problem of a Mertonian trader can be stated as follows:

$$\begin{aligned}
L = & \min_{\{\chi, \nu, \varphi\}} \max_{\{c\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^t)} \frac{1}{1-\gamma} c(z^t, \eta^t)^{1-\gamma} \pi(z^t, \eta^t) \\
& + \chi \left\{ \sum_{t=1}^{\infty} \sum_{(z^t, \eta^t)} \tilde{P}(z^t, \eta^t) [\alpha I(z^t) \eta_t - c(z^t, \eta^t)] + a_0(z^0) \right\} \\
& - \sum_{t=1}^{\infty} \sum_{(z^t, \eta^t)} \varphi_t(z^t, \eta^t) \left\{ \sum_{s \geq t} \sum_{(z^s, \eta^s) \succeq (z^t, \eta^t)} \tilde{P}(z^s, \eta^s) [\alpha I(z^s) \eta_s - c(z^s, \eta^s)] \right\}.
\end{aligned}$$

The first-order condition with respect to consumption is given by

$$\beta^t c(z^t, \eta^t)^{-\gamma} = \zeta(z^t, \eta^t) P(z^t) \text{ for all } (z^t, \eta^t), \quad (11)$$

where $\zeta(z^t, \eta^t)$ is defined recursively as

$$\zeta_t(z^t, \eta^t) = \zeta_{t-1}(z^{t-1}, \eta^{t-1}) - \varphi_t(z^t, \eta^t),$$

with initial $\zeta_0 = \chi$. It is easy to show that this is a standard convex constraint maximization problem. Therefore, the first-order conditions are necessary and sufficient.

A.2.2 Non-Mertonian Traders

Non-Mertonian traders face additional restrictions on their portfolio choices. Let $\nu_t(z^t, \eta^t)$ denote the multiplier on portfolio restrictions. Given the same definition of other multipliers as in the active trader problem, the saddle-point problem of a nonparticipant trader whose asset in the end of the period is $\hat{a}_{t-1}(z^{t-1}, \eta^{t-1})$ in the home country can be stated as

$$\begin{aligned}
L = & \min_{\{\chi, \nu, \varphi\}} \max_{\{c, \hat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t, \eta^t)} \frac{1}{1-\gamma} c_t(z^t, \eta^t)^{1-\gamma} \pi(z^t, \eta^t) \\
& \chi \left\{ \sum_{t=1}^{\infty} \sum_{(z^t, \eta^t)} \tilde{P}(z^t, \eta^t) [\alpha I(z^t) \eta_t - c(z^t, \eta^t)] + a_0(z^0) \right\} \\
& + \sum_{t=1}^{\infty} \sum_{(z^t, \eta^t)} \nu_t(z^t, \eta^t) \left\{ \begin{array}{l} \sum_{s \geq t} \sum_{(z^s, \eta^s) \succeq (z^t, \eta^t)} \tilde{P}(z^s, \eta^s) [\alpha I(z^s) \eta_s - c(z^s, \eta^s)] \\ + \tilde{P}(z^t, \eta^t) R_{t,t-1}^p(z^t) \hat{a}_{t-1}(z^{t-1}, \eta^{t-1}) \end{array} \right\} \\
& - \sum_{t=1}^{\infty} \sum_{(z^t, \eta^t)} \varphi_t(z^t, \eta^t) \left\{ \sum_{s \geq t} \sum_{(z^s, \eta^s) \succeq (z^t, \eta^t)} \tilde{P}(z^s, \eta^s) [\alpha I(z^s) \eta_s - c(z^s, \eta^s)] \right\}.
\end{aligned}$$

The first-order condition with respect to consumption is given by

$$\beta^t c(z^t, \eta^t)^{-\gamma} = \zeta(z^t, \eta^t) P(z^t) \text{ for all } (z^t, \eta^t),$$

where $\zeta(z^t, \eta^t)$ is defined as

$$\zeta_t(z^t, \eta^t) = \zeta_{t-1}(z^{t-1}, \eta^{t-1}) + \nu_t(z^t, \eta^t) - \varphi_t(z^t, \eta^t).$$

Therefore, the first-order condition with respect to consumption is independent of trading restrictions. The first-order condition with respect to total asset holdings at the end of period $t-1$,

$\widehat{a}_{t-1}(z^{t-1}, \eta^{t-1})$, is

$$\sum_{(z^t, \eta^t)} R_{t,t-1}^p(z^t) \nu_t(z^t, \eta^t) P(z^t) \pi(z^t, \eta^t) = 0 \text{ for all } z^t, \eta^t.$$

This condition varies according to different trading restrictions.

A.2.3 First-Order Condition of Foreign Households

Similarly, the first-order condition (FOC) respect to consumption for all foreign investors is

$$\beta^t c^*(z^t, \eta^{*,t})^{-\gamma} = \zeta^*(z^t, \eta^{*,t}) P^*(z^t) \text{ for all } (z^t, \eta^{*,t}),$$

and the first-order condition with respect to the asset choice for foreign non-Mertonian traders is

$$\sum_{(z^t, \eta^{*,t})} R_{t,t-1}^{p,*}(z^t) \nu_t^*(z^t, \eta^{*,t}) P^*(z^t) \pi(z^t, \eta^{*,t}) = 0 \text{ for all } z^t, \eta^{*,t}.$$

A.3 Stochastic Discount Factor

By summing the first-order conditions with respect to consumption, equation (11), across all domestic households at period t , we can obtain the consumption sharing rule (equation (5) in the main text) as follow:

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{-\frac{1}{\gamma}}}{h_t(z^t)},$$

where $h_t(z^t)$ is defined as $h_t(z^t) \equiv \sum_{\eta^t} \zeta(z^t, \eta^t)^{-\frac{1}{\gamma}} \pi(\eta^t)$. In addition, by plugging back the consumption sharing rule back to the first order condition with respect to consumption, equation (11), we

can obtain the price of home consumption basket at state z^t :

$$P(z^t) = \beta^t C(z^t)^{-\gamma} h_t(z^t)^\gamma$$

Therefore, the home stochastic discount factor is given by the Breeden-Lucas stochastic discount factor (SDF) with a multiplicative adjustment:

$$Q_{t+1}(z^{t+1}|z^t) \equiv \frac{P(z^{t+1})}{P(z^t)} = \beta \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\gamma} \left(\frac{h_{t+1}(z^{t+1})}{h_t(z^t)} \right)^\gamma.$$

Similarly, we can derive the stochastic discount factor of the foreign country:

$$Q_{t+1}^*(z^{t+1}|z^t) \equiv \frac{P^*(z^{t+1})}{P^*(z^t)} = \beta \left(\frac{C^*(z^{t+1})}{C^*(z^t)} \right)^{-\gamma} \left(\frac{h_{t+1}^*(z^{t+1})}{h_t^*(z^t)} \right)^\gamma.$$

A.4 Price of the Final Consumption Good

We start by analyzing the home country's price index. Let $P(z^t)$ be the price level in the home country. The price level represents the minimum expenditure on $c(z^t) = c_n(z^t, \eta^t)^\theta c_x(z^t, \eta^t)^{1-\theta}$. To find the price level, we solve the cost minimization problem:

$$\max_{Y_n(z^t), Y_x^*(z^t)} P(z^t) Y_n(z^t)^\theta Y_x^*(z^t)^{1-\theta} - P_n(z^t) Y_n(z^t) - P_x(z^t) Y_x^*(z^t).$$

The first-order condition implies that

$$\frac{P_n(z^t)}{P(z^t)} = \theta Y_n(z^t)^{\theta-1} Y_x^*(z^t)^{1-\theta} = \theta \frac{C(z^t)}{Y_n(z^t)}$$

$$\frac{P_x(z^t)}{P(z^t)} = (1 - \theta) Y_n(z^t)^\theta Y_x^*(z^t)^{-\theta} = (1 - \theta) \frac{C(z^t)}{Y_x^*(z^t)}$$

Next, we analyze the same problem for the foreign country:

$$\frac{P_n^*(z^t)}{P^*(z^t)} = \theta Y_n^*(z^t)^{\theta-1} Y_x(z^t)^{1-\theta} = \theta \frac{C^*(z^t)}{Y_n^*(z^t)}$$

The first-order condition implies that

$$\frac{P_x^*(z^t)}{P^*(z^t)} = (1 - \theta) Y_n^*(z^t)^\theta Y_x(z^t)^{-\theta} = (1 - \theta) \frac{C^*(z^t)}{Y_x(z^t)}.$$

B Computational Algorithm

In the spirit of Chien, Cole, and Lustig (2011), we develop the following computational algorithm.

Algorithm 1. *Computational algorithm:*

1. Guess functions for the exchange rates $e(z_k)$ as well as for the growth rate of h and h^* ,
 $h_g(z'_k; z_k)$ and $h_g^*(z'_k; z_k)$.

2. Since the consumption process is governed by the endowment shock process, the state-contingent price is determined by

$$\begin{aligned} Q(z'_k; z_k) &\equiv \frac{P(z'_k)}{P(z_k)} = \beta \left(\frac{C(z'_k)}{C(z_k)} \right)^{-\gamma} h_g(z'_k; z_k)^\gamma \\ Q^*(z'_k; z_k) &\equiv \frac{P^*(z'_k)}{P^*(z_k)} = \beta \left(\frac{C^*(z'_k)}{C^*(z_k)} \right)^{-\gamma} h_g^*(z'_k; z_k)^\gamma. \end{aligned}$$

and the level of the exchange rate is given by

$$e(z_k) = \frac{P(z_k)}{P^*(z_k)} = \frac{C(z_k)^{-\gamma} h_t(z_k)^\gamma}{C^*(z_k)^{-\gamma} h_t^*(z_k)^\gamma} = \left(\frac{m_t^*(z_k)}{m_t(z_k)} \right)^{-\gamma(1-2\theta)} b(z_k).$$

3. Given prices, solve the optimization problem of each individual investor.
4. Simulate the economy and derive the implied new exchange rate function, $e(z_k)$, and the new updating rule for $h_g(z'_k; z_k)$ and $h_g^*(z'_k; z_k)$
5. Compare the original guess with the new implied value. If they are the same, we have an equilibrium; otherwise, we iterate $e_t(z_k)$, $h_g(z'_k; z_k)$ and $h_g^*(z'_k; z_k)$.

In our simulation panel, we cannot observe h and h^* in levels since they are nonstationary. However, h and h^* have to share a common stochastic trend, given that exchange rate is stationary. From equation (4), the following equation holds:

$$\ln h_g(z'_k; z_k) - \ln h_g^*(z'_k; z_k) = b(z'_k) - b(z_k) \equiv \Delta(z'_k; z_k). \quad (12)$$

If we know $b(z_k)$ for all possible $z_k \in \mathcal{A}$, then we know the exchange rate $e(z_k)$ and $\Delta(z'_k; z_k)$.

We use $\overline{H}(z'_k, z_k)$ to denote the average growth rate:

$$\overline{H}(z'_k, z_k) \equiv \frac{\ln h_g(z'_k; z_k) + \ln h_g^*(z'_k; z_k)}{2}. \quad (13)$$

From equations (12) and (13), we know that the growth rate of h in a particular country is

given by:

$$\begin{aligned}\ln h_g(z'_k; z_k) &= \overline{H}(z'_k, z_k) + \frac{\Delta(z'_k, z_k)}{2} \\ \ln h_g^*(z'_k; z_k) &= \overline{H}(z'_k, z_k) - \frac{\Delta(z'_k, z_k)}{2}.\end{aligned}$$

Given this investment in notation, we can now describe the updating process $h_g(z'_k; z_k), h_g^*(z'_k; z_k)$,

and $e(z_k)$

Algorithm 2. *Update the guess of $e(z_k)$, $h_g(z'_k; z_k)$ and $h_g^*(z'_k; z_k)$ functions:*

1. For each pair of z_k and z'_k , update the growth rate of h and h^* , denoted by $\ln h_g(z'_k; z_k)_{sim}$ and $\ln h_g^*(z'_k; z_k)_{sim}$, respectively.

2. From equation (13), the updated guess of \overline{H} is given by

$$\overline{H}_{new}(z'_k; z_k) = \frac{\ln h_g(z'_k; z_k)_{sim} + \ln h_g^*(z'_k; z_k)_{sim}}{2}.$$

3. The updated guess of $b_{new}(z_k)$ is the solution of the following minimization problem:

$$b_{new}(z_k) = \arg \min_{b(z_k)} \left(E [h_g(z'_k; z_k) - h_g(z'_k; z_k)_{sim}]^2 + E [h_g^*(z'_k; z_k) - h_g^*(z'_k; z_k)_{sim}]^2 \right),$$

such that

$$\begin{aligned}\ln h_g(z'_k; z_k) &= \overline{H}_{new}(z'_k; z_k) + \frac{\Delta(z'_k; z_k)}{2} \\ \ln h_g^*(z'_k; z_k) &= \overline{H}_{new}(z'_k; z_k) - \frac{\Delta(z'_k; z_k)}{2}\end{aligned}$$

and

$$\Delta(z'_k; z_k) = b(z'_k) - b(z_k).$$

4. The updated guess of $e(z_k)$ and the growth rate of h and h^* are therefore

$$\ln e_{new}(z_k) = \gamma(2\theta - 1) \ln \left(\frac{m^*(z_k)}{m(z_k)} \right) + \gamma b_{new}(z_k)$$

$$\begin{aligned} \ln h_g(z'_k; z_k)_{new} &= \overline{H}_{new}(z'_k; z_k) + \frac{\Delta_{new}(z'_k; z_k)}{2} \\ \ln h_g^*(z'_k; z_k)_{new} &= \overline{H}_{new}(z'_k; z_k) - \frac{\Delta_{new}(z'_k; z_k)}{2}, \end{aligned}$$

where

$$\Delta_{new}(z'_k; z_k) = b_{new}(z'_k) - b_{new}(z_k).$$