Incentives in the Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure for Global Warming*

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Abstract

This paper designs a planning procedure for optimally adjusting the quality of goods and the global atmosphere which can be considered as a complex of Gorman-Lancasterian attributes in the New Consumer Theory. This process called the *Hedonic MDP (Malinvaud-Drèze-de la ValléePoussin) Procedure* is constructed by using the necessary conditions for the efficient combination of *tangible attributes* embedded in the goods produced by the countries, and greenhouse gases (GHGs) as *gaseous attributes* emitted by them. Sen's capability approach is also used, since the impacts of global warming upon each country affect its *functionings à la Sen*, which determine its national beings. It is shown that any country maximizes its national happiness function by consuming and producing goods and emitting GHGs. *A Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure* or the *ζMDP Procedure* is proposed and the existence of a solution is proved. In a local game associated with each iteration of the procedure, it is verified that each country truthfully reveals its dynamic hedonic marginal willingness-to-pay for GHGs as gaseous attributes and tangible attributes embodied in the goods.

Key Words: Drèze-Hagen's hedonic theory, dynamic hedonic marginal willingness-to-pay, Gorman-Lancasterian attributes or characteristics, Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure, national happiness function, New Consumer Theory, Pantaleoni effect, Sen's capability and functionings, ζMDP Procedure

JEL Classification: D6, D11, D13, D62, H31, I31, Q25

1 INTRODUCTION

Never before has there been a global alert with so many serious, worldwide environmental problems. The alarming rise of global warming has become an increasingly more crucial issue in recent decades. Not only is global warming a problem, but acid rain, desertification, deforestation of the tropical rainforests, and stratospheric ozone layer depletion are as well. The Earth's environment is now perceived to be a pure common patrimonial public good that we have to protect. Countries are involved in the fatal problem of the global warming which is now confirmed to be caused by an increase of greenhouse gases' concentration: viz., all the countries on the globe are polluters as well as victims of the gradually warming global climate.

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"Integral" and "exponential" are two key adjectives in consideration of the present state of the world on the brink: e.g., population explosion, buildup of greenhouse gases, increase in the extinction rates of endangered species, and exhaustion rates of natural resources.[1] All of these phenomena in the noosphere, which have been induced by human activities, share common integral and exponential features, especially after the Industrial Revolution. The global atmosphere has been revealing the integral and exponential trend most prominently in recent decades. The Earth's atmosphere is made up of 78% nitrogen, 21% oxygen, water vapor and trace gases. It is a composite of N_2, O_2, H_2, H_2O, and GHGs such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), sulfur hexafluoride (SF_6), perfluorocarbon (PFC) and hydrofluorocarbon (HFC), those of which reduction was agreed by the Kyoto Protocol. These gases can be considered as *gaseous attributes à la Gorman-Lancaster,* since they infinitesimally compose the global atmosphere as a global public good. In their theory goods are regarded as a complex or a composition of characteristics. The New Consumer Theory was advocated by Gorman (1956/1980) and made be well-known by Lancaster (1966).[2] The two terms, attributes and characteristics, are used interchangeably throughout this paper.

The issue of this paper is to design a *hedonic planning procedure* for adjusting the quality which can be represented as a combination of characteristics embedded in the goods as well as in the global atmosphere. For this, I adopt an analytical framework of the New Consumer Theory taken the initiative by Gorman and Lancaster, rigorously analyzed by Drèze and Hagen (1978), and Sen's (1985) Capability Theory. In this context I present a procedure for quality and quantity adjustments by applying the planning method employed in the MDP Procedure proposed by Drèze and de la Vallée Poussin (1971) and Malinvaud (1970-1971), generalized by Fujigaki and Sato (1981) and (1982), Sato (1983), (2009), (2012) and (2016b). This paper develops a theoretical framework involving the countries whose consumption and production activities result in emitting GHGs. Then, it shows the necessary conditions for the countries to consume Pareto efficient quality of goods represented by attributes. Moreover, it characterizes the conditions for the countries to maximize their national happiness function by consuming goods and to maximize their profit function by providing Pareto efficient quality and quantity of products.

The paper proceeds as follows. Section 2 introduces the model of global warming developed by Sato (2015) in the attributes/functionings framework *à la* Gorman/Lancaster/Sen. Section 3 solves optimization problems of the countries to produce and consume goods, and to emit GHGs as gaseous attributes. The hedonic optimality conditions for a complex of gaseous and tangible attributes are derived in Section 3. Section 4 proposes a tâtonnement hedonic procedure for tangible attributes of goods and a non-tâtonnement hedonic procedure for gaseous attributes of the global atmosphere. Section 5 presents a *Drèzian mixed tâtonnement/non-tâtonnement hedonic MDP Procedure* or *the ζMDP Procedure* for determining and revising the total emissions of GHGs and tangible attributes in the goods, and enumerates the properties of the procedure. The existence of a solution is proved. This section introduces the Pantaleoni effect to be explained later. Also explored is that countries' strategic manipulability in a local game associated with each iteration of the procedure. It is verified that each country truthfully reveals its dynamic hedonic marginal willingness-to-pay for GHGs and tangible attributes embedded in goods. Finally, some concluding remarks follow.

[1] Sato (2017) analyses the impacts on biodiversity of countries in the framework of Gorman-Lancaster-Sen.

[2] Gorman's "hidden", but well-known classic paper was written in 1956 and finally published in 1980. To my knowledge, he was the first to employ the term, "characteristics" to represent ingredients of goods. See also Gorman and Myles (1987), Lancaster (1971) and (1991).

2 THE GORMAN-LANCASTER-SEN's THEORY APPLIED TO GLOBAL WARMING

2.1 *Global Atmosphere as a Complex of GHGs as Gaseous Attributes*

This section introduces the Gorman-Lancaster's new consumer theory and Sen's capability theory that are applied to the global warming problem. Also intoduced are some basic knowledge of environmental science. The global atmosphere is regarded as a complex of gaseous attributes including GHGs, which are to be mainly generated by the production and consumption activities, and deforestations all over the world. An index q_{jg} is used to identify the gth gaseous attribute, when one unit of good j is produced. Let $\mathbf{G} = \{C+1, ..., C+G\}$ be the set of GHGs. Each country jointly produces a good with its tangible attributes, as well as GHGs as gaseous attributes as annoying by-products.

Let there be N countries indexed by $i \in \mathbf{N} = \{1, ..., N\}$: the set of countries as producers, each of which is assumed to supply only one good indexed by the same symbol. Let $\mathbf{J}$ be the set of goods. Any good j is composed of C characteristics indexed by $c \in \mathbf{C} = \{1, ..., C\}$, which is the set of *tangible attributes*. Denote q_{jc} as the amount of attribute c embodied in one unit of good j. Let x_{ij} be country i's consumption of good j, then $x_i = (x_{i1}, ..., x_{iJ})$ is country i's consumption vector.

Let z_{i0} be country i's labor force, which is called *characteristic* 0. By utilizing z_{i0}, the amount of each characteristic embodied in the goods and the global atmosphere consumed by country i is given by

$$z_i = (z_{i0}, z_{i1}, ..., z_{iC}, z_{C+1}, ..., z_{C+G})$$

where

$$z_{ic} = \sum_{j \in \mathbf{J}} q_{jc} x_{ij}, \ \forall c \in \mathbf{C}$$

and

$$z_g = \sum_{i \in \mathbf{N}} z_{ig} = \sum_{i \in \mathbf{N}} \sum_{j \in \mathbf{J}} q_{jg} x_{ij}, \ \forall g \in \mathbf{G}.$$

z_{ic} means the consumption of tangible attribute c, while z_{ig} represents the amount of each GHG emitted by each country. Every country is made to consume not only its emissions but also the quantity emitted by the rest of the world. Both equations may be interpreted as *characteristics availability functions* which convert commodities into attributes. They can be regarded as parameters that are objective and common to all consumers, i.e., they have a property of public goods. Thus, the countries as consumers must behave as "quality takers", and they can only change their consumption of z_{ic} and z_{ig} via the choice of x_{ij}.

Let a non-negative vector $\omega_i = (\omega_{i0}, \omega_{i1} ..., \omega_{iC})$ define the initial resources of country i, where $\omega_{i0} > 0$ is an initial endowment of z_{i0}, by which other attributes are utilized, and $\omega_{ic} \geq 0$ is the amount of each attribute that country i has. As in Drèze and Hagen (1978), every good is assumed to have at least one characteristic indexed by j' which differs among goods. Hence, other attributes embedded in good j can be measured per unit of characteristic j' with $q_{jj'} = 1$ for size normalization. Our analysis generalizes their theory to also have gaseous attributes, for some of which evaluation could be minus on account of global warming.

In order to take global warming into consideration, let us introduce the basic framework. When country j produces one unit of good j, it does not choose but to jointly emit GHGs as by-products, $q_{jg} \geq 0$ which is country j's unit emission of the gth trace gas. Thus, $q_{jg}x_j$ is country j's amount emitted of the gth GHG when it produces x_j units of the good. Moreover, z_g is the total quantity of each GHG released all over the world.

2.2 *Concentration of GHGs and the Global Temperature Increase*

A part, $\alpha_g z_g$, $0 < \alpha_g < 1$, $\forall g \in \mathbf{G}$, of emissions of GHG g, is observed to go to the atmosphere and the rest, $(1-\alpha_g)z_g$, is perceived to be absorbed by the oceans and forests as carbon sinks, if g is carbon dioxide. Of this amount, about 43% of CO_2 emissions are observed to be absorbed. The mass of the gth GHG staying in the atmosphere is $\alpha_g z_g$, $\forall g \in \mathbf{G}$. A disintegration rate or an inverse of a lifetime of each trace gas is denoted as μ_g, $0 < \mu_g < \alpha_g$, $\forall g \in \mathbf{G}$. An annual value of z_g can be scientifically deduced and concentration ζ_g of each GHG is annually reported. The problem about GHGs is that they are not flows (emissions), but stocks (concentration). Let $t \in [0, \infty)$ be the set of time argument and all variables be a function of time t, however, let us omit this argument if no confusion would arise. Let $\nu_g > 0$, $\forall g \in \mathbf{G}$, be a conversion parameter from mass (GtC/year) to concentration (ppm) of GHGs, then the latter is represented by the time derivative of concentration

$$\dot{\zeta}_g(t) = \nu_g \alpha_g z_g(t) - \mu_g \zeta_g(t), \ \forall g \in \mathbf{G}.$$

One observes therefore a vector of GHGs' concentration as a stock

$$Z(t) = (\zeta_1(t), ..., \zeta_G(t)).$$

The following notation is used in the sequel.

T : temperature (°C)

Al : planetary albedo (0.3) determining how much of the incoming energy is reflected by the atmosphere

Ω : solar constant (1372Wm^{-2})

ε : emissivity (0.95) of the surface of the earth being its effectiveness in emitting energy as thermal radiation

σ : Stefan-Boltzmann constant ($5.67 \times 10^{-8}\mathrm{Wm}^{-2}\mathrm{K}^{-4}$)

[the outgoing flux is $\varepsilon\sigma T^4$ by the Stefan-Boltzmann Law]

c_h : specific heat capacity

Ab : a coefficient (0.3) determining how much of the energy which is not absorbed by the surface of the earth.

Following Greiner (2004), the difference between the outgoing flux and the incoming radiative flux is given by

$$F = \frac{21}{109}.$$

Next we incorporate the effect of GHGs' concentration to global warming. Let $\zeta^t_{\mathrm{CO}_2}$ be a concentration of CO_2 at time t and $\zeta^0_{\mathrm{CO}_2}$ its concentration at the Pre-industrial Revolution; e.g., the former is 400ppm in 2014 and the latter is 280ppm in 1750 as a reference year. Let ln denote the natural logarithm, and IPCC (1990) supposed the radiative forcing (Wm^{-2}) of CO_2 as

$$\Xi(\zeta^t_{\mathrm{CO}_2}) = 6.3 \ln \frac{\zeta^t_{\mathrm{CO}_2}}{\zeta^0_{\mathrm{CO}_2}}.$$

Sato (2015) took two main GHGs: CO_2 and N_2O into consideration. Let β_g be a Global Warming Potential and μ_g is an inverse of an atmospheric lifetime compared with CO_2 ($\beta_{CO_2} = 1$ and $\mu_{CO_2} = 1$). Following Michaelis (1990), N_2O's contribution to global warming is $\beta_g \mu_g = 29/6$ compared with CO_2.

Remark that the global warming is due to a buildup of concentration as a stock, $Z(t)$ of GHGs

$$\Xi(Z(t)) = 6.3 \sum_{g \in \mathbf{G}} \beta_g \mu_g \ln \frac{\zeta^t_{CO_2}}{\zeta^0_{CO_2}}.$$

The temperature increase of the earth with global warming is calculated by

$$\begin{aligned}
\dot{T}(t)c_h &= \frac{\Omega(1 - A\ell)Ab}{4} - \varepsilon\sigma F T^4 + 6.3 \sum_{g \in \mathbf{G}} \beta_g \mu_g \ln \frac{\zeta^t_{CO_2}}{\zeta^0_{CO_2}} \\
&= \frac{1372}{4} 0.21 - 0.95(5.67 \times 10^{-8}) \frac{21}{109} T^4 + 6.3 \sum_{g \in \mathbf{G}} \beta_g \mu_g \ln \frac{\zeta^t_{CO_2}}{\zeta^0_{CO_2}} \\
T(0) &= T_0.
\end{aligned}$$

The average surface temperature of the earth with global warming is a function of a vector of GHGs' concentration.

$$T(t) = T(Z(t)), \ \forall t \in [0, \infty).$$

To obtain a desired result, an assumption is needed.

Assumption 1. $T(t)$ is concave and twice continuously differentiable, with $\partial T(t)/\partial \zeta_g(t) > 0$ and $\partial^2 T(t)/\partial \zeta_g^2(t) > 0$, $\forall g \in \mathbf{G}$, $\forall t \in [0, \infty)$.

This assumption means that the more GHGs emitted into the global atmosphere, the more accelerated global warming. Global warming is a typical example of a pure public good which is both non-rival and non-excludable. The CO_2 concentration is observed higher in high latitudes than the low ones. Global warming has now also been regarded as an intertemporal negative externality which has been "indirectly and privately provided" by burning fossil fuels and emitting GHGs into the global atmosphere.

2.3 *Beings and Functionings of Countries under Global Warming*

The Gorman-Lancasterian characteristics theory is suitable to analyze goods which are perfectly divisible and decomposable into elements as attributes. The characteristics availability functions are applied to any country whose utilizations, however, differ from country to country. Let the countries have *national functionings*, by extending one of the important concepts *à la* Sen (1985) to fully appraise the value of goods and characteristics consumed by countries. Each country's physical and demographical situations differ, so I must introduce national functionings. Any country's beings are representable as a function of a vector of its functionings. The set of country i's functionings is denoted $\mathbf{K}_i$. Not to mention, the number of functionings differs among countries. If it chooses a vector of functionings, then its *being* is generated by its functionings,

$f_{ik}, \forall k = 1, ..., K_i \in \mathbf{K}_i$.[3] Functionings of any country are affected by its consumption and that of other countries.

National beings as a function of the vector of functionings may be represented by

$$b_i = b_i \left(f_{i1} \left(z, T(Z) \right), ..., f_{iK_i} \left(z, T(Z) \right) \right)$$

where $z = (z_1, ..., z_N)$.

Here I need another assumption.

Assumption 2. For any $i \in \mathbf{N}$, $f_{ik}, \forall k \in \mathbf{K}_i$, is twice continuously differentiable.

Remark 1. It is natural to consider that changing the use of z_{i0} can vary country i's functionings. The signs of $\partial f_{ik}/\partial z_{ic}$ and $\partial^2 f_{ik}/\partial z_{ic}^2$ depend upon what characteristic c is, i.e., they can be $\{+, 0, -\}$ according to attribute c which is good, irrelevant, or bad, respectively, for country i's well-being. If c is CO_2, then the sign may be positive for many countries with high consumption or negative for many countries with low consumption because of enormous damages due to an accelerating global warming. The sign may be positive for some countries that are located in cold areas with a temporal increase of agricultural products as in the northern hemisphere. Hence, the signs of $\partial f_{ik}/\partial T$ and $\partial^2 f_{ik}/\partial T^2$ can be $\{+, 0, -\}$ subject to the climatic situation of each country.[4] In the above equation, I recognize that the gaseous attributes $(z_{C+1}, \ldots, z_{C+G})$ are doubly counted both in z and Z, since CO_2 is ambivalent. Remark that z_{ig} directly affects the national functionings and indirectly affects them through a temperature increase due to global warming. Hence, CO_2 has an ambivalent value, because consumptions and productions emit it and accelerate the tendency of global warming. Consuming a huge amount of goods in high consumption societies in developed countries can be supported from combusting an enormous quantity of fossil fuels which can be acquired by utilizing some functionings, i.e., emitting CO_2 in the ambient air. However, the amount of CO_2 accumulated in the atmosphere has been causing global warming which is now the greatest menace to the human beings. f_{ik} therefore requires z_i as an argument which includes gaseous attributes, and T is a function of the concentration of GHGs.

2.4 *National Happiness Function and Valuing National Well-Being*

Let us personify the countries and let any country i have its *National Happiness Function* which is assumed to depend upon its being and those of other countries, thus, one observes

$$h_i = h_i \left(b_i, b_{-i} \right)$$

where $b_{-i} = (b_1, ..., b_{i-1}, b_{i+1}, ..., b_N)$.

[3] $\mathbf{K}_i$ includes country i's cropping ability which heavily depends upon its regional climate. Shore-protection works against the rise of the sea level and preservation of flora and fauna may also be involved in the set $\mathbf{K}_i$. Humanitarian assistance to poor countries and technology transfers to submerging countries, e.g., Kiribati and Maldives, and developing methods to help alleviate the possible spread of tropical infectious diseases, such as malaria and dengue or breakbone fever, are examples of national functionings.

[4] Climate damages can be measured in physical units, which are in order: (i) increasing sea level due to temperature changes, (ii) coastal erosion and storms, (iii) loss in agricultural yield, (iv) spread of tropical epidemics such as malaria, dengue or breakbone fever, etc. It is natural to differentiate greenhouse damages among several areas in a country that has a large territory, such as Australia, Canada, China, India, Russia, and the United States. However, this paper treats each country as one area for the sake of simplicity.

Every countries' use of functionings could vary country i's national happiness. In order to obtain the desired results, another differentiability assumption is needed.

Assumption 3. For any $i \in \mathbf{N}$, h_i is strictly quasi-concave and twice continuously differentiable with $(\partial h_i/\partial b_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0}) \neq 0$ for at least one $k \in \mathbf{K}_i$.

Total differentiation of h_i gives a definition of country i's *hedonic marginal willingness-to-pay* (*HMW*) or *hedonic shadow price* of any tangible attribute $c \in \mathbf{C}$ and gaseous attribute $g \in \mathbf{G}$: $\forall i \in \mathbf{N}$

$$\pi_{ic} = \frac{\sum_{k \in \mathbf{K}_i} (\partial h_i/\partial b_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{ic})}{\sum_{k \in \mathbf{K}_i} (\partial h_i/\partial b_i)(\partial b_i/\partial f_{ik})\,(\partial f_{ik}/\partial z_{i0})}$$

and

$$\pi_{ig} = \frac{\sum_{k \in \mathbf{K}_i} (\partial h_i/\partial b_i)(\partial b_i/\partial f_{ik}) \left\{\partial f_{ik}/\partial z_{ig} + (\partial f_{ik}/\partial T)(\partial T/\partial \zeta_g)(d\zeta_g/dz_g)(\partial z_g/\partial z_{ig})\right\}}{\sum_{k \in \mathbf{K}_i} (\partial h_i/\partial b_i)(\partial b_i/\partial f_{ik})\,(\partial f_{ik}/\partial z_{i0})}.$$

Remark 2. π_{ic} is characteristic c's marginal contribution to country i's *marginal happiness* through its functionings in terms of the consumption of characteristic 0, z_{i0}. It may correspond to a "marginal rate of substitution (MRS)" between attribute c and z_{i0} in the utility theoretical context. Note that our "MRS" is different from Drèze and Hagen (1978), since it involves the concepts of national functionings and happiness functions *à la* Sen, and moreover, it can also represent attributes with negative evaluations. Hence, I have had to replace a happiness function for a utility function. Note that the analysis of Drèze and Hagen allows for negative MRSs of some consumers, with an additional assumption that $\sum_i \pi_{ig} > 0$ as expressed in our notation. This means that even if some countries put negative marginal evaluation $\pi_{ig} < 0$ on some GHG g, the aggregate value of MRSs over countries can still be positive, i.e., that they admit this attribute. However, $d\zeta_g/dz_g$ has a measure zero for any country, thus, the second term in the brackets is zero. This fact entails that any country is apt to ignore the impacts of own emissions of GHGs upon global warming. Whether any country considers a commodity as good, irrelevant, or bad to its functionings is verified by checking the sign of $\sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} x_{ij}$ for each good j. Moreover, whether a good is globally optimal or not may also be examined by summing the value as $\sum_i \sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} x_{ij}$ for any good j.

The set of feasible functionings vector for any country is the *national capability set*, i.e., opportunities to achieve any national well-being. Let $\mathbf{X}_i$ be the set of country i's entitlements of goods. Given x_i and T, one can represent the set of feasible beings vector, or the national capability set of country i

$$\mathbf{B}_i\,(x_i, T) = \{b_i |\ b_i = b_i\,(f_{i1}\,(z, T(Z)), ..., f_{iK_i}\,(z, T(Z))), \forall x_i \in \mathbf{X}_i\}.$$

In our context a country enhances its national happiness by employing its labor force and functionings. The health of the population is not an ultimate objective, but just a means which permits people to experience agreeable lifestyles on a not too hot earth. The crucial problem is that many countries' national capability set would shrink due to accelerating global warming in the future.[5]

[5] See note 4.

3 HEDONIC OPTIMALITY CONDITIONS UNDER GLOBAL WARMING

3.1 *Efficiency Conditions for Countries as Consumers*

Let me characterize the conditions for a nationally optimal consumption of goods as well as the global ambient air in terms of characteristics including gaseous attributes. I have added above "climatical constraints" to confirm an optimal composition of the global atmosphere to maximize a national happiness function, which depends upon its functionings[6].

Consider that countries are well-informed about man-made future climate changes due to a buildup of GHGs, and that they have an incentive to optimize the composition of the global atmosphere in order to aim at achieving "national best-being", by maximizing their national happiness function. It is assumed that all international markets are perfectly competitive and cleared for all goods. Let any country produce one good by using its labor force x_{i0} whose price is normalized to unity. Each country solves its optimization problem.

$$\begin{aligned}
&\text{Max } h_i = h_i(b_i, b_{-i}) \\
&s.t. \;\; b_i = b_i\left(f_{i1}\left(z, T(Z)\right), ..., f_{iK_i}\left(z, T(Z)\right)\right) \in \mathbf{B}_i\left(x_i, T\right) \\
&\qquad z_{i0} = x_{i0} + \sum_{j \in \mathbf{J}} p_j x_{ij}.
\end{aligned}$$

The characterization reads as follows.

Proposition 1. *A nationally optimal consumption of goods as a composition of Gorman-Lancasterian tangible and gaseous attributes is characterized for any $i \in \mathbf{N}$*

$$\sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} q_{jc} \leq p_j, \;\; \left(\sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} q_{jc} - p_j \right) x_{ij} = 0, \;\; \forall j \in \mathbf{J}$$
$$z_{i0} = x_{i0} + \sum_{j \in \mathbf{J}} p_{ij} x_{ij}.$$

Remark 3. These conditions are not only necessary but also sufficient from the assumptions on the functions. In the equations π_{ic} signifies a hedonic shadow price of attribute c acquired by utilizing country i's labor force and functionings. The left-hand side of the first equation is the sum of country i's marginal evaluations of the tangible attributes embodied in one unit of a good, as well as those of the gaseous characteristics emitted when producing one unit of the good. The first equation means that the unit price of the good is equal to the sum of marginal contributions of attributes to country i's marginal happiness through its labor force and functionings. The condition assures a Pareto optimality for a quantity of each good, and gives a basis upon which goods a country chooses to consume.

[6] The model presented here can also be applied to the problems of urban warming and heat island The most important intangible attribute is heat in that case. For this interesting theme, see Sato (2006), (2008), (2015) and (2016a,b).

3.2 *Efficiency Conditions for Countries as Producers*

The framework developed by Sato (2015) is employed to involve the phenomenon of global warming due to the concentration of GHGs released by the countries all over the world. As in Tulkens (1978, 3.2), it is assumed that every country is characterized not only by its national happiness function, but also by production possibilities, represented by a specific production function. This subsection is devoted to present the optimization by profit maximizing countries to supply one good with an optimal product quality to other countries as consumers. Country i's input-output vector is denoted as $y_i = (x_{i0}, x_i, q_{i1}, ..., q_{iC}, q_{iC+1}, ..., q_{iC+G})$. For the sake of simplicity, it is assumed that each country produces only one output to maximize its profit subject to the production function

$$\upsilon_i = \upsilon_i\left(x_{i0}, x_i, q_{i1}, ..., q_{iC}, q_{iC+1}, ..., q_{iC+G}\right) \leq 0$$

where q_{jg} is an amount of any GHG emitted into the atmosphere when producer produces one unit of good j, and $(q_{iC+1}, ..., q_{iC+G})$ is a vector of gaseous attributes emitted by country j. The production function may not be convex, but the difficulties arising from nonconvexities are not treated here, so I make an assumption.

Assumption 4. For any $i \in \mathbf{N}$, υ_i is convex and twice continuously differentiable.

When $x_{ij} > 0$, p_j could be computed as $\sum_c \pi_{ic} q_{jc}$ from **Proposition 1**, the profit maximization problem for the countries as producers are given by[7]

$$Max \;\; P_j = \sum_{i \in \mathbf{N}} \sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} q_{jc} x_{ij} - x_{j0}, \;\; \forall j \in \mathbf{J}.$$

The first term is the revenue, and the second one is the cost: x_{j0} is country j's labor force as an input used to produce the amount of good j, x_j with tangible attributes and emitted gaseous attributes. By assuming an interior solution, I have the following result.

Proposition 2. *Necessary conditions for Pareto optimal product quality in terms of tangible and gaseous attributes are*

$$\sum_{i \in \mathbf{N}} \pi_{ic} x_{ij} = \frac{\partial x_{j0}}{\partial q_{jc}}, \;\; \forall j \in \mathbf{J}, \;\; \forall c \in \mathbf{C} \cup \mathbf{G}, \;\; c \neq j'.$$

Remark 4. This equation establishes a Pareto optimality for an amount of each attribute and determines a vector of optimal tangible characteristics embodied in the goods supplied by the countries that simutaneously emit GHGs as gaseous attributes. The LHS of the equation is the marginal revenue which is the aggregate of the countries' marginal evaluations of a change of attribute c embedded in x_j. The R.H.S. is the marginal cost to produce q_{jc} in terms of x_{j0}.

Countries also change the amount of good that they supply, i.e., $x_j = \sum_i x_{ij}$, then we have the next result.

[7] To construct a producer's profit function, I followed Drèze and Hagen (1978) who wrote that "the implicit price could be computed... and they would in equilibrium be the same for all consumers. So we do not have to make price differentiation among consumers".

Proposition 3. *Necessary conditions for Pareto optimal quantity are*

$$\sum_{c \in \mathbf{C} \cup \mathbf{G}} \pi_{ic} q_{jc} = \frac{\partial x_{j0}}{\partial x_j}, \ \forall j \in \mathbf{J}.$$

Remark 5. This equation signifies a Pareto optimality for an amount of each good. The L.H.S. is the marginal revenue which is the aggregate of the countries' marginal evaluations of changing amount of good, x_j. The R.H.S. is the marginal cost to produce x_j. From **Proposition 1** and **3**,

$$p_j = \frac{\partial x_{j0}}{\partial x_j}, \ \forall j \in \mathbf{J}.$$

Denote $\gamma_j \equiv \partial x_{j0} / \partial x_j$.

4 THE HEDONIC MDP PROCEDURE FOR GLOBAL WARMING

4.1 *Hedonic Optimality Conditions Applied to the MDP Procedure*

The procedure which I present below can achieve both qualitative and quantitative Pareto optimality. Let $\mathbf{P}$, $\mathbf{P}_0$, and $\mathbf{B}$ be the sets of Pareto, individually rational hedonic Pareto, and boundary optima, respectively. Assume $\mathbf{P}_0 \cap \mathbf{B} = \phi$ and the issue is the same as one in which the boundary optima has been elaborately avoided. To reach a point in $\mathbf{P} \backslash \mathbf{P}_0$ is not a task given to the MDP Procedure, so I may confine myself to focus on the set $\mathbf{P}_0$. In order to achieve any limit point in $\mathbf{P}_0 \cap \mathbf{B} \neq \phi$, an alternative approach is needed. Conventional mathematical notation is used throughout in the same manner as in Sato (1983), (2009), (2012) and (2016a).

An allocation $a = (x_1, ..., x_N, y_1, ..., y_N)$ is a $2N$ tuple of vectors $\{x_i\}, \{y_i\}$, and the set of feasible allocations is denoted $\mathbf{A}$. Denote $x^* = (x_1^*, ..., x_N^*)$ and $y^* = (y_1^*, ..., y_N^*)$. The following definitions are used.

Definition 1. An allocation a is feasible if it satisfies the following conditions:

$$\sum_{i \in \mathbf{N}} x_{i0} = \sum_{i \in \mathbf{N}} \left\{ \sum_{j \in \mathbf{J}} x_{ij} + \upsilon_i(y_i) \right\}.$$

Our procedure is to adjust the private goods' quality and quantity, and the atmospheric quality as a global public good. Let a and a^* support $z = (z_1, \ldots, z_N)$ and $z^* = (z_1^*, \ldots, z_N^*)$, respectively. Denote $b_i(z) = b_i(z, T(Z))$, $b(z) = (b_1(z), ..., b_N(z))$ and $b(\omega) = (b_1(\omega_1), ..., b_N(\omega_N))$.

Definition 2. An allocation a is individually rational if and only if

$$h_i(b(z)) \geq h_i(b(\omega)), \ \forall i \in \mathbf{N}.$$

Definition 3. A hedonic Pareto optimum for this economy is an allocation $a^* \in \mathbf{A}$ such that there exists no feasible allocation a with

$$h_i(b(z)) \geq h_i(b(z^*)), \ \forall i \in \mathbf{N}$$

and

$$h_\ell(b(z)) > h_\ell(b(z^*)),\ \exists \ell \in \mathbf{N}.$$

Suppose that $\pi_{ijc} \equiv \pi_{ic} x_{ij}$ is country i's *hedonic marginal willingness-to-pay* for a quantity of the cth tangible attribute embodied in x_{ij} units of jth good. Let $\gamma_{jc} \equiv \partial x_{j0}/\partial q_{jc}$, $c \neq j'$, be the marginal cost (MC) to produce attribute c embedded in one unit of good $j \in \mathbf{J}$. For any $g \in \mathbf{G}$, $\pi_{ijg} \equiv \pi_{ig} x_{ij}$ is country i's *HMW* for an amount of the gth GHG as a gaseous attribute emitted when x_{ij} units of good j are produced. $\gamma_{jg} \equiv \partial x_{j0}/\partial q_{jg}$, $g \neq j'$, is the MC to reduce one unit of the gth GHG. This is the MC to abate one unit of gaseous attribute g.

Then, one observes the *Hedonic Samuelson's Conditions.*

Lemma 1. *Necessary and sufficient conditions for any hedonic Pareto optimum are*

$$\sum_{i \in \mathbf{N}} \pi_{ijc} \leq \gamma_{jc} \ \text{ and } \ \left(\sum_{i \in \mathbf{N}} \pi_{ijc} - \gamma_{jc} \right) q_{jc} = 0,\ \forall j \in \mathbf{N},\ \forall c \in \mathbf{C} \cup \mathbf{G}.$$

Remark 6. Note that Hedonic Samuelson's Conditions are reminiscent of the original Samuelson's Conditions, which may tacitly assume public goods with fixed quality, i.e., q_{jc}, $\forall j \in \mathbf{N}$, $\forall c \in \mathbf{C} \cup \mathbf{G}$. Alternatively, they are supposed to be some constant, or it can be interpreted that their quality has already been determined otherwise.

4.2 *The Hedonic MDP Procedure for Tangible and Gaseous Attributes*

The process that I design belongs to a family of quantity-guided processes. The MDP Procedure is the best-known member in the class of quantitative processes, in which a planning center asks individual agents to announce their MRSs between each public good and a private good as a numéraire good. Then the center revises the allocation according to the discrepancy between the reported MRSs and the MC which is a technological information assumed to be known to the center. The relevant information exchanged between the center and the periphery is in the form of quantity.

Let us introduce here a Hedonic MDP Procedure for adjusting attributes embodied in the global ambient air as well as in the goods that countries produce. Let us design a planning algorithm based on the hedonic optimality conditions for the goods and the global atmosphere. Remark that the countries do not necessarily reveal their true HMW, π_{ijc}, instead, they may announce ψ_{ijc}, which need not to be equal to the true one.

Using these local information announced by the countries, a *Tâtonnement Hedonic MDP Procedure* for tangible attributes embodied in the goods reads, $\forall j \in \mathbf{J}, \forall c \in \mathbf{C}$, $c \neq j'$:

$$\begin{cases} \dot{q}_{jc}(t) = \xi_{jc}(t) \left\{ \sum_{i \in \mathbf{N}} \psi_{ijc}(t) - \gamma_{jc}(t) \right\} \\ \\ \xi_{jc}(t) \equiv \begin{cases} 0 & \text{if } \ q_{jc}(t) = 0 \ \text{ and } \ \dot{q}_{jc}(t) < 0 \\ \\ 1 & \text{otherwise.} \end{cases} \end{cases}$$

If c is a new characteristic, the operator $\xi_{jc}(t)$ is used to avoid any reduction of this attribute in the negative direction.

As vexing by-products, producing goods entails to emit GHGs, whose evolution dynamics is represented by a *Non-Tâtonnement Hedonic MDP Procedure* for GHGs as gaseous attributes embodied in the global atmosphere, $\forall j \in \mathbf{J}, \forall g \in \mathbf{G}$, $g \neq j'$:

$$\dot{q}_{jg}(t) = \sum_{i \in \mathbf{N}} \psi_{ijg}(t) - \gamma_{jg}(t).$$

Each amount of GHG is increased (resp., decreased), i.e., global atmosphere is warmed (resp., cooled), for which any country is asked to contribute to (resp., is compensated for) global warming. The case with $q_{jg} = 0$ is unnecessary to be treated because there is already enough of each GHG that can be reduced, hence the operator such as $\xi_{jg}(t)$ is not needed for any $g \in \mathbf{G}$. If $\sum_{i \in \mathbf{N}} \psi_{ijg}(t)$ takes a negative sign, then $\dot{q}_{jg}(t) < 0$, i.e., the gth GHG is reduced. It is, however, terribly difficult to decrease the GHGs' emissions, since many countries are "fatally addicted to" consume products and fossil fuels, thus, they keep continuing to emit GHGs as usual. Therefore, $\dot{q}_{jg}(t)$ cannot easily be decreased, and unfortunately, the global warming is to proceed.

From **Proposition 3**, country i's consumed amount of any good j is adjusted by the following formula: $\forall i \in \mathbf{N}, \forall j \in \mathbf{J}$

$$\begin{cases} \dot{x}_{ij}(t) = \xi_{ij}(t) \left\{ \sum_{c \in \mathbf{C} \cup \mathbf{G}} \psi_{ic}(t) q_{jc}(t) - \gamma_j(t) \right\} \\ \\ \xi_{ij}(t) \equiv \begin{cases} 0 & \text{if} \quad x_{ij}(t) = 0 \quad \text{and} \quad \dot{x}_{ij}(t) < 0 \\ \\ 1 & \text{otherwise.} \end{cases} \end{cases}$$

Country i's amount consumed of attribute c embodied in the goods increases or reduces by the formula

$$\dot{z}_{ic}(t) = \sum_{j \in \mathbf{J}} q_{jc}(t) \dot{x}_{ij}(t), \ \forall i \in \mathbf{N}, \ \forall c \in \mathbf{C} \cup \mathbf{G}$$

and its work force is adjusted according to the equation

$$\dot{z}_{i0}(t) = -\sum_{j \in \mathbf{J}} \sum_{c \in \mathbf{C} \cup \mathbf{G}} \left\{ \psi_{ijc}(t) \dot{q}_{jc}(t) + \psi_{ic}(t) \dot{x}_{ij}(t) \right\} + \delta_i \sum_{j \in \mathbf{J}} \left\{ \sum_{c \in \mathbf{C} \cup \mathbf{G}} \dot{q}^2_{jc}(t) + \dot{x}^2_{ij}(t) \right\}$$

where $\delta_i > 0$, $\forall i \in \mathbf{N}$ and $\sum_{i \in \mathbf{N}} \delta_i = 1$.

Remark 7. $\delta = (\delta_1, ..., \delta_N)$ is a vector of distributional coefficients determined by the planner prior to the beginning of the procedure's operation. Its role is to share among the countries the "global surplus", $\delta_i \sum_{j \in \mathbf{J}} \left\{ \sum_{c \in \mathbf{C} \cup \mathbf{G}} \dot{q}^2_{jc}(t) + \dot{x}^2_{ij}(t) \right\}$ which is always nonnegative except in equilibrium.

5 A DRÈZIAN MIXED TÂTONNEMENT/NON-TÂTONNEMENT HEDONIC MDP PROCEDURE

5.1 *The ζMDP Procedure with the Pantaleoni Effect*

Non-tâtonnement procedures are of concern in real economic life as well as in our environmental issues, since greenhouse gas emissions and the resulting global atmospheric quality vary incessantly overtime. Hence, in view of obvious practical relevance, this section formalizes a non-tâtonnement version of the continuous-time process. Von dem Hagen (1991) introduced the concept of the "Pantaleoni effect" which signifies a change in the status quo for subsequent periods according to a change in the level of public-good provision. Hence, the Pantaleoni effect necessarily requires a procedure modeled as a non-tâtonnement.

Drèze (1974) proposed a Mixed Tâtonnement/Non-Tâtonnement Procedure; a tâtonnement for a production/investment decision and a non-tâtonnement for a portfolio decision under uncertainty.[8] Following Drèze (1974), let us mix the above Tâtonnement and Non-Tâtonnement Hedonic MDP Procedures for quality and quantity adjustment of attributes and goods.

A Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure for quality/quantity adjustments reads for any $t \in [0, \infty)$, $\forall i \in \mathbf{N}$:

$$
\begin{cases}
\dot{q}_{jc}(t) = \xi_{jc}(t) \left\{ \sum_{i \in \mathbf{N}} \psi_{ijc}(t) - \gamma_{jc}(t) \right\}, \ \forall j \in \mathbf{J}, \ \forall c \in \mathbf{C}, \ c \neq j' \\[2ex]
\xi_{jc}(t) \equiv \begin{cases} 0 \ \text{ if } \ q_{jc}(t) = 0 \ \text{ and } \ \dot{q}_{jc}(t) < 0, \forall j \in \mathbf{J}, \forall c \in \mathbf{C} \\[1ex] 1 \ \text{ otherwise} \end{cases} \\[2ex]
\dot{q}_{jg}(t) = \sum_{i \in \mathbf{N}} \psi_{ijg}(t) - \gamma_{jg}(t), \ \forall j \in \mathbf{J}, \ \forall g \in \mathbf{G}, \ g \neq j' \\[2ex]
\dot{x}_{ij}(t) = \xi_{ij}(t) \left\{ \sum_{c \in \mathbf{C} \cup \mathbf{G}} \psi_{ic}(t) q_{jc}(t) - \gamma_j(t) \right\}, \ \forall i \in \mathbf{N}, \ \forall j \in \mathbf{J} \\[2ex]
\xi_{ij}(t) \equiv \begin{cases} 0 \ \text{ if } \ x_{ij}(t) = 0 \ \text{ and } \ \dot{x}_{ij}(t) < 0, \forall i \in \mathbf{N}, \forall j \in \mathbf{J} \\[1ex] 1 \ \text{ otherwise} \end{cases} \\[2ex]
\dot{z}_{ic}(t) = \sum_{j \in \mathbf{J}} q_{jc}(t) \dot{x}_{ij}(t), \ \forall i \in \mathbf{N}, \ \forall c \in \mathbf{C} \cup \mathbf{G} \\[2ex]
\dot{z}_{i0}(t) = - \sum_{j \in \mathbf{J}} \sum_{c \in \mathbf{C} \cup \mathbf{G}} \left\{ \psi_{ijc}(t) \dot{q}_{jc}(t) + \psi_{ic}(t) \dot{x}_{ij}(t) \right\} + \delta_i \left\{ \sum_{c \in \mathbf{C} \cup \mathbf{G}} \dot{q}_{jc}^2(t) + \sum_{j \in \mathbf{J}} \dot{x}_{ij}^2(t) \right\}.
\end{cases}
$$

Call the process defined by the above equations the *ζMDP Procedure* which optimizes the composition of gaseous attributes in the global atmosphere, as well as that of tangible attributes in the goods. In the family of MDP Procedures, the ζMDP Procedure preserves the properties that the MDP Procedure enjoys; i.e., feasibility, monotonicity, Pareto efficiency and neutrality. Moreover, the incentive properties pertained to maximin and Nash strategies can be also proved as the MDP Procedure. The ζMDP Procedure generates in the allocation space of attributes and stops when the hedonic optimality conditions hold. Since our ζMDP Procedure is decentralized, the adjustment of attributes can be individually made by the countries under their truthful revelation of HMWs. However, if there could be international free riding behaviors, the planner requires $\psi_{ic}(t)$ and $\psi_{ijc}(t), \forall i \in \mathbf{N}, \forall j \in \mathbf{J}, \forall c \in \mathbf{C} \cup \mathbf{G}$, as relevant information to decide optimal composition of characteristics embodied in the goods and the global atmosphere. Technological information, $\gamma_{jc}(t)$ and $\gamma_j(t)$ are assumed to be available, which permits us to narrow down the incentive problem of eliciting the private information, $\psi_{ijc}(t), \forall i \in \mathbf{N}$.

Next, the problem to be solved is the existence of solutions to the ζMDP Procedure. Before proving the theorem, let us modify the differential system to that with continuous RHS. Extend

[8] See also Drèze (1972) and (1993, Part II: Non-tâtonnement) for tâtonnement and non-tâtonnement procedures under uncertainty which is one of the significant factor of global warming. However, a model involving uncertainty is postponed to a future research.

the differential system of the ζMDP Procedure defined on $\mathbf{R}_+^{\mathbf{2N}}$ to that on $\mathbf{R}_+^{\mathbf{N}} \times \mathbf{R}_+^{\mathbf{N}}$. Denote $\chi_{jc}(q_{jc}(t)) = \max\,[0,\, q_{jc}(t)]$ and $\Upsilon(x_{ij}(t)) = \max\,[0,\, x_{ij}(t)]$.

The ζMDP Procedure reads for any $a \in \mathbf{A}$:

$$\begin{cases} \dot{q}_{jc}\,(t) = \sum_{i\in\mathbf{N}} \psi_{ijc}\left(\chi_{jc}(q_{jc}(t))\right) - \gamma_{jc}\left(\chi_{jc}(q_{jc}(t))\right),\ \forall j \in \mathbf{J},\ \forall c \in \mathbf{C},\ c \neq j' \\ \\ \dot{q}_{jg}\,(t) = \sum_{i\in\mathbf{N}} \psi_{ijg}\,(t) - \gamma_{jg}\,(t)\,,\ \forall j \in \mathbf{J},\ \forall g \in \mathbf{G},\ g \neq j' \\ \\ \dot{x}_{ij}(t) = \sum_{c\in\mathbf{C}\cup\mathbf{G}} \psi_{ic}(\Upsilon(x_{ij}(t)))q_{jc}(t) - \gamma_j(t),\ \forall i \in \mathbf{N},\ \forall j \in \mathbf{J} \\ \\ \dot{z}_{ic}\,(t) = \sum_{j\in\mathbf{J}} q_{jc}\,(t)\,\dot{x}_{ij}\,(t)\,,\ \forall i \in \mathbf{N},\ \forall c \in \mathbf{C}\cup\mathbf{G} \\ \\ \dot{z}_{i0}\,(t) = -\sum_{j\in\mathbf{J}}\sum_{c\in\mathbf{C}\cup\mathbf{G}} \left\{\psi_{ijc}\left(\chi_{jc}(q_{jc}(t))\right)\dot{q}_{jc}\,(t) + \psi_{ic}\left(\Upsilon(x_{ij}(t))\right)\dot{x}_{ij}(t)\right\} \\ \qquad + \delta_i\left\{\sum_{c\in\mathbf{C}\cup\mathbf{G}} \dot{q}_{jc}^2(t) + \sum_{j\in\mathbf{J}} \dot{x}_{ij}^2(t)\right\},\ \forall i \in \mathbf{N}. \end{cases}$$

With these formulae, the following theorem can be presented.

Theorem 1. *There exists at least one solution to the ζMDP Procedure.*

Proof: The existence of solution paths of the ζMDP Procedure can be shown by the method of Henry (1972) and Champsaur, Drèze and Henry (1977). As their proof requires some additional assumptions on the utility and production functions, let me provide a simpler proof. As the RHS of the formulae are all continuous, the fundamental existence theorem assures the existence of at least one continuous trajectory. Suppose that a_0 is any initial point, $a\,(t, \delta)$ is a trajectory starting at a_0. Consider a solution path $\{a'(t)|\ a' = (x'(t), y'(t)), t \geq 0\}$ starting from $a_0 \in \mathbf{A}$, and let $\Pi = \min\{t|a'(t) \leq 0\}$. Let us define a new trajectory $\{a(t)|t \geq 0\}$ as

$$a(t) = \begin{cases} a^*(t),\ t \leq \Pi \\ a^*(\Pi),\ t > \Pi, \end{cases}$$

then it can be easily proved that this new path is a solution to the formulae above. Consequently, it is concluded that the ζMDP Procedure has as least one continuous solution. ||

Next section presents the main theorems of this paper.

5.2 *Truthful Revelation for Attributes in the ζMDP Procedure*

A local game played at each iteration of the procedure is formally defined as the normal form game $(\mathbf{N}, \mathbf{\Psi}, \mathbf{U})$. $\mathbf{N}$ is the set of players, $\mathbf{\Psi} = \times_{i\in\mathbf{N}}\mathbf{\Psi}_i$ is the Cartesian product of $\mathbf{\Psi}_i$ which is the set of country i's strategies, and $\mathbf{U} = (U_1, ..., U_N)$ is the N-tuple of payoff functions.

Definition 4. A Nash equilibrium is an N-tuple of strategies ϕ_{ijc} such that for every $i \in \mathbf{N}$

$$U_i\left(\phi_i, \phi_{-i}\right) \geq U_i\left(\psi_i, \phi_{-i}\right),\ \forall \psi_i \in \mathbf{\Psi}_i$$

where $\phi_i = (\phi_{i1}, ..., \phi_{iJ})$, $\phi_{ij} = (\phi_{ij1}, ..., \phi_{ijC+G})$, $\phi_{-i} = (\phi_1, ..., \phi_N)$ and $\psi_i = (\psi_{i1}, ..., \psi_{iJ})$.

The following conditions are imposed for the procedure.

Condition F. Feasibility

$$\sum_{i\in\mathbf{N}}\dot{z}_{i0}\left(\psi\left(t\right)\right)+\sum_{j\in\mathbf{J}}\sum_{c\in\mathbf{C}\cup\mathbf{G}}\left\{\gamma_{jc}\left(t\right)\dot{q}_{jc}\left(\psi_{jc}\left(t\right)\right)+\gamma_{j}(t)\dot{x}_{ij}(\psi_{ic}\left(t\right))\right\}=0,\ \forall\psi\in\mathbf{\Psi},\ \forall t\in[0,\infty).$$

Condition M. Monotonicity

$$\dot{U}_i\left(\psi\left(t\right)\right)\geq 0,\ \forall i\in\mathbf{N},\ \forall\psi\in\mathbf{\Psi},\ \forall t\in[0,\infty).$$

Condition N. Neutrality

$$\forall a^*\in\mathbf{A},\ \forall a_0\in\mathbf{A},\ \exists\delta\text{ and }a\left(t,\delta\right),\ \forall t\in[0,\infty).$$

Condition HPE. Hedonic Pareto Efficiency

$$\dot{q}_{jc}(\psi_{jc}\left(t\right))=0\iff\sum_{i\in\mathbf{N}}\psi_{ijc}(t)=\gamma_{jc}(t),\ \forall j\in\mathbf{N},\ \forall c\in\mathbf{C}\cup\mathbf{G},\ \forall\psi\in\mathbf{\Psi}$$

Conditions except HPE must be fulfilled for any $t\in[0,\infty)$, and HPE is defined for the announced values, which implies that a hedonic Pareto optimum (HPO) reached is not necessarily equal to one achieved under the truthful revelation of HMWs for tangible and gaseous attributes. It was Champsaur (1976) who first advocated the notion of neutrality for the MDP Procedure. Neutrality depends on the distributional coefficient vector δ. Remember that the role of δ is to attain any individually rational HPO by redistributing the global surplus generated during the operation of the procedure: δ varies the trajectory to reach every HPO. In other words, the global organization as a planner can guide the allocation via the choice of δ, but it cannot predetermine a final allocation to be reached. This is a very important property for settling the issues of transboundary pollution problems such as global warming, since the equity considerations among countries matter.

Let me examine the properties of the ζMDP Procedure defined above. Condition F is easily checked to be satisfied, since it has been already used to formulate the procedure. Condition M is verified by the construction of the procedure with the correct revelation. The properties of the process are as follows: a solution $a(t,\delta)$ defining the procedure is the function which associates a program with every iteration t. If an initial program is feasible, then every succeeding ones are also feasible. I can demonstrate with the above assumptions that the process is stable and always converges monotonically from any initial point to a HPO. Under Assumptions 3 and 4, the ζMDP Procedure always monotonically converges to an individually rational hedonic Pareto optimum. Hence, I am now in a position to present a theorem, the proof of which immediately follows from feasibility and monotonicity.[9] Above arguments lead us to the following.

Theorem 2. *The ζMDP Procedure satisfies Conditions F, M, N and HPE.*

For any time horizon τ the countries are assumed to maximize their discounted national happiness integral, i.e., they consider the outcome functional as the payoff function with possibly different time preference rate, ρ_i^t.

$$U_i=\int_0^{\tau}e^{-\rho_i^t}h_i\left(b_i\left(t\right),b_{-i}\left(t\right)\right)dt.$$

[9] For the desirable properties of the MDP Procedure, see Champsaur et al. (1977), Tulkens (1978), Henry (1979), Champsaur and Laroque (1981), Fujigaki and Sato (1981) and (1982), Champsaur and Rochet (1983), Laffont and Maskin (1983), Sato (1983), (2009), (2012) and (2016c).

Let λ_i, $\eta_{i\ell}$, ε_g and θ be the costate variables for $\dot{q}_{jc}$, $\dot{z}_{i0}$, $\dot{\zeta}_g$ and $\dot{T}$. Dropping the argument of time t, it is a necessary condition for a Nash equilibrium that the current value Hamiltonian

$$\begin{aligned} H_i =& h_i\left(b_i, b_{-i}\right) + \lambda_i \sum_{j\in\mathbf{J}} \sum_{c\in\mathbf{C}\cup\mathbf{G}} \left(\sum_{\ell\in\mathbf{N}} \psi_{\ell jc} - \gamma_{jc} \right) \\ &+ \sum_{\ell\in\mathbf{N}} \eta_{i\ell} \sum_{j\in\mathbf{J}} \left\{ - \sum_{c\in\mathbf{C}\cup\mathbf{G}} \left(\psi_{\ell jc}\dot{q}_{jc} + \psi_{\ell c}\dot{x}_{\ell j}\right) + \delta_\ell \left(\sum_{c\in\mathbf{C}\cup\mathbf{G}} \dot{q}_{jc}^2 + x_{\ell j}^2 \right) \right\} \\ &+ \sum_{g\in\mathbf{G}} \varepsilon_g(\nu_g\alpha_g z_g - \mu_g\zeta_g) + \theta(c_h)^{-1} \left(72.03 + \frac{1.04T^4}{10^8} + 6.3 \sum_{g\in\mathbf{G}} \beta_g\mu_g \ln\frac{\zeta_{\mathrm{CO}_2}^t}{\zeta_{\mathrm{CO}_2}^0} \right) \end{aligned}$$

is maximized. It is easy to verify for open-loop solutions that the shadow values placed on other country's consumption of $z_{\ell 0}$ ($\eta_{i\ell},\ i \neq \ell$) is zero. The above equation can then be simplified to

$$\begin{aligned} H_i =& h_i\left(b_i, b_{-i}\right) + \lambda_i \sum_{j\in\mathbf{J}} \sum_{c\in\mathbf{C}\cup\mathbf{G}} \left(\sum_{\ell\in\mathbf{N}} \psi_{\ell jc} - \gamma_{jc} \right) \\ &+ \eta_{ii} \sum_{j\in\mathbf{J}} \left\{ - \sum_{c\in\mathbf{C}\cup\mathbf{G}} \left(\psi_{\ell jc}\dot{q}_{jc} + \psi_{\ell c}\dot{x}_{\ell j}\right) + \delta_\ell \left(\sum_{c\in\mathbf{C}\cup\mathbf{G}} \dot{q}_{jc}^2 + x_{ij}^2 \right) \right\} \\ &+ \sum_{g\in\mathbf{G}} \varepsilon_g(\nu_g\alpha_g z_g - \mu_g\zeta_g) + \theta(c_h)^{-1} \left(72.03 + \frac{1.04T^4}{10^8} + 6.3 \sum_{g\in\mathbf{G}} \beta_g\mu_g \ln\frac{\zeta_{\mathrm{CO}_2}^t}{\zeta_{\mathrm{CO}_2}^0} \right). \end{aligned}$$

Necessary conditions for a maximum of U_i with respect to open-loop strategies ψ_{ijc} are: $\forall i \in \mathbf{N}$, $\forall j \in \mathbf{J}$, $\forall c \in \mathbf{C} \cup \mathbf{G}$, $\forall g \in \mathbf{G}$

$$\begin{aligned} \psi_{ijc} &= \frac{1}{2(\delta_i - 1)} \left\{ \frac{\lambda_i}{\eta_{ii}} + (2\delta_i - 1) \left(\sum_{\ell\neq i} \psi_{\ell jc} - \gamma_{jc} \right) \right\} \\ \dot{\lambda}_i &= \rho_i\lambda_i - \frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right) \left(\frac{\partial f_{ik}}{\partial z_{ic}}\right) x_{ij} + \eta_{ii}\dot{q}_{jc}\gamma_{jc} \\ \dot{\eta}_{ii} &= \rho_i\eta_{ii} - \frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right) \left(\frac{\partial f_{ik}}{\partial z_{i0}}\right) \\ \dot{q}_{jc} &= \frac{1}{N-1} \left(\sum_{i\in\mathbf{N}} \frac{\lambda_i}{\eta_{ii}} - \gamma_{jc} \right) \\ \dot{z}_{i0} &= \sum_{j\in\mathbf{N}} \left\{ - \sum_{c\in\mathbf{C}\cup\mathbf{G}} \left(\psi_{\ell jc}\dot{q}_{jc} + \psi_{\ell c}\dot{x}_{\ell j}\right) + \delta_i \left(\sum_{c\in\mathbf{C}\cup\mathbf{G}} \dot{q}_{jc}^2 + x_{ij}^2 \right) \right\} \\ \dot{\zeta}_g &= -\frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right) \left(\frac{\partial f_{ik}}{\partial T}\right) \frac{\partial T}{\partial \zeta_g} + \frac{\theta T^3}{10^7 c_h} + \varepsilon_g\mu_g - \frac{6.3\theta\beta_g\mu_g}{\zeta_g\zeta_g^0 c_h} \\ \dot{\theta} &= -\frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right) \left(\frac{\partial f_{ik}}{\partial T}\right) + \frac{\theta T^3}{10^7 c_h} \end{aligned}$$

and the transversality conditions are

$$
\begin{aligned}
\lambda_i(\tau) &= 0 \\
\eta_{i\ell}(\tau) &= 0 \\
\varepsilon_g(\tau) &= 0 \\
\theta(\tau) &= 0.
\end{aligned}
$$

A solution to these equations satisfying the initial conditions is an open-loop Nash equilibrium of the differential game. Note that the strategy $\psi_{ij\ell}$ is not a function of static HMW, $\pi_{ij\ell}$, but of λ_i/η_{ii}, which may be called the *dynamic hedonic marginal willingness-to-pay.* As in Von dem Hagen (1991), the difference between static and dynamic HMWs is the Pantaleoni effect.

The costate variables, $\eta_{i\ell}$, $\forall \ell \neq i$, may be non-zero at a closed-loop Nash equilibium, since the effects of other countries' strategies are taken into consideration. Optimizing the Hamiltonian requires: $\forall i \in \mathbf{N}, \forall j \in \mathbf{J}, \forall c \in \mathbf{C} \cup \mathbf{G}$

$$
\begin{aligned}
\dot{\lambda}_i &= \rho_i \lambda_i - \frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right)\left(\frac{\partial f_{ik}}{\partial z_{ic}}\right) x_{ij} - \eta_{ii}\dot{q}_{jc} \sum_{\ell \neq i} \frac{\partial \psi_{\ell jc}}{\partial q_{jc}} \\
\dot{\eta}_{ii} &= \rho_i \eta_{ii} - \frac{\partial h_i}{\partial b_i} \sum_k \left(\frac{\partial b_i}{\partial f_{ik}}\right)\left(\frac{\partial f_{ik}}{\partial z_{i0}}\right) - \eta_{ii}\dot{q}_{jc} \sum_{h \neq i} \frac{\partial \psi_{hjc}}{\partial z_{i0}} \\
\dot{\eta}_{i\ell} &= \rho_i \eta_{i\ell} - \eta_{ii}\dot{q}_{jc} \sum_{h \neq i} \frac{\partial \psi_{hjc}}{\partial z_{\ell 0}}, \ \forall i, \ell \in \mathbf{N}, \ i \neq \ell.
\end{aligned}
$$

At a stationary state of the dynamical system, $\dot{q}_{jc} = 0$ holds, so that country i's dynamic HMW becomes $\psi_{ijc} = \lambda_i/\eta_{ii}$, $\forall c \in \mathbf{C} \cup \mathbf{G}$ from the above equations. Discussions so far yield the main results.

Theorem 3. *A stationary state of the ζMDP Procedure defined by a closed-loop and an open-loop Nash equilibria is a hedonic Pareto optimum.*

Theorem 4. *Each country truthfully reveals its dynamic hedonic marginal willingness-to-pay for gaseous and tangible attributes at iterations of the ζMDP Procedure.*

6 CONCLUDING REMARKS

In summary, this paper has presented the Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure or the ζMDP Procedure for providing an optimal composition of characteristics as public goods with the following properties:

i) The ζMDP Procedure monotonically converges to an individually rational hedonic Pareto optimum, even if countries do not report their true valuation, i.e., their dynamic hedonic marginal willingness-to-pay for attributes as public goods.

ii) Revealing its true HMW for any attribute is a Nash equilibrium strategy for each myopically behaving country.

iii) The ζMDP Procedure generates in the feasible allocation space similar trajectories as the MDP Process by distributing the instantaneous surplus generated at each iteration.

This paper has provided at least an economic theoretical solution to the problem of global warming. For that purpose, I have used both the theory of planning procedures for public goods, advocated by Drèze and de la Vallée Poussin (1971) and Malinvaud (1970-1971), and the new consumer theory initiated by Gorman (1956/1980) and Lancaster (1966), and developed by Drèze and Hagen (1978). Moreover, Sen's concepts of capability and functionings are also employed to differentiate the countries. I have shown the necessary conditions for the global atmosphere as a complex of gaseous attributes, and for the products as a composition of tangible attributes that are produced by the countries. It is also shown that each country maximizes its national happiness function by producing one product and consuming goods with a Pareto optimal composition of attributes, and by simultaneously emitting greenhouse gases as gaseous attributes. This paper has applied the economic way of thinking to an environmental problem of growing importance and has proposed a theoretical possibility to moderate global warming by presenting the Drèzian Mixed Tâtonnement/Non-Tâtonnement Hedonic MDP Procedure or the ζMDP Procedure. This procedure simultaneously achieves efficiency and incentive compatibility. That is to say, it converges to an individually rational hedonic Pareto optimum and truthful revelation of HMWs for tangible characteristics embedded in the goods and gaseous attributes composing the global atmosphere is the Nash equilibrium strategy for any country.

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References

[1] Champsaur P (1976) Neutrality of planning procedures in an economy with public goods. Rev Econ Stud 43: 293-300

[2] Champsaur P, J Drèze, C Henry (1977) Stability theorems with economic applications. Econometrica 45: 273-294

[3] Champsaur P, G Laroque (1981) Le plan face aux comportements stratégiques des unités décentralisées. Annales de l'INSEE 42, avril-juin: 3-30

[4] Champsaur P, J-C Rochet (1983) On planning processes which are locally strategy proof. J Econ Th 30: 353-369

[5] Drèze J (1972) A tâtonnement process for investment under uncertainty in private ownership economy. In Szegö G, K Shell (eds) Mathematical methods in investment and finance. North-Holland, Amsterdam, 3-23

[6] Drèze J (1974) Investement under private ownership: optimality, equilibrium and stability. In Drèze J (ed) Allocation under uncertainty: equilibrium and optimality. Macmillan, New York, 129-166

[7] Drèze J (1993) Underemployment equilibria, essays in theory, econometrics and policy. Cambridge University Press, London

[8] Drèze J, D de la Vallée Poussin (1971) A tâtonnement process for public goods. Rev Econ Stud 38: 133-150
[9] Drèze J, K Hagen (1978) Choice of product quality: equilibrium and efficiency. Econometrica 46: 493-513
[10] Fujigaki Y, K Sato (1981) Incentives in the generalized MDP procedure for the provision of public goods. Rev Econ Stud 48: 473-485
[11] Fujigaki Y, K Sato (1982) Characterization of SIIC continuous planning procedures for the optimal provision of public goods. Econ Stud Q 33: 211-226
[12] Gorman W (1956/1980) A possible procedure for analyzing quality differentials in the egg market. Journal Paper No 3129 Iowa Agricultural Experiment Station 1956; Rev Econ Stud 47: 843-856
[13] Gorman W, D Myles (1987) Characteristics. In Eatwell J, M Milgate, P Newman (eds) The new Palgrave dictionary of economics. 403-406, Macmillan, London
[14] Greiner A (2004) Global warming in a basic endogenous growth model. Envi Econ Pol Stud 6: 49-73
[15] Henry C (1979) On the free rider problem in the M.D.P. process. Rev Econ Stud 46: 293-303.
[16] IPCC (1990) Climate change: the IPCC scientific assessment. Cambridge University Press, Cambridge
[17] Laffont J-J, E Maskin (1983) A characterization of strongly locally incentive compatible planning procedures with public goods. Rev Econ Stud 50: 171-186
[18] Lancaster K (1966) A new approach to consumer theory. J Pol Econ 74: 132-157; reprinted in Lancaster (1991) Chapter 2.
[19] Lancaster K (1971) Consumer demand: a new approach. Columbia University Press, New York
[20] Lancaster K (1991) Modern consumer theory. Edward Elgar, Hants
[21] Malinvaud E (1970-1971) "Procedures for the determination of collective consumption. Euro Econ Rev 2: 187-217
[22] Michaelis P (1990) Global warming: efficient policies in the case of multiple pollutants. Envi Res Econ 2: 61-78
[23] Pantaleoni M (1883) Contributo alla teoria del riparto delle spese pubbliche. Rassegna Italiana, Ottobre, translated as: Contribution to the theory of the distribution of public expenditure. In Musgrave R, A Peacock (eds) (1958) Classics in the theory of public finance. Macmillan, London, 16-27
[24] Sato K (1983) On compatibility between neutrality and aggregate correct revelation for public goods. Rev Econ Stud 34: 97-109
[25] Sato K (2006) Urban heat island: an environmental economic modeling. J Heat Island Institute International 1: 40-45
[26] Sato K (2007) Incentives in the hedonic MDP procedure for the global atmosphere as a complex of gaseous attributes. presented at the Workshop of the Environment of CORE held at l'Université Catholique de Louvain on April 19, 2007
[27] Sato K (2008) Cooling the metropolis: an economic analysis to alleviate urban heat island. J Heat Island Institute International 3: 1-15
[28] Sato K (2009) Locally strategy proof planning procedures as algorithms and game forms. RIMS Kôkyûroku 1654, Research Institute for Mathematical Sciences, Kyoto University, June 2009, 129-155
[29] Sato K (2012) Nonmyopia and incentives in the piecewise linearized MDP procedures with variable step-sizes. J Econ Struct 1: 1-22

[30] Sato K (2015) Anthropogenic climate change in an integrated energy balance model of global and urban warming. J Econ Struct 3: 1-21

[31] Sato K (2016a) Biohazards and health risks due to urban warming: an economic-ecological model. presented at the Departmental Seminar held at Musashi University, February 19, 2016

[32] Sato K (2016b) Biohazards and health risks due to urban warming: an economic-ecological model and Japanese mischiefs. presented at the 6th Asian seminar in regional science held at the graduate school of information science, Tohoku University, September 25, 2016

[33] Sato K (2016c) The MDP procedure for public goods and local strategy proofness. J Econ Struct 5: 1-31

[34] Sato K (2017) Solving the Noah's ark problem to preserve biodiversity. presented at the Public Economics Seminar held at Keio University, December 18; revised version presented at the spring meeting of the Japanese Economic Association held at Chiba University, June 5, 2010; revised version presented at the Economics seminar held at Tohoku Gakuin University, March 6, 2017.

[35] Schneider S (1989) Global warming: are we entering a greenhouse century? Sierra Club Books, San Francisco

[36] Sen A (1985) Commodities and capabilities. Elsevier Science Publishers B.V., Amsterdam

[37] Tulkens H (1978) Dynamic processes for public goods: an institution-oriented survey. J Pub Econ 9: 163 -201; reprinted in Chander P, J Drèze, C Lovell, J Mintz (eds) (2006) Public goods, environmental externalities and fiscal competition: selected papers on competition, efficiency, and cooperation in public economics by Henry Tulkens, Springer, US, Chapter 1

[38] Von dem Hagen O (1991) Strategic behaviour in the MDP procedure. Euro Econ Rev 35: 121-138