

# A Dynamic Equilibrium Model for U-Shaped Pricing Kernels\*

Akira Yamazaki

Graduate School of Business Administration, Hosei University  
The Research Institute for Innovation Management, Hosei University

## Abstract

This paper proposes a dynamic equilibrium model that can provide a unified explanation for the stylized facts observed in stock index markets such as the fat tails of risk-neutral return distribution relative to physical distribution, negative expected returns on deep OTM call options, and negative realized variance risk premiums. In particular, we focus on the U-shaped pricing kernel against the stock index return, which is closely related to the negative call returns. We assume that the stock index return follows the time-changed Lévy process and that a representative investor has power utility over the aggregate consumption that forms a linear regression of the stock index return and its stochastic activity rate. This model offers a macroeconomic interpretation of the stylized facts from the perspective of the sensitivity of the activity rate and stock index return on the aggregate consumption as well as the investor's risk aversion.

**Keywords:** stock index, U-shaped pricing kernel, stochastic activity rate, aggregate consumption, physical distribution, risk-neutral distribution, realized variance

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