Bubbly Financial Globalization*

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Abstract

Has the recent surge in financial globalization made the world economy more prone to widespread episodes of asset price bubbles? We address this question by developing a stylized global equilibrium model of two production economies with diverse financial development, North, and South. In autarky, the financially mature North produces enough assets so to keep bubbles from being viable. In the financially undeveloped South, while bubbles can potentially offset the shortage of financial assets, the limited leverage potential of productive entrepreneurs makes the required return on the bubble unsustainable. When financial globalization takes place, bubbles become possible if two conditions are met: the financial development of the South is increased *and* the globalized financial markets display a shortage of asset supply for intermediated saving. We argue that both conditions seem to have gradually emerged over that last twenty years.

Keywords: rational bubbles, financial globalization, financial frictions, asset supply shortage

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1 Introduction

Financial globalization, generally defined as the process of integration of countries with the global financial system, has expanded over the last forty years, with a substantial acceleration particularly after the mid 1990's.[1] Figure 1 reports an index of global capital mobility due to Reinhart and Rogoff (2009), together measures of incidence of real estate and equity bubbles for a set of seventeen OECD countries as calculated by Jorda, Schularick, and Taylor (2015).[2] The Figure suggests a remarkable correlation between the level of financial globalization, and the extent to which bubbles have been present over the years in most of today's advanced economies.

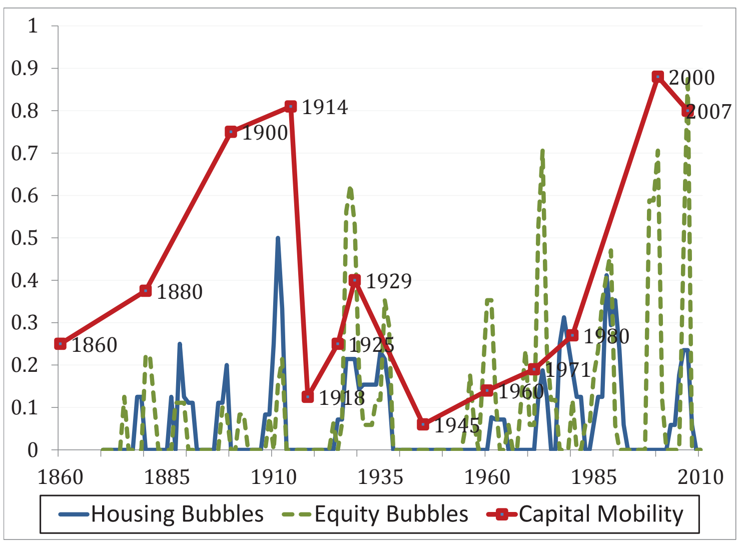


Figure 1: Financial Globalization and Bubbles

While capital mobility is usually accompanied by growth opportunities, especially for emerging economies, it also comes at the cost of financial instability. A growing empirical literature has been studying episodes of financial crises in emerging economies with limited financial development as they

[1]For a comprehensive analysis of the variety of measures of financial globalization, see Quinn, Schindler, and Toyodoa (2013)

[2]The capital mobility index was originally developed by Obstfeld and Taylor (2003), and subsequently updated by Reinhart and Rogoff (2009), see their page 159. Equity and Housing Bubbles incidence are compiled using Jorda, Schularick, and Taylor (2015)'s bubbles dates, and are computed as the number of countries with a bubble over the total number of countries for which data is available at that particular time period. The seventeen countries considered are Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Italy, Japan, Netherlands, Norway, Portugal, Sweden, United States.

open up to capital inflows from the rest of the world. For instance, Reinhart and Rogoff (2009) document that episodes of financial crises for middle income economies are usually preceded by sharp increases in stock market prices and/or real estate prices, followed by a sharp decline.[3] Because it is difficult to relate the sudden asset price changes to a change in the beliefs about underlying fundamentals, theoretical explanations of financial crises in emerging economies, such as those put forth by Caballero and Krishnamurthy (2006) and Ventura (2012), have suggested the existence of a non-fundamental component in the asset prices, usually referred to as a “bubble”.

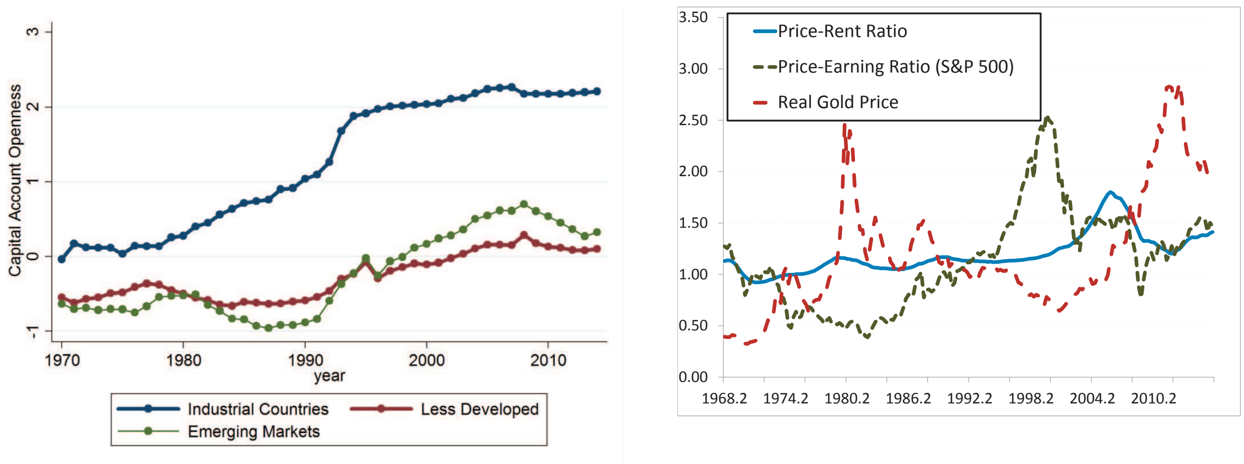
In agreement with Figure 1, the global financial distress of 2008 has indicated that bubble-like episodes may not be confined only to emerging economies, which typically display limited financial development, but they may involve financially mature economies as well. Figure 2a reports a second measure of financial globalization, the Chinn-Ito index, which is based on the “de-jure” capital account openness, aggregated for industrial, emerging and less developed economies. The plot confirms the acceleration in financial globalization observed in Figure 1 after the mid-1990’s, and further reveals that the upward trend has affected all economies across the income spectrum. Figure 2b plots the price dynamics of three assets with globalized markets: the price-to-earnings ratio for the S&P 500 index, the price-to-rent ratio of the U.S. median house, and the London Bullion market gold price.[4] Since the mid-1990’s, all three of them feature periods of sharp increase, followed by a sharp fall, a pattern typically associated with bubbly dynamics.[5] The combined interpretation of the two charts would suggest that, as the world underwent a major structural change in terms of capital markets integration, bubble-like dynamics began to affect prices of assets in the most financially globalized economies, like the U.S. In this paper, we take on this argument and we explore what theoretical underpinnings such argument would need to be plausible.

More precisely, we interpret the above stylized facts as pointing towards a connection between the financial globalization of economies that are diversely financially developed and the emergence of bubbly dynamics in globalized asset markets. Our goal is to provide a theoretical framework where such connection can be formally studied. In particular, the central question that we ask is whether, and

[3]For detailed evidence see Tables 10.7 and 10.8 and Figure 10.2 in Reinhart and Rogoff (2009)

[4]The price-earning ratio are taken from Shiller’s data set http://www.econ.yale.edu/~shiller/data.htm; the rent-to-price ratio is from Davis, Lehnert, and Martin (2009) and located at Land and Property Value in the U.S., Lincoln Institute of Land Policy, at http://www.lincolninst.edu/resources/; the gold price data is taken from FRED-St. Louis Federal Reserve, and deflated using the GDP Deflator.

[5]The dynamics of gold price around 1980 is usually explained by the increased uncertainty due to geo-political events at that time, and so it can arguably be regarded as not being a standard bubble episode.



(a) Chinn-Ito Index of Global Financial Openness

(b) U.S. Stock Market, Real Estate and Gold

Figure 2: Bubbly Financial Globalization

under what conditions, the globalization of financial markets can increase the proneness of the entire global economy to episodes of asset price bubbles.

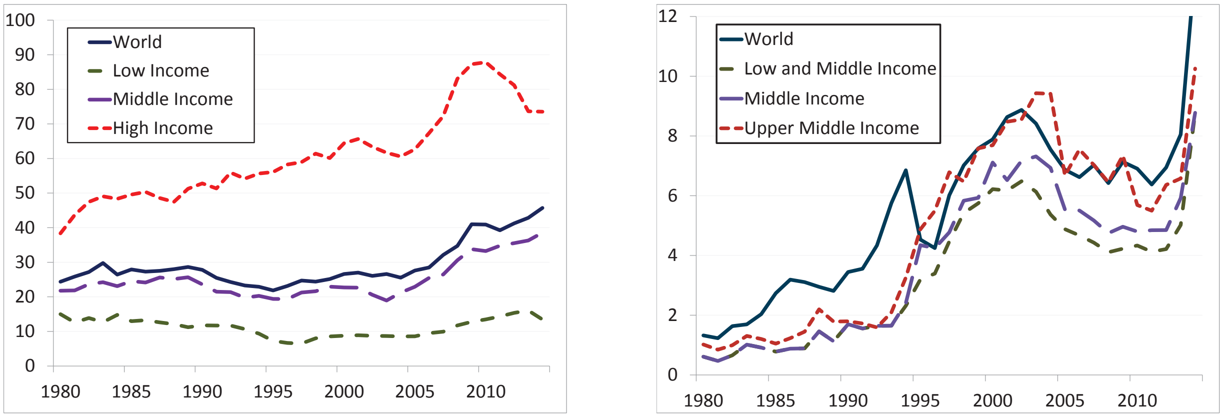
Summary of Model and Results. We address such question by developing a global equilibrium neo-classical growth model in which a financially mature economy and a financially undeveloped economy integrate their financial markets. While under financial autarky both economies do not allow for bubbles, as a result of market integration equilibria with bubbly dynamics become possible in the global economy. Interestingly, the bubbly asset can be held by depositors of both economies, thereby exposing both economies, each one in measure of the amount of bubbly asset held, to the risk of a financial crisis due to the sudden reversal of market expectations.

A bubble in our framework refer to situations in which an asset is valued not because it is expected to provide a stream dividends or interest payments, but because it is expected to be sold at a competitive value in the future when more attractive investment or consumption opportunities arise. In presence of heterogeneous investment opportunities and financial frictions, the bubble operates a transfer of resources from less productive to more productive users that would not be possible otherwise, thereby increasing the efficiency of production in the economy. If the increase in production efficiency raises the income of future savers enough to keep the bubble affordable, the bubble can be rationally sustained in equilibrium. We model bubbles as arising from aggregate shocks to investors' sentiments, as in Martin and Ventura (2012). Investors' sentiments are always present in the economy, but their implications for asset prices and investment/consumption decisions might be inconsistent with optimal strategies, rational beliefs and market clearing. Under some conditions, however, the same sentiments can affect asset prices and investment in a way that still satisfies all the requirements of a rational expectations equilibrium. In this sense our model provides discipline as to when investors' sentiments have the potential to drive aggregate dynamics. We characterize the conditions under which sentiments cannot affect the dynamics of economies in autarky, but they can become a source of aggregate fluctuations in the integrated global economy.

The crucial insight gained from our model is that the conditions for the emergence of a bubble are affected by the degree of financial development in a non-monotonic fashion. On the one hand, a financially mature economy harbors a financial system that is capable of producing a supply of assets that satisfies the saving demand and allows the funds to flow to the most efficient users. In such a context a bubble cannot arise since the resources that would be liberated would not generate an excess saving demand required for the bubble to be purchased. On the other hand, a financially undeveloped economy might be so financially constrained that the bubble would have to grow at a rate that could not possibly be matched by the growth of the income of future savers. A bubble would still provide savers with additional internal funds once the investment opportunity arises, but if the ability to leverage those internal funds is low, only a limited amount of resources would be channeled to the most productive users and the increase in efficiency in production would be severely limited. A larger bubble would correspond to more resources transferred to productive users, but the size needed to transfer enough resources would be too large to remain affordable for future savers.

As the two economies integrate their financial markets, the internal funds of the investors in the undeveloped economy inherit some of the leverage potential of the financially mature economy. This raises the efficiency gain in production for any given value of the bubble and can thus make the bubble affordable. This is not enough for a bubble to be sustained in a global equilibrium. The saving demand of the financially undeveloped economy is now satisfied by the asset supply of the financially mature economy. If the asset supply is large enough, there would be no excess saving demand to absorb the bubble. However, if the asset supply falls short of the excess saving demand coming from the undeveloped region, i.e. if financial integration creates *asset supply shortage*, a bubble becomes sustainable in the equilibrium of the global economy.

The main message from our model is that financial globalization can be bubbly if two conditions



(a) Private Credit to GDP

(b) Loans from Non-Resident Banks

Figure 3: Global Financial Development

are met: (i) the financial development of emerging and low income economies expands as they integrate with the rest of the world, and (ii) the globalized financial markets display a shortage of asset supply for intermediated saving. Figures 3a-3b represent evidence for condition (i).[6] Figure 3a reports the private credit as a percentage of GDP ratio for economies grouped by income, a measure that is typically used to evaluate a country's financial development. One can clearly observe the upward trend beginning in the mid-1990's across all income levels, and remaining sustained for both low and middle income countries through the most recent years. Figure 3b reports the amount of credit outstanding extended by banks that are non-residents with respect to their depositors as a percentage of GDP. As an example, this would capture the case of a credit line extended to a Chinese firm by a Bank of America branch in Shanghai that issues deposits instruments to Chinese residents. An increase in the amount of credit outstanding to GDP can then be interpreted as a measure of the increase in the leverage potential of entrepreneurs, specifically due to financial globalization – the opening of the BofA branch in Shanghai. The chart shows a sharp increase in the mid-1990's across all income level, and then a further acceleration in the recent years after what seems to be a temporary slow-down in the later part of the early 2000's. As for condition (ii), Caballero, Farhi, and Gourinchas (2008) forcefully argue that the global economy has experienced an increasing shortage of saving instruments over the last 20 years. While there exists no

[6] The data is taken from the Global Financial Development Data Set at the IMF, and it corresponds to the series "Private Credit by Deposits Banks and Other Financial Institutions to GDP" for Figure 3a, and to the series "Loans from Non-Resident Banks (amount outstanding over GDP)" for Figure 3b.

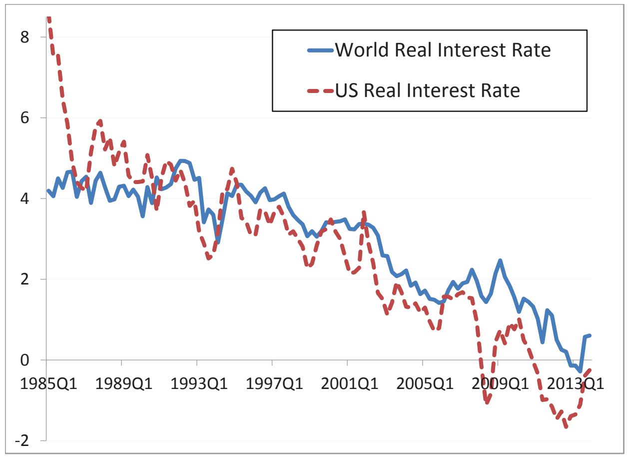


Figure 4: Long Term Real Interest Rate

direct reliable measure for the supply of assets in the economy, an indirect indication of a shortage can be found in the dynamics of the price savers are willing to pay to obtain assets with low-risk of default. Figure 4 shows the behavior for the long term real interest rate at the global level, as estimated by King and Low (2014), and the real yield on a 10-year U.S. Treasury Note.[7] Both measures show a clear downward trend beginning in the mid-1990's, a sign that global savers have been willing to receive an increasingly lower return to park their savings into safe assets, which is consistent with a shortage of asset supply. While this is far from being conclusive evidence for our model, it is consistent with the key conditions required for a bubbly financial globalization.

Related Literature. Our paper is closely related to Caballero, Farhi, and Gourinchas (2008). CFG are primarily interested on the consequences of asset supply shortage in a global equilibrium model in terms of current account balance, gross cross-country assets holdings and long run interest rates. Unlike our paper, the focus of their paper is not in isolating the effects of bubbles in asset prices, and in their analysis all the equilibria are fundamental equilibria. However, CFG main exercise consists in studying a drop in the supply of assets in emerging economies, which the authors interpret as possibly the bursting of a financial bubble. Therefore, the presence of bubbles, while not formalized, is central to the interpretation of their analysis.[8] In this paper we formally study bubbles and the main focus

[7]The World Real Rate is the weighted estimate in King and Low (2014), pages 16-18. The U.S. Real Rate is obtained by subtracting the Michigan Survey Inflation Expectations from the 10-year U.S. Treasury Constant Maturity rate.

[8] The asset supply side of the CRG model is built so that in the extreme case in which the pledgeability of income is

is on the question of *when* an asset supply shortage in the integrated global economy can create the conditions for bubbles to appear and *how* such episodes affect fluctuations in macroeconomic aggregates both as they appear and as they collapse.[9]

The way we model bubbles is closely related to Martin and Ventura (2012) which builds upon the work on rational bubbles in general equilibrium of Tirole (1985). As MV we use the modeling device of introducing an asset with a zero fundamental value, and allow a positive new supply to randomly appear in each period and benefit productive agents. Differently from Martin and Ventura (2012) we allow for a market for intermediated savings to coexist with a market for the bubble asset. This allows us to study how the existence conditions for a bubble are affected by the change in the pledgeability of future income, a property that is central in understanding the equilibrium of the integrated global economy.[10]

Hirano and Yanagawa (2016) show, in the context of an endogenous growth model, how an increase in the level of financial development can facilitate the existence of a rational bubble by disproportionately redirecting resources towards more productive borrowers. The exact same mechanism is at work in our model, with the difference that ours is an environment without endogenous growth, and the marginal return on capital has the usual declining shape. One contribution of our paper is to show that the important insight of Hirano and Yanagawa (2016) extends to settings with no endogenous growth and with overlapping generations.

Bengui and Phan (2016) also show that financial development can relax the conditions for existence of rational bubbles, accompanied by credit booms. Their mechanism relies on borrowers buying the bubbly asset by issuing debt collateralized by the asset itself, and with the option to default if the value of the assets falls below the face value of the debt. Savers find profitable to buy the collateralized debt, which lowers the interest rate, and makes leveraging to issue collateralized debt even more attractive for borrowers. At the heart of their mechanism lies a risk-shifting behavior whereas the price at which one is willing to buy an asset is higher than it would otherwise be if the buyer were to internalize the

zero, a bubble equilibrium is the only possible equilibrium (see also Caballero (2006)). The reason for this is that there is no direct investment option for agents in the model and so the demand for savings can be only satisfied by intermediated savings. In our model agents always have the option to use their savings to generate capital goods - a form of storage technology - which implies that there always exists a fundamental equilibrium where market clearing is reached without the necessity of bubbles.

[9]See Caballero and Krishnamurthy (2001), Caballero and Krishnamurthy (2006), Caballero and Krishnamurthy (2009), Maggiori (2011), Gourinchas and Rey (2007) for additional works on global asset supply shortage and implications for macroeconomic fluctuations.

[10]The role of bubble asset in our economy is also very similar to the bubbly liquidity modeled in Farhi and Tirole (2012).

risk of the asset entirely.

Finally, our paper is also related to Ikeda and Phan (2015), which consider a two-country model of the global economy with rational bubbles, and show that in presence of diverse financial development capital flows from South to North in normal times, and then reverts course when a bubble bursts.

The rest of the paper is organized as follows. Section 2 introduces the model and defines an equilibrium for the closed economy. Section 3 studies the conditions under which bubbly dynamics can emerge in the closed economy, vis-a-vis the level of financial development. Section 4 characterizes the equilibrium for the global economy. Section 5 studies the conditions under which a bubbly equilibrium, while not possible in autarky, can emerge under financial globalization. Section 6 performs a numerical simulations of a bubbly equilibrium for the financially globalized economy. Section 7 concludes. The proofs of the main results are reported in Appendix A.

2 The Model

Preferences and Technology. The individual economy consists of an infinite sequence of overlapping generations each of measure 1. An individual agent $i \in [0,1]$ born at time $t-1$ lives for three periods: young (period $t-1$), adult (period t) and old (period $t+1$).[11] When young, each agent i is endowed with one unit of time that she supplies inelastically to the labor market at the unitary wage w_{t-1}. The objective of agent i born at time $t-1$ is to maximize her expected consumption when old, $\mathbb{E}_{i,t-1}\big(c_{it+1}\big)$, where c_{it+1} denotes the amount of output good consumed at time $t+1$. Agents in the economy are risk neutral and their savings demand when young and adult is inelastic and equal to their total wealth.[12]

The output good is produced by a perfectly competitive final good sector where each firm employs labor from the young and capital via a constant return to scale technology

$$y_t = k_t^{\alpha}\ell_t^{1-\alpha}, \quad \alpha \in (0,1), \tag{2.1}$$

[11] The overlapping generations structure is chosen for analytical convenience. A model with infinitely lived agents with stochastic investment opportunities would also allow the derivation of our results, at a cost of a more burdensome notation and less analytical transparency. For alternative frameworks for the analysis of bubbly dynamics see Rondina (2012)

[12] The two assumptions are admittedly a simplification of reality, but both risk non-neutrality and intertemporal consumption decision are not essential for the basic mechanism that the model aims at capturing. They are nonetheless relevant, the former in particular, to understand the change in the composition of the external balance sheet of emerging and industrialized economy following financial globalization. See Gourinchas (2012).

where k_t denotes capital and ℓ_t labor. The labor market and the rental market for capital are both perfectly competitive, so that each factor is always paid its marginal return. Because agents supply labor inelastically, under any equilibrium $\ell_t = 1$ and the factor prices are given by

$$w_t = (1-\alpha)k_t^{\alpha} \quad \text{and} \quad R_t = \alpha k_t^{\alpha-1} \tag{2.2}$$

Capital depreciates completely after use. New capital for production at $t+1$ is obtained by investing output good at time t. Let x_{it} denote the output good invested at time t by agent i, the investment technology is

$$k_{it+1} = A_{it+1}x_{it}.$$

Both young and adult agents can operate the direct investment technology, but they differ in terms of their investment productivity. For the individual young agent i at time $t-1$ the investment productivity A_{it} is constant and equal to $\underline{a} > 0$. When adult, investment productivity A_{it+1} is drawn from the continuous distribution with cumulated density G over the support $[\underline{a}, \bar{a}] \subset \mathbb{R}$, independently across time and agents. Both young and adult agents at time t know their own investment productivity for the current period. Young agents, however, do not know their future productivity at adult age. Output produced at period t, y_t, is either consumed or invested, so output market clearing is

$$y_t = c_t + x_t^A + x_t,$$

where c_t stands for aggregate consumption and x_t^A and x_t stand for the aggregate investment of adults at time t and young at time t, respectively.

In addition to directly investing in the capital investment technology, agents have access to intermediated saving and to borrowing. In particular, they can deposit or borrow funds through a representative intermediary which operates in a competitive market with free entry, and offers the same gross financial interest rate R_{t+1}^a on both loans and deposits from period t to $t+1$.[13] The asset position of the agent i with the intermediary at the end of period t is denoted by a_{it+1}, with $a_{it+1} > 0$ if the agent is depositing and $a_{it+1} < 0$ if the agent is borrowing. The optimization problem for the young agent at time $t-1$

[13] For simplicity, we restrict our attention to an environment with one-period debt contracts only.

can then be written as

$$\max_{c_{it+1},k_{it},k_{it+1},a_{it},a_{it+1}} \mathbb{E}_{it-1}\big(c_{it+1}\big) \tag{2.3}$$

subject to

$$w_{t-1} \geq a_{it} + \frac{k_{it}}{\underline{a}}, \tag{2.4}$$

$$R^a_t a_{it} + R_t k_{it} \geq \frac{k_{it+1}}{A_{it+1}} + a_{it+1}, \tag{2.5}$$

$$c_{it+1} \leq R_{t+1}k_{it+1} + R^a_{t+1}a_{it+1}. \tag{2.6}$$

Constraint (2.4) requires the total wealth of the young at time $t-1$, equal to the wage earned in that period, to be either invested directly with productivity $\underline{a}$ or to be deposited with the intermediary. In period t, the total beginning of period wealth available to the adult agent is equal to the return on direct investment in capital, as capital is rented out to the final output sector, plus the return on intermediated savings (or minus the re-payment of any borrowing). The wealth can be allocated to direct investment into capital with a productivity A_{it+1}, or deposited with the intermediary once again. In the final period of her life the agent collects the return from her portfolio and uses it to consume.

Financial Intermediation. The representative intermediary collects deposits and extends loans to agents that find it optimal to directly invest in excess of their internal funds. We refer to the assets representing loans as "fundamental assets". The intermediary can also invest the funds deposited in an asset that, contrary to loans, does not promise any stream of payments but it is held only for the purpose of reselling it at some point in the future, were the need for funds to arise. We refer to the value of such asset as a "bubble". Let the total value of the asset held by the intermediary at the beginning of period t in terms of output good at time t be denoted by b_t. The value b_t will be assumed to have a stochastic structure related to investors' sentiments, or the coordinated willingness of depositors to buy the asset through the intermediary. The specifics of this structure will be given below. The asset can be freely disposed with, which bounds its value to be weakly above zero. The value of the asset is always taken as given by market participants, which implies that the supply of the asset is out of the control of agents and intermediaries. We follow Martin and Ventura (2012) and we assume that a new supply of the bubble asset can be randomly obtained by any individual adult agent that directly invests into capital. The individual investors cannot anticipate the additional supply of the bubble asset. Under certain conditions, the newly created supply is purchased by the intermediary, and the relative funds accrue to the investing agent in addition to her internal funds and the amount borrowed from the intermediary. The total value of the new asset created at time t and purchased by the intermediary is denoted by b_t^N. The balance sheet of the intermediary at the end of period t can be then written as

$$b_t + b_t^N + l_t = d_t,$$

where l_t denotes the total amount of loans outstanding and d_t the total value of the liabilities of the intermediary to the depositors. Notice that, indirectly, the bubble asset $b_t + b_t^N$ is always held by depositors in the economy.[14] The expected return on the bubble asset, denoted by R_{t+1}^b, consists of the capital gain from the asset between period t and $t+1$,

$$R_{t+1}^b = \frac{\mathbb{E}_t(b_{t+1})}{b_t + b_t^N}, \tag{2.7}$$

where the expectation is conditional on an information set that is common knowledge across agents at time t. Risk neutrality of agents together with the zero-profit condition for the intermediaries implies that the expected return on the bubble is equal to the return on deposits and loans, i.e. $R_{t+1}^b = R_{t+1}^a$. The risk-neutrality assumption masks an important difference between assets representing loans and b_t. The fundamental assets l_t are "safe" in the sense that their return is exactly known at the time of issuance. The bubble asset is instead "risky": if the value of the asset is higher (lower) than expected the unexpected capital gain (loss) is immediately distributed to (taken out from) depositors.[15]

Financial Frictions. When financial markets function smoothly, funds are channeled towards their more productive use. In our economy this means that young agents and adult agents with a low investment productivity transfer their wealth at time t to the adults agents with high productivity.

[14] An alternative modeling choice would be to allow agents to participate directly in the market for loans, deposits and bubble asset, under price taking conditions. While equivalent in terms of results, using a representative intermediary allows a more compact representation of the role of financial markets.

[15] The modeling choice of separating the fundamental and the bubble asset is made for convenience and is meant to capture the idea that some assets contain a component that is not related to the expected value of their stream of payments. For instance, Martin and Ventura (2012) interpret b_t as the value of a firm after production has taken place and capital has depreciated. The fundamental value of the firm is clearly zero, but if the intermediary can "sell" the firm to depositors willing to believe that the firm will be sold in the future at b_{t+1}, the firm assumes a bubbly value and becomes a tradable asset. The intermediary is therefore purchasing loans in the credit market, l_t, and old firms in the stock market b_t. Occasionally, new firms are created by investors and sold to the intermediary in the stock market with value b_t^N.

Productive adult agents invest in the capital technology, rent out the capital in the next period and repay the loans with interest.[16]

We assume that in the economy there is a financial friction. In particular, borrowing agents have limited commitment and can only pledge a fraction θ of their future income which imposes an upper bound on their borrowing of the form

$$R^a_{t+1} a_{it+1} \geq -\theta R_{t+1} k_{it+1}, \tag{2.8}$$

where $\theta \in [0, 1]$. Constraint (2.8) is to be included in the description of the maximization problem of the young agent (2.3)-(2.6). As pointed out in Caballero, Farhi, and Gourinchas (2008), the parameter θ is interpretable as an index of financial development, in the sense that it measures the extent over which property rights over earnings are well defined in the economy and can be exchanged on financial markets.

Optimal Portfolio Strategies. The solution to the problem of the young agent at time $t-1$ boils down to choosing a portfolio allocation strategy that maximizes the expected wealth once old at $t+1$. At the beginning of each period, given the total wealth available, the agent will decide whether to directly invest into the capital technology by borrowing up to the limit, or whether to deposit the funds with the financial intermediary for intermediated saving. The choice between the two portfolio strategies depends on their relative returns given the productivity of the individual agent. It is useful to define the variable ρ as

$$\rho_{t+1} \equiv \frac{R^a_{t+1}}{R_{t+1}}.$$

An agent at time t with an investment productivity draw A_{it+1} can directly invest in capital and receive $A_{it+1} R_{t+1}$ in return, or she can invest in intermediated savings and receive R^a_{t+1}. Because of the linearity of the objective function, the optimal portfolio strategy of the agent will always be a corner solution: if $A_{it+1} > \rho_{t+1}$ it is optimal to directly invest in capital all the internal funds plus the maximum amount that can be borrowed from the intermediary, which is $a_{it+1} = -\theta \frac{k_{it+1}}{\rho_{t+1}}$; if $A_{it+1} < \rho_{t+1}$ it is optimal to

[16] Alternatively, we could have assumed that the loan is issued and repaid all within the same period, so that the lending agents receive capital to carry into period $t+1$ and rent it out in the capital market. The two assumptions are equivalent in our setting, but the former streamlines the presentation of the equilibrium conditions in the asset market in presence of an intermediary.

deposit all the wealth with the intermediary. At equality the agent will be indifferent between direct investment or intermediated saving. The value of ρ in equilibrium provides an indication of the severity of financial frictions in the economy. With no financial frictions ($\theta = 1$) ρ would be equal to the highest productivity level $\bar{a}$, while under the most severe of financial frictions ρ would be stuck at $\underline{a}$.

Let us consider first the portfolio choice of the young agent at time $t-1$. The young agent will be indifferent between directly investing or depositing with the intermediary when $\rho_t = \underline{a}$. In this situation, the amount of deposits will be determined by the value of assets that the intermediary holds. If the deposits issued by the intermediary are not enough to satisfy the demand of the young agents, they will be forced to directly invest the remaining funds. Denote by the direct investment of the young agent in such case as $\delta_t \in [0, 1]$. Then the total capital produced by young agents is

$$k_t^Y = \underline{a}\left(\frac{\delta_{t-1}}{1-\theta}\right) w_{t-1}. \tag{2.9}$$

It follows that the total borrowing from the intermediary and the total savings demand are

$$a_t = -\theta\left(\frac{\delta_{t-1}}{1-\theta}\right) w_{t-1} \quad \text{and} \quad (1-\delta_{t-1}) w_{t-1}. \tag{2.10}$$

The net wealth that the young generation at $t-1$ expects to carry into t is[17]

$$w_{t|t-1}^A = R_t^a w_{t-1}. \tag{2.11}$$

If the intermediary holds a bubble asset, the return R_t^a cannot be exactly guaranteed, and any discrepancy between b_t and $\mathbb{E}_{t-1}(b_t)$ might affect the actual available wealth at time t. The crucial question is who, between the young turning adult and the adult turning old, is holding the risk of changes in the value of b_t. While not changing the strategies of risk neutral agents, the holding allocation does matter for the aggregate dynamics. We let φ measure the exposure of the young generation turning adult to bubble shocks, and $1-\varphi$ the exposure of the adult generation when turning old.[18] The realized wealth

[17] To see this note that the evolution of the wealth of the young generation can be conditioned on two cases according to

$$w_{t|t-1}^A = \begin{cases} R_t^a w_{t-1}, & \text{if} \quad \rho_t > \underline{a} \\ (1-\delta_{t-1}) R_t^a w_{t-1} + \delta_{t-1} R_t \underline{a} w_{t-1}, & \text{if} \quad \rho_t = \underline{a} \quad \text{and} \quad \delta_t \in (0, 1]. \end{cases}$$

However, when $\rho_t = \underline{a}$ it means that $R_t \underline{a} = R_t^a$ by definition, and relationship (2.11) follows.

[18] For simplicity we assume that the exposure φ is time-invariant.

for the young turning adult is then given by

$$w_t^A = w_{t|t-1}^A + \varphi\big(b_t - \mathbb{E}_{t-1}(b_t)\big).$$

Consider next the adult agent at time t. At the beginning of time t the agent receives the draw of investment productivity A_{it+1} and faces a portfolio choice with internal funds given by w_t^A and with relative return ρ_{t+1}. If $A_{it+1} < \rho_{t+1}$ the agent deposits all her funds with the intermediary. If $A_{it+1} \geq \rho_{t+1}$ the agent borrows to the limit and invests all the funds in the capital technology. The investing adult agents also receive a random new supply of the bubble asset equal to b_{it}^A that she can immediately sell to the intermediary.[19] For simplicity we assume that all the adult investing agents receive the same bubble asset shock, so that $b_{it}^A = b_t^A$. The capital produced by the investing adult i is

$$k_{it+1} = A_{it+1} \frac{w_t^A + b_t^A}{1 - \theta \frac{A_{it+1}}{\rho_{t+1}}}. \tag{2.12}$$

and her borrowing from the intermediary is $a_{it+1} = -\theta k_{it+1}/\rho_{t+1}$. To facilitate the representation of the aggregate behavior of the investing adult generation it is convenient to define an aggregate "leverage" function U as

$$U_\theta(\rho) \equiv \int_{A>\rho} \frac{A}{\rho} \frac{1}{1 - \theta \frac{A}{\rho}} dG \tag{2.13}$$

where G is the cumulative density function of the distribution for the productivity of capital investment. The leverage function is decreasing in ρ: the higher the relative cost of borrowing, the lower the total borrowing that can be done against the existing internal funds. On the other hand, U is increasing in θ: the more the fraction of future income that can be pledged, the higher the borrowing that can be done against the existing internal funds. A more subtle, but nonetheless crucial, property of U is that when θ is increased the leverage of the more productive agents is increased relatively more compared to that of the less productive ones. This is a consequence of the argument of the integral in (2.13) being non-linearly increasing in $A\theta$. The relevance of this property will become clear in our equilibrium analysis.

[19] The unexpected new fundamental asset supply plays the role of a random relaxation of the borrowing constraint which does not result in an increase in the amount borrowed, but rather in internal funds available, courtesy of the depositors buying the new asset.

Aggregating across all investing adults at time t the total capital produced is

$$k^A_{t+1} = \rho_{t+1} U_\theta(\rho_{t+1})\big(w^A_t + b^A_t\big),$$

while the total borrowing and the total intermediate saving demands for adults at time t are

$$a^A_{t+1} = -\theta U_\theta(\rho_{t+1})\big(w^A_t + b^A_t\big) \quad \text{and} \quad G(\rho_{t+1}) w^A_t.$$

The total wealth that the adult generation expects to bring into their adult age is finally given by

$$w^O_{t+1|t} = R^a_{t+1}\Big[w^A_t G(\rho_{t+1}) + (1-\theta)\big(w^A_t + b^A_t\big)U_\theta(\rho_{t+1})\Big].$$

The old agent at time $t+1$ consumes all the wealth delivered by her investments at time t. As in the case of the young agent turning adult, the adult agent turning old is facing the uncertainty related to the value of the bubble asset held by the intermediary, according to the measure $1-\varphi$. Aggregate consumption at time $t+1$ is then

$$c_{t+1} = w^O_{t+1|t} + (1-\varphi)\big(b_{t+1} - \mathbb{E}_t(b_{t+1})\big).$$

Financial Assets Demand and Supply. The portfolio strategies just described provide a description of the financial assets demand and supply in the economy. The aggregate demand for intermediated savings is

$$d_t(\rho_{t+1}) = (1-\delta_t)w_t + G(\rho_{t+1})w^A_t \tag{2.14}$$

Deposits demand is an increasing function of the relative return ρ_{t+1} and of the wealth available to young and adult agents. Note that w^A_t depends on the return on the bubble asset if any is held by the intermediary, so an increase (decrease) in the value of b_t has a positive (negative) effect on deposits demand, everything else equal.

The aggregate supply of fundamental financial assets consists of the total borrowing of the investing young and adult agents at time t, namely

$$l_t(\rho_{t+1}) = \theta\left[\frac{\delta_t}{1-\theta}w_t + U_\theta(\rho_{t+1})\big(w_t^A + b_t^A\big)\right]. \tag{2.15}$$

When $\theta = 0$ the supply of fundamental assets is zero. As θ is increased the supply becomes positive. Given the properties of U the increase in fundamental asset supply is non-linear in θ.

Equilibrium. Any equilibrium of the economy is a function of the non-negative stochastic process $\{b_t, b_t^A\}_{t=0}^{\infty}$. Let ω_t be a specific realization of the process at time t, and define $\omega^t = \{\omega_0, \omega_1, ..., \omega_t\}$ as the history of the bubble shocks up to time t, with Ω^t being the set of all possible histories, so that $\omega^t \in \Omega^t$. The specific realization of the history ω^t combined with the optimization and market clearing conditions for the output good, capital, labor and financial assets implies that the equilibrium path of aggregate capital and the financial interest rate are a function of the history ω^t, more formally $k_t = k_t\big(\omega^t\big)$ and $R_t^a = R_t^a\big(\omega^t\big)$. An equilibrium for the closed economy is defined as follows.

Definition. *Given a non-negative stochastic process $\{b_t, b_t^N\}_{t=0}^{\infty}$ an equilibrium for the economy is a sequence for aggregate capital allocation $\{k_{t+1}\}_{t=0}^{\infty}$ and financial interest rate $\{R_{t+1}^a\}_{t=0}^{\infty}$ such that individual optimization is achieved, all markets clear and the bubble remains affordable.*

When $\{b_t, b_t^A\} = 0$ we say that the economy is in a fundamental equilibrium at time t. When not in a fundamental equilibrium the economy is experiencing bubbly dynamics. Under some conditions, the fundamental equilibrium is the only possible equilibrium in the economy. In this case, the allocation of capital, output and asset prices will be deterministic. Under other conditions, the bubbly dynamics can be a possibility in equilibrium. In this case the allocation of capital, output and asset prices reflect the behavior of investors' sentiments and become subjected to random fluctuations and sudden changes.

Bubble, Leverage and Crowding In/Out of Capital. Before characterizing an equilibrium it is useful to describe the transfers of funds engineered by the bubble asset and their effect on capital accumulation. Let us consider first the case of $\theta = 0$, so that the only saving options available are direct investment and the bubble asset. In the market for b_t at time t the buyers are δ_t of the the young agents when $\rho_{t+1} = \underline{a}$ and all the young agents plus the less productive adult agents when $\rho_{t+1} > \underline{a}$. On the other hand, the sellers of the asset are all the adult agents when $\rho_{t+1} = \underline{a}$ and the most productive adult agents plus the less productive adult agents from the previous period that are now old if $\rho_t > \underline{a}$ in the previous period. Therefore b_t transfers funds from the least productive young and adults to the most productive adults and some old consumers that have not directly invested when adult. Both a crowding out and a crowding in effect are contemporaneously present in such transfers. When the buying of b_t draws funds from young and adult agents that do not directly invest it operates a crowding out effect on capital. But when the selling of b_t channels funds to productive adult agents it operates a crowding in effect on capital. If the latter effect is large enough, a bubble can be rationally sustained in the economy.

Suppose now that $\theta > 0$. In this case the transfer of funds from the least productive young and adult to the most productive adult is already happening because of the market for the fundamental asset l_t. The average efficiency level at which funds are invested is a function of the internal funds available to productive adult agents. In presence of the bubble b_t, productive adult agents selling the asset accumulate more internal funds that increase their borrowing potential. The efficiency enhancement due to the bubble is higher compared to the $\theta = 0$ case, and, given the same b_t, the crowding-in effect is stronger. There is, however, a limiting factor in the circulation of b_t when θ gets larger, which is represented by the supply of the fundamental assets. The increased borrowing capacity of productive adults creates a supply of fundamental assets that compete with b_t in capturing the savings of young and unproductive adult agents. This imposes an upper bound on the attainable value of b_t in equilibrium as θ is increased. The working of these effects in equilibrium are formalized in the next section.

3 Bubbly Dynamics in the Closed Economy

The objective of this section is to study the conditions under which a bubbly equilibrium can emerge in a closed economy of the type described in Section 2. In particular, we want to understand how financial development, measured by θ, affects the possibility of bubbly dynamics. To facilitate the analysis it is convenient to express the dynamics of the economy recursively. We re-scale all the variables by the "size" of the economy, represented by the total net wealth in the economy at time t after production of output and consumption of the old agents took place, but the portfolio choice and the capital investment did not. Let the total realized net wealth be denoted by $W_t = w_t + w_t^A$, then its distribution across adult and young is denoted by $n_t \equiv \frac{w_t^A}{W_t}$ and $1 - n_t \equiv \frac{w_t}{W_t}$. Let $W_{t|t-1}$ denote the total wealth in the economy at t as expected at the end of period $t-1$. We define the value of the bubble relative to such wealth as

$$z_t \equiv \frac{b_t}{W_{t|t-1}}, \quad \text{and} \quad z_t^A \equiv \frac{b_t^A}{W_{t|t-1}}.$$

The difference between the expected and realized wealth is due to the difference in the expected and realized value of the bubble. Let $\sigma_t \equiv z_t - \mathbb{E}_{t-1}(z_t)$, then the relationship between expected and realized wealth in the economy is

$$W_t = (1 + \varphi\sigma_t)W_{t|t-1},$$

where φ is the fraction of the change in the asset value that is accrued or sustained by young depositors turning adults at time t. Let $e_{t+1|t}$ denote the expected wealth of the young at $t+1$ in terms of the net realized wealth at t, this can be written (see Appendix for details) as

$$e_{t+1|t} \equiv \frac{1-\alpha}{\alpha}\left[\frac{\delta_t}{1-\theta}(1-n_t) + U_\theta(\rho_{t+1})\left(n_t + \frac{z_t^A}{1+\varphi\sigma_t}\right)\right],$$

The expected wealth of the young at time $t+1$ is equal to the wage payment they receive from supplying their unit of labor. The level of the wage is a function of the capital available at time $t+1$, which was determined by the portfolio allocation chosen at t by the then young and adult generations. The larger is the fraction of output that remunerates labor, $1-\alpha$, the larger the wealth of the young at $t+1$ for any level of capital available. The expected wealth of the adult at $t+1$ in terms of wealth at t is always equal to the fraction of wealth held when young at t, $1-n_t$ (see Appendix). The following proposition provides a recursive representation of the equilibrium in the closed economy.

Proposition 1. *The non-negative stochastic process* $\{z_t, z_t^A\}_{t=0}^{\infty}$ *and the sequence* $\{\delta_t \in [0,1], n_t, \rho_{t+1}\}_{t=0}^{\infty}$, *with* $\rho_{t+1} = \underline{a}$ *when* $\delta_t < 1$, *constitute an equilibrium of the closed economy if the following conditions are satisfied:*

(a) expected return of bubble

$$\mathbb{E}_t(z_{t+1}) = \frac{z_t + z_t^A\big(1 - G(\rho_{t+1})\big)}{1 - n_t + e_{t+1|t}}\frac{1}{1+\varphi\sigma_t}, \tag{3.1}$$

(b) asset market clearing

$$\theta \frac{\alpha}{1-\alpha} e_{t+1|t} + \left[z_t + z_t^A\big(1 - G(\rho_{t+1})\big)\right] \frac{1}{1+\varphi\sigma_t} = (1-n_t)(1-\delta_t) + n_t G(\rho_{t+1}), \tag{3.2}$$

(c) intergenerational wealth distribution

$$n_{t+1}(1+\varphi\sigma_{t+1}) = \frac{1-n_t}{1-n_t+e_{t+1|t}} + \varphi\sigma_{t+1}, \tag{3.3}$$

Proof. See Appendix. □

Along the fundamental dynamics, described by Equations (3.2)-(3.3) with $z_t = z_t^A = 0$, funds are transferred from young agents to adult agents through loans l_t and the equilibrium relative return ρ_{t+1} ensures that the demand for intermediated savings is equal to the supply of loans. A fundamental equilibrium always exists.

The bubbly dynamics display by definition at least one strictly positive realization for z_t. In this case equation (3.1) has to be satisfied as well. In the bubbly equilibrium funds are transferred from the young to the adult through z_t as well as l_t, but for that to be possible z_t must offer a return that is competitive with that of the loans. Equation (3.1) provides the restriction on the bubble dynamics for this to happen. The total value of the bubble asset is equal to the sum of the current value of the existing asset, z_t, and that of the new bubble asset issued by investing adult agents $z_t^A(1-G(\rho_{t+1}))$. Since both z_t and z_t^A are channeling funds from unproductive to productive investors, the average capital investment efficiency is increased in the economy, which helps to keep the purchase of the bubble affordable. However, the necessary return on z_t might be such that its value eventually exceeds the resources available to the young. At this point the relative return ρ_{t+1} increases above $\underline{a}$ in order to attract the least productive adult agents to purchase the asset. The available funds are no longer channeled only to productive adults, but they begin to be channeled also to old agents - those that were the least productive when adult - for consumption. As this happens, the original crowding-in effect is contrasted by a crowding-out effect due to the increasing size of the bubble. Eventually, the crowding-out effect might slow down capital accumulation enough to make conditions (3.1)-(3.2) jointly unattainable at some point in the future. Note that $e_{t+1|t}$ measures the income that the next generation of young is expected to receive at time $t+1$, which depends on the total capital accumulated at time t into $t+1$, and the share of the income produced with that capital that goes to young agents, represented by $\frac{1-\alpha}{\alpha}$. As $e_{t+1|t}$ is reduced, condition (3.1) implies a higher expected change in z_t, which can eventually result in (3.2) being violated.

In summary, for any given level of z_t, factors that increase the crowding-in effect or mitigate the crowding-out effect makes conditions (3.1)-(3.2) easier to achieve for all t. To isolate the role of such factors we turn the attention to a particular resting point of the dynamic system above.

Stationary Stochastic Equilibrium (SSE) The equations in Proposition 1 entirely describe the dynamics of the economy given some initial conditions on n_0, ρ_1 (or δ_0 in case $\rho_1 = \underline{a}$) and $\mathbb{E}_0(z_1)$, and as such identify a set of stochastic processes for the bubble that can be part of an equilibrium. We are interested in analyzing the sufficient conditions for such set to be non-empty, and characterize some basic features of the elements belonging to this set, such as the upper bound on the bubble. We follow Weil (1987) and Kocherlakota (2009) and study a *stationary stochastic* bubbly equilibrium (SSE) of the dynamic system (3.1)-(3.3). More precisely, we focus on the case of a bubbly z^* which is believed to disappear each period with some probability p, but which instead remains at the same stationary level z^*. This is a useful benchmark because it provides an upper bound for the expected value of a candidate stochastic process, conditional on the realization z_t, to be an equilibrium.[20]

To be more specific about the stochastic structure of the bubble, suppose that the state of the world at t, ω_t, can take two values, F and B. If $\omega_t = F$ then the non-fundamental asset has no value and $z_t = z_t^A = 0$, irrespectively of whether the expected value from $t-1$, $\mathbb{E}_{t-1}(z_t)$, was positive or equal to zero. If the expected value was positive at $t-1$, at t the non-fundamental equilibrium collapses if $\omega_t = F$. If $\omega_t = B$ then the bubble can take a positive value, so that $z_t > 0$ and/or $z_t^A > 0$. The transition probabilities from the two states are such that the process is Markovian and they are defined as follows,

$$Pr(\omega_t = B|\omega_{t-1} = F) = r \quad \text{and} \quad Pr(\omega_t = F|\omega_{t-1} = B) = p.$$

Therefore, if $\omega_t = B$ and $\mathbb{E}_t(z_{t+1}) > 0$, the probability of remaining in bubbly equilibrium in $t+1$ is

[20] An alternative approach would be to study the derivative of $E_t(z_{t+1})$ with respect to z_t and make sure that it is smaller than 1 for $z_t = 0$. By continuity then there exist a z^* that provides an upper bound on the stochastic process for z_t to be a bubbly equilibrium. This is the approach taken by Martin and Ventura (2012). It can be showed that in our setting the conditions on the structural parameters for the derivative of $E_t(z_{t+1})$ being smaller than 1 is equivalent to a $z^* > 0$ existing. We choose the SSE approach because it allows a cleaner analysis of the effect of θ on equilibrium existence.

equal to $1-p$, which gives

$$z_{t+1} = \frac{\mathbb{E}_t(z_{t+1})}{1-p}.$$

where $\mathbb{E}_t(z_{t+1})$ is restricted by the equilibrium return condition (3.1), and therefore a function of $\mathbb{E}_{t-1}(z_t)$. The SSE z^* is then the solution of the fixed point $\mathbb{E}_t(z_{t+1}) = \Phi(\mathbb{E}_{t-1}(z_t))$, where Φ is the mapping implied by the equilibrium conditions of Proposition 1.

We assume no continuous creation of new assets in steady state, so that eventually $z_t^A = 0$ for every t.[21] The SSE is then characterized by a vector $(z^*, \rho^*(\delta^* \text{ if } \rho^* = \underline{a}), n^*)$ from which all the other relevant variables can be derived. The following corollary provides the description of the SSE.

Corollary 1. *The Stationary Stochastic Equilibrium (SSE) of the closed economy is the solution to*

$$\frac{\delta^*}{1-\theta} + (1-p)U_\theta(\rho^*) = \frac{\alpha}{1-\alpha}\frac{1}{1-p}, \tag{3.4}$$

and

$$z^* = \left(\frac{1-p}{2-p}\right)\left[(1-\delta^*) + (1-p)G(\rho^*) - \theta\frac{\alpha}{1-\alpha}\frac{1}{1-p}\right] > 0, \tag{3.5}$$

with $\rho^* = \underline{a}$ *when* $\delta^* \in (0,1]$.

For a SSE equilibrium to exist two conditions must be met. First, the bubble must eventually grow at a rate that in expectations is the same rate at which wealth grows, and, for this to happen while providing a competitive return with other assets, equation (3.4) has to be satisfied. The left hand side of (3.4) is decreasing in ρ^* (and increasing in δ^*) so a SSE would fail to exist when

$$\frac{1}{1-\theta} + (1-p)U_\theta(\underline{a}) < \frac{\alpha}{1-\alpha}\frac{1}{1-p}. \tag{3.6}$$

Equation (3.6) suggests three reasons for failure of existence. First, the fraction of income that is appropriated by the new young generation might be too small, α too high, to allow them to keep purchasing the bubble. Second, the probability of the bubble collapsing to zero, p, might be too high, which means that z has to grow faster to compensate for the event of a total loss of value. Third, the

[21] Appendix B studies the conditions under which $z_t^A > 0$ facilitates the existence of a bubbly equilibrium.

"leverage potential" of the economy, as measured by θ, might be too low. We will return to the role of θ in the next section.

Even if condition (3.4) holds, a bubbly equilibrium might still fail to exist if equation (3.5) is not satisfied. For a bubble to have positive value there must be a saving demand in excess of loans assets. If loans as a share of the wealth in the economy, represented by $\theta\frac{\alpha}{1-\alpha}\frac{1}{1-p}$, are large enough, then there is no room in the financial market for the bubble and a SSE will fail to exist.

Degree of Pledgeability θ and Existence of SSE

Corollary 1 suggests that the degree of pledgeability θ plays an ambiguous role in the existence of a bubbly equilibrium. In this section we show that the existence of a SSE and the size of z^* have indeed a non-monotonic relationship with θ.[22]

For simplicity we assume that p is arbitrarily small (i.e. we set $p = 0$).[23] We first consider the case of θ being too small to allow for a bubbly equilibrium. Suppose that $\theta = 0$ and that condition (3.4) cannot be satisfied. This requires

$$1 + U_0(\underline{a}) < \frac{\alpha}{1-\alpha}. \tag{3.7}$$

As θ is increased the left hand side of (3.7) is also increased since both young and adult agents can leverage off their internal funds. For a high enough θ the LHS can overcome the RHS, providing the first necessary condition for a SSE to exist. The intuition for this result lies on the role of θ for the crowding-in effect of $z^* > 0$ on capital accumulation. For any given $z^* > 0$, when $\theta = 0$ inefficient investment by the young generation is reduced and funds are transferred to the productive adult. When $\theta > 0$, the same $z^* > 0$ is enhancing the crowding-in effect. The reason for this is that adult agents with higher productivity are able to attract relatively more external funds for a given level of internal funds, a property that is a consequence of the non-linearity of U with respect to θA. In Section 2 we described the role of b_t as eliminating the inefficient investments and substituting them with efficient ones. An increase in the pledgeability parameter θ makes this selection process more effective, which

[22] Hirano and Yanagawa (2016) derive a similar result in a model with endogenous growth - output technology linear in capital - and infinitely lived agents. One key difference with their analysis is that while they focus on conditions for dynamic inefficiency in their economy, i.e. the relationship between the growth rate and the interest rate, we focus on the tension between the fundamental vs the bubble asset supply, which turns out to be a more general notion in our setting.

[23] This assumption is not crucial for the qualitative nature of the results since all the conditions are continuous in p at $p = 0$

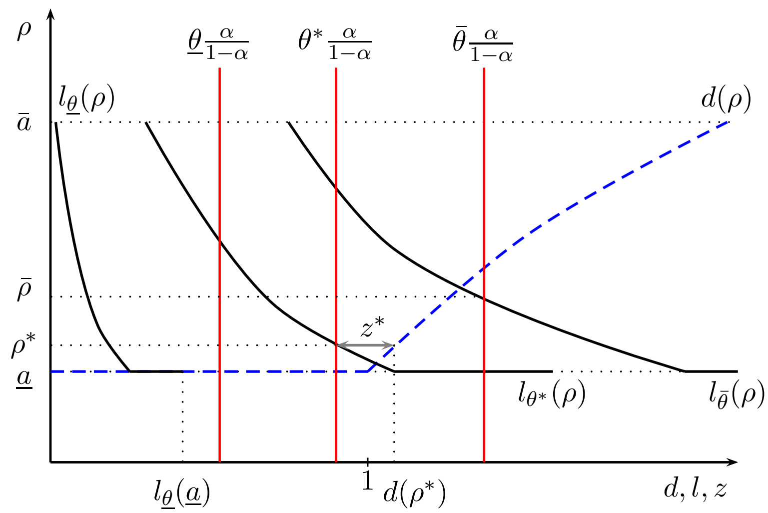


Figure 5: Metzler Diagram and Bubbly Equilibrium

means that for the same b_t a larger crowing-in effect is generated.

The increase in the efficiency by which z^* turns low productivity investments into high productivity ones, however, simultaneously generates an effect that works against the existence of a SSE. This is to be found on the *asset supply* side in equation (3.5). As θ is increased more loans assets are generated, which compete with the bubble for the savings of the young and adult agents. Eventually, there will be enough fundamental asset supply that a $z^* > 0$ will no longer be possibly demanded by savers, and a bubbly equilibrium will cease to exist.

Figure 5 depicts the Metzler diagram for the closed economy that shows the case of three different increasing values for θ: $0 < \underline{\theta} < \theta^* < \bar{\theta}$. At $\underline{\theta}$ there is an excess savings demand that in principle could absorb the bubble since the fundamental asset supply is at $l_{\underline{\theta}}(\underline{a}) < 1$, while savings demand can be anywhere between 0 and 1 as long as $\rho = \underline{a}$. However, any value for z will not be able to generate a crowding-in effect strong enough to remain affordable in the economy, which corresponds to the lack of an intersecting point for the fundamental supply curve $l_{\underline{\theta}}(\rho)$ and the minimal level of average investment efficiency necessary to sustain a bubbly equilibrium $\underline{\theta}\frac{\alpha}{1-\alpha}$ (recall that $l_\theta(\rho) = \theta\big(\frac{\delta}{1-\theta} + U_\theta(\rho)\big)$). When $\theta = \theta^*$, the crowding-in effect of bubble is larger and the return on the bubble can be sustained. As the graph shows, the fundamental supply curve $l_{\theta^*}(\rho)$ intersects $\theta^*\frac{\alpha}{1-\alpha}$ at $\rho^* > \underline{a}$. This is still not enough to guarantee the existence of an SSE, as the bubble must find a demand from savers not satisfied by loans assets. From the diagram we see that at the relative return ρ^* there is an excess demand for deposits, $d(\rho^*) > \theta^* \frac{\alpha}{1-\alpha}$, and so a bubble $z^* > 0$ can indeed clear the financial market. Finally, when $\theta = \bar{\theta}$ the crowding-in effect is still strong enough for a bubbly equilibrium and the relative return is raised to $\rho = \bar{\rho}$. However, at that return the supply of fundamental assets is higher than the demand for deposits, $l_{\bar{\theta}}(\bar{\rho}) > d(\bar{\rho})$, which means that the financial market is already saturated by the fundamental supply and a SSE ceases to exist.

4 Equilibrium under Financial Globalization

The global economy consists of two regions, "North" and "South", whose individual economies have the structure of the economy presented in Section 2. In what follows the variables with *tilde* refer to the South region. The two regions produce the same output good employing identical technologies. The only difference between the two economies is in their level of financial development measured by the degree of pledgeability: θ for the borrowers who operate the capital investment technology in the North and $\tilde{\theta}$ for those located in the South. We assume that $\theta > \tilde{\theta}$. We think of the degree of pledgeability as capturing the institutional environment in which loans are generated together with the ability of the financial intermediaries to evaluate investment projects. In this sense, a loan extended by an intermediary from a developed financial environment to a borrower located in a less developed one is subjected to some of the pledgeability restrictions of the institutions where the borrower is located since the recovery of the loan, in case of lack of repayment, has to take place in the location with weaker institutions. We capture this feature of the financial environment by assuming that for a financial intermediary from the North to extend a loan to an agent operating the investment technology in the South the degree of pledgeability is $\phi\theta$ with $\phi \in [0, 1]$. Thus, when financial markets are integrated, an investor from the South will be able to pledge the fraction $\theta_s = max\{\tilde{\theta}, \phi\theta\}$ of her future income from investment when borrowing from the financial intermediary.

Financial integration corresponds to the situation where the two regions can freely trade in financial assets and output. Once the investment in physical capital is made in a specific region, the capital obtained can only be used in the output technology of that region, so capital goods are not directly tradable. The market for investment is nevertheless open, one unit of output good from the North can be directly invested in obtaining capital in the South, and viceversa. We assume that labor is not a mobile factor across regions. Therefore, under financial integration wages will not be equated across regions unless financial markets function smoothly and capital reaches the same level in the North and in the South.

Financial integration also means that the markets for the bubble asset are integrated: intermediaries from both North and South can buy new bubble assets from investing adults and can trade bubble assets among them. We let the total value of the global bubble at the beginning of period t be denoted by b_t^*.

Under financial integration the market clearing for the output good in the global equilibrium is

$$y_t + \tilde{y}_t = c_t + \tilde{c}_t + x_t + x_t^A + \tilde{x}_t + \tilde{x}_t^A$$

where $\tilde{y}_t$, $\tilde{c}_t$ denote output produced and consumption in the South, respectively, and $\tilde{x}_t$, $\tilde{x}_t^A$ is aggregate investment of young and adult agents in the South. Financial markets integration implies that intermediaries from the North and the South freely compete for deposits and loans. Free entry and the zero profit condition essentially mean that there is a representative global intermediary holding all the deposits and extending all the loans. The balance sheet of the global intermediary is

$$b_t^* + b_t^A + \tilde{b}_t^A + l_t + \tilde{l}_t = d_t + \tilde{d}_t, \tag{4.1}$$

where $\tilde{l}_t$ are the total loans extended to directly investing agents in the South and $\tilde{d}_t$ are deposits of saving agents from the South. The values of the specific components of the balance sheet of the global intermediary can be used to determine the flow of funds across regions, and, as a consequence, the current accounts of the two economies. Suppose that we are in a fundamental equilibrium so that the bubble is zero. In the closed economy it must be $\tilde{l}_t = \tilde{d}_t$, so that all the domestic savings are channeled towards domestic investments. However, because of the financial frictions the asset supply from loans are limited and agents are forced to directly invest instead of depositing funds in intermediated savings. When financial markets are open, the saving demand of the South can be satisfied by the asset supply of the more financially developed North so that $\tilde{l}_t < \tilde{d}_t$, and funds are now channeled from the South to the North. In the presence of a bubble, the difference between the expected return and the realized return is assumed to be distributed across depositors from the North in fraction μ and from the South in fraction $1-\mu$. Within each region, the difference is distributed across young and adult agents according to the fraction φ and $1-\varphi$ in the North (resp. $\tilde{\varphi}$ and $1-\tilde{\varphi}$ in the South).

The global equilibrium is characterized by a world financial interest rate R^*_{t+1} that clears the market for financial assets. Symmetrically to the closed economy analysis we define the relative financial returns for the two regions as $\rho_{t+1} = \frac{R^*_{t+1}}{R_{t+1}}$ and $\tilde{\rho}_{t+1} = \frac{R^*_{t+1}}{\tilde{R}_{t+1}}$. The presence of financial frictions in both regions, if severe enough, can prevent the return on capital from being equated across the economies. For example, if $\rho_{t+1} > \tilde{\rho}_{t+1}$, financial frictions are keeping funds from flowing from the North to the South, or, alternatively, they are channeling savings from the South to the North. We let q_{t+1} capture the tension between North and South in terms of net flow of funds, where

$$q_{t+1} \equiv \frac{\tilde{\rho}_{t+1}}{\rho_{t+1}} = \frac{R_{t+1}}{\tilde{R}_{t+1}}. \tag{4.2}$$

If a unit of output could be freely allocated across the regions, the value of q_{t+1} would provide the information necessary for an efficient allocation: when $q_{t+1} > 1$ the extra unit should be allocated to the North and when $q_{t+1} < 1$ to the South.

The definition of an equilibrium for the global economy is analogue to the definition of an equilibrium for the individual economy of Section 2, and so we omit its statement. To characterize the global equilibrium conditions recursively we define the fractions of wealth of each economy held by the adults as in Section 3 and denote them by n_t and $\tilde{n}_t$. In addition, we let v_t represent the relative size of net wealth of the South economy with respect to the North, where $v_t = \frac{W_t}{\tilde{W}_t}$. Next, we express the bubble in terms of the net wealth in the North at time t as expected at $t-1$, so that $z^*_t \equiv \frac{b^*_t}{W_{t|t-1}}$, $z^A_t \equiv \frac{b^A_t}{W_{t|t-1}}$, $\tilde{z}^A_t \equiv \frac{\tilde{b}^A_t}{W_{t|t-1}}$. In presence of a positive bubble, the global economy is subjected to global stochastic fluctuations defined as $\sigma^*_t \equiv z^*_t - \mathbb{E}_{t-1}(z^*_t)$. The definition for $e_{t+1|t}$ is as in Section 3. Here we define the analogue variable for the South economy,

$$\tilde{e}_{t+1|t} \equiv \frac{1-\alpha}{\alpha}\left[\frac{\tilde{\delta}_t}{1-\theta_s}(1-\tilde{n}_t)v_t + U_{\theta_s}(\rho_{t+1}q_{t+1})\left(\tilde{n}_t v_t + \frac{\tilde{z}^A_t}{1+\mu\varphi\sigma^*_t}\right)\right], \tag{4.3}$$

which represents the wealth of the young agents at time $t+1$ as expected at time t in terms of the North wealth at time t. The wealth of the adult agents at time $t+1$ as expected at time t in terms of the North wealth at time t is $v_t(1-\tilde{n}_t)$. Finally, the expected relative size of wealth in the two regions is

$$v_{t+1|t} = \frac{v_t(1-\tilde{n}_t) + \tilde{e}_{t+1|t}}{1 - n_t + e_{t+1|t}}.$$

The following proposition characterizes the dynamic equilibrium of the global economy.

Proposition 2. *The non-negative stochastic process* $\{z_t^*, z_t^A, \tilde{z}_t^A\}_{t=0}^{\infty}$ *and the sequence* $\{n_t, \rho_t, \tilde{n}_t, q_t, v_t\}_{t=0}^{\infty}$ *constitute an equilibrium of the global economy with integrated financial markets if the following conditions are satisfied:*

(a) expected return of the bubble

$$\mathbb{E}_t(z_{t+1}^*) = \frac{z_t^* + z_t^A(1 - G(\rho_{t+1})) + \tilde{z}_t^A(1 - G(\rho_{t+1}q_{t+1}))}{e_{t+1|t} + 1 - n_t} \frac{1}{1 + \mu\varphi\sigma_t^*}, \tag{4.4}$$

(b) asset market clearing

$$\frac{1}{1+\mu\varphi\sigma_t^*}\left[z_t^* + z_t^A(1 - G(\rho_{t+1})) + \tilde{z}_t^A(1 - G(\rho_{t+1}q_{t+1}))\right] + \frac{\alpha}{1-\alpha}\left[\theta e_{t+1|t} + \theta_s \tilde{e}_{t+1|t}\right] = \tag{4.5}$$

$$(1-n_t)(1-\delta_t) + n_t G(\rho_{t+1}) + v_t\left[(1-\tilde{n}_t)(1-\tilde{\delta}_t) + \tilde{n}_t \tilde{G}(\rho_{t+1}q_{t+1})\right], \tag{4.6}$$

(c) capital return inequality and size inequality

$$q_{t+1} = \left(\frac{\tilde{e}_{t+1|t}}{e_{t+1|t}}\right)^{\frac{1-\alpha}{\alpha}}, \qquad (1 + \mu\varphi\sigma_{t+1}^*)v_{t+1} = v_{t+1|t} + (1-\mu)\tilde{\varphi}\sigma_{t+1}^* \tag{4.7}$$

(d) intergenerational realized wealth distribution

$$(1 + \mu\varphi\sigma_{t+1}^*)n_{t+1} = \frac{1 - n_t}{1 - n_t + e_{t+1|t}} + \mu\varphi\sigma_{t+1}^*, \tag{4.8}$$

$$\left(1 + \frac{1-\mu}{v_{t+1|t}}\tilde{\varphi}\sigma_{t+1}^*\right)\tilde{n}_{t+1} = \frac{v_t(1-\tilde{n}_t)}{\tilde{e}_{t+1|t} + v_t(1-\tilde{n}_t)} + (1-\mu)\tilde{\varphi}\sigma_{t+1}^*, \tag{4.9}$$

Proof. See Appendix. ☐

The conditions of Proposition 2 parallel those for the closed economy case. Aside from the evolution of

q_{t+1} and v_{t+1}, which are essentially equilibrium accounting, the key difference lies in the asset market clearing condition, which now equates the global demand for savings with the global supply of both fundamental and bubble assets.

5 Bubbly Equilibrium in the Global Economy

In this Section we formally address the question raised in the introduction: can financial globalization create the conditions for the emergence of bubbly dynamics in the global economy? To answer this question we proceed as we did in Section 3 and focus on the characterization of the stochastic stationary steady state of the global economy (GSSE), under the assumption that no new bubble assets are created in steady state and that $p = 0$. To show that financial markets integration can open the door to bubbly dynamics in the global economy, we consider the case in which a bubbly steady state does not exist for the individual economies under autarky. We know from Section 3 that this is possible if the degree of pledgeability is high enough, e.g. in the North economy, or low enough, e.g. in the South economy. Next we show that under the same parameter values that ensure non-existence of a bubble in autarky, a bubbly steady state is indeed possible when financial markets integrates.

The next corollary characterizes the GSSE for the global economy.

Corollary 2. *Suppose that $\phi\theta \geq \tilde{\theta}$. The GSSE of the global economy (z^*, ρ^*, q^*, v^*) is the solution to*

$$\frac{\delta^*}{1-\theta} + U_\theta(\rho^*) = \frac{\alpha}{1-\alpha} \tag{5.1}$$

where $\rho^* = \underline{a}$ when $\delta^* \in (0, 1]$,

$$\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*) = \frac{\alpha}{1-\alpha} \tag{5.2}$$

where $q^*\rho^* = \underline{a}$ when $\tilde{\delta}^* \in (0, 1]$, and

$$z^* = \frac{1}{2}\left[(1-\delta^*) + G(\rho^*) + v^*\left[(1-\tilde{\delta}^*) + G(q^*\rho^*)\right] - \theta\frac{\alpha}{1-\alpha}\left(1+\phi v^*\right)\right] > 0, \tag{5.3}$$

with $v^* = q^{*\frac{\alpha}{1-\alpha}}$.

Proof. See Appendix. □

Conditions (5.1) and (5.3) parallel those of Corollary 1 for the closed economy. Condition (5.2) is, however, not immediately comparable to Corollary 1 since the dynamics of z_t^* are defined in terms of wealth in the North. The reason for equation (5.2) to be necessary in a GSSE is that we are imposing a constant ratio for the wealth of the two regions, v^*, in steady state. At z^*, for the bubble to remain affordable and the relative size of the two economies to remain constant, (5.2) must necessarily hold.

In Proposition 2 we have assumed that the financial development of the North is strictly higher compared to the South. To simplify the analysis further without much loss of generality we assume that investing agents in the South cannot pledge any portion of their future income.

Assumption 1. $\tilde{\theta} = 0$.

An immediate consequence of Assumption 1 is that, in order for the South region to not allow bubbly equilibria in autarky, the following condition is sufficient

Assumption 2. $1 + U_0(\underline{a}) < \frac{\alpha}{1-\alpha}$.

In the North economy $\theta > 0$ and we know from Section 3 that to ensure that a bubbly equilibrium does not exist it is sufficient to assume that at the relative return that would ensure a sustainable bubble, the supply of fundamental assets is already enough to satisfy the intermediated saving demand. Formally,

Assumption 3. $1 + G(\rho) \leq \theta \frac{\alpha}{1-\alpha} \quad \textit{for} \quad \rho : \theta U_\theta(\rho) = \frac{\alpha}{1-\alpha}\theta$.

Can the conditions in Corollary 2 be satisfied with $z^* > 0$ under Assumptions 1-3? Condition (5.1) is implied by Assumption 3, and so it will hold for some $\rho^* \geq \underline{a}$. Condition (5.2) for q^* requires some additional elaboration. Consider first the case where $\phi = 0$, so that the degree of pledgeability of investment returns from the South is not changed by financial integration. Then under Assumption 2, the condition in (5.2) cannot possibly hold, since by construction $q^*\rho^* \geq \underline{a}$. As $\phi > 0$ the left hand side of (5.2) is increased for any value of $q^*\rho^*$ and so for ϕ large enough condition (5.2) will eventually hold. Intuitively, for a positive bubble to remain sustainable in the global economy, there needs to be a minimal level of leverage potential for productive investors in the South region so to avoid the relative size of the region becoming arbitrarily small, i.e. $v^* \to 0$. For $\phi < 1$ conditions (5.1) and (5.2) imply that $q^* < 1$. In a GSSE of the global economy, as long as there is financial development heterogeneity, there will remain a difference in the marginal return on physical capital across regions. Note that this statement is conditional on a GSSE existing, which means that the financial development overall is already not enough to channel funds from savers to investors. The final requirement for a GSSE to exist for the global economy is that $z^* > 0$. Under Assumption 3 the total demand for savings of the North in the GSSE is entirely satisfied by the North supply of fundamental assets. For the bubble to have a positive value then it is necessary that the financial integration generates a demand for intermediated savings from the South and that such demand is high enough that it absorbs the supply of fundamental assets in excess of the demand for savings of the North *and* the supply of fundamental assets due to the borrowing of the South, which was not possible in autarky.

Intuitively, for a bubbly equilibrium of the global economy to exist, the North region must be already close to a situation of asset supply "shortage". To make this as transparent as possible, suppose that the North economy is parameterized with θ, α and $\underline{a}$ such that $U_\theta(\underline{a}) = \frac{\alpha}{1-\alpha}$ and $\theta \frac{\alpha}{1-\alpha} = 1$. Then in autarky the North region would have a bubble exactly equal to zero in SSE since $\rho = \underline{a}$ and $\theta U_\theta(\underline{a}) = 1$. As financial integration happens, for ϕ high enough it will be possible that $\tilde{\delta}^* > 0$ such that (5.2) is satisfied. The higher is ϕ the higher will be the fraction of young agents in the South demanding intermediated savings, so that $\tilde{\delta}^*(\phi)$ defined by (5.2) is a decreasing function of ϕ. The condition for a GSSE in this specific case then boils down to

$$1 - \tilde{\delta}^*(\phi) > \phi. \tag{5.4}$$

Equation (5.4) captures the tension between asset demand and supply for the existence of a bubbly equilibrium in the global economy. The left hand side of the inequality is the saving demand from the South, which is increasing in ϕ. The right hand side represents the fundamental asset supply generated by loans to the South, which is also increasing in ϕ. As long as the saving demand is larger than the supply, there is room for a bubble. The inequality is not satisfied for $\phi = 0$ and $\phi = 1$, while it can hold for intermediate values. The same non-monotonic relationship between the degree of pledgeability and the existence of a bubbly equilibrium in the closed economy directly extends to the global economy, where it takes the form of non-monotonicity with respect to the degree of financial integration across regions ϕ. The following proposition summarizes our result.

Proposition 3. *Suppose that Assumptions 1-3 hold. Then, when in autarky both the North and the South economy do not allow for episodes of bubbly dynamics in financial assets. Episodes of bubbly dynamics become possible when financial markets are integrated if: (i) the North economy is relatively*

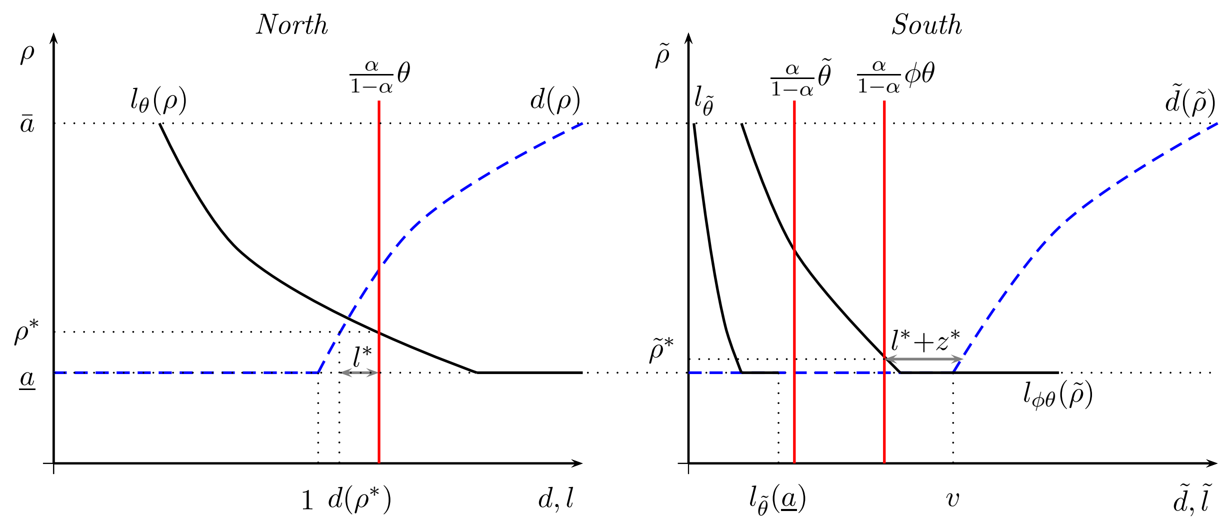


Figure 6: Metzler Diagram and Global Economy SSE

close to a condition of asset supply shortage in autarky; (ii) the degree of financial integration, measured in terms of the increase in pledgeability of investment income in the South, is intermediate.

Figure 6 is the Metzler diagram for the global economy. In autarky a SSE is not possible in the North since at ρ^* the asset supply is bigger than the savings demand, while it is not possible in the South since $\tilde{l}_{\tilde{\theta}}(\underline{a}) < \frac{\alpha}{1-\alpha}\tilde{\theta}$. In a fundamental equilibrium of the integrated economy the Metzler diagram instructs one to look for the values of ρ and $\tilde{\rho}$ that ensure equality between the excessive supply of fundamental asset in the North economy, $l_\theta(\rho) - d(\rho) > 0$, and the shortage of fundamental assets in the South economy, $l_{\phi\theta}(\tilde{\rho}) - \tilde{d}(\tilde{\rho}) < 0$. In presence of different financial developments, the fundamental equilibrium happens at $\rho > \tilde{\rho}$ since the marginal return on capital is higher in the South. In a GSSE with $z^* > 0$ the values for ρ^* and $\tilde{\rho}^*$ are determined by conditions (5.1) and (5.2), and they correspond to the points in the figure where the fundamental asset supply intersects $\frac{\alpha}{1-\alpha}\theta$ in the North and $\frac{\alpha}{1-\alpha}\phi\theta$ in the South. The Metzler diagram can then be used to check whether the excess fundamental asset supply in the North at ρ^*, given by $l^* = \frac{\alpha}{1-\alpha}\theta - d(\rho^*)$, is smaller than the excess savings demand from the South at $\tilde{\rho}^*$, given by $\tilde{d}(\tilde{\rho}^*) - \frac{\alpha}{1-\alpha}\phi\theta$. This is the case in the figure where a GSSE with $z^* > 0$ indeed exists.

We have focused on purpose on the starkest case of non-existence of bubbly dynamics in autarky and existence once the regions are financially integrated. A more general version of our result holds in terms of the maximum bubble that can be sustained in equilibrium in autarky as opposed to the case of financial integration. Suppose that both the North and the South allow for episodes of bubbly dynamics in autarky with upper bounds of $z_a^* \geq 0$ and $\tilde{z}_a^* \geq 0$ respectively, both measured in terms of steady state output of the North in autarky. It is then possible to show that under financial integration the maximum bubble can be higher than the sum of the two, namely $z^* > z_a^* + \tilde{z}_a^* \geq 0$.

6 Global Equilibrium Dynamics: Numerical Simulation

In this section we study by numerical simulations the dynamics of asset values, output and interest rates in the global economy under financial integration. We are interested in understanding whether episodes of asset price fluctuations qualitatively similar to those reported in Figure 2b can be generated within the model we have presented once financial markets are integrated, even though the same episodes were not possible in autarky. When financial integration happens, the steady state capital levels of the individual regions in the global economy change as the incidence of financial frictions is different in the open economy. For the purpose of this section we do not focus on the adjustment path towards the new fundamental steady state, but we assume that as the global economy is integrated the new fundamental steady state is immediately achieved and bubbles happen around the new steady state. We assume that the productivity distribution for adult agents is uniform between $[\underline{a}, \bar{a}]$.[24] While in autarky, investors both in the North and in the South experience sentiment shocks that would allow them to sell a bubble at a positive value. However, the structure of the economy is such that the bubble will never be rationally believed to be feasible in equilibrium and nobody will be willing to buy the asset. In the global economy the increased financial development of the South and the more stringent asset supply shortage provide the conditions for the sentiment shocks to affect asset prices, as they are now rationally believed to be sustainable. The sentiment shocks that were dormant in autarky can now actively affect the aggregates of the global economy.

We simulate the equilibrium dynamics of the global economy under the following parameterization: $\theta = .36$, $\alpha = .73$, $\phi = .6$, $\underline{a} = .5$, $\bar{a} = 1.5$. We assume that the probability of going from the fundamental state to the bubbly state in every period is $r = .05$, while the probability of going from the bubbly state back to the fundamental state is set at $p = .15$. We also assume that during a bubble episode there is

[24] By setting $G(\rho) = \frac{\rho - \underline{a}}{\bar{a} - \underline{a}}$ the leverage function becomes

$$U_\theta(\rho) = -\frac{1}{(\bar{a} - \underline{a})\theta}\left[\bar{a} - \rho + \frac{\rho}{\theta}\log\left(\frac{1 - \bar{a}\theta/\rho}{1 - \theta}\right)\right].$$

These are also the functional forms used to draw the diagrams in Figures 5 and 6.

a new bubble assets supply of $z_t^A = 0.002$ and $\tilde{z}_t^A = 0.001$ every period. This is not necessary for the existence of a bubbly equilibrium, but it generates some interesting properties of the dynamics of the key macroeconomic aggregates.

The choice of the functional forms and parameterization is not done with the intention to calibrate the numerical simulation to the actual economy. Our main interest is in understanding whether the model can generate the qualitative features outlined in the introduction.[25] Under the chosen parameterization both the North and the South regions in autarky would not allow for bubbly episodes, so $z_a^* = \tilde{z}_a^* = 0$. The financially integrated global economy would instead allow bubbly episodes up to $z^* = .08$, or 8% of total wealth in the North region.[26]

Figure 7 shows the behavior of the simulated global equilibrium under a realization for the stochastic process of investors sentiments that is within the existence conditions of Proposition 2. The global economy is at the fundamental state for the first part of the simulation. At period $t = 33$ a bubble appears and the new asset begins to be exchanged in the global financial markets. The increased total availability of assets draws funds from young and possibly adult agents in the South to investors in the North. Note that the bubble is held by both North and South savers in measure μ and $1 - \mu$. For this simulation we set $\mu = .5$. The funds drawn into the intermediated savings market are channeled towards the North and so output and investment fall in the South. As expected, the global financial interest rate increases as the supply of assets is increased. The savers in both regions take advantage of the higher financial return and increase their consumption when old. The funds from the savers of the South are invested by productive adults in the North, where a boom in both investment and output takes place. The boom in output makes the increase in consumption in both regions feasible. The trade balance of the North is deteriorating during the bubbly episode because resources are being transferred from the South to the North. The flip side of the deficit in the North is that the South is running an increasing trade balance surplus. The relative return on capital q is smaller than 1 in the fundamental steady state, due to the lower financial development in the South. During the bubble episode it falls even further, as funds are pulled from investment in the South and invested in capital in the North

[25] For instance, both the assumption of uniform distribution of productivity for adult agents, and the symmetry of this distribution across regions are imposing unnecessary strong restrictions on the quantitative potential of our model.

[26] To provide a ballpark figure for the size of the bubble, households net worth in the United States in 2012 was estimated to be $66 Trillion (Federal Reserve Flow of Funds), making the North region correspond with the US Economy would give an upper bound of around $6 Trillion for the bubble. The values for z_a^* and $\tilde{z}_a^*$ have been calculated using the expressions in Corollary 1, the value for z^* has been computed using a version of the equations in Corollary 2 with $p > 0$.

region.

When the bubble episode ends, in period $t = 41$, there is a "forced" transfer of wealth from the holders of the bubble to the next young generation, which cannot buy the asset anymore. The direction of the wealth transfer depends on who is holding the asset at the time of the collapse. In the numerical simulation it is assumed that the adjustment is equally sustained by North and South savers, which means a drop in consumption in both regions. The disappearance of the asset also drastically reduces the transfer of resources from the South to the North, which creates a sudden but transitory reversal in the trade balance. Both output and consumption fall sharply in the North at the time of the collapse and then begin a slow recovery towards the fundamental steady state level. Finally, output (and thus investment) in the South experiences a sudden upward reversal during a bubble collapse since the funds of the young generation that would have been transferred to the North now have to be invested in the South. Accordingly, the global financial interest rate drops back to the fundamental steady state level and the capital return differential increases back to the initial value. In period $t = 63$ another bubble episode starts and a patter similar to the first episode is once again observed.

7 Conclusion

We presented a stylized equilibrium model of the global economy and studied the conditions for shocks to investors sentiments to translate into bubbles that can have real effects on output and consumption when financial markets integrate.

In the global equilibrium with integrated financial markets the bubble is held by the intermediaries and owned, indirectly, by the savers of the North and of the South. Our model captures the allocation of the risk related to the sudden loss of value of the asset in very reduced-form through the parameter μ. We can think of μ as identifying the holders or the "location" of the bubble. For example, when $\mu = 1$ any unexpected change in the value of the asset will affect the value of the deposits that North savers can claim on the intermediary, while the South savers are shielded by such direct risk. In this case we can say that the bubble is located in the North region. Note that the location of the asset has nothing to do with where the asset originated in the first place. A new bubble sold by a productive adult from the South is effectively located in the North if held by a saver in the North.

In our model a bubble is fueled by the demand of savers from the South, but in the end the savers in the North could end up holding the asset. Arguably, this is the sort of global financial imbalance that developed during the real estate bubble of the early 2000's in the global economy. The strong surge in demand for safe assets from emerging economies, such as China, provided the resources to make a bubble affordable, but because such demand was mainly satisfied by purchase of fundamental assets from the US, primarily in the form of US Treasuries, the US savers ended up holding the bubble, in the form of overvalued real estate. The version of the model presented in this paper abstracts from risk considerations, but it already contains the components to formally analyze episodes of global financial imbalances and bubbles, such as the one just described, if some degree of risk aversion is introduced. We leave this to future work.

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8 Appendix

8.1 Derivation of Recursive Representation Using the definitions in the text it is easy to see that

$$\frac{b_t}{W_t} = \frac{b_t}{W_{t|t-1}} \frac{W_{t|t-1}}{W_t} = \frac{z_t}{1+\varphi\sigma_t}. \tag{8.1}$$

The evolution of total capital in the economy is

$$k_{t+1} = k^A_{t+1} + k^Y_{t+1} = w_t \underline{a} \frac{\delta_t}{1-\theta} + \rho_{t+1}(w^A_t + b^A_t)U_\theta(\rho_{t+1}), \tag{8.2}$$

where $\delta_t = 0$ when $\rho_{t+1} > \underline{a}$. Using the definition for n_t one has that

$$\frac{k_{t+1}}{W_t}\frac{1}{\rho_{t+1}} = (1-n_t)\frac{\delta_t}{1-\theta} + \left(n_t + \frac{z^A_t}{1+\varphi\sigma_t}\right)U_\theta(\rho_{t+1}). \tag{8.3}$$

Adult agents wealth at $t+1$ as expected at time t is $w^A_{t+1|t} = R^a_{t+1}w_t$ which results in

$$\frac{w^A_{t+1|t}}{R^a_{t+1}}\frac{1}{W_t} = 1-n_t \tag{8.4}$$

The next period total wealth expected at time t can be written as

$$W_{t+1|t} = R^a_{t+1}\left(\frac{w^A_{t+1|t}}{R^a_{t+1}} + \frac{w_{t+1}}{R^a_{t+1}}\right) = R^a_{t+1}\left(\frac{w^A_{t+1|t}}{R^a_{t+1}} + \frac{w_{t+1}}{R_{t+1}}\frac{1}{\rho_{t+1}}\right) \tag{8.5}$$

Using the output technology under competitive returns

$$\frac{R_{t+1}}{w_{t+1}} = \frac{\alpha}{1-\alpha}\frac{1}{k_{t+1}}. \tag{8.6}$$

Combining expressions it follows that

$$\frac{W_{t+1|t}}{W_t} = R^a_{t+1}\left(1-n_t + \frac{k_{t+1}}{W_t}\frac{1-\alpha}{\alpha}\frac{1}{\rho_{t+1}}\right) \tag{8.7}$$

This suggests the definition used in the main text for $e_{t+1|t}$,

$$e_{t+1|t} \equiv \frac{k_{t+1}}{W_t}\frac{1-\alpha}{\alpha}\frac{1}{\rho_{t+1}} = \frac{1-\alpha}{\alpha}\left[\frac{\delta_t}{1-\theta}(1-n_t) + U_\theta(\rho_{t+1})\left(n_t + \frac{z^A_t}{1+\varphi\sigma_t}\right)\right]. \tag{8.8}$$

8.2 Proof of Proposition 1 The return on the bubble, R^b_{t+1}, has to be equal to the return on the fundamental asset R^a_{t+1} in a bubbly equilibrium, which means that

$$\mathbb{E}_t(b_{t+1}) = R^a_{t+1}\big(b_t + b^A_t(1 - G(\rho_{t+1}))\big). \tag{8.9}$$

Multiply both sides by $W_{t|t-1}W_t/W_{t+1|t}$ and using the expressions above one can immediately obtain (3.1). The asset market clearing condition is

$$l_t(\rho_{t+1}) + b_t + (1 - G(\rho_{t+1}))b^A_t = d_t(\rho_{t+1}). \tag{8.10}$$

Dividing both sides by W_t and using expressions (2.14) and (2.15) together with (8.1) expression (3.2) follows. Finally, the evolution of n_t is obtained using

$$n_{t+1} = \frac{w^A_{t+1}}{W_{t+1}} = \frac{w^A_{t+1|t} + \varphi(b_{t+1} - \mathbb{E}_t(b_{t+1}))}{W_{t+1|t} + \varphi(b_{t+1} - \mathbb{E}_t(b_{t+1}))} = \frac{\frac{w^A_{t+1|t}}{W_{t+1|t}} + \varphi(z_{z+1} - \mathbb{E}_t(z_{t+1}))}{1 + \varphi(z_{t+1} - \mathbb{E}_t(z_{t+1}))}, \tag{8.11}$$

and recognizing that

$$\frac{w^A_{t+1|t}}{W_{t+1|t}} = \frac{w^A_{t+1|t}}{W_t}\frac{W_t}{W_{t+1|t}} = \frac{1 - n_t}{1 - n_t + e_{t+1|t}} \tag{8.12}$$

Combining the expressions (3.3) follows.

8.3 Proof of Corollary 1 In SSE $\sigma_t = 0$ and $z^A_t = 0$ so the conditions of Proposition 1 under $z^* > 0$ become

$$(\rho^*): \quad \frac{1-\alpha}{\alpha}\left[(1-n^*)\frac{\delta^*}{1-\theta} + n^* U_\theta(\rho^*)\right] + (1-n^*) = \frac{1}{1-p} \tag{8.13}$$

$$(z^*): \quad \frac{z^*}{1-p} + \theta\left[(1-n^*)\frac{\delta^*}{1-\theta} + n^* U_\theta(\rho^*)\right] = (1-n^*)(1-\delta^*) + n^* G(\rho^*) \tag{8.14}$$

$$(n^*): \quad \left[\frac{\delta^*}{1-\theta} + \frac{n^*}{1-n^*}U_\theta(\rho^*)\right] = \frac{\alpha}{1-\alpha}\frac{1-n^*}{n^*} \tag{8.15}$$

(8.16)

where $\delta^* = 0$ when $\rho^* > \underline{a}$. Substituting (ρ^*) into (n^*) one obtains

$$\frac{1}{1-n^*}\left(\frac{1}{1-p} - (1-n^*)\right) = \frac{1-n^*}{n^*}. \tag{8.17}$$

Rearranging gives

$$\frac{n^*}{1-n^*} = 1-p \quad \text{and} \quad n^* = \frac{1-p}{2-p}. \tag{8.18}$$

Substituting n^* back into (ρ^*) expression (3.4) follows. Expression (3.5) is similarly obtained by substituting n^* and (8.18) into (z^*).

Fundamental Steady State For completeness we report the characterization of a fundamental steady state. Setting $z^* = 0$ and disregarding (ρ^*), the fundamental steady state variables $\hat{n}$ and $\hat{\rho}$ are given by

$$(\hat{\rho}): \qquad \theta\left((1-\hat{n})\frac{\hat{\delta}}{1-\theta} + \hat{n}U_\theta(\hat{\rho})\right) = (1-\hat{n})(1-\hat{\delta}) + \hat{n}G(\hat{\rho}) \tag{8.19}$$

$$(\hat{n}): \qquad \frac{\hat{n}}{1-\hat{n}}\left(\frac{\hat{\delta}}{1-\theta} + \frac{\hat{n}}{1-\hat{n}}U_\theta(\hat{\rho})\right) = \frac{\alpha}{1-\alpha}, \tag{8.20}$$

where $\hat{\delta} = 0$ when $\hat{\rho} > \underline{a}$.

8.4 Proof of Proposition 2

Under the definition given in the main text we have that

$$\frac{\tilde{b}_t^A}{\tilde{W}_t} = \frac{\tilde{z}_t^A}{v_t(1+\mu\varphi\sigma_t^*)}. \tag{8.21}$$

The following expressions can then be derived

$$\frac{\tilde{k}_{t+1}}{\tilde{W}_t}\frac{1}{\tilde{\rho}_{t+1}} = \left((1-\tilde{n}_t)\frac{\tilde{\delta}_t}{1-\theta_s} + \left(\tilde{n}_t + \frac{\tilde{z}_t^A}{v_t(1+\mu\varphi\sigma_t^*)}\right)U_{\theta_s}(\tilde{\rho}_{t+1})\right) \quad \text{and} \quad \frac{\tilde{w}_{t+1|t}^A}{R_{t+1}^*}\frac{1}{\tilde{W}_t} = (1-\tilde{n}_t), \tag{8.22}$$

where $\tilde{\delta}_t = 0$ when $\tilde{\rho}_{t+1} > \underline{a}$. The expression on the left can be used to define $\tilde{e}_{t+1|t}$ as we did for the closed economy case, so that (4.3) follows. For v_t one has

$$v_t \equiv \frac{\tilde{W}_t}{W_t} = \frac{\tilde{W}_{t|t-1} + (1-\mu)\tilde{\varphi}(b_t - \mathbb{E}_{t-1}(b_t))}{W_{t|t-1} + \mu\varphi(b_t - \mathbb{E}_{t-1}(b_t))} = \frac{\tilde{W}_{t|t-1}}{W_{t|t-1}}\left(\frac{1 + (1-\mu)\tilde{\varphi}\frac{W_{t|t-1}}{\tilde{W}_{t|t-1}}\sigma_t^*}{1+\mu\varphi\sigma_t^*}\right). \tag{8.23}$$

It is straightforward to show that

$$v_{t+1|t} \equiv \frac{\tilde{W}_{t+1|t}}{W_{t+1|t}} = \frac{\tilde{e}_{t+1|t} + v_t(1-\tilde{n}_t}{e_{t+1|t} + 1 - n_t}, \tag{8.24}$$

and the equation for the evolution of v_{t+1} immediately follows. For q_{t+1}, given the function form of the output technology $q_{t+1} = \left(\frac{k_{t+1}}{\tilde{k}_{t+1}}\right)^{\alpha-1}$. Using the expressions for k_{t+1} and $\tilde{k}_{t+1}$ derived above and recalling that

$q_{t+1}\rho_{t+1} = \tilde{\rho}_{t+1}$ one gets

$$q_{t+1} = \left(\frac{e_{t+1|t}}{q_{t+1}\tilde{e}_{t+1|t}}\right)^{\alpha-1}, \tag{8.25}$$

from which the left hand expression in (4.7) follows. The bubble return and the market clearing condition are immediate consequences of the definition of the variables for the integrated economy and so we omit their derivation. The evolution of n_t and $\tilde{n}_t$ can be also derived by using the above expressions for $W_{t|t-1}$ and $\tilde{W}_{t|t-1}$ and so on, as was done for the closed economy case.

8.5 Proof of Corollary 2

Setting $z_t^A = \tilde{z}_t^A = \sigma_t^* = 0$ and $z_t = z^*$ in (4.4) one gets

$$\frac{1-\alpha}{\alpha}\left(\frac{\delta^*}{1-\theta} + U_\theta(\rho^*)\frac{n^*}{1-n^*}\right) = 1. \tag{8.26}$$

From the steady state version of the equation for v_{t+1} the above expression implies

$$\frac{1-\alpha}{\alpha}\left(\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*)\frac{\tilde{n}^*}{1-\tilde{n}^*}\right) = 1. \tag{8.27}$$

Using the steady state version of (4.8) gives $\frac{n^*}{1-n^*} = 1$, or $n^* = \frac{1}{2}$, and similarly $\frac{\tilde{n}^*}{1-\tilde{n}^*} = 1$. Substituting these relationships in (8.26) and (8.27), conditions (5.1) and (5.2) follow. Using these two conditions the steady state version of the equation for q_{t+1} immediately simplifies to $q^* = (v^*)^{\frac{1-\alpha}{\alpha}}$ which gives $v^* = (q^*)^{\frac{\alpha}{1-\alpha}}$. Finally, the asset market clearing condition in steady state is

$$z^* = \frac{1}{2}\Big((1-\delta^*) + G(\rho^*) + v^*(1-\tilde{\delta}^* + G(q^*\rho^*))\Big) - \frac{1}{2}\left[\theta\left(\frac{\delta^*}{1-\theta} + U_\theta(\rho^*)\right) + \phi\theta v^*\left(\frac{\tilde{\delta}^*}{1-\phi\theta} + U_{\phi\theta}(q^*\rho^*)\right)\right]. \tag{8.28}$$

Substituting conditions (5.1) and (5.2), equation (5.3) is obtained.

Appendix B: New Asset Supply z_t^A and Bubbly Dynamics

The issuance of a new bubble asset z_t^A by investing adult agents has a direct crowding-in effect at time t as funds are channeled from low efficiency young agents to higher efficiency adult agents. However, the new bubble asset will have to be sold by young agents in the future and so it will become part of the existing bubble z_{t+1}, which requires a continuous demand from future savers. So a natural question to ask is whether the initial crowding in effect for a newly supplied bubble asset can compensate the future crowding-out effect enough to allow for a bubbly equilibrium to exist. We address this question using the dynamic equations of Proposition (1) and following the method of Martin and Ventura (2012).[27] In order for z_t^A to relax the condition for an

[27] The stationary steady state analysis for the case of the existence of a bubbly equilibrium and new asset creation is

equilibrium it must shift the $E_t(z_{t+1})$ schedule downward. Assuming that $\rho_{t+1} = \underline{a}$ it is possible to show that downward shift happens if and only if

$$\frac{1 - n_t + n_t(1-\alpha)U_0(\underline{a})}{(1-\alpha)U_0(\underline{a})} < z_t < 1 - n_t. \tag{8.29}$$

This condition can be satisfied if the interval exists in the first place, which happens for n_t small enough compared to $1 - n_t$. This is intuitive, the smaller is the share of wealth available to the productive adult agents, the more effective would a new bubble asset be in crowding-in funds towards efficient investments. Second, a crowding-in is possible only if an old bubble asset has already a non-negligible size in the economy. For a bubbly equilibrium not to exists when $z_t^A = 0$ it has to be that $E_t(z_{t+1}) > z_t$ so that the a competitive return is not affordable. If condition (8.29) is satisfied, a new bubble asset can create the condition for an equilibrium to exists if there is a $z_t^A > 0$ with $z_t + z_t^A \leq 1$ such that the inequality is reversed, i.e. $E_t(z_{t+1}) < z_t$. Suppose we want to give the best chance to the new asset by letting $z_t^A = 1 - n_t - z_t$, then together with (8.29) it must be that

$$z_t\left(1 - n_t + \frac{1-\alpha}{\alpha}U_0(\underline{a})(1 - z_t)\right) < 1. \tag{8.30}$$

Characterizing the conditions for the case of $\rho_{t+1} > \underline{a}$ is more involved and we do not report it here, but similar conditions on z_t can be derived, together with less stringent conditions on the relationship between n_t and $1 - n_t$.

not appropriate since it would require to assume that a new asset is continuously created. Given the diminishing return to capital in the economy it is possible to show that such a steady state will not exist unless a bubbly equilibrium was already possible without the new asset being continuously created.

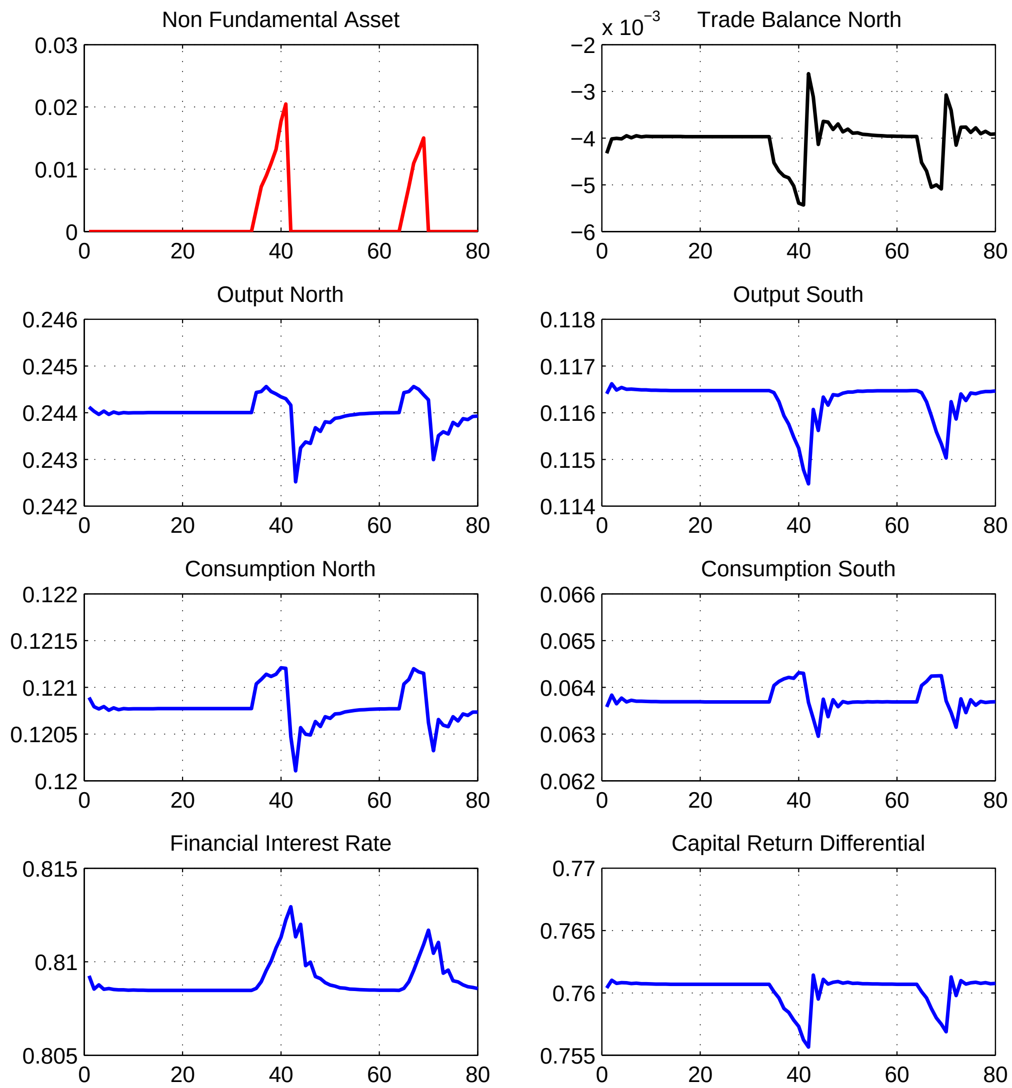

Figure 7: Bubbly Dynamics in the Global Economy: Numerical Simulation