Risk Measures on Orlicz spaces: some new characterisation of convex closed sets

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The usual definition for monetary utility functions is given on the space L^∞. For dual spaces, L^Φ, of Orlicz$-\Delta_2$ spaces, L^Ψ, there are two generalisations. One uses norm bounded sets, the other one uses order intervals. We show that a monetary utility function has a dual representation with a penalty function defined on L^Φ, if the utility function is upper semi continuous for the convergence in probability on order intervals. More precisely we show that a convex set $C \subset L^\Phi$ is $\sigma(L^\Phi, L^\Psi)$ closed if for each order interval, $[-\eta, \eta] = \{\xi \mid -\eta \le \xi \le \eta\}$ $(0 \le \eta \in L^\Phi)$, the intersection $C \cap [-\eta, \eta]$ is closed for the convergence in probability. The result is based on the following technical lemma. For a norm bounded sequence ξ_n in L^Φ, which converges in probability to 0, there exist *forward* convex combinations $\zeta_n \in conv\{\xi_n, \xi_{n+1}, \ldots\}$ as well as an element $\eta \in L^\Phi$, such that $\zeta_n \to 0$, almost surely and $|\zeta_n| \le \eta$.