Subsidy competition, imperfect labor markets, and the endogenous entry of firms*

Tadashi Morita† Yukiko Sawada‡ Kazuhiro Yamamoto §

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Abstract

This paper constructs a model of subsidy competition for manufacturing firms under labor market imperfection. Because subsidies affect the distribution of firms, they influence unemployment rates, the number of firms, and welfare. In our model, governments always provide inefficiently high subsidy rates to manufacturing firms. When labor market friction is high, subsidy competition is beneficial, although subsidies under subsidy competition are inefficiently high. We show that an increase in labor market friction always lowers welfare, while trade liberalization always improves welfare. Finally, we find that a rise in labor market friction in a country raises the equilibrium subsidy rate, affects unemployment rates, and lowers welfare.

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Key words: Labor market friction, Unemployment, Subsidy competition.

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†Faculty of Economics, Kindai University, E-mail address: t-morita@kindai.ac.jp

‡Graduate School of Economics, Osaka University, E-mail address:nge012sy@student.econ.osaka-u.ac.jp

§Graduate School of Economics, Osaka University. E-mail address: yamamoto@econ.osaka-u.ac.jp

1 Introduction

In recent years, Active Labor Market Policy (ALMP) involving subsidies to private sector employments have been executed in many EU countries (Kluve (2010)) and OECD countries (Card, et al. (2010) and Martin (2015)). On the one hand, in many countries, governments provide subsidies to private firms, which have an objective to lower unemployment rates and been considered to have only indirect effects on foreign countries.[1] On the other hand, the Subsidies and Countervailing Measures (SCM) Agreement aims to discipline subsidies granted by WTO members, since subsidies may be harmful for other countries.[2] Does the subsidy to improve the condition of a domestic labor market influences on the welfare of foreign countries? We points out that such a type of subsidy may be harmful for foreign countries, which induces subsidy competition. Is subsidy competition wasteful or beneficial for two countries? We show whether subsidy competition under imperfect labor markets are beneficial or wasteful for two countries depends on the size of labor market frictions.

In this paper, we show that when labor market is imperfect, governments have a strong incentive to give subsidies to firms, since internalizing distortion generated by labor market imperfection improves the welfare. However, a subsidy in a country may provides the other country with an externality.[3] ALMP may bring about negative externality to other countries, although its objective is to internalize the distortion generated by labor market imperfection. This type of subsidy policy may thus be prohibited by the WTO, since it may be harmful for other countries.[4] This paper investigates whether WTO's prohibition of subsidy competition is beneficial or harmful. The analysis shows that subsidy competition is beneficial (wasteful), when labor market friction is large (small). The SCM agreements which prohibits subsidy competition will be harmful, when labor market friction is large.

We construct a two-country, two-sector (manufacturing and agriculture) model in which markets for manufactured goods are segmented between two countries and the total number of manufacturing firms is endogenous.[5] [6] One

[1] OECD (2010) states that labor market interventions are indirect bearing on international trade.

[2] Mavroidis (2016) states that the Subsidies and Countervailing Measures (SCM) Agreement aims to discipline subsidies granted by WTO members. To this end, it requires that WTO members avoid using two types of prohibited subsidies (local content and export subsidies) and other subsidies that may adversely affect other WTO members. The current SCM Agreement does not condition the treatment of subsidies on their rationale. Subsidies can nowadays be counteracted regardless of their rationale.

[3] We will see the explanation of negative externality by subsidy competition in the later part of Introduction.

[4] The SCM Agreements prohibits an export subsidy or a subsidy contingent on the use of domestic over imported goods. If the subsidy policy in our paper can be interpreted as an export subsidy or a subsidy contingent on the use of domestic over imported goods, the SCM Agreement would prohibit such a subsidy.

[5] Davies and Eckel (2010) and Pflüger and Suedekum (2013) construct models of tax (subsidy) competition with an endogenous number of heterogeneous firms. In those models, labor markets are perfect.

[6] Several works study segmented product markets in which the total number of manufac-

specific feature of our model is that the labor market in the manufacturing sector is assumed to be imperfect. Workers who enter the manufacturing sector search for a job and pay opportunity costs equal to wages in the agriculture sector. Firms entering the manufacturing sector search for workers to employ, and these search activities are assumed to incur a positive search cost. Individual firms' entries into the search market raise the probability of unemployed searching workers finding a job. Since the search activities by firms incur positive search costs, a finite number of firms enter in a period and thus the search duration for a worker becomes positive. In this duration, they pay opportunity costs which equals to the agricultural wage. Therefore, the wage in the manufacturing sector should be higher than that in the agriculture sector. If firms' search costs are zero, an infinite number of firms enter in one period, which makes the expected search duration for a worker zero. In this case, the equilibrium wage in the manufacturing sector equals the wage in the agriculture sector, which means that the labor market is perfect.[7] Since search costs for firms are positive, matched firms and workers receive positive rents in the absence of policy intervention by governments. Specifically, labor market imperfection brings about inefficiency, which may be internalized by government intervention. Thus, under the existence of a positive search cost, governments have an incentive to provide subsidies to manufacturing firms to internalize the inefficiency induced by labor market imperfection.[8]

Each government is assumed to provide a subsidy to maximize the welfare in its own country. In our model, there is an externality generated by subsidy competition, as in previous subsidy competition studies, like Borck, et al. (2012) and Pflüger and Suedekum (2013). The increase in subsidies speeds up the entry of firms. This entry of firms in a country then intensifies competition, which induces the exit of firms from the other country. The decrease in firms in the other country lowers welfare in the other country through three channels. First, the decrease in domestic firms raises the equilibrium price of manufactured goods.[9] We call this effect, which can be observed in studies of segmented markets, the *consumer surplus effect*. Second, the decrease in domestic firms reduces the number of matched firms and workers in the manufacturing sector. In our model, matched firms and workers in the manufacturing sector receive

turing firms is exogenous. Baldwin and Krugman (2004), Borck and Pflüger (2006), Haufler and Wooton (2010), Kind et al. (2000), and Ludema and Wooton (2004) construct models of tax competition under segmented markets. In those models, the total number of firms is exogenous.

[7]If search costs for firms are zero, firms incur no costs in the search duration, since firms do not pay opportunity costs. Thus, when search costs are zero, there is no inefficiency.

[8]This type of subsidy can be interpreted as an employment subsidy as in Harris and Todaro (1970).

[9]In our model, we assume that the total number of firms is endogenous and increases with a rise in the subsidy rate in a country. Thus, the increase in the number of firms in the subsidy country is larger than the decrease in the number of firms in the other country. This dampens the fiscal externality compared with models in which the number of firms is exogenous. However, subsidy competition always results in a race to the bottom, even in a perfect labor market.

rents brought about by labor market imperfection.[10] Thus, the decrease in the number of matched firms and workers in the manufacturing sector lowers welfare in the country, which we term the *labor market imperfection effect*. Third, the decrease in the number of firms reduces the total expenditure for subsidy, which raises the welfare. We call this as the *fiscal externality effect*.

We show that when labor market friction is small, subsidy competition lowers welfare compared with the case without subsidy competition; hence, subsidy competition is wasteful. Therefore, in the perfect labor market case (i.e., when labor market friction is zero), subsidy competition is always wasteful. When labor market friction is large, subsidy competition is beneficial, although subsidy rates under subsidy competition are inefficiently high.[11]

We study how an increase in labor market friction affects unemployment rates and welfare. Our analysis shows that the increase in the labor market friction reduces the equilibrium number of matched firms, which lowers the unemployment rate. In our model, equilibrium unemployment rates increase with the number of manufacturing firms, since a rise in the number of manufacturing firms increases the number of workers searching for jobs in the manufacturing sector. The increase in the labor market friction also lowers the tightness of labor market, which raises the unemployment rate. When the former effect is stronger (weaker) than the latter effect, the increase in the labor market friction lowers (raises) the unemployment rate. When the market size for manufactured goods is small (large), an increase in labor market friction lowers (raises) unemployment rates. An increase in labor market friction decreases the welfare level monotonically in the symmetric equilibrium. An increase in labor market friction decreases the number of matched firms. The decrease in matched firms reduces the number of matched workers, which lowers welfare through labor market imperfection effect. In addition, the decrease in the number of matched firms raises the price level of manufactured goods, which also lowers welfare.

Effects of trade liberalization on welfare under tax competition are investigated by some recent papers like Egger and Seidel (2011), Exbrayat et al. (2012), and Haufler and Mittermaier (2011). Our simple model enables us to derive clear results about effects of trade liberalization on unemployment rates and welfare under subsidy competition. When trade liberalization occurs, competition among manufacturing firms becomes intensive, which decreases the number of firms, while exports increase, which increases the number of firms. When trade liberalization increases the number of firms, it also raises unemployment rates. We further show that when trade costs are high, trade liberalization

[10] Workers employed in the manufacturing sector earn a higher wage than workers in the agriculture sector. Unemployed individuals in the manufacturing sector that are searching for a job have the same utility as workers in the agriculture sector. Thus, workers in the manufacturing sector receive rents brought about by labor market imperfection. The stock market value of matched firms becomes higher than unmatched vacancies, which also provides rents to matched firms.

[11] Boadway et al. (2002) show a case in which tax competition improves welfare in a model with labor market imperfection. Wilson (1999) and Wilson and Wildasin (2004) introduce models in which tax competition improves efficiency in their comprehensive surveys of tax competition studies.

raises unemployment rates. In addition, when trade costs are low, the influence of trade liberalization on unemployment rates depends on market size: when market size is large (small), trade liberalization lowers (raises) unemployment rates. As seen above, trade liberalization can therefore raise or lower unemployment rates. However, our model shows that trade liberalization always improves welfare. Thus, policies that facilitate trade liberalization improve welfare when two countries are under subsidy competition.

Finally, we study the case that two countries are asymmetric with respect to labor market friction. Our analysis shows that in the country with higher labor market friction, the equilibrium subsidy rate is higher than that in the other country.[12] Moreover, an increase in labor market friction in a country raises its subsidy rate and lowers the subsidy rate in the other country. We analyze how an increase in labor market friction in a country affects unemployment rates and welfare, finding that it lowers its welfare, while it reduces unemployment rates and improves welfare in the other country.

Related works include Baldwin and Krugman (2004), Borck and Pflüger (2006), Haufler and Wooton (2010), Kind et al. (2000), and Ludema and Wooton (2004), all of which present tax competition models with segmented markets. In all these models, however, the number of manufacturing firms is exogenous and the labor market is perfect, in contrast to our model. Similarly, while Davies and Eckel (2010) and Pflüger and Suedekum (2013) construct models of tax (subsidy) competition with an endogenous number of firms, they focus on the effects of heterogeneous firms on tax competition rather than on the effects of labor market friction.

Some studies examine the effects of an imperfect labor market on the results of tax competition. Fuest and Huber (1999), Ogawa et al. (2006, 2016), and Sato (2009), for instance, study how labor market friction influences the results of tax competition in a model with perfect product markets. Fuest and Huber (1999) introduce wage bargaining, Ogawa et al. (2006) introduce a minimum wage, Ogawa et al. (2016) introduce labor unions, and Sato (2009) introduces search friction to study the effects of labor market imperfection on tax competition. In these papers, the number of firms is exogenous and product markets are perfectly integrated. We assume that the number of firms is endogenous and that markets are segmented between two countries.

Egger and Seidel (2011), Exbrayat et al. (2012), and Haufler and Mittermaier (2011) construct tax competition models of imperfect product markets and labor markets. In the latter two studies, the presence of a labor union brings about labor market imperfection, while in the former, a fair-wage preference produces labor market imperfection. In our model, search friction a la Pissarides (2000) brings about labor market friction. In Egger and Seidel (2011), Exbrayat et al. (2012), and Haufler and Mittermaier (2011), the number of firms is exogenous; however, none of these studies investigates whether tax competition is beneficial or wasteful.

[12] Egger and Seidel (2011), Exbrayat et al. (2012), and Haufler and Mittermaier (2011) also show that the equilibrium tax rate is lower in the high labor market friction country than in the other country.

Some studies point out that tax competition may be beneficial. Ottaviano and van Ypersele (2005) present a tax competition model with monopolistic competition, showing that under certain conditions, tax competition is efficiency enhancing. Borck et al. (2012) present a model in which the inefficiency lock-in of agglomeration may be removed by subsidy competition. Boadway et al. (2002) construct a tax and redistributive policy competition model with search friction in which governments compete by implementing inefficient redistributing policies. They find that tax competition reduces such inefficient redistributive policy competition, which improves welfare. Although these papers show that tax competition may be beneficial, how tax competition improves efficiency is different from our model. In our model, the entry of manufacturing firms becomes inefficiently scarce in the case without subsidy competition because of the existence of positive search costs. Positive subsidies under subsidy competition thus increase the number of firms, which improves welfare. In Ottaviano and van Ypersele (2005) and Borck and Pflüger (2012), the labor market is perfect and number of firms is exogenous. In Boadway et al. (2002), inefficiency is not induced by positive search costs, while the number of firms is exogenous. Based on the foregoing, our paper thus adds a new channel that brings about beneficial tax competition.

The seminal paper of Harris and Todaro (1970) presents a model of urban unemployment in developing countries. Our model has a similar structure to theirs, in which the labor market in the rural agriculture sector is assumed to be perfect, while that in the urban manufacturing sector is imperfect. Workers therefore migrate from rural to urban areas since expected real wages in urban areas are higher than those in rural areas, although unemployment also exists in the former. In the equilibrium, expected real wages in urban areas thus equal real wages in rural areas.[13] Our model analyzes the effect of subsidies in the urban manufacturing sector in developing countries. In our model, governments provide urban manufacturing subsidies to improve welfare. Such a subsidy induces firms' entry, which brings about the externality to the other country and the equilibrium subsidy rate is too high. This paper shows that subsidy competition is beneficial, when the labor market friction in urban manufacturing sector is large.

The remainder of this paper is organized as follows. Section 2 presents the model and derives the equilibrium conditions. Section 3 studies the case of perfect labor markets. Section 4 analyzes the case of imperfect labor markets. Section 5 investigates the case of asymmetry between two countries with respect to labor market friction. Section 6 concludes.

[13] Harris and Todaro (1970) and studies following their tradition such as Krichel and Levine (1999), Yabuuchi (1993), and Zenou (2011) analyze the welfare effects of urban employment subsidies.

2 The model

2.1 Basic setup

There are two countries, 1 and 2. The variables that refer to country 1 have the subscript 1 and those that refer to country 2 have the subscript 2. Each country is endowed with a fixed amount of labor $L_1 = L_2 = 1$.[14] We assume that the agents in both countries obtain utility from the consumption of agricultural goods and homogeneous manufactured goods. In the agricultural goods sector, there is no labor market friction, whereas in the manufactured goods sector, there is labor market friction. While labor can be mobile between sectors in the same country, it cannot be mobile between different countries. The utility function of the agent in country i is given by

$$\tilde{U}_i = z_i + Aq_i - \frac{q_i^2}{2},$$

where z_i and q_i represent the consumption level of agricultural goods and homogeneous manufactured goods in country i, respectively. The budget constraint of the agent in country i is

$$z_i + p_i q_i = y_i,$$

where y_i is the total income. In this model, agricultural goods are chosen to be the numéraire. By maximizing the utility function, the demand function for manufactured goods becomes

$$q_i = A - p_i.$$

Then, the indirect utility level in country i is $\tilde{U}_i = y_i + \frac{(A-p_i)^2}{2}$.

Technology in the agriculture sector requires one unit of labor to produce one unit of output. With free trade in agriculture, the choice of this good as the numéraire implies that the equilibrium wage is equal to one in both regions, $w_1 = w_2 = 1$.

Our focus lies on the market for manufactured good q_i, which is served by n firms. Following Haufler and Stähler (2013), we assume that a manufactured goods firm can produce a fixed amount of goods. We assume that a manufactured goods firm can produce one unit of a good for the domestic market and $t < 1$ units of goods for the foreign market with one unit of labor. We interpret t as trade costs. When t is small (large), trade costs are high (low).[15]

The inverse demand functions in country i are given by

$$p_i = A - (n_i + tn_j), \tag{1}$$

[14] We assume that both countries have the same market size. Then, when the level of labor market imperfection is the same in both countries, they are perfectly symmetric.

[15] Under the assumption of manufacturing firms having fixed outputs, we can derive the explicit forms of the equilibrium subsidy rates and social welfare functions. In the variable output case, however, we cannot derive the explicit forms of the equilibrium subsidy rates and social welfare functions. In the Appendix, numerical methods are thus used to show that in the variable output case, we can derive the same main result as in the fixed output case.

Therefore, the revenue of manufacturing firms in country i is given by

$$R_i = [A - (n_i + tn_j)] + t\left[A - (tn_i + n_j)\right], \tag{2}$$

2.2 Matching

The search and matching setting in this paper has a similar model to that presented by Pissarides (2000, Ch. 1). In the manufactured goods sector, there are search and matching frictions. Let the matching function be $M_i = g(u_i, v_i)$ where M_i denotes the number of job matches, u_i denotes unemployed workers, and v_i denotes job vacancies engage in matching. The probability of a manufactured goods firm finding a worker is $q(\theta_i) = M_i/v_i$ where $\theta_i = v_i/u_i$ and θ_i represents the tightness of labor market. An increase in θ_i decreases the probability of a firm finding worker. The probability of a worker finding a job is $M_i/u_i = q(\theta_i)\theta_i$. An increase in θ_i raises the probability of a worker finding a job.

Next, we focus on the value of workers and manufacturing firms. Let W_i and U_i be the present value of the expected income of an employed and unemployed worker, respectively. The unemployed worker receives unemployment benefit b. For analytical simplicity, we assume that $b = 0$. Then, U_i is

$$\rho U_i = (\overline{z} + a_i - T_i + \frac{(A - p_i)^2}{2}) + q(\theta_i)\theta_i\left(W_i - U_i\right), \tag{3}$$

where ρ is the discount rate, a_i is the asset revenue, and T_i represent the lump-sum head tax, respectively, in country i. We assume that $\overline{z}$ units of agricultural goods are distributed to each agent in each period. [16]The second term represents the capital gain from succeeding in matching. The value of W_i is given by

$$\rho W_i = (\overline{z} + w_{Mi} + a_i - T_i + \frac{(A - p_i)^2}{2}) + \delta(U_i - W_i), \tag{4}$$

where w_{Mi} denotes the wage rate in the manufactured goods sector in country i and δ denotes the rate of job destruction, which is an exogenous variable. The second term represents the capital loss to workers from losing their jobs.

Next, we describe firms' activities. Let J_i and V_i be the present-discounted value of the expected profit of an occupied job and a vacant job, respectively. The value of a vacant job is given by

$$\rho V_i = -k + q(\theta_i)(J_i - V_i), \tag{5}$$

where k denotes the search cost, which is identical in both countries. The second term represents the capital gain from succeeding in matching. The value of an occupied job is given by

$$\rho J_i = (R_i - w_{Mi} + s_i) + \delta(V_i - J_i). \tag{6}$$

[16]We assume that $\overline{z}$ is sufficiently large, which ensures that the income without lum-sum head tax of all agents becomes positive at the equilibrium.

s_i represents the lump-sum subsidy rates in country i.[17]

We assume that workers and firms engage in wage bargaining. Specifically, the wage rate in the manufactured goods sector is determined by Nash bargaining. The worker's share of the total surplus is β and the firm's share of the total surplus is $1-\beta$. Then, the following equation must hold:

$$W_i - U_i = \beta(J_i + W_i - V_i - U_i). \tag{7}$$

2.3 Equilibrium

In the equilibrium, the number of workers finding a job equals to the number of workers lose a job, the following equation must hold:

$$q(\theta_i)\theta_i u_i = \delta n_i. \tag{8}$$

L_{Mi} denotes the supply of workers in the manufactured goods sector in country i. Some workers succeed in matching and others become unemployed. Then, the labor market equilibrium condition in the manufactured goods sector is given by

$$L_{Mi} = u_i + n_i, i = 1, 2. \tag{9}$$

In this paper, we assume that the agents engaged in the agricultural goods sector cannot search for manufactured goods firms. When the agents move from the agricultural goods sector to the manufactured goods sector, the agents become unemployed. The value of an unemployed worker is equal to the value of a worker engaged in the agricultural goods sector. Thus, the following equation can be obtained:

$$\rho U_i = 1 + \overline{z} + a_i - T_i + \frac{(A - p_i)^2}{2}, i = 1, 2. \tag{10}$$

From (3) and (4), $W_i - U_i$ is given by

$$W_i - U_i = \frac{w_{Mi}}{\rho + \delta + q(\theta_i)\theta_i}. \tag{11}$$

Then, by substituting (10) and (11) into (3), we can obtain the wage rate in the manufactured goods sector as follows:

$$w_{Mi} = 1 + \frac{\rho + \delta}{q(\theta_i)\theta_i}. \tag{12}$$

The first term of 1 represents the outside option of the worker and the second term is the risk premium. By using (5) and (6), we can obtain $J_i - V_i$ as follows:

$$J_i - V_i = \frac{(R_i - w_{Mi} + s_i) + k}{\rho + \delta + q(\theta_i)}. \tag{13}$$

[17]In this paper, we assume that governments provide subsidies to manufacturing firms. In the case that governments provide subsidies to matched workers (wage subsidies), we can dervie the same results to the case of subsidies to manufacturing firms.

By substituting (7), (11), and (12) into (5), the value of a vacant job is given by

$$\rho V_i = -k + \frac{1-\beta}{\beta\theta_i}. \tag{14}$$

Therefore, the value of a vacant job is decreasing in the tightness of labor market. When the value of a vacant job is positive, firms enter the market and the tightness of the labor market becomes severe. When the value of a vacant job is negative, firms exit the market and this alleviates labor market tightness. Therefore, the value of a vacant job becomes zero, $V_i = 0$, and the tightness of the labor market in each country is given by

$$\theta_1^* = \theta_2^* = \theta^* = \frac{1-\beta}{\beta k}. \tag{15}$$

Then, in this setting, equilibrium labor market tightness θ_i^* is independent of subsidy rate s_i, which simplifies the analysis. Because of the free-entry of manufacturing firms and the arbitrage of workers between the manufacturing sector and agricultural sector, the subsidy to matched manufacturing firms increases both the number of vacant firms and the labor supply in the manufacturing sector, which makes the tightness of labor market to be independent of the subsidy rate. When the search cost is large or the worker's share of total output is large, the tightness of the labor market becomes small.[18]

Lemma 1 *When a labor market friction (search cost) is large, the tightness of the labor market becomes small. The tightness of the labor market is independent of the lump-sum subsidy rate.*

By substituting (11), (12), (13), and $V_i = 0$ into (7), we can obtain the profits of manufactured goods firms in country i as follows:

$$\beta q(\theta^*)\theta^* \frac{R_i + s_i - 1}{\rho + \delta} = 1, \tag{16}$$

where the left-hand side of this equation is the expected benefit for workers once they can match with firms and the right-hand side is the benefit when workers engage in the agricultural goods sector.[19] Then, from the above equation, the profit level in country i can be obtained as follows:

$$R_i + s_i = 1 + \frac{\rho + \delta}{\beta q(\theta^*)\theta^*} \equiv r, \tag{17}$$

where r represents the after-subsidy profit rate and $\partial r(\theta^*)/\partial\theta^* < 0$. Then, when search costs are large, the entry of firms becomes small and the profit level in country i becomes large.

[18] Policies such as subsidies to unemployed workers and for the search costs of firms affect equilibrium labor market tightness θ_i^*, which complicates the analysis. In the Appendix, we thus analyze the case that governments subsidize the search costs of firms.

[19] We substitute θ_i^* into (16) as follows:

$$(1-\beta)q(\theta_i^*)\frac{R_i - t_i - 1}{\rho + \delta} = k.$$

This equation means that the expected benefit of firms equals the search costs.

Here, we focus on the interior equilibrium in which there are a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Equations (16) determine the equilibrium number of firms in both countries. By substituting (17) into (2), we get

$$[A - (n_i + tn_j)] + [A - (tn_i + n_j)]\, t + s_i = r. \tag{18}$$

Thus, the equilibrium number of firms in country i is

$$n_i = \frac{A\,(1+t)\,(1-t)^2 - (1+t^2)(r - s_i) + 2t(r - s_j)}{(1-t^2)^2}. \tag{19}$$

Then, $\partial n_i / \partial s_i > 0$,and $\partial n_i / \partial s_j < 0$. The subsidy rates in one country influences on the number of firm in the other country, which is the externality by subsidy. We define $\varepsilon \equiv \left| -\frac{\partial n_i}{\partial s_j} \frac{s_j}{n_i} \right|$, as the elasticity of the number of firms in a country to the subsidy rate in the other country. When ε is large, a small increase in the subsidy rate in the other country brings about a large decrease in the firms' number, which generates a large externality by subsidy. Hence, $\left| \frac{\partial \varepsilon}{\partial r} \right| < 0$. Thus, when r increases, the increase in the number of firm induced by the rise in the subsidy rate in the other country is small. When r is large, the externality generated by subsidy competition becomes small. In addition, $\left| \frac{\partial \varepsilon}{\partial A} \right| > 0$, it means that when the size of manufactured goods market, A is large, the externality caused by subsidy competition is large.

From (19), the total number of firms in this economy is given by

$$n_1 + n_2 = \frac{2A(1+t) + s_1 + s_2 - 2r}{(1+t)^2}. \tag{20}$$

Then, an increase in the subsidy rate raises the total number of firms because of their free entry.

In this paper, we assume that the agents in country i own the firms located in that country. Then, the capital market equilibrium condition in country i is given by

$$a_i = \rho n_i J_i,$$

where a_i is the aggregate asset value in country i. [20]The government budget constraint is $T_i = s_i n_i$, where the left-hand side represents the tax revenue and the right-hand side represents the government expenditure on the subsidy.

The government chooses its subsidy rate to maximize welfare in each country:[21]

$$SW_i = \rho n_i W_i + \rho(1 - n_i)U_i = 1 + \overline{z} + n_i(\eta - s_i) + \frac{(n_i + tn_j)^2}{2}, \tag{21}$$

[20] We assume that all asset in a country are equally holded by all agents in a country. In our assumpetion of symmetric countries, the results are the same, if we assume that all agents in the world hold equal amounts of assets in the world.

[21] Note that unemployed workers and workers producing homogeneous goods have the same instantaneous utility ρU_i in the equilibrium.

where $\eta \equiv \rho(J_i - V_i) + \rho(W_i - U_i) = \frac{\rho}{\beta q(\theta^*)\theta^*}$ represents the rents of matched workers and firms in the manufacturing sector brought about by labor market imperfection. Thus, η is the size of labor market friction. The term $n_i\eta$ is the aggregate rents in country i. The term $s_i n_i$ represents the total subsidy expenditure, and the third term represents the consumer surplus.

Note that $r = 1 + \frac{\rho+\delta}{\rho}\eta$. Further, $\frac{\partial\varepsilon}{\partial r} < 0$. Thus, we find that $\frac{\partial\varepsilon}{\partial\eta} < 0$, which means that the externality by subsidy decreases with the size of labor market friction. When labor market friction is large, the entry of firms incurs high costs for firms. Thus, the elasticity of the number of firms to the subsidy rate decreases with an increase in labor market friction. This finding shows that when η is large, the externality by subsidy becomes small.

We can see that η is an increasing function of the search k. When the search costs is zero, $k = 0$, $\eta = 0$. Thus, the rents of matched workers and firms increase with labor market friction, whereas rents do not exist in the perfect labor market. In our model, we assume that firms incur positive search costs to search for workers. Under the condition of positive search costs, k, the number of firms' entering the manufacturing sector becomes inefficiently small. In this circumstance, a matched firm can generate an inefficiently high revenue involving rent η, which is divided between a matched worker and a matched firm.

The government sets its subsidy rate to maximize (21). When η is large, the government has a large concern about total rents $n_i\eta$ relative to the consumer surplus. On the contrary, when η is small, the government has a large concern about the consumer surplus.

3 Subsidy competition

The reaction function of the government in country i is given by

$$s_i = s_i(s_j) = \frac{-t(r - s_j) + t(1-t^2)A + (1+t^2)\eta + t^2 r}{1+2t^2}. \tag{22}$$

Then, because $0 < \partial s_i/\partial s_j < 1$, subsidy rates are strategic complements and the competitive equilibrium is stable.

From (19) and (22), the equilibrium number of firms in each country is

$$n^* = \left(1+t^2\right)\frac{A(1+t) - \left(1 + \frac{\delta}{\rho}\eta\right)}{(1+t)^2(1-t+2t^2)}. \tag{23}$$

An increase in labor market friction decreases the number of firms in both countries. Large labor market friction prevents manufacturing firms from entering the market. The equilibrium price in each country thus becomes

$$p^* = \frac{(1+t^2)(1+\frac{\delta}{\rho}\eta) - t(1-t^2)A}{(1+t)(1-t+2t^2)}.$$

An increase in labor market friction raises the price level. In the equilibrium, the condition that ensures the price in both countries is positive ($p_i > 0$) is given by

$$A < \frac{1+t^2}{t(1-t)} \frac{1+\frac{\delta}{\rho}\eta}{1+t} \equiv \overline{A}.$$

In addition, from (23), the condition that there exists a positive number of manufacturing firms in both countries ($n_i > 0$) is

$$A > \frac{1+\frac{\delta}{\rho}\eta}{1+t} \equiv \underline{A}.$$

We can observe that $\underline{A} < \overline{A}$. Hereafter, we assume that $\underline{A} < A < \overline{A}$.

The equilibrium lump-sum subsidy rates are

$$s_i^* = \eta + \frac{t(1-t)\Gamma}{1-t+2t^2}, \tag{24}$$

where

$$\Gamma = A(1+t) - (1+\frac{\delta}{\rho}\eta) > 0,$$

from $\underline{A} < A < \overline{A}$, which means that $s^* > 0$. Further, we can see that $\partial s^*/\partial A > 0$.

Proposition 1 *1) When $\underline{A} < A < \overline{A}$, governments subsidize manufacturing firms. 2) The subsidy rate is an increasing function of the market size for manufactured goods.*

As we saw, the externality by subsidy increase with A, which is expressed in $\partial\Gamma/\partial A > 0$. Thus, the increase in the market size raises the equilibrium subsidy rate.

The first term on the right-hand side of (24) is the inefficiency induced by labor market imperfection (search costs), and equals the rent of a matched firm and a matched worker. The second term represents the externality by subsidy, which decreases with an increase in η. As we saw earlier, the externality by subsidy becomes small when η increases, which induces $\partial\Gamma/\partial\eta < 0$. When the labor market friction increases, the first term in (24) (labor market inefficiency) increases, while the second term (the externality by subsidy) decreases.

Substituting (19) into (21), and differentiating it with s_j, we can derive

$$\frac{\partial SW_i}{\partial s_j} = \frac{t}{(1-t)^2}\left[-(1-t)\Gamma + s_i + ts_j - (1+t)\eta\right]. \tag{25}$$

In the symmetric equilibrium ($s_i = s_j$), when $s_i = s_j < \eta + \frac{(1-t)\Gamma}{1+t}$, $\frac{\partial SW_i}{\partial s_j} < 0$, which means that the rise of subsidy in a country brings about the negative externality to the other country. We can observe that $s_i^* = s_j^* < \eta + \frac{(1-t)\Gamma}{1+t}$.

Thus, in the subsidy competition equilibrium, the rise of subsidy in a country brings about the negative externality to the other country.

The rise in the subsidy decrease the number of domestic firms, which influences on the domestic welfare through three channels: the consumer surplus effect, the labor market imperfection effect, and the fiscal externality effect. With the rise of the subsidy rate in the country, the consumer surplus effect and the labor market imperfection effect lowers the welfare in the foreign country, while the fiscal externality effect lower the welfare. In the subsidy competition equilibrium, the sum of the consumer surplus effect and the labor market imperfection effect overwhelms the fiscal externality effect, and the rise in subsidy in a country brings about the negative externality to the other country.[22]

3.1 Coordinated subsidy rate

Global welfare is the sum of the welfare of the two countries, which can be described as

$$\begin{aligned} SW_W &= SW_1 + SW_2 \\ &= 2 + 2\overline{z} + n_1(\eta - s_1) + n_2(\eta - s_2) + \frac{(n_1 + tn_2)^2}{2} + \frac{(tn_1 + n_2)^2}{2} \end{aligned} \quad (26)$$

where the number of firms is given by (19). By substituting (19) into global welfare and differentiating it with s_1 and s_2, we find the first-order conditions for this problem as follows

$$\frac{\partial SW_W}{\partial s_i} = \frac{2t(s_j - \eta) - (1 + t^2)(s_i - \eta)}{(1 - t^2)^2} = 0, i, j \in \{1, 2\}, i \neq j.$$

We also derive the subsidy rate that maximizes global welfare as follows:

$$s_i^c = \eta > 0, \quad (27)$$

where the superscript c stands for the coordinated equilibrium. From (27), the coordinated equilibrium subsidy level equals η, namely the rent of a matched firm and a matched worker. [23]

[22] The consumer surplus effect can be represented by

$$\frac{\partial\left(\frac{(n_i+tn_j)^2}{2}\right)}{\partial s_j} = -\frac{1}{(t^2-1)^2}\left(ts_i - At^3 + \left(1 + \frac{\delta}{\rho}\eta + \eta\right)t^2 - t^2 s_j + At - \left(1 + \frac{\delta}{\rho}\eta + \eta\right)t\right),$$

the labor imperfection effect can be expressed as

$$\frac{\partial n_i \eta}{\partial s_j} = -\frac{2t\eta}{(t^2-1)^2}$$

and the fiscal externality effect can be described as

$$-\frac{\partial n_i s_i}{\partial s_j} = 2t\frac{s_i}{(t^2-1)^2}.$$

[23] In our model, when firms enter into the manufacturing market, they consider the value of $\beta(J - V + W - U)$ while the value generated by a match is $J - V + W - U$. The workers who

The marginal benefit of subsidy can be decomposed into

$$MB = \frac{\eta}{(t+1)^2} + \frac{\left(A - r + s_i - 2ts_j - At^2 + At^3 - rt^2 + t^2 s_i - At + 2rt\right)}{(t^2-1)^2}. \tag{28}$$

The first part of the right hand side of (28) is the increase in the rent generated by the increase in matched workers and firms The second part represents the increase in consumer surplus generated by the increase in firms. The marginal cost of subsidy is

$$MC = -\frac{\left(A - r + 2s_i - 4ts_j - At^2 + At^3 - rt^2 + 2t^2 s_i - At + 2rt\right)}{(t^2-1)^2}. \tag{29}$$

From (28) and (29), when $\eta = 0$, $s_i = s_j = 0$ is the subsidy rate which makes the marginal befit to be the same to the marginal costs. Thus, the marginal value of increase in consumer surplus equals to the marginal costs. If $k = 0$, $\eta = 0$, which means that when labor market is perfect, the coordinated subsidy rate becomes zero.

By comparing the subsidy rates of the competitive equilibrium with the coordinated equilibrium subsidy rates, we find that s^* is always larger than s_i^c. These inefficiently high subsidy rates are caused by subsidy at which governments subsidize the manufacturing firms in their country. Each government subsidizing these manufacturing firms ignores the externality in the other country caused by the entry and exit of firms. If subsidized firms start to operate, they export their goods, which intensifies competition and induces exits of firms in the other country. These exits of firms lower welfare in the other country by two channels: the consumer surplus effect and labor market imperfection effect. Therefore, in the competitive equilibrium, each government subsidizes its firms above the coordinated level.

Comparing the number of firms in the competitive equilibrium with the coordinated equilibrium number of firms yields

$$n^* - n^c = \frac{\left(2 - t + 3t^2\right)\left[(1+t)A - (1 + \frac{\delta}{\rho}\eta)\right]}{(1+t)^2\,(1 - t + 2t^2)} > 0,$$

because $\underline{A} < A < \overline{A}$ and $0 < t < 1$. Therefore, in the competitive equilibrium, the number of firms is larger and market competition is fiercer than that when governments provide coordinated equilibrium subsidy rates. From (32), the unemployment rates in the competitive equilibrium are higher than those when the subsidy rates are coordinated because the number of firms and probability

choose the sector where they work consider the value of $\beta(J - V + W - U)$, while the value generated by a match is $J - V + W - U$, These may induces the too large or small number of manufacturing firms and workers. In addition, when firms enter into the manufacturing market, they do not consider the effect of their entry into manufacturing market on the domestic consumer surplus and the foreign consumer surplus. The workers who choose the sector where they work do not consider the effects of their choice on consumer surplus. These may also induces the too large or small number of manufacturing firms and workers.

of a worker finding a job are higher. Then, the number of workers entering the manufactured goods sector and unemployment rates become larger.

Summarizing these results, we can obtain the following proposition.

Proposition 2 *Subsidy competition results in an inefficiently high subsidy rate (race to the bottom) and high unemployment rates.*

In our model, subsidy competition always results in a race to the bottom. The existence of the negative externality by subsidy under subsidy competition therefore generates a race to the bottom in our model.

3.2 Unemployment rates and welfare with or without subsidy competition

Here, we study the case that neither of the two countries provides a subsidy and as a result, neither engages in subsidy competition ($s_i = s_j = 0$). From (19), the equilibrium number of manufacturing firms becomes

$$n_1^n = n_2^n = n^n = \frac{A(1-t) - (1 + \frac{\rho+\delta}{\rho}\eta)}{(1+t)^2},$$

where the superscript n represents the economy when neither government subsidizes the manufacturing sector. We see that under our assumption of $\underline{A} < A < \overline{A}$, $n^n < n^*$ holds. From (32) the equilibrium unemployment rate is an increasing function of the number of manufacturing firms. Thus, subsidy competition raises unemployment rates.

Lemma 2 *Subsidy competition raises the equilibrium number of firms and unemployment rates.*

In our model, the increase in the number of manufacturing firms raises unemployment rates, since the number of workers searching for jobs in the manufacturing sector increases. Under subsidy competition, governments provide positive subsidies to manufacturing firms, which increases the equilibrium number of firms. Thus, unemployment rates are higher with subsidy competition than without subsidy competition.

The welfare level in country i in the subsidy competition equilibrium as a function of the subsidy rate can be written as

$$SW_i(s) = 1 + \overline{z} + n(s)(\eta - s) + \frac{(1+t)^2\, n(s)^2}{2}, \tag{30}$$

where $n(s)$ is the number of firms as a function of s in the subsidy competition equilibrium, which is given by (19). Following some calculation, we can derive the following equation:

$$\frac{\partial SW_i(s)}{\partial s} = \frac{-s + \eta}{(1+t)^2}.$$

Thus, $SW_i(s)$ is a quadratic function of s and has the maximum value at $s^o = \eta$. From (30), we can recognize that

$$SW_i(s)|_{s=0} = SW_i(s)|_{s=2\eta}$$

Thus, if $2\eta > (<) s^*$, $SW_i(s)|_{s=0} < (>) SW^*$. See Figure 1a and 1b. From (24), we see that if $2\eta > s^*$, the next inequality holds:

$$\eta > \frac{\rho t(A(1+t^2) - (1-t))}{\rho(1-t+2t^2) + \delta t(1-t^2)} \equiv \overline{\eta}. \tag{31}$$

If the labor market is perfect ($k = \eta = 0$), welfare with subsidy competition is always lower than that without subsidy competition. If (31) is satisfied, subsidy competition improves welfare compared with the case without subsidy competition.

Proposition 3 *When $\eta > \overline{\eta}$ ($\eta < \overline{\eta}$), subsidy competition is beneficial (wasteful).*

This proposition states that when labor market friction is large, subsidy competition is beneficial. In the case without subsidy competition, the number of entries of manufacturing firms becomes inefficiently small, because the search activity by unmatched firms incurs a positive search cost. In our model, the inefficiency induced by labor market imperfection is internalized in the subsidy competition equilibrium, since the government maximizes social welfare in a country given in (21), which involves this inefficiency captured by the term $n_i(\eta - s_i)$. However, subsidy competition brings about the externality by subsidy, which lowers social welfare; further, the externality by subsidy becomes small with an increase in η. In the equilibrium without subsidy competition, no externality by subsidy exists, while the inefficiency induced by labor market imperfection is not internalized. When η is large, the inefficiency induced by labor market imperfection is large, while the externality by subsidy is small. Thus, welfare under subsidy competition is higher than that without subsidy competition. On the contrary, when η is small, the inefficiency induced by labor market imperfection is relatively small compared with the externality by subsidy, and welfare under subsidy competition is lower than that without subsidy competition.

Note that when no labor market friction exists ($k = 0$ and $\eta = 0$), the coordinated subsidy rate becomes zero ($s^c|_{k=0} = 0$), while the equilibrium subsidy rate is positive, that is $s|_{k=0} = -\frac{t(1-t)[1-A(1+t)]}{1-t+2t^2} > 0$ because $\underline{A} < A < \bar{A}$. Thus, when labor markets are perfect, subsidy competition always lowers welfare to below that in the case without subsidy competition. Our results show that since there is labor market imperfection, subsidy competition may be beneficial.

3.3 Effects of labor market friction

From (24), we can derive that

$$\frac{\partial s^*}{\partial k} = \frac{\rho(1-t+2t^2) - t\delta(1-t)}{\rho\,(1-t+2t^2)} \frac{\partial \eta}{\partial k},$$

where $\partial\eta/\partial k > 0$. Then, we can obtain the following lemma.

Lemma 3 *The subsidy rate increases (decreases) with labor market friction, when* $\rho(1-t+2t^2) - t\delta(1-t) > (<)0$.

Lemma 3 shows that there is a case that a rise in labor market friction raises (reduces) the equilibrium subsidy rate. From (22), the rise in labor market friction in a country raises the equilibrium subsidy rate in that country. However, the rise in labor market friction in a foreign country lowers the equilibrium subsidy rate in the home country. Thus, there are both cases that a rise in labor market friction raises and lowers the subsidy rate.

The unemployment rate in the symmetric country is given by

$$u^* = \frac{\delta n^*}{q(\theta^*)\theta^*}. \tag{32}$$

Note that unemployment rates are an increasing function of the number of firms. By substituting $\frac{1}{q(\theta^*)\theta^*} = \frac{\beta\eta}{\rho}$ from the definition of η into (32) and differentiating it with respect to k, the following equation can be obtained:

$$\frac{\partial u^*}{\partial k} = -\frac{\beta\delta(1+t^2)\left(1+\frac{2\delta\eta}{\rho} - A(1+t)\right)}{\rho^2\left(1+t\right)^2\left(1-t+2t^2\right)}\frac{\partial\eta}{\partial k}.$$

When $A < (>) \frac{1+\frac{2\delta\eta}{\rho}}{1+t} \equiv A_1$, $\frac{\partial u}{\partial k} < (>) 0$ holds because of $\frac{\partial\eta}{\partial k} > 0$ and $\underline{A} < A_1 < \overline{A}$ holds.[24] In our model, an increase in labor market friction affects unemployment rates in two opposite ways. On the one hand, it decreases the probability of a worker finding a job, which transfers migrants from the agriculture sector to the manufacturing sector and reduces unemployment rates. On the other hand, it reduces the entry of firms, and thus the probability of a worker finding a firm becomes small, which raises equilibrium unemployment rates. When market size is sufficiently small, the former effect is stronger than the latter effect. Therefore, an increase in labor market friction decreases unemployment rates.

Lemma 4 *When* $A < (>) A_1$*, the increase in labor market friction lowers (raises) equilibrium unemployment rates in each country.*

By differentiating the welfare level with respect to k, we can obtain the following equation:

$$\frac{\partial SW_i^*}{\partial k} = \frac{\delta\left[1+\frac{\delta}{\rho}\eta - A(1+t)\right]}{\rho\left(1+t^2+2t^3\right)^2}\left(1+t^2\right)\left(1-2t+3t^2\right)\frac{\partial\eta}{\partial k} < 0,$$

because $\underline{A} < A < \overline{A}$ and $1-2t+3t^2 > 0$ in $0 < t < 1$. An increase in labor market friction decreases the welfare level monotonically in the equilibrium. Then, by summarizing the above results, the following proposition can be obtained.

[24] By subtracting from A_1 to $\underline{A}$, the following equation can be obtained:

$$A_1 - \underline{A} = \frac{\rho(1-t+2t^2) + \delta\eta(1-2t+3t^2)}{t\rho\left(1-t^2\right)} > 0,$$

because $0 < t < 1$ and $1-2t+3t^2 > 0$.

Proposition 4 *An increase in labor market friction decreases the welfare level monotonically.*

An increase in labor market friction decreases the number of matched firms. The decrease in matched firms reduces the number of matched workers, which lowers welfare through labor market imperfection effect. In addition, the decrease in the number of matched firms raises the price level of manufactured goods, which also lowers welfare through consumer surplus effect.

3.4 Effects of trade costs

In this subsection, we investigate how trade liberalization affects unemployment rates and welfare. We interpret t as trade costs, and an increase in t means a decline in trade costs. We define such a decline in trade costs as trade liberalization. The effects of trade liberalization on unemployment rates can be expressed as $\frac{\partial u^*}{\partial t} = \frac{\delta}{q(\theta^*)\theta^*}\frac{\partial n^*}{\partial t}$. Then, the sign of $\frac{\partial u^*}{\partial t}$ is the same as that of $\frac{\partial n^*}{\partial t}$. Here, we define $\widehat{t} = 0.144427$ and $A_2 \equiv \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+2t^4\right)}{2t^2(1+t)(3+t^2)}$ and obtain the following lemma (see the Appendix for the proof).

Lemma 5 *When $0 < t < \widehat{t}$, trade liberalization always increases unemployment rates. When $\widehat{t} < t < 1$, trade liberalization increases unemployment rates in $\underline{A} < A < A_2$ and decreases unemployment rates in $A_2 < A < \overline{A}$.*

Trade liberalization has opposite effects on the unemployment rates. The negative effect is that it intensifies competition among manufacturing firms, reducing the number of firms and lowering unemployment rates. The positive effect is that a reduction in trade costs means that firms grow their volume of exports and this increases profits. Then, the number of firms increases and some workers move from the agricultural goods sector to the manufactured goods sector. Therefore, unemployment rates rise. When trade costs are sufficiently high $(0 < t < \widehat{t})$, trade liberalization increases the number of firms and raises unemployment rates. When trade costs are sufficiently low, the effects of trade liberalization on unemployment rates depend on market size. When market size is small (large), trade liberalization raises (lowers) unemployment rates.

When market size is small in both countries, the number of firms is small and the manufactured goods market becomes less competitive. Then, the positive effect is stronger than the negative effect and trade liberalization increases the number of firms and raises unemployment rates. When market size is large, the number of firms is large and the market is competitive. Then, the negative effect overcomes the positive one and trade liberalization decreases the number of firms and lowers unemployment rates.

We also find that trade liberalization always improves welfare, as shown in the following proposition (see the Appendix for the proof).

Proposition 5 *Trade liberalization always increase the welfare level.*

In our model, trade liberalization may increases (decreases) the number of firms and raises (lowers) unemployment rates. The increase in the number of firms improves the welfare, while the decrease in it worsens the welfare. From (30), trade liberalization raises the consumer surplus, since consumer can get imported goods with lower trade costs. In our model, the effect of the rise in consumer surplus because of the low imported goods price is strong enough that trade liberalization always improves the welfare.

4 Asymmetric labor market friction

In this section, we study the effects of asymmetric labor market friction on subsidy rates. Without loss of generality, we assume that the labor market in country 2 is more efficient than that in country 1, namely $k_1 \geqq k_2$ and $\eta_1 \geqq \eta_2$. The difference between the subsidy rate is given by[25]

$$s_1^{*a} - s_2^{*a} = \left[1 + \frac{\delta s}{\rho}\frac{1+t^2+2t^3}{1+3t^2+4t^4}\right](\eta_1 - \eta_2) > 0,$$

because $\eta_1 \geqq \eta_2$. Thus, the country with the more inefficient labor market provides a higher subsidy rate than the other country. In addition, we see that $\partial\left(s_1^{*a} - s_2^{*a}\right)/\partial t > 0$. Thus, a decline in trade costs raises the difference in equilibrium subsidy rates. Form the analysis in Appendix, we can derives the next lemma:

Lemma 6 *In the case of asymmetric countries, the subsidy rate is higher in the larger labor market friction country than in the other country. Subsidy competition always results in a race to the bottom.*

We next analyze the effects of labor market friction on unemployment rates and welfare. Since deriving clear results in the general case of asymmetric countries is difficult, we focus our attention on the neighborhood of symmetric countries. In the Appendix, we show that $\left.\frac{\partial u_2}{\partial k_1}\right|_{k_1=k_2} < 0$ is always satisfied. In the Appendix, we also show that when $\underline{A}_a < A < A_a$, $\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} < 0$ and when $A_a < A < \overline{A}_a$, $\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} > 0$. Then, we can obtain the following lemma.

Lemma 7 *Suppose that the two countries are symmetric and labor market friction in country 1 increases, while that in country 2 is constant.*

1) The unemployment rate in country 2 always rises.

2) When $\underline{A} < A < A_a$, the unemployment rate in country 1 lowers.

3) When $A_a < A < \overline{A}$, the unemployment rate in country 1 rises.

With the increase in labor market friction in a country, the number of manufacturing firms in the own country decreases, while that in the other country increases. Then, unemployment rates in the other country rise. When A is small, the manufacturing sector is small. Hence, the equilibrium profits of firms and workers' wages are low. In this case, with an increase in labor market friction, a large number of workers searching for jobs in the manufacturing sector migrate

[25] See the Appendix for the analysis of asymmetric countries.

to the agriculture sector. This migration lowers unemployment rates. When A is large, equilibrium profits and wages are high. Thus, only a small number of workers switch from the manufacturing sector to the agriculture sector when labor market friction increases, which raises unemployment rates.

We now study the effect of search costs on welfare. We differentiate welfare thus:

$$\left.\frac{\partial SW_1}{\partial k_1}\right|_{k_1=k_2} = -C\left(A\rho(1+t)-\rho-\delta\eta\right),$$

$$\left.\frac{\partial SW_2}{\partial k_1}\right|_{k_1=k_2} = D\left(A\rho(1+t)-\rho-\delta\eta\right),$$

and $C>0$ and $D>0$.[26] Since $\underline{A}<A<\overline{A}$, $A\rho(1+t)-\rho-\delta\eta>0$. Then, we can observe that

$$\left.\frac{\partial SW_1}{\partial k_1}\right|_{k_1=k_2} < 0 \text{ and } \left.\frac{\partial SW_2}{\partial k_1}\right|_{k_1=k_2} > 0.$$

We can summarize these results in the next proposition.

Proposition 6 *Suppose that the two countries are symmetric and labor market friction in country* 1 *increases, while that in country* 2 *is constant. Welfare in country* 1 *lowers, while that in country* 2 *rises.*

The increase in labor market friction in a country reduces the number of firms in that country, which lowers the welfare though the consumer surplus effect and the labor market imperfection effect. The rise in labor market friction in the other country increases the number of firms in that country, which raises the welfare through the consumer surplus effect and the labor market imperfection effect.

5 Conclusion

In this paper, we construct a two-country model with labor market friction in the labor market to investigate how subsidy competition affects welfare. Our analysis shows that governments engaged in subsidy competition provide positive subsidies to manufacturing firms. In our model, the externality by subsidy lowers welfare in the other country through two channels. Since the markets for manufactured goods are segmented, the increase in the number of firms in a country lowers welfare in the other country. In addition, the decrease in the number of firms reduces the number of workers employed in the manufacturing sector, which lowers welfare. Because of the negative externality by subsidy in a country to the other country, subsidy competition always results in a race to the bottom in our model.

[26] We define $C \equiv \frac{\delta}{\rho^2}\left(1+t^2\right)\frac{(1-t)^2}{\left(1+2t^2+t^4-4t^6\right)^2}\left(1+t+4t^2+3t^3+9t^4+6t^5+8t^6\right)$ and $D \equiv \frac{t\delta}{\rho^2}\left(1-t\right)^3\left(1+3t^2+2t^4\right)\frac{1+t+2t^2}{\left(1+2t^2+t^4-4t^6\right)^2}$.

We also show that subsidy competition is beneficial when labor market friction is large. In the case without subsidy competition, the number of entries of manufacturing firms becomes inefficiently small, because the search activity by unmatched firms involves positive search costs. In our model, the inefficiency induced by labor market imperfection is internalized in the subsidy competition equilibrium. However, subsidy competition brings about the externality by subsidy that lowers social welfare. In the equilibrium without subsidy competition, no externality by subsidy exists, while the inefficiency induced by labor market imperfection is not internalized. When the labor market friction is large, the inefficiency induced by labor market imperfection is large and welfare under subsidy competition is higher than that without subsidy competition. On the contrary, when the labor market friction is small, the inefficiency induced by labor market imperfection is relatively small compared with the externality by subsidy, and welfare under subsidy competition is lower than that without subsidy competition.

Further, we show that the increase in labor market friction always reduces welfare, whereas trade liberalization always improves welfare. The increase in labor market friction raises labor market inefficiency and reduces the total number of matched firms and workers in the manufacturing sector as well as the consumer surplus. Trade liberalization lowers the equilibrium price of imported goods, which raises the consumer surplus. Hence, trade liberalization reduces the number of workers employed in the manufacturing sector, which lowers welfare. Even in this case, however, the rise in the consumer surplus exceeds the decrease in the number of matched firms and workers.

Finally, in terms of asymmetric labor market friction between countries, we show that the equilibrium subsidy rate is lower in the country with larger labor market friction. In addition, the increase in labor market friction in a country lowers its subsidy rate and raises the subsidy rate in the other country. Further, the increase in labor market friction in a country in the neighborhood of symmetric countries lowers its welfare and raises welfare in the other country.

The presented model can be extended in a number of directions. One is that firm productivity is heterogeneous. If manufacturing firms are heterogeneous and governments provide subsidies to firms, competition among firms becomes intensive, which may lower or raise cutoff productivities. Then, when firms are heterogeneous, new fiscal externalities can be observed, which enriches the model.

Future research could aim to analyze labor market or redistribution policies under subsidy competition. For example, unemployment fees could be financed by the corporate tax. Hence, we could study the effect of redistribution policies on employed and unemployed workers.

References

[1] R. Baldwin and P. Krugman (2004) "Agglomeration, integration and tax harmonization", *European Economic Review* 48, 1-23.

[2] R. Boadway, K, Cuff, and N. Marceau (2002) "Inter-jurisdictional competition for firms", *International Economic Review* 43, 761-782.

[3] R. Borck, H. Koh, and M. Pflüger (2012) "Inefficiency lock-in and subsidy competition", *International Economic Review* 53, 1179-1204.

[4] R. Borck and M. Pflüger (2006) "Agglomeration and tax competition", *European Economic Review* 50, 647-668.

[5] Card, D., Kluve, J. and Weber, A. (2010). "Active Labour Market Policy Evaluations: A Meta-Analysis." *Economic Journal*, 120: F452–F477

[6] R. Davies and A. Eckel (2010) "Tax competition for heterogenous firms with endogenous entry", *American Economic Journal: Economic Policy* 2, 77-102.

[7] P. Dicken (2003) *Global Shift. Reshaping the Global Economic Map in the 21st Century*, 4th edition, London: Sage Publications.

[8] P. Egger and T. Seidel (2011) "Tax competition, trade liberalization, and imperfect labor market", *Oxford Economic Papers* 63, 722-739.

[9] N. Exbrayat, C. Gaigné, and S. Riou (2012) "The effects of labor unions on international capital tax competition", *Canadian Journal of Economics,* 45:4, 1480-1503.

[10] C. Fuest and B. Huber (1999) "Tax coordination and unemployment", *International Tax and Public Finance* 6, 7-26.

[11] J. R. Harris and M. P. Todaro (1970) "Migration, unemployment and development: A two-sector analysis", *American Economic Review* 60, 126-142.

[12] A. Haufler and F. Stähler (2013) "Tax competition in a simple model with heterogenous firms: How larger markets reduce profit taxes", *International Economic Review* 54, 665-692.

[13] A. Haufler and F. Mittermaier (2011) "Unionization triggers tax incentives to attract foreign direct investment", *Economic Journal* 121, 793-818.

[14] A. Haufler and I. Wooton (2010) "Competition for firms in an oligopolistic industry: The impact of economic integration", *Journal of International Economics* 80, 239-248.

[15] Hosios A. J. (1990) "On the efficiency of matching and related models search unemployment" *Review of Economic Studies* 57, 279-198.

[16] H. Kind, K. Knoarvik, and G. Schjelderup (2000) Competing for capital in a 'lumpy' world", *Journal of Public Economics* 78, 253-274.

[17] Kluve, J. (2010), "The effectiveness of European active labor market programs." *Labour Economics* 17, 904-918

[18] T. Krichel and P. Levine (1999) "The welfare economics of rural-to-urban migration: The Harris-Todaro model revisited", *Journal of Regional Science* 39, 429-447.

[19] R. Ludema and I. Wooton (2000) "Economic geography and the fiscal effects of regional integration", *Journal of International Economics* 52, 331-357.

[20] Martin, J. P. (2015) "Activation and Active Labour market policies in OECD countries: Stylized Facts and Evidence on their Effectiveness," *IZA Journal of Labor Policy* 4:4. http://izajolp.springeropen.com/articles/10.1186/s40173-015-0032-y

[21] P. C. Mavroidis (2016) *The Regulation of International Trade, Volume 2: The WTO Agreements on Trade in Goods*, Cambridge, MA: MIT Press.

[22] OECD (2010) *Trade and Economic Effects of Responses to the Economic Crisis*, OECD Publishing.

[23] H. Ogawa, Y. Sato, and T. Tamai (2006) "A note on unemployment and capital tax competition", *Journal of Urban Economics* 60, 350-356.

[24] H. Ogawa, Y. Sato, and T. Tamai (2016) "Who gains from capital market integration: Tax competition between unionized and non-unionized countries", forthcoming in *Canadian Journal of Economics.*

[25] G. I. P. Ottaviano and T. van Ypersele (2005) "Market size and tax competition", *Journal of International Economics* 67, 25-46.

[26] C. Pissarides (2000) *Equilibrium Unemployment Theory*, second edition, Cambridge, MA: MIT Press.

[27] M. Pflüger and J. Suedekum (2013) "Subsidizing firm entry in open economies", *Journal of Public Economics* 97, 258-271.

[28] Y. Sato (2009) "Capital tax competition and search unemployment", *Papers in Regional Science* 88, 749-764

[29] J. Wilson (1999) "Theories of tax competition", *National Tax Journal* 52, 269-304.

[30] J. Wilson and D. Wildasin (2004) "Capital tax competition: bane or boon", *Journal of Public Economics* 88, 1065-1091.

[31] S. Yabuuchi (1993) "Urban unemployment, international capital mobility and development policy", *Journal of Development Economics* 41, 399-403.

[32] Y. Zenou (2011) "Rural-urban migration and unemployment: Theory and policy implications", *Journal of Regional Science* 51, 65-82.

6 Appendix

6.1 Proof of Lemma 5

Differentiating the number of firms with respect to s yields

$$\frac{\partial n^*}{\partial s} = \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+4t^4\right) - 2t^2A(1+t)\left(3+t^2\right)}{(1+t)^3\left(1-t+2t^2\right)^2}.$$

When $A < (>) \frac{\left(1+\frac{\delta}{\rho}\eta\right)\left(1-t+9t^2-t^3+2t^4\right)}{2t^2(1+t)(3+t^2)} \equiv A_2$, $\frac{\partial n^*}{\partial t} > (<) 0$ holds. By subtracting from A_2 to $\underline{A}$, the following equation can be obtained:

$$A_2 - \underline{A} = \frac{1-t+5t^2-t^3-2t^5}{2t^2}\frac{1+\frac{\delta}{\rho}\eta}{(3+t^3)(1+t)} > 0,$$

because $1-t+5t^2-t^3-2t^5 > 0$ in $0<t<1$. By subtracting from $\bar{A}$ to A_2, we get the following equation:

$$\overline{A} - A_2 = -\frac{\rho+\delta\eta}{2t^2\rho}\frac{F(t)}{3-2t^2-t^4},$$

where $F(t) \equiv 1-8t+10t^2-18t^3+3t^4-4t^5$ and $3-2t^2-t^4 > 0$ in $0<t<1$. When $0 < t < \widehat{t}$, where $F(\widehat{t}) = 0$ and $\widehat{t} = 0.144427$, $\overline{A} < A_2$ holds. When $\widehat{t} < t < 1$, $\overline{A} > A_2$ holds. Therefore, when $0 < t < \widehat{t}$, $\frac{\partial n^*}{\partial t} > 0$ holds. When $\widehat{t} < t < 1$, $\frac{\partial n^*}{\partial t} > 0$ holds in $\underline{A} < A < A_2$ and $\frac{\partial n^*}{\partial t} < 0$ holds in $A_2 < A < \overline{A}$.

6.2 Proof of Proposition 5

By differentiating the welfare level with respect to t, we obtain the following equation:

$$\frac{\partial SW_i}{\partial t} = \frac{\left[A(1+t)-(1+\frac{\delta}{\rho}\eta)\right]}{(1+t^2+2t^3)^3}G(A),$$

where

$$G(A) = (1+t\Phi)(1+\frac{\delta}{\rho}\eta) - tA(1+t^2)(1-3t+t^2+t^3),$$

and $\Phi \equiv -2+3t(1-2t(1-t))(2+t^2)$ and $1+t\Phi > 0$ in $0<t<1$. Because $\underline{A} < A < \overline{A}$, $A(1+t)-(1+\frac{\delta}{\rho}\eta)$ is positive and then the sign of $\frac{\partial SW_i}{\partial t}$ is the same sign as that of $G(A)$. When $\widetilde{t} < t < 1$, where $1-3\widetilde{t}+\widetilde{t}^2+\widetilde{t}^3 = 0$ and $\widetilde{t} = 0.414214$, $G(A)$ is positive. When $0 < t < \widetilde{t}$, $G(A)$ is also positive in $\underline{A} < A < \overline{A}$. Therefore, $\frac{\partial SW_i}{\partial t} > 0$ in $0<t<1$.

6.3 Asymmetric countries

6.3.1 Proof of Lemma 6

Substituting (24) into (19) yields

$$n_i^{*a} = \frac{\left(1+t^2\right)\left[\left(1+t+2t^2\right)(1-t)^2(A(1+t)-1) - \frac{\delta}{\rho}\eta_i\left(1+t^2+2t^4\right) + \frac{\delta}{\rho}t\eta_j\left(1+3t^2\right)\right]}{\left(1-t^2\right)^2\left(1+3t^2+4t^4\right)}. \tag{33}$$

We can observe that $n_1^{*a} < n_2^{*a}$. For $n_1^{*a} > 0$, we assume that

$$A > \frac{1}{1+t} + \frac{\delta}{\rho}\frac{\eta_1\left(1+t^2+2t^4\right) - t\eta_2\left(1+3t^2\right)}{(1-t)^2(1+t)\left(1+t+2t^2\right)} \equiv \underline{A}_a.$$

The equilibrium prices in country 1 and 2 are

$$p_i^{*a} = \frac{\rho(1-t)\left(1+t+2t^2\right)\left[1+t^2-tA\left(1-t^2\right)\right] + \delta\left(1+t^2\right)\left[\eta_i(1+t^2) - 2t^3\eta_j\right]}{\rho\left(1-t^2\right)\left(1+3t^2+4t^4\right)}.$$

Since $\eta_1 > \eta_2$, $p_1^{*a} > p_2^{*a}$. For $p_2^{*a} > 0$, the following inequality should hold:

$$A < \frac{\left(1+t^2\right)}{s\rho\left(1-s^2\right)}\frac{\delta\left(\eta_2(1+t^2) - 2t^3\eta_1\right) + \rho(1-t)\left(1+t+2t^2\right)}{(1-t)\left(1+t+2t^2\right)} \equiv \overline{A}_a. \tag{34}$$

By comparing $\overline{A}$ with $\underline{A}$, we can obtain the following equation:

$$\overline{A}_a - \underline{A}_a = \frac{\left(1-t+2t^2\right)\left[\frac{\delta}{\rho}(\eta_2 - \eta_1 t) + (1-t)\right])}{t(1-t)^2(1+t)}.$$

For the existence of the asymmetric equilibrium, we assume that $\frac{\delta}{\rho}(\eta_2 - \eta_1 t) + (1-t) > 0$.

We can see that $\partial\Gamma_1(A)/\partial A > 0$ and $\Gamma_1(\underline{A}_a) = -\frac{t\delta}{\rho}\frac{\eta_1-\eta_2}{1-t}\left(1+t^2+2t^3\right) > 0$. In addition, $\partial\Gamma_2(A)/\partial A > 0$ and $\Gamma_2(\underline{A}_a) = -\frac{t^2\delta}{\rho}\frac{\eta_1-\eta_2}{1-t}\left(1+t^2+2t^3\right) > 0$. Therefore, $\Gamma_1(A) > 0$ and $\Gamma_2(A) > 0$. These results prove Lemma 6.

6.3.2 Proof of Proposition 6

By differentiating the unemployment rate in countries 1 and 2 with respect to search costs in the case of symmetric countries, the following equations can be obtained:

$$\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} = \frac{A(1-t)^2(1+t)(1+t+2t^2) - (1-t)^2(1+t+2t^2) - (2-t+2t^2-3t^3+4t^4)\frac{\delta}{\rho}\eta}{(1-t^2)^2(1+3t^2+4t^4)} \times\frac{(1+t^2)\beta\delta}{\rho}\frac{\partial\eta_1}{\partial k_1},$$

$$\left.\frac{\partial u_2}{\partial k_1}\right|_{k_1=k_2} = \frac{1+4t^2+3t^4}{(1-t^2)^2(1+3t^2+4t^4)} \frac{\beta t \eta_2 \delta^2}{\rho^2} \frac{\partial \eta_1}{\partial k_1} > 0,$$

where $2 - t + 2t^2 - 3t^3 + 4t^4 > 0$ in $0 < t < 1$. When $A > (<) \frac{1}{1+t} + \frac{(2-t+2t^2-3t^3+4t^4)\frac{\delta}{\rho}\eta}{(1-t)^2(1+t)(1+t+2t^2)} \equiv A_a$, $\left.\frac{\partial u_1}{\partial k_1}\right|_{k_1=k_2} > (<) 0$ holds. By subtracting from A_a to $\underline{A}_a$, the following equation can be obtained:

$$A_a - \underline{A}_a = \frac{\delta\eta}{\rho\,(1-t)^2} \frac{1+t^2+2t^4}{1+2t+3t^2+2t^3} > 0.$$

Then, A_a is larger than $\underline{A}_a$. By subtracting from $\overline{A}_a$ to A_a, the following equation can be obtained:

$$\overline{A}_a - A_a = \frac{(1-t)(1+3t^2+4t^4) + \frac{\delta\eta}{\rho}\left(1-2t+3t^2-4t^3+4t^4-6t^5\right)}{t(1-t)^2(1+t)(1+t+2t^2)}.$$

When $\frac{\delta\eta}{\rho} > (<) B$, $\overline{A}_a > (<) A_a$ holds, where we define $\frac{(1-t)(1+3t^2+4t^4)}{-1+2t-3t^2+4t^3-4t^4+6t^5} \equiv B$. In addition, $\frac{\partial B}{\partial t} = -\frac{1+4t^3+t^4+8t^5+14t^6-4t^7+8t^8}{(1-2t+3t^2-4t^3+4t^4-6t^5)^2} < 0$ and $B|_{t=0} = -1$. Thus, $B < 0 < \frac{\delta\eta}{\rho}$ is always satisfied. Therefore, $\underline{A}_a < A_a < \overline{A}_a$ always holds.

6.4 Variable outputs of firms

In the basic model of this paper, we assume that the outputs of the firms are constant, for analytical simplicity. In this subsection, we extend the model by making the outputs of the firms variable. The setup of the model involving the utility function, the agriculture sector, and the matching process in the manufacturing sector is assumed to be the same as in our basic model. In this subsection, the firms, which are under Cournot competition, can choose their optimal amounts of domestic and export outputs. We assume that firms employ one unit of a worker and share revenue with that worker if they are matched. For simplicity, the marginal costs incurred to produce manufactured goods are assumed to be zero. We also assume that the export of manufactured goods incurs trade costs. To export one unit of manufactured goods, firms incur t units of numéraire goods. Under these conditions, the equilibrium price of manufactured goods in country i is

$$p_i = A - n_i q_{ii} - n_j q_{ji},$$

where q_{ii} represents the domestic supply of manufactured goods produced by a firm in country i and q_{ji} is the exported manufactured goods produced by a firm in country j. The revenue for a firm in a country can be described as

$$R_i = p_i q_{ii} + (p_j - t) q_{ij},$$

where t represents trade costs. We can derive the equilibrium amount of outputs and substitute them into the above revenue functions to find the equilibrium revenue of firms. Since we assume that the matching process is the same as that in our basic model, the condition of (17) should also hold:

$$R_i + s_i = r \equiv 1 + \frac{\rho + \delta}{\rho}\eta.$$

These equations determine the equilibrium number of firms in the two countries, n_i and n_j. We thus substitute these values into the next social welfare function:

$$SW_i = \rho n_i W_i + \rho(1 - n_i)U_i = 1 + \overline{z} + n_i(\eta - s_i) + \frac{(A - p_i)^2}{2}.$$

The government in country i sets its subsidy rate to maximize the country's welfare.

Since the calculations in the variable output case are complex, we cannot derive the explicit form of the equilibrium subsidy rate or social welfare in a country. We thus apply numerical methods to compare the equilibrium social welfare with and without the case of subsidy competition ($s_i = s = 0$). Figure A1 describes the results of these numerical methods. This figure shows that when labor market friction is large (η is large), subsidy competition becomes beneficial. Thus, we show that our main result that subsidy competition is beneficial in the case of large labor market friction can be derived in the general model of variable firms' outputs.

6.5 Subsidy for firms' search costs

In this subsection, we study the case that governments provide a subsidy to cover firms' search activities. In this case, the net search costs of a firm become $k - s_i$ in country i. The value of a vacant job is given by

$$\rho V_i = -k + s_i + q(\theta_i)(J_i - V_i). \tag{35}$$

The value of an occupied job is given by

$$\rho J_i = (R_i - w_{Mi}) + \delta(V_i - J_i). \tag{36}$$

By using (35) and (36), we can obtain $J_i - V_i$ as follows:

$$J_i - V_i = \frac{(R_i - w_{Mi}) + k - s_i}{\rho + \delta + q(\theta_i)}. \tag{37}$$

By substituting (7), (11), and (12) into (35), the value of a vacant job is given by

$$\rho V_i = -k + s_i + \frac{1 - \beta}{\beta\theta_i}. \tag{38}$$

In the equilibrium, the value of a vacant job becomes zero $V_i = 0$, and the tightness of the labor market in each country is given by

$$\theta_i^* = \frac{1 - \beta}{\beta(k - s_i)}. \tag{39}$$

Thus, the increase in the subsidy rate raises equilibrium labor market tightness, θ_i^*. When $s_i = k$, $\theta_i^* = \infty$, which means that labor market imperfection vanishes.

By substituting (11), (12), (37), and $V_i = 0$ into (7), the profit level in country i can be obtained as follows:

$$R_i = 1 + \frac{\rho + \delta}{\beta q(\theta_i^*)\theta_i^*} \equiv r_i. \tag{40}$$

Here, we focus on the interior equilibrium in which there are a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Equations (16) determine the equilibrium number of firms in the two countries. By substituting (16) into (2), we get

$$[A - (n_i + tn_j)] + [A - (tn_i + n_j)]\,t = r_i. \tag{41}$$

Thus, the equilibrium number of firms in country i is

$$n_i = \frac{A\,(1+t)\,(1-t)^2 - (1+t^2)r_i + 2tr_j}{(1-t^2)^2}. \tag{42}$$

Equations (39), (40), and (42) show that $\partial n_i/\partial s_j < 0$. Thus, when governments subsidize the search costs of firms, there exists the externality by subsidy.

The government chooses its subsidy rate to maximize welfare in each country:

$$\begin{aligned} SW_i &= 1 + \overline{z} + \rho n_i J_i - (s_i v_i) \\ &\quad + n_i \left(\frac{\rho + \delta + q(\theta_i^*)\theta_i^*}{q(\theta_i^*)\theta_i^*} - \frac{\delta}{q\,(\theta_i^*)\,\theta_i^*} - 1 \right) + \frac{(n_i + tn_j)^2}{2} \\ &= 1 + \overline{z} + n_i \left(\frac{(1-\beta)\rho}{\beta q\,(\theta_i^*)\,\theta_i^*} + \frac{\rho}{q(\theta_i^*)\theta_i^*} - s_i \frac{\delta}{q(\theta_i^*)} \right) + \frac{(n_i + tn_j)^2}{2} \\ &= 1 + \overline{z} + n_i (\frac{\rho}{\beta q\,(\theta_i^*)\,\theta_i^*} - s_i \frac{\delta}{q(\theta_i^*)}) + \frac{(n_i + tn_j)^2}{2}, \end{aligned} \tag{43}$$

where we use $v_i^* = \theta_i^* u_i^* = \frac{\delta n^{i*}}{q(\theta_i^*)}$ and the government's budget constraint becomes $T_i = s_i v_i$. We can see that when $s_i = k$, $\theta_i^* = \infty$. In this case, $r_i = 1$ and n_i has a finite value. Thus, when $s_i = k$, $n_i \left(\frac{\rho}{\beta q(\theta_i^*)\theta_i^*} - s_i \frac{\delta}{q(\theta_i^*)} \right) = -\infty$. This means that the equilibrium value of the subsidy rate is lower than k, $s_i^* < k$.

We specify $q(\theta_i) = \theta_i^{-\gamma}$, where $0 < \gamma < 1$. In this case,

$$SW_i = 1 + \overline{z} + n_i (\frac{\rho}{\beta \left(\frac{1-\beta}{\beta}\right)^{1-\gamma} \left(\frac{1}{k-s_i}\right)^{1-\gamma}} - s_i \frac{\delta}{\left(\frac{1-\beta}{\beta}\right)^{-\gamma} \left(\frac{1}{k-s_i}\right)^{-\gamma}}) + \frac{(n_i + tn_j)^2}{2}.$$

It is impossible to derive an explicit solution of s_i^*. We use numerical methods with $A = 20$, $t = 1/2$, $\rho = 1/2$, $\delta = 1/2$, $\beta = 1/2$, and $\gamma = 1/2$. We show that there is a case that subsidy competition is beneficial (see Figure A2).

7 Appendix (Not for publication)

7.1 Subsidy for employed worker

When we assume that the government provides subsidy for employed workers, the value of W_i is given by

$$\rho W_i = (\overline{z} + w_{Mi} + s_{Mi} + a_i - T_i + \frac{(A - p_i)^2}{2}) + \delta(U_i - W_i), \tag{44}$$

where s_{Mi} denotes the wage subsidy to the employed worker. Under the wage subsidy, from (3) and (44), $W_i - U_i$ is given by

$$W_i - U_i = \frac{w_{Mi} + s_{Mi}}{\rho + \delta + q(\theta_i)\theta_i}. \tag{45}$$

Then, by substituting (10) and (45) into (3), we can obtain the wage rate in the manufactured goods sector as follows:

$$w_{Mi} + s_{Mi} = 1 + \frac{\rho + \delta}{q(\theta_i)\theta_i}. \tag{46}$$

The first term of 1 represents the outside option of the worker and the second term is the risk premium. By substituting (7), (45), and (46) into (5), the value of a vacant job becomes the same as (14). Then, the labor market tightness in the case of the subsidy rate for employed worker is

$$\theta_1^* = \theta_2^* = \theta^* = \frac{1 - \beta}{\beta k}. \tag{47}$$

Thus, this is the same to that in the case of subsidy to manufacturing firms, which is independent of subsidy rates.

By substituting (45), (46), (13), and $V_i = 0$ into (7), we can obtain the profits of manufactured goods firms in country i as follows:

$$\beta q(\theta^*)\theta^* \frac{R_i + s_{Mi} - 1}{\rho + \delta} = 1. \tag{48}$$

Then, from the above equation, the profit level in country i can be obtained as follows:

$$R_i + s_{Mi} = 1 + \frac{\rho + \delta}{\beta q(\theta^*)\theta^*} \equiv r_M, \tag{49}$$

Here, we focus on the interior equilibrium in which there are a positive number of firms in both countries ($n_1 > 0$ and $n_2 > 0$). Thus, the equilibrium number of firms in country i is

$$n_i = \frac{A\,(1 + t)\,(1 - t)^2 - (1 + t^2)(r - s_{Mi}) + 2t(r - s_{Mj})}{(1 - t^2)^2}. \tag{50}$$

The government chooses its subsidy rate to maximize welfare in each country:

$$SW_i = \rho n_i W_i + \rho(1-n_i)U_i = 1 + \overline{z} + n_i(\eta - s_{Mi}) + \frac{(n_i + tn_j)^2}{2}. \tag{51}$$

Thus, we saw that the equilibrium number of subsidy rates in the case of subsidy for employed workers is the same to the that in the subsidy for manufacturing firms. In addition, the social welfare function in the case of subsidy for employed workers is the same to the case in the subsidy to manufacturing firms. Therefore, in the case of subsidy for employed workers, we can derive the same results to the case of subsidy for manufacturing firms.

7.2 Derivation of the welfare level

The welfare level in country i is given by

$$\begin{aligned} SW_i &= \rho n_i W_i + \rho(1-n_i)U_i \\ &= n_i\left(\overline{z} + w_{Mi} + a_i - T_i + \frac{(A_i - p_i)^2}{2} + \delta(U_i - W_i)\right) \\ &\quad +(1-n_i)\left(1 + \overline{z} + a_i - T_i + \frac{(A_i - p_i)^2}{2}\right). \end{aligned}$$

By substituting (11), (12), (14), (15), (17), $a_i = \rho n_i J_i$, and the government budget constraint into the above equation, we can obtain the following equation:

$$\begin{aligned} SW_i &= 1 + \overline{z} + \rho n_i J_i - (s_i n_i) \\ &\quad + n_i\left(\frac{\rho + \delta + q(\theta_i^*)\theta_i^*}{q(\theta_i^*)\theta_i^*} - \frac{\delta}{q(\theta_i^*)\theta_i^*} - 1\right) + \frac{(n_i + tn_j)^2}{2} \\ &= 1 + \overline{z} + n_i\left(\frac{(1-\beta)\rho}{\beta q(\theta_i^*)\theta_i^*} + \frac{\rho}{q(\theta_i^*)\theta_i^*} - s_i\right) + \frac{(n_i + tn_j)^2}{2} \\ &= 1 + \overline{z} + n_i\left(\frac{\rho}{\beta q(\theta_i^*)\theta_i^*} - s_i\right) + \frac{(n_i + tn_j)^2}{2}. \end{aligned}$$

Then, we can obtain the welfare level in country i.

7.3 Derivations of $n_1^{*a} < n_2^{*a}$ and $p_1^{*a} > p_2^{*a}$ in asymmetric labor market friction

Subtracting n_1^{*a} from n_2^{*a} yields

$$n_2^{*a} - n_1^{*a} = \frac{\delta}{\rho}\frac{(1+t^2)(\eta_1 - \eta_2)}{(1-t)^2(1+t+2t^2)} > 0,$$

because $\eta_1 > \eta_2$. Then, an efficient labor market attracts more firms. In addition, we investigate the difference in the price level as follows:

$$p_1^{*a} - p_2^{*a} = \frac{\delta}{\rho}\frac{(1+t^2)(\eta_1 - \eta_2)}{1 + t^2 - 2t^3} > 0,$$

because $1 + t^2 - 2t^3 > 0$ in $0 < t < 1$.

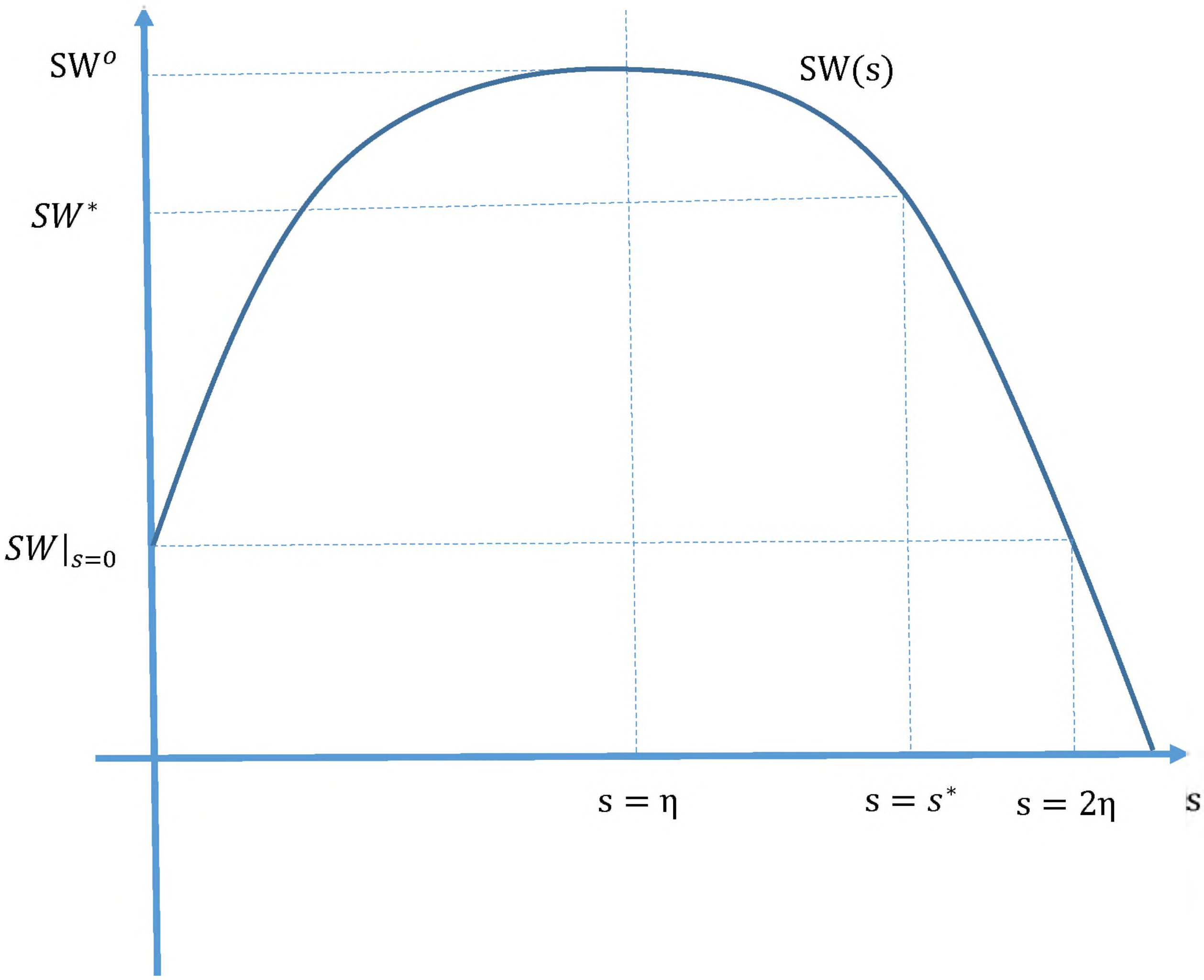


Figure 1A: Subsidy competition is beneficial ($\eta > \bar{\eta}$).

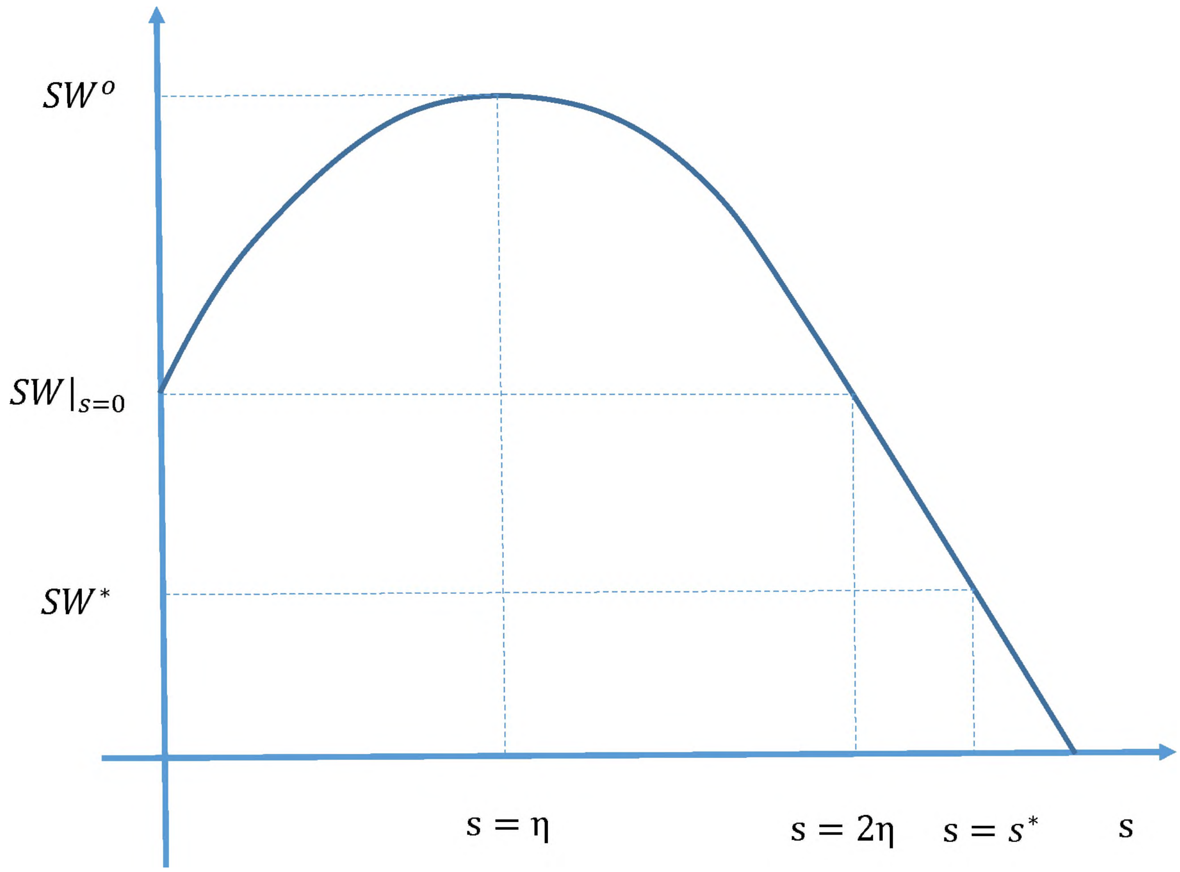
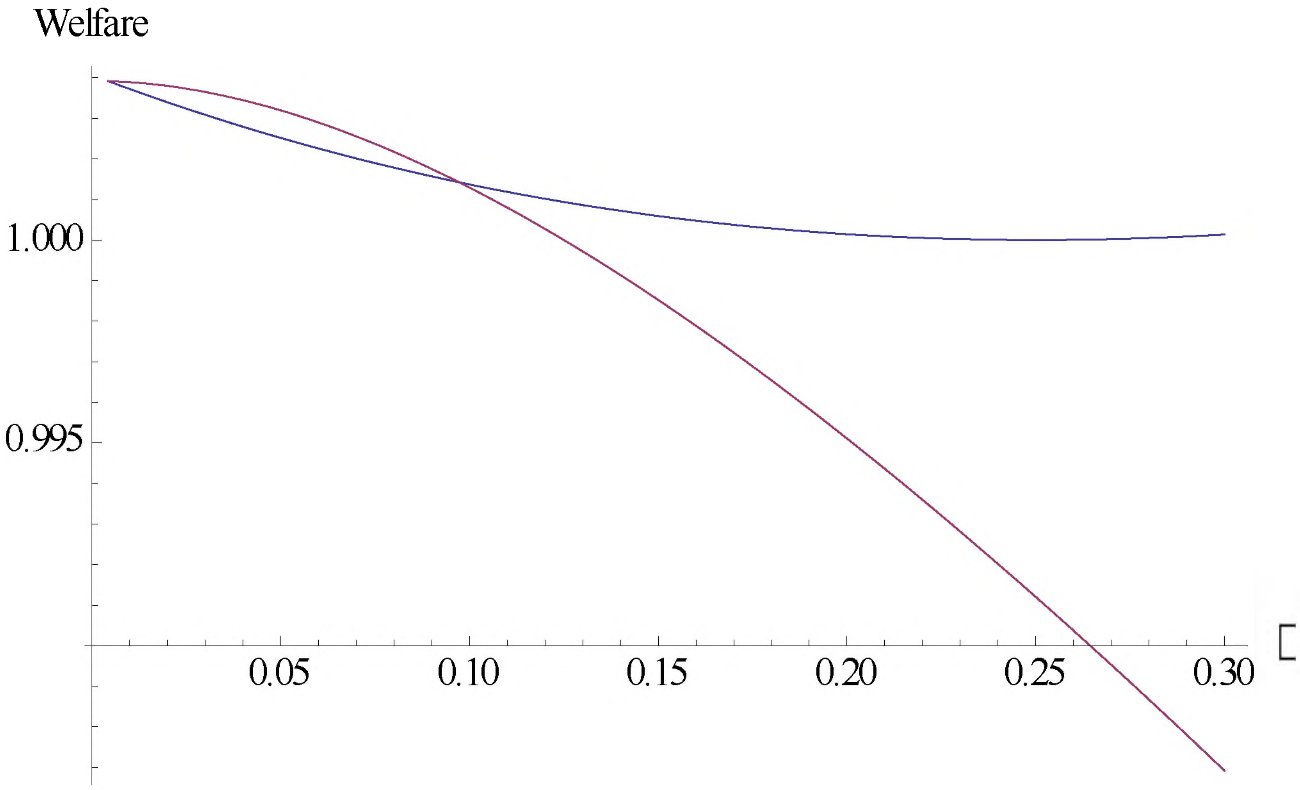


Figure 1B: Subsidy competition is wasteful (η < $\bar{\eta}$).

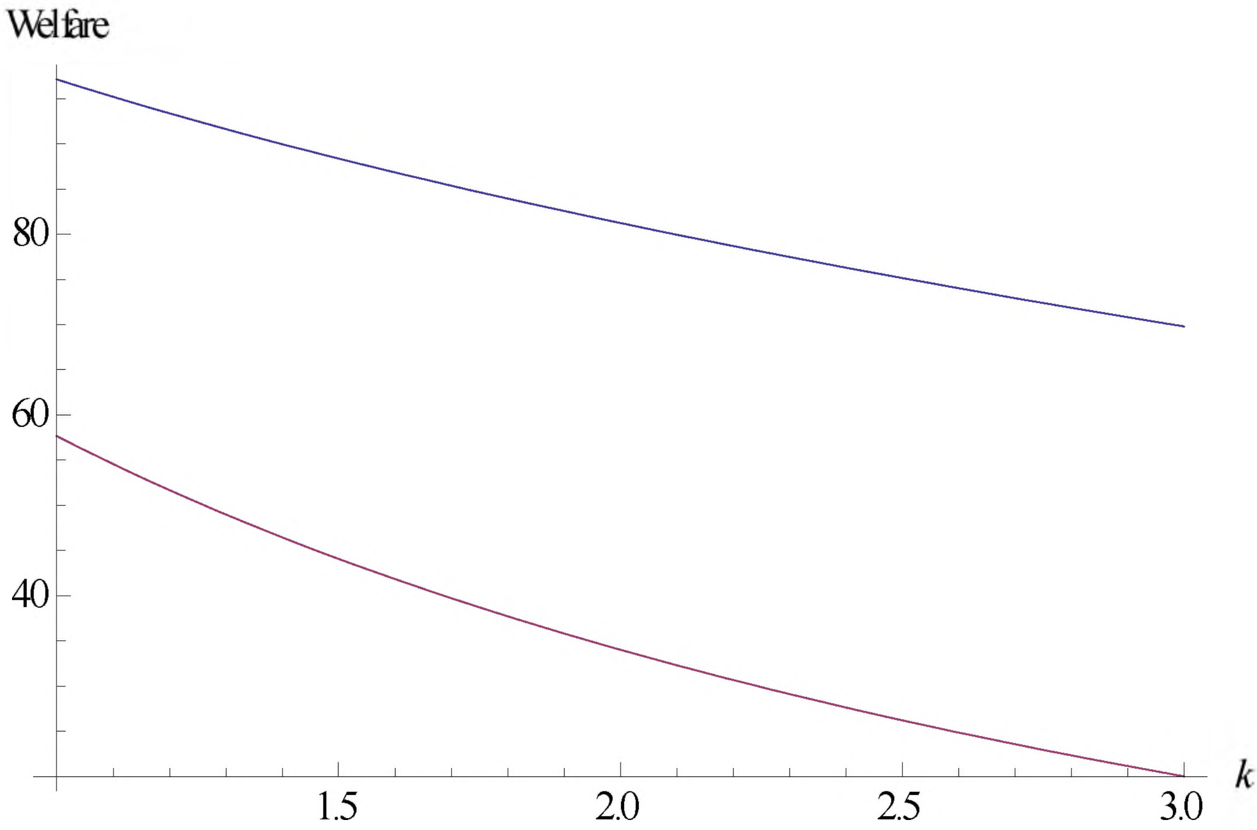


$A = 1, t = \frac{1}{2}, \rho = 0.3, \delta = 0.3.$

(The condition $\eta > 0.00425484$ is necessary to get a real value solution of the subsidy rate).

Blue line: Welfare with subsidy competition, Red line: Welfare without subsidy competition

Figure A1: Welfare with or without subsidy competition



$A = 20, t = \frac{1}{2}, \beta = \frac{1}{2}, \gamma = \frac{1}{2}, \delta = \frac{1}{2}$ and $\mu = \frac{1}{2}$,

Red line: welfare without subsidy competition

Blue line: welfare with subsidy competition

Figure A2: Subsidy to firms' search costs