Homoclinic orbit and stationary sunspot equilibrium in a three-dimensional continuous-time model with a predetermined variable

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Abstract

If for a given deterministic model, there exists a continuum of perfect foresight equilibria, equilibrium is said to be indeterminate. Suppose that fundamental characteristics of an economy are deterministic, but that economic agents believe nevertheless that equilibrium dynamics is affected by random factors apparently irrelevant to the fundamental characteristics (sunspots). This prophecy could be self-fulfilling, and one will get a sunspot equilibrium, if the resulting equilibrium dynamics is subject to a nontrivial stochastic process. A sunspot equilibrium is called a stationary sunspot equilibrium, if an equilibrium stochastic process is stationary. For a large class of models whose fundamental characteristics are deterministic, if equilibrium is indeterminate, there exists a sunspot equilibrium.

In the present study, we treat a three-dimensional continuous-time model that includes one predetermined variable and two non-predetermined variables. We assume (1) that the model has a two-dimensional well-located invariant manifold and (2) that the manifold includes a one-dimensional closed curve that could be either a homoclinic orbit or a closed orbit. Equilibrium is globally indeterminate on a subset of the two-dimensional invariant manifold enclosed by either the homoclinic orbit or the closed orbit. We construct a stationary sunspot equilibrium in this three-dimensional model by means of generalizing the methods due to Nishimura and Shigoka, Int J Econ Theory 2: 199-2016, (2006) and Benhabib et al. Int J Econ Theory 4: 337-355, (2008).

By appealing to the same argument as in Nishimura and Shigoka (2006), we can apply our result to endogenous growth models, such as due to the Matta et al., Int J Econ Theory 5: 25-47, (2009) and Bella et al., Homoclinic bifurcations and global indeterminacy in Chamley's model of endogenous growth, mimeo, (2009), the transitional dynamics of which is three-dimensional and undergoes homoclinic bifurcation. Our result is also applicable to endogenous growth models, due to Lucas, J Monet Econ 22: 3-42, (1988), and Benhabib et al, Ricerche Economiche 48: 279-298 (1994), when the transitional dynamics of these models undergoes Hopf bifurcation whether Hopf bifurcation is supercritical or subcritical.