Individual bounded response of social choice functions*

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May 11, 2016

Abstract

We introduce a new axiom called *individual bounded response* which states that for each "smallest" change of a preference profile, the change of the social choice must be the "smallest", if any, for the agent who induces the change of a preference profile. We show that *individual bounded response* is weaker than *strategy-proofness*, and that *individual bounded response* and *efficiency* imply dictatorship. This impossibility has a far-reaching negative implication. On the universal domain of preferences, it is hard to find a nonmanipulability condition which leads to a possibility result.

*This research is supported by JSPS KAKENHI 16K17082 (Nozomu Muto) and JSPS KAKENHI 16K03571 (Shin Sato).

1 Introduction

We consider a society which is to choose one alternative based on the agents' preferences on a finite set of alternatives. A social choice function (SCF) maps each profile of agents' preferences to an alternative. We propose an axiom called *individual bounded response*. A SCF satisfies *individual bounded response* if for each "smallest" change of a preference profile, the change of the social choice must be the "smallest", if any, for the agent who induces the change of a preference profile.

We explain *individual bounded response* in detail. Given a preference profile $\boldsymbol{R} = (R_1, \ldots, R_n)$, assume that an alternative x is chosen at $\boldsymbol{R}$. Consider that one agent, say agent i, exchanges the positions of one pair of consecutively ranked alternatives in R_i. We regard this as the "smallest change" of a preference profile. Let y be the social choice after agent i changes his preference. Then, *individual bounded response* requires that either $x = y$ or x and y are consecutively ranked in R_i.

Our main result is simple; A SCF satisfies *individual bounded response* and *efficiency* if and only if the SCF is dictatorial. This impossibility has interesting and important implications.

First, our main result shows that the impossibility of the Gibbard–Satterthwaite theorem (Gibbard, 1973; Satterthwaite, 1975) is not necessarily due to an incentive requirement of *strategy-proofness*. By the Gibbard–Satterthwaite theorem, it is well-known that *strategy-proofness* and *efficiency* lead to dictatorship. It can be seen that *individual bounded response* is weaker than *strategy-proofness*. Thus, *individual bounded response*, which is a "side effect" of *strategy-proofness*, is sufficient for the impossibility. Note that *individual bounded response* is not a condition on incentives to misreport preferences. *Individual bounded response* just says how much the social choice can vary corresponding to changes of agents' preferences. Thus, when agent i changes his preference from R_i to R'_i, it is possible under *individual bounded response* that the social choice at R'_i is preferable (according to R_i) to the social choice at R_i.

Second, our result readily leads to a new interesting impossibility theorem. Following recent researches on weakening *strategy-proofness* (for example, Reffgen, 2011; Carroll, 2012; Sato, 2013; Cho, 2016), we consider a new incentive condition, called *weak AM-proofness*. Assume that the options of misrepresentation are restricted to the adjacent preferences to the true one as in Sato (2013). Given a preference profile $\boldsymbol{R}$, let x be the chosen alternative at $\boldsymbol{R}$, and R'_i be a false preference of agent i which is adjacent to R_i. Let y and z be the alternatives whose ranks are exchanged in the passage from R_i to R'_i. *Weak AM-proofness* requires that (i) if y and z are "near" x in R_i, then the social choice at

R_i' cannot be preferred to x according to R_i, and (ii) if y and z are "far" from x in R_i, then the social choice at R_i' can be preferred to x according to R_i, but in that case, the social choice at R_i' and x should be consecutively ranked in R_i. As a straightforward corollary of our main result, we can see that *weak AM-proofness* and *efficiency* lead to dictatorship. This result is a surprising one. Even when we allow profitable misrepresentation, when the degree of the profit is restricted, we cannot deviate from the impossibility.

The remainder of the paper is organized as follows. In Section 2, we introduce notations and definitions, including our main axiom *individual bounded response*. In Section 3, we present a number of results. In Section 3.1 we show our main theorem after introducing a technical condition called *flipping-wall*. In Section 3.2, we present an application to *weak AM-proofness*. In Section 3.3, we discuss results when *efficiency* is weakened to *unanimity*. In Section 3.4, we discuss whether our impossibility result holds on restricted domains of preferences. In Section 4, we provide a complete proof of the main theorem. Section 5 concludes.

2 Model

We consider a society consisting of n *agents* in $N = \{1, \ldots, n\}$ where $n \geq 2$. Let X be a finite set of feasible *alternatives* with $|X| = m \geq 3$, and $\mathcal{L}$ be the set of all *linear orders* on X.[1] By definition, $x\ R\ x$ for each $R \in \mathcal{L}$ and each $x \in X$. Each agent $i \in N$ has a *preference* $R_i \in \mathcal{L}$. For each pair of distinct alternatives $x, y \in X$, $x\ R_i\ y$ means that i (strictly) prefers x to y. If each agent i has a preference $R_i \in \mathcal{L}$, the n-tuple $(R_1, \ldots, R_n)$ is denoted by $\boldsymbol{R}$, and if some agent i changes the preference from R_i to R_i', the new preference profile is written as $(R_i', \boldsymbol{R}_{-i})$. For each preference $R \in \mathcal{L}$ and each integer k ($1 \leq k \leq m$), let $r^k(R) \in X$ be the kth-ranked alternative according to R. For each preference $R \in \mathcal{L}$ and each alternative $x \in X$, let $\rho_R(x)$ be the rank of x with respect to R, i.e., $\rho_R(x) = \big|\{y \in X \mid y\ R\ x\}\big|$. Two alternatives x and y are *adjacent* in $R \in \mathcal{L}$ if they are consecutively ranked in R, i.e., $|\rho_R(x) - \rho_R(y)| = 1$. Two preferences R and R' are *adjacent* if the only difference between them is the ranks of one pair of adjacent alternatives. If R and R' are adjacent and two distinct alternatives $x, y \in X$ satisfy $x\ R\ y$ and $y\ R'\ x$, the set of two alternatives $\{x, y\}$ is denoted by $A(R, R')$.

A *social choice function* (*SCF*) f is a function from the set of preference profiles $\mathcal{L}^n$ to the set of alternatives X. A SCF is *dictatorship* if there exists $i \in N$ such that $f(\boldsymbol{R}) = r^1(R_i)$ for each $\boldsymbol{R} \in \mathcal{L}^n$. This agent i is called a *dictator*. We introduce a few properties of a SCF. A SCF f satisfies

[1] A binary relation is a *linear order* if it is complete, transitive, and antisymmetric.

(i) *strategy-proofness* if $f(\boldsymbol{R})\ R_i\ f(R'_i, \boldsymbol{R}_{-i})$ for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$, and each $R'_i \in \mathcal{L}$.

(ii) *monotonicity* if $f(R'_i, \boldsymbol{R}_{-i}) = f(\boldsymbol{R})$ for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$, and each $R'_i \in \mathcal{L}$ such that $\{x \in X \mid f(\boldsymbol{R})\ R_i\ x\} \subseteq \{x \in X \mid f(\boldsymbol{R})\ R'_i\ x\}$.

(iii) *efficiency* if $f(\boldsymbol{R}) \neq x$ for each $\boldsymbol{R} \in \mathcal{L}^n$ and each $x \in X$ such that there exists $y \in X \setminus \{x\}$ satisfying $y\ R_i\ x$ for each $i \in N$.

(iv) *individual bounded response* if for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$, and each $R'_i \in \mathcal{L}$ which is adjacent to R_i, $f(R_i, \boldsymbol{R}_{-i})$ and $f(R'_i, \boldsymbol{R}_{-i})$ are adjacent in R_i or the same, i.e.,

$$\left|\rho_{R_i}(f(R_i, \boldsymbol{R}_{-i})) - \rho_{R_i}(f(R'_i, \boldsymbol{R}_{-i}))\right| \leq 1.$$

Strategy-proofness ensures that reporting the true preference is always the optimal strategy regardless of what the other agents report. *Monotonicity* says that expanding the lower contour set of the social choice does not change the social choice. Muller and Satterthwaite (1977) show that, as long as strict preferences are considered, *monotonicity* is a necessary and sufficient condition of *strategy-proofness*. *Efficiency* is the standard axiom saying that an alternative cannot be a social choice if it is Pareto dominated by some other alternative. *Individual bounded response* is our main axiom in the paper.[2] It states that if an agent i changes the report from R_i to R'_i, and this change is the smallest in the sense that R_i and R'_i are adjacent, then the change of the social choice must be the smallest, if any. Here, the change of the social choice is measured by the difference in the ranks according to the initial preference R_i of agent i, and thus this condition imposes no requirement on the change of the ranks according to the other agents' preferences. This is why we call the axiom "individual". *Individual bounded response* may not seem an incentive condition because it allows agent i to be either better off or worse off after the change of i's preference. Nevertheless, we will observe that *individual bounded response* is weaker than *strategy-proofness* in the next section.

[2] Muto and Sato (2016a) introduce an axiom called (*weak*) *individual bounded response*: for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$, and each $R'_i \in \mathcal{L}$ which is adjacent to R_i, $\left|\rho_{R_i}(f(R_i, \boldsymbol{R}_{-i})) - \rho_{R'_i}(f(R'_i, \boldsymbol{R}_{-i}))\right| \leq 1$. Note that the rank of $f(R'_i, \boldsymbol{R}_{-i})$ is measured according to R'_i in *weak individual bounded response* whereas it is measured according to R_i in *individual bounded response*. It is readily shown that *weak individual bounded response* follows from *individual bounded response* in this paper, and there exists a nondictatorial SCF satisfying *weak individual bounded response* and *efficiency*. An example of a nondictatorial SCF satisfying *weak individual bounded response* and *efficiency* is the following; For each $\boldsymbol{R} \in \mathcal{L}^n$, $f(\boldsymbol{R}) = r^1(R_1)$ if $r^1(R_1)\ R_2\ r^2(R_1)$, and $f(\boldsymbol{R}) = r^2(R_1)$ if $r^2(R_1)\ R_2\ r^1(R_1)$.

3 Result

In Section 3.1, we show our main theorem: *individual bounded response* and *efficiency* imply dictatorship. Then, in Section 3.2, we propose a new incentive condition and show that our main theorem readily implies the impossibility with the new incentive condition. In Sections 3.3 and 3.4, we examine robustness of our impossibility result.

3.1 Main theorem

First, we show that *individual bounded response* follows from *strategy-proofness*.

Proposition 3.1. Strategy-proofness *implies* individual bounded response.

Proof. Suppose that a SCF f satisfies *strategy-proofness*. Take preference profile $\boldsymbol{R} \in \mathcal{L}^n$, agent $i \in N$, and $R'_i \in \mathcal{L}$ which is adjacent to R_i, arbitrarily. By *strategy-proofness*, we have

$$f(\boldsymbol{R})\ R_i\ f(R'_i, \boldsymbol{R}_{-i}), \text{ and} \tag{1}$$

$$f(R'_i, \boldsymbol{R}_{-i})\ R'_i\ f(\boldsymbol{R}). \tag{2}$$

It is obvious that (at least) one of the following three conditions is true: (i) $f(\boldsymbol{R}) \notin A(R_i, R'_i)$, (ii) $f(R'_i, \boldsymbol{R}_{-i}) \notin A(R_i, R'_i)$, or (iii) $f(\boldsymbol{R}) \in A(R_i, R'_i)$ and $f(R'_i, \boldsymbol{R}_{-i}) \in A(R_i, R'_i)$. We show $\left|\rho_{R_i}(f(\boldsymbol{R})) - \rho_{R_i}(f(R'_i, \boldsymbol{R}_{-i}))\right| \leq 1$ in each case. First, suppose (i). Then, the lower contour set of $f(\boldsymbol{R})$ remains the same after the change from R_i to R'_i. By (1), $f(\boldsymbol{R})\ R'_i\ f(R'_i, \boldsymbol{R}_{-i})$, and by (2), we have $f(\boldsymbol{R}) = f(R'_i, \boldsymbol{R}_{-i})$. Thus, $\left|\rho_{R_i}(f(\boldsymbol{R})) - \rho_{R_i}(f(R'_i, \boldsymbol{R}_{-i}))\right| = 0 \leq 1$. Second, suppose (ii). Then, the lower contour set of $f(R'_i, \boldsymbol{R}_{-i})$ remains the same after the change from R'_i to R_i. By (2), $f(R'_i, \boldsymbol{R}_{-i})\ R_i\ f(\boldsymbol{R})$, and by (1), we have $f(\boldsymbol{R}) = f(R'_i, \boldsymbol{R}_{-i})$. Thus, $\left|\rho_{R_i}(f(\boldsymbol{R})) - \rho_{R_i}(f(R'_i, \boldsymbol{R}_{-i}))\right| = 0 \leq 1$. Third, suppose (iii). Then, the conclusion is immediate because for each $x, y \in X$, if $x \in A(R_i, R'_i)$ and $y \in A(R_i, R'_i)$, then $\left|\rho_{R_i}(x) - \rho_{R_i}(y)\right| \leq 1$. $\square$

Next, we introduce a comprehensive condition, which turns out to be weaker than *individual bounded response* (and also *strategy-proofness* by Proposition 3.1). For each pair of adjacent preferences $R_i, R'_i \in \mathcal{L}$, the following partition on the set of alternatives is induced: (a) $U(R_i, R'_i)$, the alternatives (strictly) preferred to those in $A(R_i, R'_i)$ with respect to R_i or R'_i, (b) $A(R_i, R'_i)$, the pair of alternatives whose ranks are exchanged between R_i and R'_i, and (c) $L(R_i, R'_i)$, the alternatives (strictly) less preferred to those in $A(R_i, R'_i)$ with respect to R_i or R'_i. More formally, for each pair of adjacent preferences $R_i, R'_i \in \mathcal{L}$, let $U(R_i, R'_i) = \{x \in X \setminus A(R_i, R'_i) \mid x\ R_i\ y \text{ for each } y \in A(R_i, R'_i)\}$, and

$$
\begin{array}{r|cc}
 & R_i & R'_i \\
\hline
U(R_i, R'_i)\left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{c}\vdots \\ \vdots\end{array} & \begin{array}{c}\vdots \\ \vdots\end{array} \\
A(R_i, R'_i)\left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{c}x \\ y\end{array} & \begin{array}{c}y \\ x\end{array} \\
L(R_i, R'_i)\left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{c}\vdots \\ \vdots\end{array} & \begin{array}{c}\vdots \\ \vdots\end{array}
\end{array}
$$

Figure 1: A partition of X given by a pair of adjacent preferences (R_i, R'_i).

$L(R_i, R'_i) = \{x \in X \setminus A(R_i, R'_i) \mid y \; R_i \; x \text{ for each } y \in A(R_i, R'_i)\}$. (We note that $U(R_i, R'_i)$ or $L(R_i, R'_i)$ may be empty.)

This partition is illustrated by Figure 1, in which each column presents a preference, and each column with dots represent the identical ordering between two preferences. The following condition, called *flipping-wall*, states that even if the social choice changes by the change of agent i's preference from R_i to R'_i, these social choices should belong to the same partition element. Thus, the flipping part $A(R_i, R'_i)$ is a "wall" which blocks the social choice from moving between the upper part $U(R_i, R'_i)$ and the lower part $L(R_i, R'_i)$.

Definition 3.1 (*Flipping-wall*)**.** A SCF satisfies *flipping-wall* if for each $\boldsymbol{R} \in \mathcal{L}^n$, each $x \in X$, each $i \in N$, and each $R'_i \in \mathcal{L}$ such that R_i and R'_i are adjacent, the following three conditions are true:

(a) $f(\boldsymbol{R}) \in U(R_i, R'_i)$ implies $f(R'_i, \boldsymbol{R}_{-i}) \in U(R_i, R'_i)$,

(b) $f(\boldsymbol{R}) \in A(R_i, R'_i)$ implies $f(R'_i, \boldsymbol{R}_{-i}) \in A(R_i, R'_i)$, and

(c) $f(\boldsymbol{R}) \in L(R_i, R'_i)$ implies $f(R'_i, \boldsymbol{R}_{-i}) \in L(R_i, R'_i)$.

This condition is weak in that if $f(\boldsymbol{R}) \in U(R_i, R'_i)$ or $f(\boldsymbol{R}) \in L(R_i, R'_i)$, and the partition element has more than two alternatives, then the difference in the ranks of $f(\boldsymbol{R})$ and $f(R'_i, \boldsymbol{R}_{-i})$ according to R_i may be larger than one. Indeed, we can show that *flipping-wall* is implied by *individual bounded response*.

Lemma 3.2. Individual bounded response *implies* flipping-wall.

Proof. Suppose that a SCF satisfies *individual bounded response*. Let $x = f(\boldsymbol{R})$ and $y = f(R'_i, \boldsymbol{R}_{-i})$. By *individual bounded response*,

$$\left|\rho_{R_i}(y) - \rho_{R_i}(x)\right| \leq 1, \text{ and} \tag{3}$$

$$\left|\rho_{R'_i}(x) - \rho_{R'_i}(y)\right| \leq 1. \tag{4}$$

First, suppose that $x \in U(R_i, R'_i)$ and $y \notin U(R_i, R'_i)$. By inequality (3), $y \in A(R_i, R'_i)$ and $\rho_{R_i}(y) - \rho_{R_i}(x) = 1$. Then, $\rho_{R'_i}(x) = \rho_{R_i}(x)$ and $\rho_{R'_i}(y) = \rho_{R_i}(y) + 1$, which contradicts inequality (4). This shows (a). Second, suppose that $x \in L(R_i, R'_i)$ and $y \notin L(R_i, R'_i)$. By inequality (3), $y \in A(R_i, R'_i)$ and $\rho_{R_i}(x) - \rho_{R_i}(y) = 1$. Then, $\rho_{R'_i}(x) = \rho_{R_i}(x)$ and $\rho_{R'_i}(y) = \rho_{R_i}(y) - 1$, which contradicts inequality (4). This shows (c). Finally, if $x \in A(R_i, R'_i)$ and $y \in U(R_i, R'_i)$, (a) is violated where the roles R_i and R'_i are exchanged. If $x \in A(R_i, R'_i)$ and $y \in L(R_i, R'_i)$, (c) is violated where the roles R_i and R'_i are exchanged. Therefore, (b) is shown. □

It is seen that *flipping-wall* together with *efficiency* are enough to imply dictatorship.

Lemma 3.3. *If a SCF f satisfies* flipping-wall *and* efficiency *then f is dictatorship.*

The proof of Lemma 3.3 is given in Section 4. Our main theorem is an immediate corollary of Lemmas 3.2 and 3.3:

Theorem 3.4. Individual bounded response *and* efficiency *imply dictatorship.*

Note that Theorem 3.4 (impossibility with *individual bounded response*) is logically weaker than Lemma 3.3 (impossibility with *flipping-wall*). Nevertheless, we present the impossibility with *individual bounded response* as our main result. This is because *individual bounded response* has a normative meaning, while *flipping-wall* is just a technical property of SCFs.

3.2 Application

We consider a new condition related to incentives to misreport preferences. We assume that the options for misrepresentation are restricted to the adjacent preferences to the true one. Let $\boldsymbol{R} \in \mathcal{L}^n$ and $i \in N$. Assume that agent i does not have a time to consider every possible candidate of misrepresentation, or he is reluctant to do so. In investigating an opportunity of profitable misrepresentation, a natural focal point is $f(\boldsymbol{R})$. Then, agent i would focus on alternatives around $f(\boldsymbol{R})$ at R_i.

1. Thus, agent i thinks carefully if he can have a better outcome by reporting a false preference R'_i such that $A(R_i, R'_i)$ is near $f(\boldsymbol{R})$ in R_i.

2. On the other hand, if a big reward is not expected, he is not willing to think carefully about a false preference R'_i such that $A(R_i, R'_i)$ is far from $f(\boldsymbol{R})$ in R_i.

We do not argue that the agents always behave in this way. However, we believe that the above setting is plausible in some cases, and it is interesting to see whether we can construct a SCF which prevents such misrepresentation. The condition ensuring that each agent reports his true preference in such a setting is the following. We say that a SCF f satisfies *weak AM-proofness* if for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$ and each R'_i which is adjacent to R_i,[3]

1. $f(\boldsymbol{R})\ R_i\ f(R'_i, \boldsymbol{R}_{-i})$, when $|\rho_{R_i}(x) - \rho_{R_i}(f(\boldsymbol{R}))| \leq 1$ or $|\rho_{R_i}(y) - \rho_{R_i}(f(\boldsymbol{R}))| \leq 1$, where $\{x, y\} = A(R_i, R'_i)$, and

2. $\rho_{R_i}(f(R'_i, \boldsymbol{R}_{-i})) - \rho_{R_i}(f(\boldsymbol{R})) \leq 1$, when $|\rho_{R_i}(x) - \rho_{R_i}(f(\boldsymbol{R}))| \geq 2$ and $|\rho_{R_i}(y) - \rho_{R_i}(f(\boldsymbol{R}))| \geq 2$.

Since it can be readily seen that *individual bounded response* implies *weak AM-proofness*, we have the following corollary.

Corollary 3.5. *If a SCF f satisfies* weak AM-proofness *and* efficiency, *then f is dictatorship.*

3.3 Unanimity

A SCF f satisfies *unanimity* if $f(\boldsymbol{R}) = x$ for each $\boldsymbol{R} \in \mathcal{L}^n$ and each $x \in X$ such that $r^1(R_i) = x$ for each $i \in N$. We note that *unanimity* follows from *efficiency*. Since the Gibbard–Satterthwaite theorem shows that *strategy-proofness* and *unanimity* imply dictatorship, it is of interest to ask whether *individual bounded response* and *unanimity* imply dictatorship. In general, we have a negative answer to this question, as the following counterexample shows.

Example 3.1. Suppose $n = 3$ and $m = 4$. Consider the following SCF f. For each $\boldsymbol{R} \in \mathcal{L}^n$,

(a) if $|\{r^1(R_1), r^1(R_2), r^1(R_3)\}| = 1$, then $f(\boldsymbol{R}) = r^1(R_1)$.

(b) if $|\{r^1(R_1), r^1(R_2), r^1(R_3)\}| = 2$, then $f(\boldsymbol{R}) = r^1(R_i)$ where there exist $i, j, k \in N$ such that $\{i, j, k\} = N$ and $r^1(R_i) \neq r^1(R_j) = r^1(R_k)$.

(c) if $|\{r^1(R_1), r^1(R_2), r^1(R_3)\}| = 3$, then $f(\boldsymbol{R}) = w$ where w is the unique alternative in $X \setminus \{r^1(R_1), r^1(R_2), r^1(R_3)\}$.

We explain this SCF by words. If the three agents agree on the best alternative, that alternative is chosen. If exactly two of them agree on the best alternative, the best alternative for the remaining agent is chosen. If the best alternatives by the three agents are distinct from each other, the alternative which is not the best for any of them is chosen.

[3] In Sato (2013), a SCF f satisfies *AM-proofness* if $f(\boldsymbol{R})\ R_i\ f(R'_i, \boldsymbol{R}_{-i})$ for each $\boldsymbol{R} \in \mathcal{L}^n$, each $i \in N$, and each $R'_i \in \mathcal{L}$ which is adjacent to R_i. Here, "AM" stands for Adjacent Manipulation.

By (a), f satisfies *unanimity*. Let us observe that f satisfies *individual bounded response*.

For each preference profile $\boldsymbol{R} \in \mathcal{L}^3$, let $T(\boldsymbol{R}) = \{r^1(R_1), r^1(R_2), r^1(R_3)\} \subset X$ be the set of top alternatives. Fix a preference profile $\boldsymbol{R} \in \mathcal{L}^3$ and an agent $i \in N$ arbitrarily. Since f depends only on the top alternatives, it suffices to consider the flip between the top alternative and the second-best one. Let $R'_i \in \mathcal{L}$ be the preference adjacent to R_i given by flipping $r^1(R_i)$ and $r^2(R_i)$. We consider three cases in order.

CASE a: Suppose that $\left|T(\boldsymbol{R})\right| = 1$. Then, $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 2$, and $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R'_i) = r^2(R_i)$. *Individual bounded response* holds in this case.

CASE b: Suppose that $\left|T(\boldsymbol{R})\right| = 2$.

SUBCASE b.1: If $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 1$, *individual bounded response* holds by Case a.

SUBCASE b.2: Suppose that $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 2$, that is, there exist $j, k \in N \setminus \{i\}$ such that $r^1(R_i) = r^1(R_j) \neq r^1(R_k) = r^1(R'_i)$. Then, $f(\boldsymbol{R}) = r^1(R_k) = r^1(R'_i) = r^2(R_i)$, and $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R_j) = r^1(R_i)$. *Individual bounded response* holds in this case.

SUBCASE b.3: Suppose that $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 3$, that is, there exist $j, k \in N \setminus \{i\}$ such that $r^1(R_i) = r^1(R_j) \neq r^1(R_k)$ and $r^1(R'_i) = r^2(R_i) \in X \setminus T(\boldsymbol{R})$. Then, $f(\boldsymbol{R}) = r^1(R_k) \in X \setminus \{r^1(R_i), r^2(R_i)\}$, and $f(R'_i, \boldsymbol{R}_{-i}) \in X \setminus \{r^1(R'_i), r^1(R_j), r^1(R_k)\} \subset X \setminus \{r^1(R_i), r^2(R_i)\}$. Thus, $\{f(\boldsymbol{R}), f(R'_i, \boldsymbol{R}_{-i})\} \subseteq \{r^3(R_i), r^4(R_i)\}$. *Individual bounded response* holds in this case.

CASE c: Suppose that $\left|T(\boldsymbol{R})\right| = 3$. Then, $\left|T(R'_i, \boldsymbol{R}_{-i})\right| \geq 2$.

SUBCASE c.1: If $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 2$. *individual bounded response* holds by Subcase b.3.

SUBCASE c.2: Suppose that $\left|T(R'_i, \boldsymbol{R}_{-i})\right| = 3$, that is, $r^1(R'_i) = r^2(R_i) \in X \setminus T(\boldsymbol{R})$. Then, $f(\boldsymbol{R}) = r^2(R_i)$ and $f(R'_i, \boldsymbol{R}_{-i}) = r^2(R'_i) = r^1(R_i)$. *Individual bounded response* holds in this case.

Therefore, f satisfies *individual bounded response* in all cases.

Let $X = \{x, y, z, w\}$. In Example 3.1, if $\boldsymbol{R} = (R_1, R_2, R_3)$ is such that $r^1(R_1) = x$, $r^1(R_2) = y$, $r^1(R_3) = z$, and $r^4(R_1) = r^4(R_2) = r^4(R_3) = w$, then $f(\boldsymbol{R}) = w$. This is somewhat curious in that the worst alternative w is chosen even if the agents unanimously agree that the best three alternatives are x, y, and z. In fact, we can show that a strengthened version of *unanimity*, which excludes such cases, is enough to obtain the impossibility result.

We say that a SCF f satisfies *strong unanimity* if f satisfies *unanimity*, and $f(\boldsymbol{R}) \in \{x, y, z\}$ for each $\boldsymbol{R} \in \mathcal{L}^n$ and each $x, y, z \in X$ such that $\{r^1(R_i), r^2(R_i), r^3(R_i)\} = \{x, y, z\}$ for each $i \in N$. We note that *strong unanimity* follows from *efficiency*.[4]

[4]Thus, Theorem 3.4 is a corollary of Proposition 3.6. We nevertheless place Theorem 3.4 as the main theorem because *efficiency* is the standard axiom while *strong unanimity* is not. Moreover, the proof with *strong unanimity* is more complicated than the proof with *efficiency*.

Proposition 3.6. *If a SCF f satisfies* flipping-wall *and* strong unanimity, *then f is dictatorship.*

Proof. See the supplementary note Muto and Sato (2016b). □

We note that by the definition of *strong unanimity*, if $m = 3$, *strong unanimity* is trivially equivalent to *unanimity*. Also, we can show that if $n = 2$, *flipping-wall* and *unanimity* implies *strong unanimity*. Hence, we have the following corollary.

Corollary 3.7. *Suppose that $n = 2$ or $m = 3$. If a SCF f satisfies* flipping-wall *and* unanimity, *then f is dictatorship.*

Proof. It suffices to show that if $n = 2$, *flipping-wall* and *unanimity* implies *strong unanimity*.

Suppose that $n = 2$. Take an arbitrary preference profile $(R_1, R_2) \in \mathcal{L}^2$ satisfying $\{r^1(R_1), r^2(R_1), r^3(R_1)\} = \{r^1(R_2), r^2(R_2), r^3(R_2)\}$, which implies $\rho_{R_1}(r^1(R_2)) \leq 3$. Let $R_1' \in \mathcal{L}$ be the preference such that $r^1(R_1') = r^1(R_2)$, and $x\ R_1'\ y$ if and only if $x\ R_1\ y$ for each $x, y \in X \setminus \{r^1(R_2)\}$. By *unanimity*, $f(R_1', R_2) = r^1(R_2)$. By Definition 3.1 (a) and (b), $\rho_{R_1}(r^1(R_2)) = \rho_{R_1}(f(R_1', R_2)) \geq \rho_{R_1}(f(R_1, R_2))$. Since $\rho_{R_1}(r^1(R_2)) \leq 3$, we have $\rho_{R_1}(f(R_1, R_2)) \leq 3$. Hence, f satisfies *strong unanimity*. □

3.4 Restricted domains

So far, we considered the universal domain of preferences $\mathcal{L}$. It may be natural to ask if the impossibility result of Theorem 3.4 holds on restricted domains. Although we have no complete answer to this question, we provide two examples of restricted domains on which the possibility result holds when $n = 3$ and $m = 4$.

The first example is a domain on which *unanimity* and *strategy-proofness* imply dictatorship. Thus, the possibility on this domain suggests a distance between *strategy-proofness* and *individual bounded response*.

Example 3.2. Suppose that X is indexed as $\{x_1, x_2, \ldots, x_m\}$. For each pair of integers ℓ, ℓ', let $x_{\ell'} = x_\ell$ if $\ell' \equiv \ell \mod m$.[5] Let $\mathcal{D} \subset \mathcal{L}$ be the restricted domain of preferences $R \in \mathcal{L}$ such that there exists an integer ℓ satisfying $r^1(R) = x_\ell$ and $r^2(R) \in \{x_{\ell-1}, x_{\ell+1}\}$. This domain $\mathcal{D}$ is a circular domain (Sato, 2010), on which *unanimity* and *strategy-proofness* imply dictatorship.

Suppose that $n = 3$ and $m = 4$. Consider the SCF f defined as follows. For each $\boldsymbol{R} = (R_1, R_2, R_3) \in \mathcal{D}^3$,

[5] For each pair of integers k, k' and each positive integer K, $k' \equiv k \mod K$ if and only if $k' - k$ is a multiple of K.

(a) if there exist $i, j \in N$ such that $i \neq j$ and $r^1(R_i) = r^1(R_j)$, then $f(\boldsymbol{R}) = r^1(R_i)$, and

(b) otherwise, there must exist an integer ℓ such that $\{r^1(R_1), r^1(R_2), r^1(R_3)\} = \{x_{\ell-1}, x_\ell, x_{\ell+1}\}$. We define $f(\boldsymbol{R}) = x_\ell$ in this case.

This SCF f depends only on the profile of top alternatives $(r^1(R_1), r^1(R_2), r^1(R_3))$. If at most two alternatives appear in this profile, then $f(\boldsymbol{R})$ is defined by the plurality rule. If not, $f(\boldsymbol{R})$ is determined by the specific tie-breaking rule which picks the "middle" one among three. Since for each $\boldsymbol{R} \in \mathcal{D}^3$ there exists $i \in N$ such that $f(\boldsymbol{R}) = r^1(R_i)$, the SCF f satisfies *efficiency*. Let us observe that f satisfies *individual bounded response*.

For each preference profile $\boldsymbol{R} \in \mathcal{L}^3$, let $T(\boldsymbol{R}) = \{r^1(R_1), r^1(R_2), r^1(R_3)\} \subset X$ be the set of top alternatives. Fix a preference profile $\boldsymbol{R} \in \mathcal{L}^3$ and an agent $i \in N$ arbitrarily. Since f depends only on the top alternatives, it suffices to consider the flip between the top alternative and the second-best one. Let $R'_i \in \mathcal{L}$ be the preference adjacent to R_i given by flipping $r^1(R_i)$ and $r^2(R_i)$. We consider two cases in order.

CASE a: Suppose that $|T(\boldsymbol{R})| \leq 2$.

SUBCASE a.1: If $|T(\boldsymbol{R})| = 1$ or $|T(R'_i, \boldsymbol{R}_{-i})| = 1$, then $f(\boldsymbol{R}) = f(R'_i, \boldsymbol{R}_{-i})$. *Individual bounded response* is trivial in this case.

SUBCASE a.2: Suppose that $|T(\boldsymbol{R})| = |T(R'_i, \boldsymbol{R}_{-i})| = 2$ and $f(\boldsymbol{R}) \neq f(R'_i, \boldsymbol{R}_{-i})$, that is, there exist $j, k \in N \setminus \{i\}$ such that $r^1(R_i) = r^1(R_j) \neq r^1(R_k) = r^1(R'_i)$. Then, $f(\boldsymbol{R}) = r^1(R_i)$, and $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R'_i) = r^2(R_i)$. *Individual bounded response* holds in this case.

SUBCASE a.3: Suppose that $|T(\boldsymbol{R})| = 2$ and $|T(R'_i, \boldsymbol{R}_{-i})| = 3$, that is, there exist $j, k \in N \setminus \{i\}$ such that $r^1(R_i) = r^1(R_j) \neq r^1(R_k)$ and $r^1(R'_i) = r^2(R_i) \in X \setminus T(\boldsymbol{R})$. Then, $f(\boldsymbol{R}) = r^1(R_i)$.

Let $r^1(R_i) = x_\ell$. By the definition of $\mathcal{D}$, $r^1(R'_i) = r^2(R_i) \in \{x_{\ell-1}, x_{\ell+1}\}$. First, suppose that $r^1(R'_i) = x_{\ell-1}$. Then, $r^1(R_k) = x_{\ell+1}$ or $x_{\ell-2}$, and $f(R'_i, \boldsymbol{R}_{-i}) = x_\ell$ or $x_{\ell-1}$. This implies that $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R_i)$, or $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R'_i) = r^2(R_i)$. *Individual bounded response* holds in either case. Next, suppose that $r^1(R'_i) = x_{\ell+1}$. Then, $r^1(R_k) = x_{\ell+2}$ or $x_{\ell-1}$, and $f(R'_i, \boldsymbol{R}_{-i}) = x_{\ell+1}$ or x_ℓ. This implies that $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R'_i) = r^2(R_i)$, or $f(R'_i, \boldsymbol{R}_{-i}) = r^1(R_i)$. *Individual bounded response* holds in either case.

CASE b: Suppose that $|T(\boldsymbol{R})| = 3$. Then, $|T(R'_i, \boldsymbol{R}_{-i})| \geq 2$.

SUBCASE b.1: If $|T(R'_i, \boldsymbol{R}_{-i})| = 2$, then *individual bounded response* holds by Subcase a.3.

SUBCASE b.2: Suppose that $|T(R'_i, \boldsymbol{R}_{-i})| = 3$, that is, $r^1(R'_i) = r^2(R_i) \in X \setminus T(\boldsymbol{R})$. Let $r^1(R_i) = x_\ell$. By the definition of $\mathcal{D}$, $r^1(R'_i) = r^2(R_i) \in \{x_{\ell-1}, x_{\ell+1}\}$. First, suppose that $r^1(R'_i) = x_{\ell-1}$. Then, $\{r^1(R_j), r^1(R_k)\} = \{x_{\ell+1}, x_{\ell+2}\}$. We have $f(\boldsymbol{R}) = x_{\ell+1}$ and

$f(R_i', \boldsymbol{R}_{-i}) = x_{\ell+2}$. Thus, $\{f(\boldsymbol{R}), f(R_i', \boldsymbol{R}_{-i})\} \subseteq \{r^3(R_i), r^4(R_i)\}$. *Individual bounded response* holds in this case. Next, suppose that $r^1(R_i') = x_{\ell+1}$. Then, $\{r^1(R_j), r^1(R_k)\} = \{x_{\ell-1}, x_{\ell-2}\}$. We have $f(\boldsymbol{R}) = x_{\ell-1}$ and $f(R_i', \boldsymbol{R}_{-i}) = x_{\ell-2}$. Thus, $\{f(\boldsymbol{R}), f(R_i', \boldsymbol{R}_{-i})\} \subseteq \{r^3(R_i), r^4(R_i)\}$. *Individual bounded response* holds in this case.

Therefore, f satisfies *individual bounded response* in all cases.

The second example is the single-peaked domain. On this domain, we provide a nondictatorial SCF which satisfies *individual bounded response* and *efficiency* but violates *strategy-proofness*. This also suggests a distance between *strategy-proofness* and *individual bounded response*.

Example 3.3. Suppose that n is odd, and $m = 4$. Let $X = \{x_1, x_2, x_3, x_4\}$, and $\mathcal{D} \subset \mathcal{L}$ be the single-peaked domain with respect to the above indexes, that is, $\mathcal{D}$ is the set of all preferences R such that there exists $k \in \{1, 2, 3, 4\}$ such that if $4 \geq k > k' > k'' \geq 1$ or $1 \leq k < k' < k'' \leq 4$, then $x_{k'} \ R \ x_{k''}$. Consider the following SCF f. For each $\boldsymbol{R} \in \mathcal{D}^n$, if there exists $y \in X$ such that $r^1(R_i) = y$ for each $i \in N$, then $f(\boldsymbol{R}) = y$. Otherwise,

(a) if $\big|\{i \in N \mid x_1 \ R_i \ x_4\}\big| \geq (n+1)/2$, then

  (i) if x_2 is Pareto efficient at $\boldsymbol{R}$, then $f(\boldsymbol{R}) = x_2$,

  (ii) otherwise, x_3 must be Pareto efficient at $\boldsymbol{R}$,[6] and $f(\boldsymbol{R}) = x_3$.

(b) if $\big|\{i \in N \mid x_1 \ R_i \ x_4\}\big| \leq (n-1)/2$, then

  (i) if x_3 is Pareto efficient at $\boldsymbol{R}$, then $f(\boldsymbol{R}) = x_3$,

  (ii) otherwise, x_2 must be Pareto efficient at $\boldsymbol{R}$, and $f(\boldsymbol{R}) = x_2$.

This SCF f satisfies *unanimity*. Suppose that at a preference profile $\boldsymbol{R}$, some agents disagree with the most-preferred alternative. In this case, $f(\boldsymbol{R})$ is defined by two steps. Either x_1 or x_4 is the worst alternative at every preference in the single-peaked domain. In the first step, agents determine the socially worst alternative by the plurality rule between x_1 and x_4. In the second step, the social alternative is chosen from $\{x_2, x_3\}$ by the rule which chooses the one "more distant" from the worst as long as it is efficient.

The SCF f satisfies *efficiency* by definition. f violates *strategy-proofness* because when $n = 3$ and a preference profile $\boldsymbol{R} \in \mathcal{L}^n$ satisfies $x_2 \ R_1 \ x_3 \ R_1 \ x_4 \ R_1 \ x_1$, $x_2 \ R_2 \ x_3 \ R_2 \ x_1 \ R_2 \ x_4$,

[6]If $\boldsymbol{R} \in \mathcal{D}^n$, either x_2 or x_3 is Pareto efficient at $\boldsymbol{R}$. Suppose that neither x_2 nor x_3 is Pareto efficient. Then, no agent ranks x_2 or x_3 at the top of his preference. Thus, for each $i \in N$, $r^1(R_i) = x_1$ or x_4. Since the agents do not agree on the best alternative, $x_2 \ R_i \ x_3 \ R_i \ x_4$ for some $i \in N$, and $x_2 \ R_j \ x_1$ for some $j \in N$. Thus, x_2 is Pareto efficient, which is a contradiction.

and $x_3\ R_3\ x_2\ R_3\ x_4\ R_3\ x_1$, agent 1 may change the reported preference to $R'_1 = R_2$ and can manipulate the social choice from $f(\boldsymbol{R}) = x_3$ to $f(R'_1, \boldsymbol{R}_{-1}) = x_2$.

Let us observe that f satisfies *individual bounded response*. By symmetry, we can focus on the cases in which $f(\boldsymbol{R}) \in \{x_1, x_2\}$. First, suppose that $f(\boldsymbol{R}) = x_1$. By definition, $r^1(R_i) = x_1$ for each $i \in N$. By the assumption of the single-peaked domain, $x_1\ R_i\ x_2\ R_i\ x_3\ R_i\ x_4$ for each $i \in N$. The only flip available in $\mathcal{D}$ is exchanging x_1 and x_2. This flip changes the social choice to x_2. Therefore, *individual bounded response* holds in this case.

Next, suppose that $f(\boldsymbol{R}) = x_2$. Fix an agent $i \in N$ and a preference $R'_i \in \mathcal{L}$ adjacent to R_i, arbitrarily. If $f(R'_i, \boldsymbol{R}_{-i}) = x_1$, then the flip between R_i and R'_i must be exchanging x_1 and x_2. Thus, *individual bounded response* holds in this case. If $f(R'_i, \boldsymbol{R}_{-i}) = x_2 = f(\boldsymbol{R})$, then *individual bounded response* is trivial. If $f(R'_i, \boldsymbol{R}_{-i}) = x_4$, then $x_4\ R'_i\ x_3\ R'_i\ x_2\ R'_i\ x_1$ and $x_4\ R_j\ x_3\ R_j\ x_2\ R_j\ x_1$ for each $j \in N \setminus \{i\}$. The only flip between R_i and R'_i available in $\mathcal{D}$ is exchanging x_3 and x_4, and thus $f(\boldsymbol{R}) = x_3$. This contradicts the assumption $f(\boldsymbol{R}) = x_2$. Thus, we assume $f(R'_i, \boldsymbol{R}_{-i}) = x_3$. We consider three cases in order.

Case 1: Suppose that either [$x_1\ R_i\ x_4$ and $x_4\ R'_i\ x_1$] or [$x_4\ R_i\ x_1$ and $x_1\ R'_i\ x_4$]. By the assumption of the single-peaked domain, x_1 and x_4 are the bottom two alternatives at R_i and R'_i. This implies that $\{r^1(R_i), r^2(R_i)\} = \{r^1(R'_i), r^2(R'_i)\} = \{x_2, x_3\}$. Thus, *individual bounded response* holds in this case. Therefore, in the following cases, we assume that either [$x_1\ R_i\ x_4$ and $x_1\ R'_i\ x_4$] or [$x_4\ R_i\ x_1$ and $x_4\ R'_i\ x_1$].

Case 2: Suppose that $\left|\{j \in N \mid x_1\ R_j\ x_4\}\right| \geq (n+1)/2$, x_3 is inefficient at $\boldsymbol{R}$, and x_3 is efficient at $(R'_i, \boldsymbol{R}_{-i})$. If x_4 Pareto dominates x_3 at $\boldsymbol{R}$, then $x_4\ R_j\ x_3\ R_j\ x_2\ R_j\ x_1$ for each $j \in N$. This contradicts $f(\boldsymbol{R}) = x_2$. If x_1 Pareto dominates x_3 at $\boldsymbol{R}$, then x_2 also Pareto dominates x_3 at $\boldsymbol{R}$ by the assumption of the single-peaked domain. Therefore, we assume that x_2 Pareto dominates x_3 at $\boldsymbol{R}$. Since x_3 is efficient at $(R'_i, \boldsymbol{R}_{-i})$, x_2 and x_3 are exchanged between R_i and R'_i, that is, x_2 and x_3 are consecutively ranked at R_i and R'_i. Thus, *individual bounded response* holds in this case.

Case 3: Suppose that $\left|\{j \in N \mid x_1\ R_j\ x_4\}\right| \leq (n-1)/2$, x_2 is efficient at $\boldsymbol{R}$, and x_2 is inefficient at $(R'_i, \boldsymbol{R}_{-i})$. If x_1 Pareto dominates x_2 at $(R'_i, \boldsymbol{R}_{-i})$, then $x_1\ R'_i\ x_2\ R'_i\ x_3\ R'_i\ x_4$ and $x_1\ R_j\ x_2\ R_j\ x_3\ R_j\ x_4$ for each $j \in N \setminus \{i\}$. This contradicts $f(R'_i, \boldsymbol{R}_{-i}) = x_3$. If x_4 Pareto dominates x_2 at $(R'_i, \boldsymbol{R}_{-i})$, then x_3 also Pareto dominates x_2 at $(R'_i, \boldsymbol{R}_{-i})$ by the assumption of the single-peaked domain. Therefore, we assume that x_3 Pareto dominates x_2 at $(R'_i, \boldsymbol{R}_{-i})$. Since x_2 is efficient at $\boldsymbol{R}$, x_2 and x_3 are exchanged between R_i and R'_i, that is, x_2 and x_3 are consecutively ranked at R_i and R'_i. Thus, *individual bounded response* holds in this case.

4 Proof

In this section, we prove Lemma 3.3 which immediately implies Theorem 3.4. We divide the proof into several steps. Namely, we prove three Lemmas 4.1, 4.2, and 4.3 as milestones of the proof, and then show Lemma 3.3. Each of Lemmas 4.1, 4.2, and 4.3 states that there exists a dictator i^* in a certain special situation.

Lemma 4.1. *Suppose that a SCF f satisfies* flipping-wall *and* efficiency*. For each $\bar{R} \in \mathcal{L}$, there exists an agent $i^* \in N$ such that for each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$ satisfying $r^1(\bar{R}_j) = r^2(\bar{R})$ and $r^m(\bar{R}_j) = r^1(\bar{R})$ for each $j \in N \setminus \{i^*\}$, we have $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = r^1(\bar{R})$.*

We utilize several figures which illustrate preference profiles. For example, the situation considered in the statement of Lemma 4.1 is illustrated by Figure 2, which is interpreted as follows. For each $\bar{R} \in \mathcal{L}$, let $x = r^1(\bar{R})$, $y = r^2(\bar{R})$, and the cells with vertical dots represent arbitrary alternatives. Then, Lemma 4.1 says that for each preference $\bar{R} \in \mathcal{L}$, there exists a dictator $i^* \in N$ when the top alternative in every other agent's preference is y, and the bottom alternative in every other agent's preference is x. Since i^* is the dictator in this situation, the social choice is x. In Figure 2 and those in the subsequent proofs, the square brackets indicate the social choice at the preference profile specified by the figure.

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
y	$\cdots$	y	$[x]$	y	$\cdots$	y
$\vdots$	$\cdots$	$\vdots$	y	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$[x]$	$\cdots$	$[x]$	$\vdots$	$[x]$	$\cdots$	$[x]$

Figure 2:

To show Lemma 4.1, we basically follow the proof strategy of Steps 1–4 in Reny (2001) who proved the Gibbard–Satterthwaite theorem. Since some manipulation in Reny (2001) is not applicable under *individual bounded response*, we focus on the top three alternatives in steps 1–3, and then consider every alternative in X. In the following proof, the numbers of the steps correspond to those in Reny (2001).

Proof of Lemma 4.1. Fix a preference $\bar{R} \in \mathcal{L}$ arbitrary. Let $x = r^1(\bar{R})$, $y = r^2(\bar{R})$, and $z = r^3(\bar{R})$.

STEP 1: We start with a preference profile in which every agent's preference is $R \in \mathcal{L}$ such that $r^1(R) = x$, $r^2(R) = z$, $r^3(R) = y$, and $r^k(R) = r^k(\bar{R})$ for each $k \geq 4$. By *efficiency*, the social choice is x. This setting is shown in Figure 3. Then, exchange x and z in agent

R	$\cdots$	R	R	R	$\cdots$	R
$[x]$	$\cdots$	$[x]$	$[x]$	$[x]$	$\cdots$	$[x]$
z	$\cdots$	z	z	z	$\cdots$	z
y	$\cdots$	y	y	y	$\cdots$	y
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 3:

1's preference. By *efficiency*, the social choice is x or z. If it is x, exchange x and z in agent 2's preference. If it is x, repeat the same procedure until for some $i^* \in N$, the social choice becomes z. We eventually obtain Figures 4 and 5.

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
z	$\cdots$	z	$[x]$	$[x]$	$\cdots$	$[x]$
$[x]$	$\cdots$	$[x]$	z	z	$\cdots$	z
y	$\cdots$	y	y	y	$\cdots$	y
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 4:

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
$[z]$	$\cdots$	$[z]$	$[z]$	x	$\cdots$	x
x	$\cdots$	x	x	$[z]$	$\cdots$	$[z]$
y	$\cdots$	y	y	y	$\cdots$	y
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 5:

STEP 2: In Figure 5, exchange z and y in the preferences of agents $i^* + 1$ to n. By Definition 3.1 (b), the social choice is z or y, and by *efficiency*, the social choice must be z. We have Figure 6. In Figure 6, exchange x and y in the preferences of agents 1 to $i^* - 1$,

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
$[z]$	$\cdots$	$[z]$	$[z]$	x	$\cdots$	x
x	$\cdots$	x	x	y	$\cdots$	y
y	$\cdots$	y	y	$[z]$	$\cdots$	$[z]$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 6:

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
$[z]$	$\cdots$	$[z]$	$[z]$	y	$\cdots$	y
y	$\cdots$	y	x	x	$\cdots$	x
x	$\cdots$	x	y	$[z]$	$\cdots$	$[z]$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 7:

and also exchange x and y in the preferences of agents $i^* + 1$ to n. By Definition 3.1 (a) and (c), the social choice is neither x nor y, and by *efficiency*, the social choice must be z. We have Figure 7.

In Figure 7, exchange z and x in agent i^*'s preference. By Definition 3.1 (b), the social choice must be x or z. We can show that it is x: If it is z, exchange y and x in the preferences of agents i^*+1 to n, exchange y and x in the preferences of agents 1 to i^*-1, and exchange y and z in the preferences of agents i^*+1 to n. The social choice remains z in this process because of *efficiency* and Definition 3.1. Since it returns to Figure 4 in which the social choice is x, this is a contradiction. Therefore, we have Figure 8.

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
z	$\cdots$	z	$[x]$	y	$\cdots$	y
y	$\cdots$	y	z	$[x]$	$\cdots$	$[x]$
$[x]$	$\cdots$	$[x]$	y	z	$\cdots$	z
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 8:

STEP 3: In Figure 8, exchange z and y in the preferences of agents 1 to i^*-1, and also i^*. The social choice is neither z nor y by Definition 3.1 (a) and (c), and by *efficiency*, the social choice remains x. We have Figure 9.

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
y	$\cdots$	y	$[x]$	y	$\cdots$	y
z	$\cdots$	z	y	$[x]$	$\cdots$	$[x]$
$[x]$	$\cdots$	$[x]$	z	z	$\cdots$	z
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$

Figure 9:

R_1	$\cdots$	R_{i^*-1}	R_{i^*}	R_{i^*+1}	$\cdots$	R_n
y	$\cdots$	y	$[x]$	y	$\cdots$	y
z	$\cdots$	z	y	z	$\cdots$	z
$\vdots$	$\cdots$	$\vdots$	z	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$[x]$	$\cdots$	$[x]$	$\vdots$	$[x]$	$\cdots$	$[x]$

Figure 10:

STEP 4: In Figure 9, lower the positions of x to the bottom in the preferences of the agents except for i^*. The social choice cannot be y by Definition 3.1 (b), and by *efficiency*, the social choice remains x in this process. We have Figure 10.

In Figure 10, shuffle the alternatives in $X \setminus \{x, y\}$ in the preferences except for agent i^*, so that for each $j \in N \setminus \{i^*\}$, the preference of agent j becomes $\bar{R}_j$. By *efficiency*, the social choice must be either x or y in the entire process of shuffling, and by Definition 3.1 (c), the social choice cannot be y. Hence, the resulting social choice must be $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x = r^1(\bar{R})$. □

The above proof of Lemma 4.1 has followed the proof strategy of Steps 1–4 in Reny (2001). The proof of Reny (2001) proceeds to his last step, which cannot be directly applied to the setting with *individual bounded response*. We instead prove the next lemma which states that for each preference $\bar{R} \in \mathcal{L}$, there exists a dictator $i^* \in N$ under an assumption that the bottom alternative in every other agent's preference equals the top in i^*'s.

Lemma 4.2. *Suppose that a SCF f satisfies* flipping-wall *and* efficiency*. For each $\bar{R} \in \mathcal{L}$, there exists an agent $i^* \in N$ such that for each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$ satisfying $r^m(\bar{R}_j) = r^1(\bar{R})$ for each $j \in N \setminus \{i^*\}$, we have $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = r^1(\bar{R})$.*

Given Lemma 4.1, the above Lemma 4.2 says that in the situation of Figure 2, the social choice remains the same if the position of y in the preference of agent $j \in N \setminus \{i^*\}$ is lowered while x stays at the bottom in the preference of j. If *monotonicity* is assumed as in Reny (2001), Lemma 4.2 is immediate because the upper contour set of the social choice is unchanged by such a manipulation. Under *individual bounded response*, however, Lemma 4.2 is fairly nontrivial.

proof of Lemma 4.2. Fix $\bar{R} \in \mathcal{L}$ arbitrarily. Let $x = r^1(\bar{R})$ and $y = r^2(\bar{R})$. Let $i^* \in N$ be the agent given in Lemma 4.1. For each preference profile $\boldsymbol{R}_{-i^*} \in \mathcal{L}^{n-1}$, let $\tau(\boldsymbol{R}_{-i^*}) = \sum_{j \in N \setminus \{i^*\}} \rho_{R_j}(y)$.

We prove the lemma by induction. The following induction base is given by Lemma 4.1:

THE INDUCTION BASE: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $r^m(\bar{R}_j) = x$ for each $j \in N \setminus \{i^*\}$, and $\tau(\bar{\boldsymbol{R}}_{-i^*}) = n-1$, then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

The induction proceeds with the following hypothesis and step.

THE INDUCTION HYPOTHESIS: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $r^m(\bar{R}_j) = x$ for each $j \in N \setminus \{i^*\}$, and $\tau(\bar{\boldsymbol{R}}_{-i^*}) = t$ (where $n-1 \leq t \leq (m-1)(n-1)-1$), then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

THE INDUCTION STEP: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $r^m(\bar{R}_j) = x$ for each $j \in N \setminus \{i^*\}$, and $\tau(\bar{\boldsymbol{R}}_{-i^*}) = t+1$, then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

Fix $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$ such that $\tau(\bar{\boldsymbol{R}}_{-i^*}) = t+1$, arbitrarily. We assume that $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) \neq x$, and derive a contradiction.

STEP 1: We show that $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) \neq y$.

Assume $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = y$. Since $t+1 \geq (n-1)+1$, there exist $j \in N \setminus \{i^*\}$ and $k \geq 2$ such that $y = r^k(\bar{R}_j)$. Let $R_j \in \mathcal{L}$ be the preference given by exchanging the ranks of $r^{k-1}(\bar{R}_j)$ and $y = r^k(\bar{R}_j)$ in $\bar{R}_j$. Since $x = r^m(\bar{R}_j) \neq r^{k-1}(\bar{R}_j)$, by Definition 3.1 (b), we have $f(\bar{R}, R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) \neq x$. This contradicts the induction hypothesis because $\tau(R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) = t$.

STEP 2: We show that for each $j \in N \setminus \{i^*\}$, $\rho_{\bar{R}_j}(y) < \rho_{\bar{R}_j}\big(f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*})\big)$.

By Step 1, this inequality is immediate if $\rho_{\bar{R}_j}(y) = 1$. Assume that there exists $j \in N \setminus \{i^*\}$ such that $\rho_{\bar{R}_j}(y) = k \geq 2$ and $k \geq \rho_{\bar{R}_j}\big(f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*})\big)$. Let $R_j \in \mathcal{L}$ be the preference given by exchanging the ranks of $r^{k-1}(\bar{R}_j)$ and $y = r^k(\bar{R}_j)$ in $\bar{R}_j$. Then by Definition 3.1 (a) and (b), $f(\bar{R}, R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) \neq x \; (= r^m(\bar{R}_j))$. This contradicts the induction hypothesis because $\tau(R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) = t$.

STEP 3: We derive a contradiction.

Since $r^m(\bar{R}_j) = x$ for each $j \in N \setminus \{i^*\}$, $\rho_{\bar{R}_j}(y) \leq m - 1$. By Step 2, $\rho_{\bar{R}_j}(y) < \rho_{\bar{R}_j}\big(f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*})\big)$ for all $j \in N \setminus \{i^*\}$. Since we assumed $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) \neq x$, we also have $\rho_{\bar{R}}(y) < \rho_{\bar{R}}\big(f(\bar{R}, \bar{\boldsymbol{R}}_{i^*})\big)$. These inequalities contradict *efficiency*.

Therefore, the induction step is shown. This completes the proof. □

Next, we show the following lemma, which states that for each preference $\bar{R} \in \mathcal{L}$, agent i^* given in Lemma 4.2 is the dictator when i^*'s preference is $\bar{R}$.

Lemma 4.3. *Suppose that a SCF f satisfies* flipping-wall *and* efficiency*. For each $\bar{R} \in \mathcal{L}$, there exists an agent $i^* \in N$ such that for each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, we have $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = r^1(\bar{R})$.*

Given Lemma 4.2, the above Lemma 4.3 says that the social choice remains the same if the position of the bottom alternative, which equals the social choice, in the preference of agent $j \in N \setminus \{i^*\}$ is raised. If *monotonicity* is assumed as in Reny (2001), Lemma 4.3 is immediate because the upper contour set of the social choice is reduced by such a change. Under *individual bounded response*, however, Lemma 4.3 needs an elaborate proof.

proof of Lemma 4.3. Fix a preference $\bar{R} \in \mathcal{L}$ arbitrarily. Let $x = r^1(\bar{R})$. Let $i^* \in N$ be the agent given in Lemma 4.2. For each $\boldsymbol{R}_{-i^*} \in \mathcal{L}^{n-1}$, let $\sigma(\boldsymbol{R}_{-i^*}) = \sum_{j \in N \setminus \{i^*\}} \rho_{R_j}(x)$.

We prove the theorem by induction. The following induction base is given by Lemma 4.2:

THE INDUCTION BASE: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $\sigma(\bar{\boldsymbol{R}}_{-i^*}) = (n-1)m$, then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

The induction proceeds with the following hypothesis and step.

THE INDUCTION HYPOTHESIS: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $\sigma(\bar{\boldsymbol{R}}_{-i^*}) = t$ (where $n \leq t \leq (n-1)m$), then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

THE INDUCTION STEP: For each $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$, if $\sigma(\bar{\boldsymbol{R}}_{-i^*}) = t - 1$, then $f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*}) = x$.

Fix $\bar{\boldsymbol{R}}_{-i^*} \in \mathcal{L}^{n-1}$ such that $\sigma(\bar{\boldsymbol{R}}_{-i^*}) = t-1$ arbitrarily. Let $y = f(\bar{R}, \bar{\boldsymbol{R}}_{-i^*})$. Let $J_1 = \{j \in N \setminus \{i^*\} \mid \rho_{\bar{R}_j}(x) \leq m-2\}$, $J_2 = \{j \in N \setminus \{i^*\} \mid \rho_{\bar{R}_j}(x) = m-1\}$, and $J_3 = \{j \in N \setminus \{i^*\} \mid \rho_{\bar{R}_j}(x) = m\}$. Since $\sigma(\bar{\boldsymbol{R}}_{-i^*}) = t-1 < m(n-1)$, $J_1 \cup J_2 \neq \emptyset$.

We assume that $y \neq x$ and derive a contradiction.

STEP 1: We show that for each $j \in J_1 \cup J_2$, if $x = r^k(\bar{R}_j)$, then $y = r^{k+1}(\bar{R}_j)$.

Assume not. Then, there exist $j \in J_1 \cup J_2$ and $z \neq y$ such that $x = r^k(\bar{R}_j)$, and $z = r^{k+1}(\bar{R}_j)$. Let R_j be the preference given by exchanging the ranks of x and z in $\bar{R}_j$. Then, $f(\bar{R}, R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) \notin \{x, z\}$ because of Definition 3.1 (a) and (c). This contradicts the induction hypothesis because $\sigma(R_j, \bar{\boldsymbol{R}}_{-(i^*,j)}) = t$.

Therefore, we have Figure 11. Since the choice of $\bar{\boldsymbol{R}}_{-i^*}$ was arbitrary, we have shown that for each $j \in N \setminus \{i^*\}$ and each $\boldsymbol{R}_{-i^*} \in \mathcal{L}^{n-1}$ such that $f(\bar{R}, \boldsymbol{R}_{-i^*}) \neq x$ and $\sigma(\boldsymbol{R}_{-i^*}) = t-1$, if there exists $k \leq m-1$ such that $r^k(R_j) = x$, then $r^{k+1}(R_j) = f(\bar{R}, \boldsymbol{R}_{-i^*})$.

i^*	J_1			J_2			J_3		
x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\cdots$	x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	x	$\cdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$[y]$	$\cdots$	$[y]$	x	$\cdots$	x

Figure 11:

i^*	J_2			J_3		
x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\cdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\cdots$	$[y]$	x	$\cdots$	x

Figure 12:

STEP 2: We show that $J_1 \neq \emptyset$.

Assume $J_1 = \emptyset$. Since $J_1 \cup J_2 \neq \emptyset$, $J_2 \neq \emptyset$. We have Figure 12. For each $j \in J_3$, lower the rank of y to the second last position in agent j's preference. By Definition 3.1 (a) and (b), the social choice cannot be x in this process. Since $J_2 \neq \emptyset$, Step 1 shows that the social choice remains y. By *efficiency*, $r^2(\bar{R}) = y$. Letting $z = r^3(\bar{R})$, we have Figure 13. In Figure 13, exchange the ranks of y and z in the preference of agent i^*. By Definition 3.1 (b), the social choice must be y or z, and by *efficiency*, the social choice is z. We have Figure 14.

In Figure 14, exchange the ranks of x and y in $\bar{R}_j$ for some $j \in J_2$. The resulting social choice cannot be x by Definition 3.1 (a). Next, exchange the ranks of z and y in the preference of agent i^*. The resulting social choice cannot be x by Definition 3.1 (b) and

i^*	J_2			J_3		
x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$[y]$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
z	x	$\cdots$	x	$[y]$	$\cdots$	$[y]$
$\vdots$	$[y]$	$\cdots$	$[y]$	x	$\cdots$	x

Figure 13:

i^*	J_2			J_3		
x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$[z]$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
y	x	$\cdots$	x	y	$\cdots$	y
$\vdots$	y	$\cdots$	y	x	$\cdots$	x

Figure 14:

(c). This contradicts to the induction hypothesis because the value of σ is t after the these manipulations. Therefore, $J_1 \neq \emptyset$ is shown.

STEP 3: We show that $J_2 = \emptyset$ and there exists $j^* \in N \setminus \{i^*\}$ such that $J_1 = \{j^*\}$ arbitrarily.

By Step 2, $J_1 \neq \emptyset$. Fix an agent $j^* \in J_1$. Let $r^k(\bar{R}_{j^*}) = x$ and $w = r^{k+2}(\bar{R}_{j^*})$. We have Figure 15, in which the left column in J_1 presents agent j^*'s preference. In Figure 15, exchange the ranks of y and w in $\bar{R}_{j^*}$. By Definition 3.1 (b), the social choice is y or w, and by Step 1, the social choice must be w. Assume that $(J_1 \cup J_2) \setminus \{j^*\} \neq \emptyset$, and fix $j \in (J_1 \cup J_2) \setminus \{j^*\}$. Since the social choice is not y, this contradicts Step 1. Therefore, $J_1 = \{j^*\}$ and $J_2 = \emptyset$. We have Figure 16.

i^*	J_1			J_2			J_3		
x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\cdots$	x	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\cdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	x	$\cdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\cdots$	$\vdots$	$[y]$	$\cdots$	$[y]$	x	$\cdots$	x

Figure 15:

i^*	j^*	J_3		
x	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	x	$\cdots$	x

Figure 16:

STEP 4: We derive a contradiction.

In Figure 16, raise the rank of y until the rank of y exceed the rank of $w = r^{k+2}(\bar{R}_{j^*})$ in the preference of each $j \in J_3$. (If y's rank exceeds w's rank in the initial preference, then do nothing.) By Step 1, the social choice is y or x in this process, and by Definition 3.1 (a) and (b), the social choice must be y. Next, lower the rank of w to the second last position in the preference of each $j \in J_3$. By Definition 3.1 (a), the social choice cannot be x during this process. Step 1 implies that the social choice remains y. As a result, we have Figure 17.

i^*	j^*	J_3		
x	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	w	$\cdots$	w
$\vdots$	$\vdots$	x	$\cdots$	x

Figure 17:

i^*	j^*	J_3		
x	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	x	$\cdots$	x
$\vdots$	$\vdots$	w	$\cdots$	w

Figure 18:

i^*	j^*	J_3		
x	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$[y]$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	x	$\cdots$	x
$\vdots$	$\vdots$	w	$\cdots$	w

Figure 19:

i^*	j^*	J_3		
x	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	x	$\vdots$	$\cdots$	$\vdots$
$\vdots$	w	$\vdots$	$\cdots$	$\vdots$
$\vdots$	y	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	w	$\cdots$	w
$\vdots$	$\vdots$	x	$\cdots$	x

Figure 20:

In Figure 17, for each $j \in J_3$, exchange the ranks of w and x in the preference of j. By Definition 3.1 (a), the social choice cannot be x or w. We can show that it is y: Suppose that the social choice changes to some alternative distinct from y. Then, exchange the ranks of x and y in $\bar{R}_{j^*}$. The social choice cannot be x by Definition 3.1 (a) and (c), and cannot be w by *efficiency*. Exchange the ranks of w and x in the preference of each $j \in J_3$. By Definition 3.1 (a), the social choice cannot be x. This contradicts the induction hypothesis. Thus, the social choice must be y after the above changes. We have Figure 18.

In Figure 18, exchange the ranks of y and w in $\bar{R}_{j^*}$. By Definition 3.1 (b), the social choice is y or w, and by *efficiency*, the social choice must be y. We have Figure 19. In Figure 19, for each $j \in J_3$, exchange the ranks of x and w in the preference of j. The resulting social choice should not be x or w because of Definition 3.1 (b). We have Figure 20. Since the value of σ in the preference profile presented in Figure 20 is $t-1$, this contradicts Step 1.

Hence, we have $y = x$. $\square$

Finally, we prove Lemma 3.3.

Proof of Lemma 3.3. By Lemma 4.3, for each $\bar{R} \in \mathcal{L}$, there exists a dictator $i^* \in N$ at $\bar{R}$, i.e., there exists an agent $i^* \in N$ such that $f(\bar{R}, \boldsymbol{R}_{-i^*}) = r^1(\bar{R})$ for each $\boldsymbol{R}_{-i^*} \in \mathcal{L}^{n-1}$. We show that such an agent i^* is determined independent of the choice of $\bar{R}$.

Suppose that $i^* \in N$ is the dictator at $\bar{R} \in \mathcal{L}$, and $j^* \in N$ is the dictator at $R \in \mathcal{L}$. Assume $i^* \neq j^*$. Then for each $\boldsymbol{R}_{-(i^*,j^*)} \in \mathcal{L}^{n-2}$, $f(\bar{R}, R, \boldsymbol{R}_{-(i^*,j^*)}) = r^1(\bar{R})$ because i^* is the dictator, and also $f(\bar{R}, R, \boldsymbol{R}_{-(i^*,j^*)}) = r^1(R)$ because j^* is the dictator. Thus, $r^1(\bar{R}) = r^1(R)$. Take a preference $R' \in \mathcal{L}$ such that $r^1(R') \neq r^1(\bar{R})$, and suppose that agent $k^* \in N$ is the dictator at R'. Since $r^1(R') \neq r^1(\bar{R})$, it must be that $k^* = i^*$, and also because $r^1(R') \neq r^1(R)$, it must be that $k^* = j^*$. This contradicts the assumption $i^* \neq j^*$.

Therefore, f is dictatorship. $\square$

5 Concluding remarks

We have introduced a new axiom called *individual bounded response*, and proved that *individual bounded response* and *efficiency* imply dictatorship. Since *individual bounded response* follows from *strategy-proofness*, the Gibbard–Satterthwaite theorem is shown as a corollary of our impossibility result. This result also suggests that even if profitable misrepresentation is permitted, the impossibility is inevitable as long as the degree of the profit is restricted.

On the universal domain, *strategy-proofness* is not a useful condition of nonmanipulability in the sense that no plausible SCF satisfies it. As we mentioned in the Introduction, there are recent researches investigating the result of weakening *strategy-proofness* in some natural or interesting ways. Our result shows that as long as we want a deterministic SCF on the universal domain, unfortunately, it is hard to find a useful nonmanipulability condition except for some extreme ones.[7] On the one hand, this might imply that we have to be satisfied with SCFs satisfying necessary conditions for *strategy-proofeness* which are not usually considered as nonmanipulability conditions. Examples of such conditions are *unanimity*, *efficiency*, and *weak monotonicity*. On the other hand, this might imply the limit of the classical social choice framework, and invite us to consider other models in which the possibility of constructing nonmanipulable SCFs is not investigated very much. For example, let us assume that agents have rankings over alternatives *and* evaluations, either "acceptable" or "unacceptable". This is the preference-approval model by Brams and Sanver (2006). Among few papers considering nonmanipulability in the preference-approval model, Erdamar et al. (2016) find some plausible rules satisfying an axiom called *evaluationwise strategy-proofness*.

References

Brams, S. J. and Sanver, M. R. (2006). Critical strategies under approval voting: Who gets ruled in and ruled out. *Electoral Studies*, 25(2):287–305.

Carroll, G. (2012). When are local incentive constraints sufficient? *Econometrica*, 80(2):661–686.

[7]For example, Muto and Sato (2016a) present a possibility result by employing *top-restricted strategy-proofness*, an axiom which requires that each agent cannot change the social choice from the second preferred one to the most preferred one. This axiom is a severe restriction of *strategy-proofness* in the sense that it considers only the top two alternatives.

Cho, W. J. (2016). Incentive properties for ordinal mechanisms. *Games and Economic Behavior*, 95:168 – 177.

Erdamar, B., Sanver, M. R., and Sato, S. (2016). Evaluationwise strategy-proofness. Mimeo.

Gibbard, A. (1973). Manipulation of voting schemes: A general result. *Econometrica*, 41(4):587–601.

Muller, E. and Satterthwaite, M. A. (1977). The equivalence of strong positive association and strategy-proofness. *Journal of Economic Theory*, 14(2):412–418.

Muto, N. and Sato, S. (2016a). A decomposition of strategy-proofness. *Social Choice and Welfare*. forthcoming.

Muto, N. and Sato, S. (2016b). Supplementary notes to "Individual bounded response of social choice functions". unpublished manuscript, available at `https://sites.google.com/site/nozomumuto/ibrsupplementary.pdf`.

Reffgen, A. (2011). Generalizing the Gibbard–Satterthwaite theorem: partial preferences, the degree of manipulation, and multi-valuedness. *Social Choice and Welfare*, 37(1):39–59.

Reny, P. J. (2001). Arrow's theorem and the Gibbard–Satterthwaite theorem: a unified approach. *Economics Letters*, 70(1):99–105.

Sato, S. (2010). Circular domains. *Review of Economic Design*, 14(3):331–342.

Sato, S. (2013). A sufficient condition for the equivalence of strategy-proofness and non-manipulability by preferences adjacent to the sincere one. *Journal of Economic Theory*, 148(1):259–278.

Satterthwaite, M. A. (1975). Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187 – 217.