Is a Big Entrant a Threat to Incumbents? The Role of Demand Substitutability in Competition among the Big and the Small*

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Abstract

We establish a model of market competition among big and small firms and investigate how demand substitutability affects the impacts of big firms' entry on incumbents. We identify two opposing effects of entry on the incumbent big firms' demand: the direct substitution effect among the big firms (negative), and the indirect feedback effect through the change in the aggregated behavior of small firms (positive). If the substitutability between big and small firms is sufficiently high, the indirect effect dominates the direct effect, and the big incumbents' equilibrium prices and profits increase. We show that the welfare effects are ambiguous, which calls for careful assessment for regulating big firms' entry.

Keywords: big firms, small firms, substitutability, entry, market impacts

JEL classification: D21, D43, L11, L13

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1 Introduction

The entry of a large firm has substantial effects on market competition and market structure. Evidence suggests that the impacts on large and small firms differ in markets. Igami (2011) studied the supermarket industry in Japan after deregulation (relaxation of the Large-Scale Retail Law) and showed that large supermarkets' entry has negative impacts on large (and medium) supermarkets, while it has neutral or positive impacts on small supermarkets. On the other hand, the 2014 IKEA entry into South Korea increased the profits of large furniture makers while substantially lowering the profits of small furniture makers.[1] Why do the impacts differ? What factors determine the differences? Does the entry of a large firm enhance overall efficiency?

To answer those questions, we consider the following model of competition with large and small firms and study the impacts of large firms' entry. A large firm is modeled as a multi-product firm, the number of whose products is a choice variable.[2] A small firm is modeled as a single-product firm with free entry and exit. Following the model of monopolistic competition, we treat each variety symmetrically. While the number of large firms is finite (exogenous), the variety of each large firm is determined through oligopolistic competition, and the measure of small firms is determined by the free entry condition. We consider a static game where all decisions, including entry, variety choices, and production, are simultaneous. The following two key features of our model are noted. First, a large firm can exert market power through variety choice and coordinated pricing of its own varieties. Second, by adopting a quadratic quasi-linear utility function of the representative consumer, demand in our model displays rich configurations of product substitutability within and across large and small firms.[3] We derive the condition

[1] After the entry of IKEA into South Korea in 2014, the large national furniture makers, such as HANSSEM and LIVART, enjoyed an increase in their revenue. After the establishment of the IKEA store in Gwangmyeong, the revenue of LIVART's Gwangmyeong branch increased 27%, while HANSSEM's Gwangmyeong store saw a 10% rise in sales over the same period of the previous year. Small furniture makers, however, suffered from more than a 70% decrease in their revenue on average, and many were at the edge of shutdown. Source: *Korea Bizwire*. March 27, 2015, "Korea's Large Furniture Makers Boost Revenues Thanks to IKEA," http://koreabizwire.com/koreas-large-furniture-makers-boost-revenues-thanks-to-ikea/32438

[2] Bernard et al. (2010) show that multi-product firms are almost omnipresent in the U.S. manufacturing industry. According to the data between 1979 and 1992, multi-product firms account for 41% of the total number of firms but supply 91% of total output. In addition, 89% of multi-product firms adjust their product range every five years.

[3] A quasi-linear quadratic form of utility function is adopted in Singh and Vives (1984), Ottaviano and Thisse (1999), Ottaviano et al. (2002), and Parenti (2015). Unlike our paper, none of these papers considered different substitutabilities among firms. Although Ottaviano and Thisse (1999) considered monopolistic competition among single-product firms and oligopolistic competition among multi-product firms, this paper unifies monopolistic competition and oligopolistic

under which a unique mixed market equilibrium exists (i.e., the coexistence of active large and small firms) and investigate the impacts of a large firm's entry on other firms' behavior and on welfare.

Our main result shows how product substitutabilities affect the impact of a large firm's entry on the incumbent large firms. Indeed, we find a necessary and sufficient condition under which the product range, the price of each product, and the profit of each incumbent all increase (Proposition 2). The key is to separate two effects of a large firm's entry on the demand for incumbent large firms' products. The first effect is the direct substitution effect: the new large firm's products are substituted for the incumbent large firms' products. This negatively affects the demand for large firms' products. The second effect is the indirect feedback effect due to the change int the number of small firms. A large firm's entry squeezes out a portion of small firms if the products are substitutes across large and small firms, whereas such entry invites more small firms if the products are complements. It is important to see that the resulting indirect feedback effect is *non-negative* on the demand for large firms' products, whether large firms' and small firms' products are substitutes or complements. If the degree of substitutability or complementarity *between* large firms' and small firms' products is relatively larger than the substitutabilities *within* large firms' products and *within* small firms' products, the indirect feedback effect outweighs the direct substitution effect, thereby resulting in a rise in demand for the large firms' products. This characterizes the condition for an increase in the product range, the price, and the profit of an incumbent large firm.

Our finding may explain the different impacts of a large firm's entry in the above evidence. In his analysis of supermarkets in Japan, Igami (2011) observed that supermarkets of different sizes "offer differentiated services from the perspective of consumers." This suggests that the substitutability among large firms' products is larger than the substitutability between large firms' and small firms' products. Therefore, the feedback effect may not be strong enough to offset the substitution effect for the large incumbents. In the South Korean furniture industry, large furniture makers have a superior ability to design furniture. This implies that the products of large furniture makers are more differentiated than those of small makers and suggests that the substitutability within large firms is weaker than that between large and small firms. Therefore, the feedback effect could outweigh the substitution effect, resulting in a rise in demand for the large firms' products.

The welfare effects of a large firm's entry are ambiguous. To fix ideas, consider the case in which large and small firms' products are substitutes. Observe that some of small firms exit after the entry of a large firm while large firms' (total) product range expands, and thus a portion of small firms' product range is replaced

competition to investigate a market with large and small firms.

by large firms'. Also, the total product range after entry may expand or shrink. We first look at the associated changes in consumer welfare. For each variety in the replacement range, the consumer welfare increases or decreases depending on the relative allocative inefficiency between large and small firms. Also, if net product range expands (shrinks), consumer welfare improves (worsens). For the associated change in producer surplus, large firms' aggregate profits always increase due to their product range expansion.[4] This argument shows that the welfare consequence depends on the sign of each effect and/or the relative strength of them. Our model thus implies that policy makers should make careful assessment for the case of a large firm's entry.

On the markets with large and small firms, there are several strands of study which differ in capturing the large firms' market power. The first strand is the so-called dominant firm model (e.g., Markham, 1951, Chen, 2003, and Gowrisankaran and Holmes, 2004). The dominant firm is large since it is the leader and the price-maker, while the price-taking followers are small. Another strand is to use the Stackelberg model (Etro, 2004, 2006, and Ino and Matsumura, 2012). In this model, a first mover is large due to the commitment power in the market. The third strand is to model the mixture of oligopoly and monopolistic competition (Shimomura and Thisse, 2012, and Parenti, 2015). In this approach, the oligopolistic firms are large due to their ability to produce a large amount (or a large number of varieties) while the monopolistically competitive firms are small, since they can produce only a negligible amount in the market (or just a variety of differentiated products). Our model falls into this strand of study but differs from the other works by considering the different substitutabilities of products within and across large and small firms.[5]

This paper's focus on the impact on a large firm's entry is shared by Shimomura and Thisse (2012) and Ino and Matsumura (2012). Ino and Matsumura (2012) studied a homogeneous-good Stackelberg game with multiple leaders and free-entry followers. They found that the impact of adding another Stackelberg leader is beneficial to social welfare since it drives out some of the excessively entering followers, while increasing the total quantity supplied. Shimomura and Thisse (2012) studied a general equilibrium model of the mixture of oligopoly and monopolistic competition. They found that the entry of a large firm increases the incumbent large firms' profit and raises consumer and social welfare. In their model, the entry has three effects: the substitution and feedback effects, as seen in our model, and the income effect from the increased aggregated profits due to

[4] Consumer welfare and producer surplus are also affected by the price change due to the entry. However, these effects are neutral on social welfare.

[5] Another strand of research differentiates between large and small firms from the perspective of firm heterogeneity in cost, represented by Lahiri and Ono (1988) and Matsumura and Matsushima (2010).

the entry. Note that their model focuses on the same substitutabilities within and across large and small firms. Hence, the first two effects offset each other, showing that the income effect is crucial for their results. Our model complements theirs by taking the following approach.[6] First, we adopt a partial equilibrium analysis, and thus no income effect arises. Second and more importantly, we consider richer substitutability among the products. Incorporating different substitutability and complementarity, our study thus sheds new light on competition among the big and the small. One advantage of our model is the generation of more flexible patterns of production behavior and welfare change than the previous studies. For example, social welfare may improve or worsen in our model while it always improves in Shimomura and Thisse (2012) and Ino and Matsumura (2012).

The rest of the paper is organized as follows. We construct the model in Section 2. In Section 3, we analyze the equilibrium of the model and explore the impact of a large firm's entry. Section 4 discusses the robustness of the established results. Section 5 concludes.

2 The Model

Consider a closed economy consisting of two sectors. Firms in sector 1 are perfectly competitive and produce a homogenous good under constant returns to scale. Sector 2 provides differentiated goods that are produced by two types of firms. The first type of firms is large in size, and the number of these firms is exogenous. The second type of firms is infinitessimal and freely enters or exits from the market.

The large and small firms differ in three respects. First, each large firm imposes a non-negligible impact on the market and competes in an oligopolistic manner, whereas each small firm is negligible in the market and behaves as a monopolistic competitor. Here we follow the approach by Shimomura and Thisse (2012). Second, each large firm produces a range of varieties and strategically chooses both the product range and the quantity of each variety, while each small firm only produces one variety of product. Third, the varieties are equally substitutable within the group of large firms and that of small firms, but the level of substitution across these two types of firms can be different.

[6] There are two more differences between our model and Shimomura and Thisse's (2012). For the utility function of the representative consumer, Shimomura and Thisse (2012) adopted the Cobb–Douglas CES nested utility function while we adopt a quadratic quasi-linear utility. For production, in Shimomura and Thisse (2012), a large firm is a single-product firm whose quantity is non-negligible, while in our model, a large firm is a multi-product firm: it chooses a variety range as well as quantities in each chosen variety. Since the equilibrium quantity level is the same, the variety range in our model effectively plays the same role as the large firm's quantity in Shimomura and Thisse (2012). We discuss the fact that these differences are not crucial for the results in Section 4.

2.1 Preferences and Demand

The utility of the representative consumer U is described by a quasi-linear utility with a quadratic subutility:

$$U = \alpha \left[\int_0^N q_S(i)di + \sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega \right] - \frac{\beta}{2}\int_0^N [q_S(i)]^2 di - \frac{\beta}{2}\sum_{m=1}^{M}\int_{\omega\in\Omega_m} [q_L^m(\omega)]^2 d\omega$$

$$-\frac{\gamma_1}{2}\left[\int_0^N q_S(i)di\right]^2 - \frac{\gamma_2}{2}\left[\sum_{m=1}^{M}\int_{\omega\in\Omega_m} q_L^m(\omega)d\omega\right]^2 - \gamma_3\left[\int_0^N q_S(i)di\right]\left[\sum_{m=1}^{M}\int_{\omega\in\Omega_m} q_L^m(\omega)d\omega\right] + q_0,$$

where $q_S(i)$ is the quantity of small firm i with $i \in [0, N]$. The output of each small firm is of zero measure, and the total mass of small firms is N, describing the competitive fringe. The set of varieties produced by the large firm m ($m = 1, ..., M$) is represented by Ω_m, and the quantity for variety $\omega \in \Omega_m$ is $q_L^m(\omega)$.[7] The total number of the incumbent large firms is $M \geq 2$. Here we treat $|\Omega_m|$ continuously. The output of sector 1 is q_0, which is treated as the numeraire. Consumer preferences are characterized by five parameters, which are α, β, and γ_i ($i = 1, 2, 3$). The intensity of preferences for the differentiated good is measured by $\alpha > 0$, which determines the size of the differentiated good market, whereas $\beta > 0$ implies the consumer's preference for a diversified consumption of products. The substitutability between varieties is characterized by γ_i ($i = 1, 2, 3$). Specifically, the substitutability among the varieties produced by small firms and that among the varieties of large firms are expressed by γ_1 and γ_2, respectively, and the cross-substitutability between the varieties of large firms and those of small firms is expressed by γ_3. The products are substitutes if $\gamma_i > 0$ and complements if $\gamma_i < 0$. The products are closer substitutes (complements) when $|\gamma_i| > 0$ ($i = 1, 2, 3$) is higher. The products of the small and large firms have the same level of substitution when $\gamma_1 = \gamma_2 = \gamma_3$ and have different substitutabilities otherwise. Finally, to ensure the concavity of the quadratic subutility, we have

(i) $\beta/N + \gamma_1 > 0$,
(ii) $\beta/(\sum_m |\Omega_m|) + \gamma_2 > 0$, and
(iii) $(\beta/N + \gamma_1)[\beta/(\sum_m |\Omega_m|) + \gamma_2] > \gamma_3^2$.[8,9]

[7] All the varieties are differentiated so that they do not overlap with each other.

[8] See Appendix **A** for details. Here γ_i ($i = 1, 2, 3$) can be positive or negative as long as the conditions for the concavity of the utility function hold. We will discuss the different signs of γ_i in the third section.

[9] We can generalize the model in that the substitutability of the varieties within a multi-product large firm may differ from the substitutability across firms. In this case, our results still hold qualitatively.

The representative consumer's budget constraint is

$$\int_0^N p_S(i)q_S(i)di + \sum_{m=1}^{M} \int_{\omega\in\Omega_m} p_L^m(\omega)q_L^m(\omega)d\omega + q_0 = I,$$

where $p_S(i)$ and $p_L^m(\omega)$ are the prices of the small firm i's and large firm m's variety ω, respectively. The representative consumer's income is I, which is exogenously given. The inverse demand functions facing small firms and large firms are determined by the maximization of the consumer's utility subject to the budget constraint:

$$p_S(i) = \alpha - \beta q_S(i) - \gamma_1 Q_S - \gamma_3 Q_L, \tag{1}$$

$$p_L^m(\omega) = \alpha - \beta q_L^m(\omega) - \gamma_3 Q_S - \gamma_2 Q_L, \tag{2}$$

where $Q_S \equiv \int_0^N q_S(i)di$ and $Q_L \equiv \sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega$ are the total output of the small firms and that of the large firms, respectively.

2.2 Firms

Both large and small firms incur variable costs and fixed costs. All firms incur a common and constant marginal cost, which is normalized to zero, whereas the fixed cost may differ across the two types of firms.

Small Firms The profit of the small firms is expressed by

$$\Pi_S(i) = p_S(i)q_S(i) - (f^e + f^p),$$

where $\Pi_S(i)$ is the profit of small firm i, and f^e and f^p are the entry cost and fixed production cost of the small firm, respectively. To simplify our denotation and explanation, we denote $f \equiv f^e + f^p$ as the total fixed cost of a small firm.

Plugging $p_S(i)$ of equation (1) into the above profit function, $\Pi_S(i)$ can be rewritten as

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f, \tag{3}$$

Each small firm maximizes its profit with respect to its quantity $q_S(i)$.

The free entry and exit of small firms pins down the equilibrium profit of the small firm to zero:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - f = 0. \tag{4}$$

Large Firms The profit of the large firm is

$$\Pi_L^m(\Omega_m, q_L^m(\cdot)) = \int_{\omega\in\Omega_m} (p_L^m(\omega)q_L^m(\omega) - F)d\omega,$$

where $\Pi_L^m(\Omega_m, q_L^m(\cdot))$ is the profit of large firm m, and F is the fixed production cost for the large firm to produce one variety.[10]

Substituting $p_L^m(\omega)$ of equation (2) into the above profit function, $\Pi_L^m(\Omega_m, q_L^m(\cdot))$ can be rewritten as

$$\begin{aligned}\Pi_L^m(\Omega_m, q_L^m(\cdot)) = &\left\{\alpha - \gamma_3 Q_S - \gamma_2 \sum_{k\neq m} \int_{\omega\in\Omega_k} q_L^k(\omega)d\omega\right\} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega \\ &- \beta \int_{\omega\in\Omega_m} [q_L^m(\omega)]^2 d\omega - \gamma_2 \left[\int_{\omega\in\Omega_m} q_L^m(\omega)d\omega\right]^2 - F\left|\Omega_m\right|.\end{aligned} \tag{5}$$

The large firm maximizes its profit with respect to both its product range Ω_m and the quantity of each variety $q_L^m(\omega)$. Note that the varieties do not overlap with each other.

2.3 Definition of Equilibrium

Since consumers are passive, an equilibrium state arises if no firm wishes to unilaterally deviate. Note that we consider a market competition by the large firms and small firms, in which all firms behave simultaneously, including entry decision by small firms. Our solution concept is Nash equilibrium.

An equilibrium is characterized by the mass of small firms, N^*, the output of each small firm, $q_S^*(i)$, $\forall i \in N^*$, the product range of each large firm Ω_m^*, $m = 1, ..., M$, and the output of each variety for the large firm $q_L^{m*}(\omega)$ $\forall \omega \in \Omega_m^*$, $\forall m = 1, ..., M$, such that each firm maximizes the profits given other firms' behavior, and no more small firms can earn positive profits due to free entry. An equilibrium is called a **mixed market equilibrium** if $Q_S^* > 0$ and $Q_L^* > 0$.

2.4 Welfare

The social welfare comprises consumer welfare and producer surplus. Consumer welfare is measured by

$$CW = U - I,$$

[10] In the analysis, we consider an exogeneous entry of a large firm, and thus the entry cost is only relevant for the entrant's profit. We normalize the entry cost for a large entrant to zero.

Hence, the change in consumer welfare is the same as that in consumer's utility.

Since small firms earn zero profit, producer surplus is given by the sum of all large firms' profits:

$$PS = \sum_{m=1}^{M} \Pi_L^m,$$

Then social welfare is the sum of consumer welfare and producer surplus:

$$SW = U - I + \sum_{m=1}^{M} \Pi_L^m. \tag{6}$$

3 Equilibrium Analysis

In this section, we derive the equilibrium results and conduct the comparative static analysis to investigate the impacts of the entry of a large firm on the other firms' behavior and welfare.

3.1 Derivation of Mixed Market Equilibrium

Small Firms' Profit Maximization and Entry A small firm only accounts for the impact of the market's total production because its own impact on the market is negligible. Thus, it does not internalize its externality in its production. The small firm maximizes its profit given by equation (3) with respect to its output $q_S(i)$, yielding the optimal quantity of the small firm for an expected total output of large firms Q_L and mass of small firms N:

$$q_S^*(Q_L, N) = \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N}. \tag{7}$$

Using equation (1), the price of the small firm can be expressed by

$$p_S^*(Q_L, N) = \beta \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N}. \tag{8}$$

Accordingly, the equilibrium price of the small firm decreases with the mass of small firms and the total output of large firms.

Entry and exit are free for small firms. Using equation (4) after plugging in (7) and (8), the equilibrium mass of small firms with a given total output of large firms Q_L is

$$N^*(Q_L) = \frac{1}{\gamma_1} \left[\sqrt{\frac{\beta}{f}} (\alpha - \gamma_3 Q_L) - 2\beta \right]. \tag{9}$$

which decreases with the total output of large firms.

Substituting (9) into (7), the optimal quantity of each small firm is

$$q_S^* = \sqrt{\frac{f}{\beta}}.$$

Owing to free entry and exit, the quantity produced by the small firm is independent of the behavior of large firms. In other words, the aggregate behavior of small firms responds to the change in the market condition only by adjusting the competitive fringe. (See also Lemma 1.)

Plugging q_S^* into (8) yields the equilibrium price of small firms:

$$p_S^* = \sqrt{\beta f}.$$

Large Firms' Profit Maximization and Variety Choice Unlike small firms, large firms impose non-negligible impacts on the market. Large firm m maximizes its profit given by equation (5) with respect to its output $q_L^m(\omega)$, yielding the optimal quantity of each variety, given the total output of small firms Q_S, the total output of other large firms $Q_L^{-m} = \sum_{j \neq m} \int_{\omega \in \Omega_j} q_L^j(\omega) d\omega$, and its own product range $|\Omega_m|$:

$$q_L^{m*}(Q_S, Q_L^{-m}, |\Omega_m|) = \frac{\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m}}{2(\beta + \gamma_2 |\Omega_m|)}. \tag{10}$$

Everything else being equal, an increase in firm m's product range (larger $|\Omega_m|$) results in a reduction in the quantity of each variety, implying cannibalization.

The product range of large firm m, $|\Omega_m^*|$, that maximizes (5) after substituting (10) satisfies

$$2(\beta + \gamma_2 |\Omega_m^*|) = \sqrt{\frac{\beta}{F}}(\alpha - \gamma_3 Q_S - \gamma_2 Q_L^{-m}). \tag{11}$$

We obtain the optimal output per variety for the large firm from equations (10) and (11):

$$q_L^{m*} = \sqrt{\frac{F}{\beta}}.$$

which is determined only by the fixed cost of large firms and the demand parameters but independent of its product range or other firms' behavior. (See also the discussion after Lemma 1.)

Substituting q_L^{m*} into equation (11), we obtain the equilibrium product range $|\Omega_m^*|$ given the expected aggregate output of small firms Q_S:

$$|\Omega_m^*|(Q_S) = \frac{\sqrt{\beta/F}(\alpha - \gamma_3 Q_S) - 2\beta}{\gamma_2(M+1)}. \tag{12}$$

Mixed Market Equilibrium In equilibrium, the total output of large firms can be expressed by $Q_L^* = M\left|\Omega_m^*\right| q_L^{m*}$, and the aggregate output of small firms is $Q_S^* = N^* q_S^*$. Plugging these two expressions into (9) and (12), the mass of small firms and the product range of each large firm are

$$
\begin{aligned}
N^* &= \sqrt{\frac{\beta}{f}} \frac{\alpha[\gamma_2(M+1) - \gamma_3 M] - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}, \\
|\Omega^*| &= \sqrt{\frac{\beta}{F}} \frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.
\end{aligned}
$$

Plugging N^*, $|\Omega^*|$, q_S^* and q_L^{m*} into equation (2), the price of the large firm in equilibrium is

$$
p_L^* = \sqrt{\beta F} + \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}.
$$

Substituting the equilibrium range of varieties $|\Omega^*|$, the output of each variety q_L^{m*} and the equilibrium price of large firms p_L^* into equation (5), we obtain the equilibrium profit of the large firm:

$$
\Pi_L^* = \gamma_2\left[\frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}\right]^2.
$$

The total output is

$$
\begin{aligned}
Q^* = {} & \frac{1}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}\{\alpha M(\gamma_1 + \gamma_2 - 2\gamma_3) + \gamma_2(\alpha - 2\sqrt{\beta f}) \\
& - 2M\sqrt{\beta}[(\gamma_2 - \gamma_3)\sqrt{f} + (\gamma_1 - \gamma_3)\sqrt{F}]\}.
\end{aligned}
$$

We focus on the market with the coexistence of large and small firms. To ensure that the market is mixed and stable in equilibrium, the following proposition establishes the conditions.

Proposition 1 *There exists a unique mixed market equilibrium if the following three conditions hold:*
(i) $\alpha(\gamma_1 - \gamma_3) > 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$*;*
(ii) $\alpha[\gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{f} - \gamma_3 M\sqrt{F}]$*;*
(iii) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*.*

Proof. See Appendix **B.1**. ■

Conditions (i) and (ii) ensure the existence of large firms and small firms, respectively. Condition (iii) is the sufficient condition to guarantee the stability of equilibrium. Conditions (i) and (ii) require that α should be sufficiently large, and conditions (ii) and (iii) imply that the range of parameters narrows with an increase in the number of large firms M.

These three conditions also impose constraints on the substitutabilities. Specifically, the three conditions imply that (a) $\gamma_i > 0$ and $i = 1, 2, 3$ or (b) $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 < 0$. We can express them parametrically by γ_1/γ_3 and γ_2/γ_3, which represent the relative substitutability for the small firm and that for the large firm, respectively. Figure 1 shows the area where a unique mixed market equilibrium exists. The horizontal axis and vertical axis are γ_1/γ_3 and γ_2/γ_3, respectively. Also note that these three conditions imply $\alpha > 2\sqrt{\beta f}$ and $\alpha > 2\sqrt{\beta F}$. In addition, if $\gamma_1 = \gamma_2 = \gamma_3$, condition (i) implies $f > F$. That is, if the varieties are equally substitutable among all firms, the existence of large firms requires that the total fixed cost of a small firm be larger than the large firm's fixed production cost of one variety. When the large and small firms share the same fixed production cost, the small firm's entry cost should be positive so that the large firm enjoys economies of scope (Parenti, 2015). Even when the small firm's entry cost is close to zero, the large firm may also exist if it incurs a lower fixed cost to produce a variety. Note that conditions (i) and (ii) are redundant if $\gamma_3 < 0$.

[Figure 1 around here]

We can also illustrate the conditions by the aggregate reaction functions of the large and small firms. The aggregate reaction of large firms to the competitive fringe is

$$Q_L(Q_S) = Mq_L^* \left|\Omega_m\right| (Q_S^*) = \frac{M}{\gamma_2(M+1)}(\alpha - 2\sqrt{\beta F} - \gamma_3 Q_S), \tag{13}$$

The aggregate reaction of the competitive fringe to large firms is

$$Q_S(Q_L) = \frac{1}{\gamma_1}(\alpha - 2\sqrt{\beta f} - \gamma_3 Q_L). \tag{14}$$

The coexistence of large and small firms in equilibrium requires that the two aggregate reaction functions intersect. Figure 2a and Figure 2b depict these two aggregate reaction functions when $\gamma_3 > 0$ and $\gamma_3 < 0$, respectively. We first explain the case when $\gamma_3 > 0$. As in the standard Cournot model, an unstable equilibrium is fragile and thus implausible. The stability of the intersection requires that the slope of $Q_L(Q_S)$ be flatter than the slope of $Q_S(Q_L)$, i.e. $\gamma_3 M/\gamma_2(M+1) < \gamma_1/\gamma_3$. This condition is equivalent to condition (iii). If condition (iii) does not hold, the mixed market equilibrium is not stable, resulting in the equilibrium with only small firms or the equilibrium with only large firms. Two more conditions are necessary to ensure that the two aggregate reaction functions intersect. On the horizontal axis, the intercept of $Q_S(Q_L)$ should be smaller than the intercept of $Q_L(Q_S)$, i.e. $(\alpha - 2\sqrt{\beta f})/\gamma_1 < (\alpha - 2\sqrt{\beta F})/\gamma_3$, which is equivalent to condition (i). On the vertical axis, the intercept of $Q_S(Q_L)$ should be larger than the intercept of $Q_L(Q_S)$, i.e. $(\alpha - 2\sqrt{\beta f})/\gamma_3 > (\alpha - 2\sqrt{\beta F})M/[\gamma_2(M+1)]$, which is equivalent to condition (ii).

When $\gamma_3 < 0$, the aggregate behavior of large firms and that of small firms act as strategic complements. (See Figure 2b.) Similarly, the three conditions ensure the existence of a unique market equilibrium in this case.

[Figure 2 around here]

Concavity Conditions Finally, the three conditions for the existence of a unique mixed market equilibrium should be consistent with the conditions for the concavity of the utility function. Recall that the concavity of the utility function requires three conditions: i) $\beta/N + \gamma_1 > 0$, ii) $\beta/(M\,|\Omega|) + \gamma_2 > 0$, and iii) $(\beta/N + \gamma_1)[\beta/(M\,|\Omega|) + \gamma_2] > \gamma_3^2$. It is readily verified that Proposition 1 ensures the first two concavity conditions because $\gamma_1 > 0$ and $\gamma_2 > 0$. The third concavity condition is also satisfied when $\gamma_1\gamma_2 \geq \gamma_3^2$.

If $\gamma_1\gamma_2 < \gamma_3^2$, the conditions in Proposition 1 may not guarantee concavity. More conditions should be imposed to satisfy the following inequality:

$$(\frac{\beta}{N} + \gamma_1)(\frac{\beta}{M\,|\Omega|} + \gamma_2) > \gamma_3^2$$

Substituting N^* and $|\Omega^*|$ into the above condition, we can yield the inequality expressed by γ_1/γ_3, γ_2/γ_3, M, f and F. Due to the complexity of the expression, we do not include the detailed calculation here, though it is available upon request. Figure 3 depicts the conditions for concavity and a unique mixed market equilibrium with variations of fixed costs when $M = 2$. Here we normalize α to 1. The concavity condition is expressed by the purple curves; the three conditions in Proposition 1 are indicated by dashed curves; and the orange curve expresses $\gamma_1\gamma_2 = \gamma_3^2$. In the purple shaded area, the concavity condition cannot be satisfied even if the conditions in Proposition 1 hold. In the blue shaded area, all the conditions hold.

[Figure 3 around here]

It is observed that the concavity condition may not hold when f and F are very small even if the conditions for the unique mixed market equilibrium are satisfied. If f and F are small, then N^* and $|\Omega^*|$ are large, and it is more difficult to satisfy the above concavity condition when $\gamma_1\gamma_2 < \gamma_3^2$. We can also observe from Figure 3 that the range of inconsistency shrinks with the increase in the fixed costs of large and small firms. In Figure 3d, the conditions in Proposition 1 are sufficient to ensure the concavity of the utility function when $\beta F = \beta f = 0.01$. The consistency also holds as βF and βf increase further.

Similarly, we can identify the area where the concavity condition and the conditions for a unique mixed market equilibrium are consistent when $\gamma_3 < 0$ (Figure 4). As shown earlier, the third condition in Proposition 1 is sufficient to guarantee a unique mixed market equilibrium in this case. Here we depict it by the blue dashed curve. The concavity condition is expressed by the purple curve, and the red dotted curve plots $\gamma_1\gamma_2 = \gamma_3^2$. Similarly, the consistent area (green shaded area) expands with the increase in F and f.

[Figure 4 around here]

In the rest of our analysis, we focus on the market where both large and small firms coexist. We first examine the case when $\gamma_3 > 0$ and then discuss the case when $\gamma_3 < 0$.

3.2 The Impacts of a Large Firm's Entry

Now we investigate the impacts of a large firm's entry. We have some preliminary results that will be useful for later analysis. Formally, based on q_S^* and q_L^*, we have

Lemma 1 *The entry of a large firm has no impact on (i) the output and price level of the small firm or (ii) the output of each variety of the large firm.*

The first outcome is in line with the traditional monopolistic competition model. As shown in Figure 5, the free entry and exit of small firms shifts the demand curve such that there is only one equilibrium quantity, at which the average cost (AC) is tangent to the average revenue (AR) and marginal revenue (MR) intersects with marginal cost.

[Figure 5 around here]

The second result can be briefly explained as follows. The profit maximization of large firm m with respect to the output of each variety q_L^m yields $p_L^m - \beta q_L^m - \gamma_2 |\Omega_m| q_L^m = 0$, where the last term on the LHS is the internalization by the large firm. Applying the envelope theorem, the profit maximization of large firm m with respect to the product range $|\Omega_m|$ yields $p_L^m q_L^m - \gamma_2 |\Omega_m| (q_L^m)^2 = F$, where the second term on the LHS is the cannibalization effect. With linear demand and the same technology across varieties within the large firm, the cannibalization and internalization effects completely offset each other, and consequently the optimal output of each variety q_L^m is independent of the product range $|\Omega_m|$. This implies that the large firm reacts to changes in the market condition by varying its product range only.

Substitution between the Big and the Small ($\gamma_i > 0$, $i = 1, 2, 3$) Based on the results in Lemma 1, we first investigate the impact of a large firm's entry on firms' behavior when products are substitutes across the big and the small. Proposition 2 establishes the results.

Proposition 2 *The entry of a large firm will exert the following impacts on firms' behavior:*

(i) the competitive fringe shrinks;

(ii) the product range, price, and profit of each large firm rise if $\gamma_1\gamma_2 < \gamma_3^2$, fall if $\gamma_1\gamma_2 > \gamma_3^2$, and remain the same if $\gamma_1\gamma_2 = \gamma_3^2$; and

(iii) the total output increases if $\gamma_1 > \gamma_3$, decreases if $\gamma_1 < \gamma_3$, and remains the same if $\gamma_1 = \gamma_3$.

Proof. See Appendix **B.2**. ■

As shown by Figure 2a, an increase in the number of large firms M generates a clockwise rotation of $Q_L(Q_S)$ around its intercept on the horizontal axis, resulting in a rise in the total output of large firms Q_L^* and a fall in the aggregate output of small firms Q_S^*. This explains the shrinkage of the competitive fringe.

The entry of a large firm may raise or reduce the prices and profits of the incumbent large firms when the substitutability across the products of large firms and those of small firms is different from the substitutabilities within the groups of large and small firms. To illustrate the mechanism, we establish the following two expressions:

$$p_S^* = \alpha - \beta q_S^* - \gamma_1 Q_S^* - \gamma_3 Q_L^*, \tag{15}$$

$$p_L^* = \alpha - \beta q_L^* - \gamma_2 Q_L^* - \gamma_3 Q_S^*. \tag{16}$$

The equilibrium conditions describing the demands for large and small firms, the profit maximization of large and small firms, and the free entry of small firms boil down to expressions (15) and (16). Here $Q_S^* = N^* q_S^*$ is the total output of small firms, and $Q_L^* = M\,|\Omega^*|\,q_L^*$ is the total output of large firms.

As indicated by Figure 2a, the entry of a large firm raises the equilibrium total output of large firms Q_L^*. Denote this increase in Q_L^* by ΔQ_L^*. Two opposing effects are generated by the entry of a large firm. First, according to equation (16), ΔQ_L^* generates a direct negative substitution effect on p_L^* by $-\gamma_2 \Delta Q_L^*$. Meanwhile, ΔQ_L^* also leads to the shrinkage of the competitive fringe, which has a positive effect on the large firms. As shown by Lemma 1, p_S^* and q_S^* are not affected by a large firm's entry. According to equation (15), an increase in the total output of large firms ΔQ_L^* squeezes out the aggregate output of small firms by $\Delta Q_S^* = -(\gamma_3/\gamma_1)\Delta Q_L^*$. Then the substitution effect of the small firms on the large firms is weakened due to the shrinkage of the competitive fringe, according to equation (16). Precisely, this indirect squeezing effect is measured by $(-\gamma_3)(-\gamma_3/\gamma_1)\Delta Q_L^* = (\gamma_3^2/\gamma_1)\Delta Q_L^*$.

Therefore, whether the entry of a large firm raises or reduces the price of large firms depends on the comparison between the direct substitution effect and the indirect squeezing effect. If $\gamma_1\gamma_2 > \gamma_3^2$, which implies $-\gamma_2\Delta Q_L^* + (\gamma_3^2/\gamma_1)\Delta Q_L^* < 0$, then the negative substitution effect dominates the positive squeezing effect, and large firms have to reduce their price. Because $\Delta\left|\Omega^*\right| = (\sqrt{\beta/F}/\gamma_2)\Delta p_L^*$, the equilibrium product range of the large firm also shrinks, and consequently, the equilibrium profit of each large firm decreases. If $\gamma_1\gamma_2 < \gamma_3^2$, on the other hand, then the positive squeezing effect dominates the negative substitution effect, and the price, product range, and profit of each large firm increase. Finally, if $\gamma_1\gamma_2 = \gamma_3^2$, the positive squeezing effect exactly offsets the negative substitution effect, and consequently the large firms do not change their behavior. The last result is consistent with Shimomura and Thisse (2012) in terms of the elimination of income effect, and Parenti (2015).

The change in total output can be directly explained from (15). Because $\Delta Q_S^* = -(\gamma_3/\gamma_1)\Delta Q_L^*$, as shown earlier, the change in total output is $\Delta Q^* = \Delta Q_S^* + \Delta Q_L^* = (\gamma_1 - \gamma_3)/\gamma_1\Delta Q_L^*$, which is positive if $\gamma_1 > \gamma_3$ and negative otherwise.

Now let us consider how the entry of a large firm influences consumer welfare, producer surplus and social welfare. Proposition 3 establishes the results.

Proposition 3 *The entry of a large firm generates the following impacts on welfare:*

(i) consumer welfare rises if

$$\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + E(\gamma_1\gamma_2 - \gamma_3^2)[\frac{M}{D(M)} + \frac{(M+1)}{D(M+1)}] > 0$$

and falls otherwise;

(ii) producer surplus rises if

$$\frac{(M+1)}{D^2(M+1)} - \frac{M}{D^2(M)} > 0$$

and falls otherwise; and

(iii) social welfare rises if

$$\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + \gamma_1\gamma_2 E(\frac{1}{D(M)} + \frac{1}{D(M+1)}) > 0$$

and falls otherwise,

where $D(M) = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$, $D(M+1) = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)(M+1) > 0$, *and* $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) > 0$ *according to the conditions in Proposition 1.*

Proof. See Appendix **B.3**. ■

Proposition 3 shows that the entry of a large firm would only conditionally raise consumer welfare, producer surplus and social welfare. The change of consumer welfare is determined by three effects. First, a portion of the small firms' product range is replaced by the large firms' in the representative consumer's consumption. This replacement effect affects consumer welfare because the quantity of one variety differs across large and small firms. Second, the entry of a large firm affects consumer welfare through the change in total product range. The consumer benefits from an increase in the total product range. Third, the change in consumer welfare depends on the interaction across large and small firms, which is determined by the relative substitution levels. These three effects determine the sign of the change in consumer welfare.

Specifically, the impact of a large firm's entry on consumer welfare can be decomposed as

$$\Delta CW = (\frac{\beta q_S^{*2}}{2} - \frac{\beta q_L^{2*}}{2})\Delta V_S^* + \frac{\beta q_L^{2*}}{2}\Delta V^* - \frac{1}{2}[Q_L^*(M) + Q_L^*(M+1)]\Delta p_L^*. \quad (17)$$

Here ΔV_S^* is the (negative) change of the product range of small firms, and ΔV^* is the change in total product range. The first term characterizes the replacement effect. The utility obtained from each variety of the small firm and that of the large firm are $\beta q_S^{*2}/2$ and $\beta q_L^{2*}/2$, respectively. Therefore, when ΔV_S^* of small firms' product range is replaced by large firms', the utility change is measured by the first term. The second term characterizes the variety change effect. If the total product range expands (ΔV^*), the representative consumer gains utility by $(\beta q_L^{2*}/2)\Delta V^*$ because (s)he consumes more varieties of large firms. The third term characterizes the interdependent effects from the interaction between large and small firms, which moves in the same direction as the change in the large firm's price. As shown in Proposition 2, the sign of this term is ambiguous, depending on the relative substitution level across large and small firms. Therefore, the total impact on consumer welfare depends on the comparison of the replacement effect, the variety change effect, and the interdependent effect.

The impact on producer surplus simply depends on the comparison between the profit change due to the increase in the total output of large firms (positive marginal effect) and the change in profits due to the price change (inframarginal effect). A sufficient condition for the increase in producer surplus is $\gamma_1\gamma_2 < \gamma_3^2$, which means the marginal and inframarginal effects are both positive. Producer surplus decreases only if the inframarginal effect is negative and outweighs the marginal effect. Formally,

$$\Delta PS = \frac{1}{2}(p_L^*(M) + p_L^*(M+1) - 2\beta q_L^*)\Delta Q_L^* + \frac{1}{2}[Q_L^*(M) + Q_L^*(M+1)]\Delta p_L^*.$$

Summing up the change in consumer welfare and that in producer surplus, the impact on social welfare can be expressed by

$$\Delta SW = (\frac{\beta q_S^{*2}}{2} - \frac{\beta q_L^{2*}}{2})\Delta V_S^* + \frac{\beta q_L^{2*}}{2}\Delta V^* + \frac{1}{2}[p_L^*(M) + p_L^*(M+1) - 2\beta q_L^*]\Delta Q_L^*. \quad (18)$$

Similar to ΔCW, the first two terms here characterize the replacement effect and variety change effect on consumer welfare. The third term, which is always positive, measures the marginal increase in producer surplus because of the rise in the total output of large firms. Note that the third effect on consumer welfare and the inframarginal increase of producer surplus offset each other. In consequence, the change in social welfare depends on the replacement effect and variety change effect on consumer welfare (the first two terms), and the marginal increase in producer surplus (the third term). Figure 6 provides a graph depiction of these three effects.

[Figure 6 around here]

We also analyze a few specific cases. First, we consider the case when $\gamma_1 = \gamma_2 = \gamma_3$, i.e., the products are equal substitutes across the big and the small. In this case, the large firm's price does not change, so the interdependent effect on consumer welfare is zero. The total output does not change either. Moreover, the condition for the coexistence of large and small firms requires $f > F$. This condition implies that $\Delta CW < 0$, $\Delta PS > 0$, and $\Delta SW > 0$. Although the total consumption does not change, the proportion of large firms' products increases in the consumption bundle, and thereby, the representative consumer suffers from a raised average price. Producer surplus rises because of the expansion of the total output of large firms. Social welfare improves because the deterioration of consumer welfare is dominated by the increase in producer surplus with the entry of a large firm.[11]

Now we consider the case when the fixed cost of a small firm is the same as the per-variety fixed cost of a large firm, i.e. $f = F$. In this case, the coexistence condition in Proposition 1 implies $\gamma_1 > \gamma_3$, so the total output (and total product range) rises, while the replacement effect is zero because per-variety output is the same across large and small firms. With the marginal increase in producer surplus, social welfare improves. A sufficient condition for the improvement of consumer welfare is $\gamma_1\gamma_2 > \gamma_3^2$, which implies a fall in the large firm's price. On the other hand, a sufficient condition for the increase in producer surplus is $\gamma_1\gamma_2 < \gamma_3^2$, as shown earlier.

We summarize these findings in the following corollary:

[11] This case has also been discussed in Parenti (2015).

Corollary 1 *(i) When the products are equally substitutable across large and small firms ($\gamma_1 = \gamma_2 = \gamma_3$), the entry of a large firm results in a deterioration of consumer welfare, an increase in producer surplus and a rise in social welfare;*

(ii) When the fixed cost of a small firm is the same as the per-variety fixed cost of a large firm ($f = F$), the entry of a large firm improves social welfare. A sufficient condition for consumer welfare to rise is $\gamma_1\gamma_2 > \gamma_3^2$, whereas a sufficient condition for producer surplus to rise is $\gamma_1\gamma_2 < \gamma_3^2$.

In the following example, we show the case when the entry of a large firm worsens consumer welfare and social welfare.

Example 1 *Consider a mixed market with 2 large firms and a host of small firms. Let the size of the differentiated goods market $\alpha = 1$, the preference for diversity $\beta = 1$, the substitutability among small firms' products $\gamma_1 = 0.4$, the substitutability among large firms' products $\gamma_2 = 0.625$, the substitutability across small and large firms' products $\gamma_3 = 0.5$, a small firm's fixed cost $f = 0.16$, and a large firm's fixed production cost of one variety $F = 0.1444$. In this case, $\gamma_1\gamma_2 = \gamma_3^2$, and the conditions for the coexistence of large and small firms are satisfied. The change in consumer welfare is -0.056, and the change in social welfare is -0.032. Here the consumer suffers from a negative replacement effect and a reduction of total product range, and these two negative effects dominate the marginal increase of producer surplus.*

Complementarity between the Big and the Small ($\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 < 0$) Having examined the case when all the products in the differentiated goods market are substitutes, now we consider the case when the products produced by the large firms are complementary to the goods of the small firms. The complementarity between the big and the small implies $\gamma_3 < 0$. In this case, the results in the previous section are mainly robust, except for the change in the number of small firms and social welfare. We establish the results in the following proposition.

Proposition 4 *When the goods of large firms are complementary to those of small firms, the entry of a large firm generates the following impacts:*

(i) The impacts on the behavior of large firms are the same as in Proposition 2, but the competitive fringe expands.

(ii) The impacts on consumer welfare and producer surplus are also ambiguous, based on the same conditions as in Proposition 3. Nevertheless, social welfare always rises.

As shown by Figure 2-b, the entry of a large firm generates a counterclockwise rotation of $Q_L(Q_S)$, so both Q_L^* and Q_S^* increase. The expansion of the competitive fringe then generates a positive effect on the price of large firms, according to equation (16). In consequence, the change in price of the large firm is determined by the comparison between the negative substitution effect $-\gamma_2$ and the positive indirect effect from the expansion of the competitive fringe, which is represented by γ_3^2/γ_1. Therefore, the impacts on the large firm are also determined by the same conditions as in Proposition 2.

Compared with the case when all products are substitutes, it is more likely that consumer welfare improves when the products are complementary between large and small firms in that both the total variety range of large firms and that of small firms increase, i.e. the first two terms in expression (17) are positive. The only possible negative effect originates from the rise in large firms' price when $\gamma_1\gamma_2 < \gamma_3^2$. In this case, the third term of expression (17) is negative.

Because the large firms' behavior is affected in the same way as in the case when products are substitutes across the big and the small, the impact on producer surplus here follows the same intuition as in the previous section.

Finally, social welfare always rises here. That is because the first two terms in expression (18) are positive when the products are complementary between large and small firms. This also implies that the negative impact on consumer welfare is dominated by the rise in producer surplus even with a rise in the price of large firms.

Our results provide several policy implications for regulations on the entry of a large firm. Many countries, such as France and the United Kingdom, enforce laws and regulations to restrict the entry of large firms. Our results partially support such restriction in terms of welfare because the entry of a large firm may reduce consumer welfare or social welfare in some cases, but we also show that the entry of large firms may improve welfare on certain conditions. With different levels of substitution across the big and the small, we show that incumbent firms react differently to the entry of a large firm. When products are substitutes across large and small firms ($\gamma_3 > 0$), if the substitutability between large and small firms is sufficiently weak ($\gamma_3^2 < \gamma_1\gamma_2$), a large firm's entry squeezes out a few small firms and intensifies the competition among large firms, and consumers benefit from reduced price. Such an entry behavior may be welcome in terms of the social benefits with modest costs paid by small firms. On the other hand, if the cross-substitution level is sufficiently strong ($\gamma_3^2 > \gamma_1\gamma_2$), more small firms will be squeezed out, and consumers are more likely to suffer due to the increased price. If the products are complementary between large and small firms ($\gamma_3 < 0$), such an entry behavior actually expands the business of small firms. Owing to the ambiguity of the impacts of a large firm's entry, our results suggest that governments carry out more meticulous and flexible policies to deal with different entry cases.

4 Discussions

In this section, we test the robustness of our results.

4.1 Single-Product Large Firms

When the varieties of large firms are exogenously given, e.g., $|\Omega_m| = 1$, we have two interpretations of the large firms' production behavior. The first is that a large firm is a multi-product one with a product range of 1, and the second to interpret is that a large firm is a single-product firm. In this case, our results are robust, and the change in each large firm's output is qualitatively the same as the change in the large firm's variety choice in our original model. Specifically, the impact of a large firm's entry generates the same impacts on firms' behavior as in Proposition 2. The welfare effects are also ambiguous, with slight changes in the conditions. The conditions for the unique mixed market equilibrium are also slightly modified. The following proposition establishes the results.

Proposition 5 *When both large and small firms are single-product firms,*

(i) there exists a unique mixed market equilibrium if the following three conditions hold:

(i-1) $\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*;*

(i-2) $\alpha[2\beta + \gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta f}[2\beta + \gamma_2(M+1)]$*; and*

(i-3) $\alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f} > [\gamma_1(2\beta + \gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]\sqrt{F/(\beta + \gamma_2)}$*.*

(ii) the impacts of a large firm's entry on firms' behavior are the same as in Propositions 2 and 4.

(iii) the entry of a large firm generates the following impacts on social welfare:

(iii-1) consumer welfare rises if $\alpha\beta(\gamma_1 - \gamma_3) - \gamma_2\gamma_3\sqrt{\beta f} + (\beta + \gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J[M/K(M) + (M+1)/K(M+1)] > 0$ *and falls otherwise;*

(iii-2) producer surplus rises if $(M+1)/K^2(M+1) - M/K^2(M) > 0$ *and falls otherwise; and*

(iii-3) social welfare rises if $\alpha\beta(\gamma_1-\gamma_3)-\gamma_2\gamma_3\sqrt{\beta f}+\gamma_1(\beta+\gamma_2)(2\beta+\gamma_2)J[1/K(M)+1/K(M+1)] > 2FK(M)K(M+1)/J$ *and falls otherwise,*

where $J = \alpha(\gamma_1-\gamma_3)+2\gamma_3\sqrt{\beta f} > 0$*,* $K(M) = \gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M > 0$*, and* $K(M+1) = \gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)(M+1) > 0$*, according to the conditions in (i).*

Proof. See Appendix **B.4**. ■

This model relates to Shimomura and Thisse (2012), who assume that large and small firms are single-product firms. Shimomura and Thisse (2012) show that the entry of a large firm shrinks the competitive fringe and thus generates the market expansion effect on large firms. This market expansion effect is amplified by the income effect, raising the profits of large firms and improving welfare. The key to their result is the income effect that amplifies the market expansion effect on large firms. We distinguish our model from theirs by excluding the income effect and explicitly introducing the different substitutabilities between large and small firms. We conclude that the entry of a large firm may result in an increase or a decrease of each large firm's output and may be harmful or beneficial to consumer welfare and social welfare, depending on the different levels of substitution across large and small firms.

4.2 Income Effect

As mentioned in Proposition 2, large firms do not change their behavior with the entry of a new large firm if $\gamma_1 = \gamma_2 = \gamma_3$, which corresponds to Shimomura and Thisse (2012) with the income effects washed out. Here we would like to elaborate more on the elimination of income effects in the CES framework.

One way to eliminate the income effect, as also mentioned by Shimomura and Thisse (2012), is to redistribute the profit to the absentee shareholders. In this case, the profits earned by large firms are not enjoyed or spent by the representative consumer, and consequently the income is exogenously given.

Another way to eliminate the income effect is to nest the CES composite good in a quasi-linear utility function:

$$U = \mathbf{Q} + q_0.$$

where $\mathbf{Q} = [\int_0^N (q_S(i))^\rho di + \sum_{j=1}^M (q_L^j)^\rho]^{1/\rho}$ is the CES composite good, and ρ $(0 < \rho < 1)$ is an inverse measure of the degree of differentiation across varieties. The numeraire good is denoted by q_0. This utility function is in the spirit of existing monopolistic competition literature, such as Helpman and Krugman (1989) and Feenstra and Ma (2007). It is readily shown that the free entry and exit of small firms fixes $\mathbf{Q}$, which is independent of the number of large firms. As a consequence, the behavior of large firms does not change with the entry of a large firm.

The income effect can also diminish in the following way. The utility in Shimomura and Thisse (2012) is expressed by a nested Cobb–Douglas function with a CES subutility of the differentiated goods market:

$$U = \mathbf{Q}^\alpha X^{1-\alpha},$$

where $\mathbf{Q}$ is the composite good as before. The consumption of the homogeneous good is represented by X, and α $(0 < \alpha < 1)$ represents the substitution between the composite good and the homogeneous good. As α falls, consumption of the composite good also goes down, and it is readily shown that the income effect diminishes. With α approaching zero, the income effect becomes negligible, and the large firms' total profits play a negligible role in the consumer's expenditure on the composite good.

4.3 Other Discussions

Finally, we find that the entry of a large firm will qualitatively exert the same impacts achieved by Propositions 1, 2, 3, and 4 if we consider the following cases.

(i) Large firms and small firms are vertically differentiated. In this case, α is replaced by α_L for the large firm and by α_S for the small firm. If $\alpha_L > (<)\alpha_S$, the products of the large firms have a higher (lower) quality than those of the small firms.

(ii) Large firms and small firms have the same or different marginal costs. In the constant marginal cost case, the variable costs of the large and small firms are $c_L q_L$ and $c_S q_S$, respectively. If firms incur increasing marginal costs, the variable costs of the large and small firms can be represented by $c_L q_L^2/2$ and $c_S q_S^2/2$, respectively.

5 Conclusion

In this paper we considered the market with large and small firms whose products may have different levels of substitution. In this market structure, we investigated the impact of a large firm's entry on incumbent firms' behavior and welfare. As we mentioned in the beginning, different industries featuring this mixed market structure see distinct impacts on large and small firms. We identify two opposing effects on the incumbent large firms. The first effect is the negative substitution effect, and the second effect is the positive squeezing (expanding) effect due to the shrinkage (expansion) of small firms when the products are substitutes (complements) across the big and the small. Which of these two effects dominates hinges on the different substitutabilities between large and small firms. The welfare effect is also ambiguous, depending on the different levels of substitution and technology.

Many countries enforce laws to restrict the entry of large firms presumably due to the protection of small- and medium-size enterprises. Our analysis shows that this policy can be justified in terms of welfare, since the entry of a large firm sometimes induces consumer welfare loss more than producer surplus gain. In particular, this may arise if the fixed cost of small firms is larger than per-variety fixed production cost of large firms (the production of each variety by a small firm is higher than that by a large firm), and the products of small firms are less differentiated than those of large firms (the entry of a large firm squeezes out many small firms).

Despite the prevalence of the market with large and small firms, few theoretical studies have been conducted so far. This paper is an attempt to explore this market structure. Future research is open in several directions. This paper assumes symmetric technology among large firms and among small firms, but heterogeneity of technology may also take place within large firms or among small firms. In addition, this paper employs a quasi-linear utility with a quadratic subutility to express the representative consumer utility. A more general framework is another interesting project. Finally, this paper investigates the change in social welfare caused by a large firm's entry, but a comprehensive welfare analysis of this mixed market structure, such as the optimal number of small firms given the number of large firms or the optimal number of large firms, can be an important project for future research.

References

[1] Bernard A. B., Redding S. J., Schott P. K. (2010). "Multiple-product firms and product switching." *American Economic Review*, 100(1), 70-97.

[2] Chen, Z. (2003). "Dominant retailers and the countervailing-power hypothesis." *RAND Journal of Economics*, 34(4): 612-625.

[3] Etro, F. (2004). "Innovation by leaders." *Economic Journal*, 114(495): 281-303.

[4] Etro, F. (2006). "Aggressive leaders." *RAND Journal of Economics*, 37(1): 146-154.

[5] Feenstra R, Ma H. (2007). "Optimal choice of product scope for multiproduct firms under monopolistic competition." National Bureau of Economic Research.

[6] Gowrisankaran, G., Holmes, T. J. (2004). "Mergers and the evolution of industry concentration: results from the dominant-firm model." *RAND Journal of Economics*, 35(3): 561-582.

[7] Helpman E, Krugman P. R. *Trade policy and market structure.* MIT press, 1989.

[8] Igami, M. (2011). "Does big drive out small?" *Review of Industrial Organization*, 38(1): 1-21.

[9] Ino, H., Matsumura, T. (2012). "How many firms should be leaders? Beneficial concentration revisited." *International Economic Review*, 53(4): 1323-1340.

[10] Lahiri, S., Ono, Y. (1988). "Helping minor firms reduces welfare." *The Economic Journal*, 98(393): 1199-1202.

[11] Markham, J. (1951). "The Nature and Significance of Price Leadership." *American Economic Review* , Vol. 41: pp. 891–905.

[12] Matsumura, T., Matsushima, N. (2010). "When small firms fight back against large firms in R&D activities." *B.E. Journal of Economic Analysis & Policy*, 10(1): Article 81.

[13] Kokovin, S., Parenti, M., Thisse, J. F., Zhelobodko, E. (2011). "Monopolistic Competition with Big Firms." mimeo.

[14] Ottaviano, G. I., Tabuchi, T., Thisse, J. F. (2002). "Agglomeration and trade revisited." *International Economic Review*, 43: 409-436.

[15] Ottaviano, G. I., Thisse, J. F. (1999). "Monopolistic competition, multiproduct firms and optimum product diversity." (Discussion Paper No. 2151, London: CEPR).

[16] Parenti, M. (2015). "David vs. Goliath." ECARES Working Papers.

[17] Shimomura, K. I., Thisse, J. F. (2012). "Competition among the big and the small." *RAND Journal of Economics*, 43(2): 329-347.

[18] Singh, N., Vives, X. (1984). "Price and quantity competition in a differentiated duopoly." *RAND Journal of Economics*, 15(4): 546-554.

A Concavity of the Quadratic Subutility Function

To ensure the concavity of the quadratic subutility function, the second-order condition should hold, i.e., the associated Hessian is negative definite. Since the product space is infinite dimensional, we discretize the product space (and take a limit).

Consider an integrable consumption bundle $q_S(i)$, $i \in [0, N]$, and $q_L^m(j)$, $j \in [0, |\Omega_m|]$, $m = 1, ..., M$. Take a positive integer, n_S, for the number of small firms, and positive integers n_L^m, $m = 1, ..., M$, for the number of varieties of large firm

m. We approximate the utility function by taking the grid points as $(i/n_S)N$, $i = 1, ..., n_S$ for small firms, and $(j/n_L^m)|\Omega_m|$, $j = 1, ..., n_L^m$, for large firm m, $m = 1, ..., M$:

$$
\begin{aligned}
U = \alpha & \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S} + \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m} \right] \\
& - \frac{\beta}{2} \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right)^2 \frac{N}{n_S} + \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right)^2 \frac{|\Omega_m|}{n_L^m} \right] \\
& - \frac{\gamma_1}{2} \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S} \right]^2 - \frac{\gamma_2}{2} \left[\sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m} \right]^2 \\
& - \gamma_3 \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S} \right] \left[\sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m} \right].
\end{aligned}
$$

We consider the associated Hessian matrix of the approximated utility. For concavity of the subutility, H is negative definite, i.e., for any $q(\cdot) \neq \mathbf{0}$, $-q^T H q > 0$, where

$$
\begin{aligned}
-q^T H q = \beta & \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right)^2 \frac{N}{n_S} + \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right)^2 \frac{|\Omega_m|}{n_L^m} \right] \\
& + \gamma_1 \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S} \right]^2 + \gamma_2 \left[\sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m} \right]^2 \\
& + 2\gamma_3 \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S} \right] \left[\sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m} \right].
\end{aligned}
$$

To simplify, let $a = \sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right) \frac{N}{n_S}$, and $b = \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right) \frac{|\Omega_m|}{n_L^m}$. Then

$$
-q^T H q = \beta \left[\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right)^2 \frac{N}{n_S} + \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right)^2 \frac{|\Omega_m|}{n_L^m} \right] + \gamma_1 a^2 + \gamma_2 b^2 + 2\gamma_3 ab.
$$

Applying Jensen's inequality, we have

$$
\sum_{i=1}^{n_s} q_S \left(\frac{i}{n_S} N \right)^2 \frac{N}{n_S} + \sum_{m=1}^{M} \sum_{j=1}^{n_L^m} q_L^m \left(\frac{j}{n_L^m} |\Omega_m| \right)^2 \frac{|\Omega_m|}{n_L^m} \geq \frac{a^2}{N} + \frac{b^2}{\sum_{m=1}^{M} |\Omega_m|}.
$$

Thus,

$$\begin{aligned}-q^T Hq &\geq \beta \left[\frac{a^2}{N} + \frac{b^2}{\sum_{m=1}^{M} |\Omega_m|} \right] + \gamma_1 a^2 + \gamma_2 b^2 + 2\gamma_3 ab \\ &= Ya^2 + Zb^2 + 2\gamma_3 ab \\ &= \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} Y & \gamma_3 \\ \gamma_3 & Z \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a & b \end{pmatrix} X \begin{pmatrix} a \\ b \end{pmatrix}\end{aligned}$$

where $Y = \beta/N + \gamma_1$, and $Z = \beta/\sum_{m=1}^{M} |\Omega_m| + \gamma_2$. This shows that H is negative definite if and only if X is positive definite, which is equivalent to $Y > 0$, $Z > 0$, and $YZ > \gamma_3^2$. Therefore, the subutility function is concave when $\beta/N + \gamma_1 > 0$, $\beta/(\sum_{m=1}^{M} |\Omega_m|) + \gamma_2 > 0$, and $(\beta/N + \gamma_1)(\beta/(\sum_{m=1}^{M} |\Omega_m|) + \gamma_2) > \gamma_3^2$.

Q.E.D.

B Proofs

B.1 Proof of Proposition 1

Given the equilibrium values of $q_S^* = \sqrt{f/\beta}$ and $q_L^* = \sqrt{F/\beta}$, the free entry condition of small firms and the profit maximization of a large firm yield the following two expressions of the dynamic adjustment process:

$$\dot{N}(N, |\Omega|) = d_1[\alpha q_S^* - \beta q_S^{*2} - (\gamma_1 N q_S^* + \gamma_3 M |\Omega| q_L^*) q_S^* - f],$$

$$\dot{|\Omega|}(N, |\Omega|) = d_2\{[\alpha - \beta q_L^* - \gamma_3 N q_S^* - \gamma_2 (M+1) |\Omega| q_L^*] q_L^* - F\}.$$

where $\dot{N} = dN/dt$, $\dot{|\Omega|} = d|\Omega|/dt$. $d_1 > 0$ and $d_2 > 0$ are the speed of dynamic adjustment. Without loss of generality, set $d_1 = d_2 = 1$. To ensure the local stability of the established model, the Jacobian matrix derived from the above two expressions is required to be negative definite:

$$\Phi = \begin{pmatrix} \partial\dot{N}/\partial N & \partial\dot{N}/\partial |\Omega| \\ \partial\dot{|\Omega|}/\partial N & \partial\dot{|\Omega|}/\partial |\Omega| \end{pmatrix} = \begin{pmatrix} -\gamma_1 q_S^{*2} & -\gamma_3 M q_S^* q_L^* \\ -\gamma_3 q_S^* q_L^* & -\gamma_2 (M+1) q_L^{*2} \end{pmatrix}.$$

$\Phi^1 = -\gamma_1 q_S^* < 0$, and $\Phi^2 = [\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M] q_S^{*2} q_L^{*2} > 0$. The stability condition thus holds if

$$\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0.$$

Q.E.D.

B.2 Proof of Proposition 2.

Let $D(M) = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M$, and $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f})$. Assuming that a unique mixed market equilibrium arises before and after the entry, we have $D(M) > 0$, $D(M+1) > 0$, and $E > 0$ (Proposition 1).

From the equilibrium, the changes of the variables when the number of large firms increase from M to $M+1$ are derived as follows:

$$\Delta q_S^* = 0, \Delta p_S^* = 0, \Delta q_L^* = 0,$$
$$\Delta N^* = \frac{-\gamma_2\gamma_3 E}{D(M)D(M+1)}\sqrt{\frac{\beta}{f}} < 0.$$

And

$$\Delta p_L^* = \frac{\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E}{D(M)D(M+1)},$$
$$\Delta |\Omega^*| = \frac{(\gamma_3^2 - \gamma_1\gamma_2)E}{D(M)D(M+1)}\sqrt{\frac{\beta}{F}},$$
$$\Delta \Pi_L^* = \frac{\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E^2}{D(M)D(M+1)}(\frac{1}{D(M)} + \frac{1}{D(M+1)}).$$

all of which are positive if $\gamma_1\gamma_2 < \gamma_3^2$, negative if $\gamma_1\gamma_2 > \gamma_3^2$, and constant if $\gamma_1\gamma_2 = \gamma_3^2$. Also

$$\Delta Q^* = \frac{\gamma_2(\gamma_1 - \gamma_3)E}{D(M)D(M+1)}.$$

which is positive if $\gamma_1 > \gamma_3$, negative if $\gamma_1 < \gamma_3$, and constant if $\gamma_1 = \gamma_3$.

Q.E.D.

B.3 Proof of Proposition 3.

Consumer welfare, producer surplus and social welfare can be expressed as

$$CW(M)^* = \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\,q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} - \frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^*,$$
$$PS(M)^* = \frac{\gamma_2 M E^2}{D^2(M)},$$
$$SW(M)^* = CW(M)^* + PS(M)^*.$$

The impact of an increase from M to $M+1$ on consumer welfare is

$$\Delta CW = \frac{\gamma_2 E}{2D(M)D(M+1)}\{\sqrt{\beta}(\gamma_1\sqrt{F} - \gamma_3\sqrt{f}) + E(\gamma_1\gamma_2 - \gamma_3^2)[\frac{M}{D(M)} + \frac{M+1}{D(M+1)}]\}.$$

which is positive if $\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f})+E(\gamma_1\gamma_2-\gamma_3^2)[M/D(M)+(M+1)/D(M+1)]>0$, and is negative otherwise.

The impact of an increase from M to $M+1$ on producer surplus is:

$$\Delta PS=\gamma_2E^2[\frac{M+1}{D^2(M+1)}-\frac{M}{D^2(M)}].$$

which is positive if $(M+1)/D^2(M+1)-M/D^2(M)>0$, and is negative otherwise.

The impact of an increase from M to $M+1$ on social welfare is

$$\Delta SW=\frac{\gamma_2E}{2D(M)D(M+1)}[\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f})+\gamma_1\gamma_2E(\frac{1}{D(M)}+\frac{1}{D(M+1)})].$$

which is positive if $\sqrt{\beta}(\gamma_1\sqrt{F}-\gamma_3\sqrt{f})+\gamma_1\gamma_2E[1/D(M)+1/D(M+1)]>0$, and is negative otherwise.

Q.E.D.

B.4 Proof of Proposition 5.

(4-i) When the large firm supplies one variety, the computation shows that

$$\begin{aligned}
q_S^* &= \sqrt{\frac{f}{\beta}},\\
q_L^* &= \frac{\alpha(\gamma_1-\gamma_3)+2\gamma_3\sqrt{\beta f}}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M},\\
p_L^* &= (\beta+\gamma_2)\frac{\alpha(\gamma_1-\gamma_3)+2\gamma_3\sqrt{\beta f}}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M},\\
N^* &= \sqrt{\frac{\beta}{f}}\frac{\alpha[2\beta+\gamma_2(M+1)-\gamma_3M]-2\sqrt{\beta f}[2\beta+\gamma_2(M+1)]}{\gamma_1(2\beta+\gamma_2)+(\gamma_1\gamma_2-\gamma_3^2)M}.
\end{aligned}$$

To ensure the existence of a unique mixed market equilibrium, we should have the stability of the equilibrium, $N^*>0$ and $\Pi_L^*>0$.

Given the equilibrium value of $q_S^*=\sqrt{f/\beta}$, the free entry condition of small firms and the profit maximization of the large firm yield the following two expressions of dynamic adjustment process:

$$\dot{N}(N,q_L)=d_1[\alpha q_S^*-\beta q_S^{*2}-(\gamma_1Nq_S^*+\gamma_3Mq_L)q_S^*-f],$$

$$\dot{q_L}(N,q_L)=d_2\{[\alpha-2\beta q_L^*-\gamma_3Nq_S^*-\gamma_2(M+1)q_L]\}.$$

where $\dot{N}=dN/dt$, $\dot{q_L}=dq_L/dt$, $d_1>0$ and $d_2>0$.

To ensure the local stability of the established model, the Jacobian matrix derived from the above two expressions is required to be negative definite:

$$\Psi = \begin{pmatrix} \partial \dot{N}/\partial N & \partial \dot{N}/\partial q_L \\ \partial \dot{q_L}/\partial N & \partial \dot{q_L}/\partial q_L \end{pmatrix} = \begin{pmatrix} -\gamma_1 q_S^{*2} & -\gamma_3 M q_S^* \\ -\gamma_3 q_S^* & -2\beta - \gamma_2(M+1) \end{pmatrix}.$$

$\Psi^1 = -\gamma_1 q_S^{*2} < 0$, and $\Psi^2 = [\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]q_S^{*2} > 0$. Hence the stability condition holds if

$$\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M > 0.$$

In addition, we should have $N^* > 0$, and $\Pi_L^* > 0$, i.e., $F < p_L^* q_L^*$. Accordingly,

$$\alpha[2\beta + \gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta f}[2\beta + \gamma_2(M+1)],$$

$$\alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f} > [\gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M]\sqrt{\frac{F}{(\beta+\gamma_2)}}.$$

(4-ii) Let $J = \alpha(\gamma_1 - \gamma_3) + 2\gamma_3\sqrt{\beta f}$, and $K(M') = \gamma_1(2\beta+\gamma_2) + (\gamma_1\gamma_2 - \gamma_3^2)M'$. By (4-i), $J > 0$, $K(M) > 0$ and $K(M+1) > 0$. From the equilibrium, the changes of the variables when the number of large firms increase from M to $M+1$ are derived as follows:

$$\begin{aligned} &\Delta q_S^* = 0, \\ &\Delta p_S^* = 0, \\ &\Delta N^* = -\frac{(2\beta+\gamma_2)\gamma_3 J}{K(M)K(M+1)}\sqrt{\frac{\beta}{f}} < 0 \text{ if } \gamma_3 > 0, \text{ and} > 0 \text{ if } \gamma_3 < 0. \end{aligned}$$

Moreover,

$$\begin{aligned} \Delta q_L^* &= \frac{(\gamma_3^2 - \gamma_1\gamma_2)J}{K(M)K(M+1)}, \\ \Delta p_L^* &= \frac{(\beta+\gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)J}{K(M)K(M+1)}, \\ \Delta \Pi_L^* &= \frac{(\beta+\gamma_2)(\gamma_3^2 - \gamma_1\gamma_2)J^2}{K(M)K(M+1)}[\frac{1}{K(M)} + \frac{1}{K(M+1)}]. \end{aligned}$$

which are positive if $\gamma_1\gamma_2 < \gamma_3^2$, negative if $\gamma_1\gamma_2 > \gamma_3^2$, and constant if $\gamma_1\gamma_2 = \gamma_3^2$. Also

$$\Delta Q^* = \frac{(2\beta+\gamma_2)(\gamma_1 - \gamma_3)J}{K(M)K(M+1)},$$

which is positive if $\gamma_1 > \gamma_3$, negative if $\gamma_1 < \gamma_3$, and constant if $\gamma_1 = \gamma_3$.

(4-iii) The associated consumer welfare, producer surplus and social welfare are

$$CW(M)^* = \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} - \frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^*,$$
$$PS(M)^* = \frac{\gamma_2 M J^2}{K^2(M)},$$
$$SW(M)^* = CW(M)^* + PS(M)^* - FM.$$

The impact of an increase from M to $M+1$ on consumer welfare is

$$\Delta CW^* = \frac{J}{2K(M)K(M+1)}[A + (\beta+\gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J(\frac{M}{K(M)} + \frac{M+1}{K(M+1)})].$$

where $A = \alpha\beta(\gamma_1 - \gamma_3) - \gamma_2\gamma_3\sqrt{\beta f}$. ΔCW^* is positive if $A + (\beta+\gamma_2)(\gamma_1\gamma_2 - \gamma_3^2)J[M/K(M) + (M+1)/K(M+1)] > 0$, and is negative otherwise.

The impact of a marginal increase of M on producer surplus is

$$\Delta PS^* = \frac{(\beta+\gamma_2)J^2}{K(M)K(M+1)}[\frac{M+1}{K^2(M+1)} - \frac{M}{K^2(M)}].$$

which is positive if $(M+1)/K^2(M+1) - M/K^2(M) > 0 > 0$, and is negative otherwise.

The impact of a marginal increase of M on social welfare is

$$\Delta SW^* = \frac{J}{2K(M)K(M+1)}[A + \gamma_1(\beta+\gamma_2)(2\beta+\gamma_2)J(\frac{1}{K(M)} + \frac{1}{K(M+1)})] - F.$$

which is positive if $A+\gamma_1(\beta+\gamma_2)(2\beta+\gamma_2)J[1/K(M)+1/K(M+1)] > 2FK(M)K(M+1)/J$, and is negative otherwise.

Q.E.D.

Figures

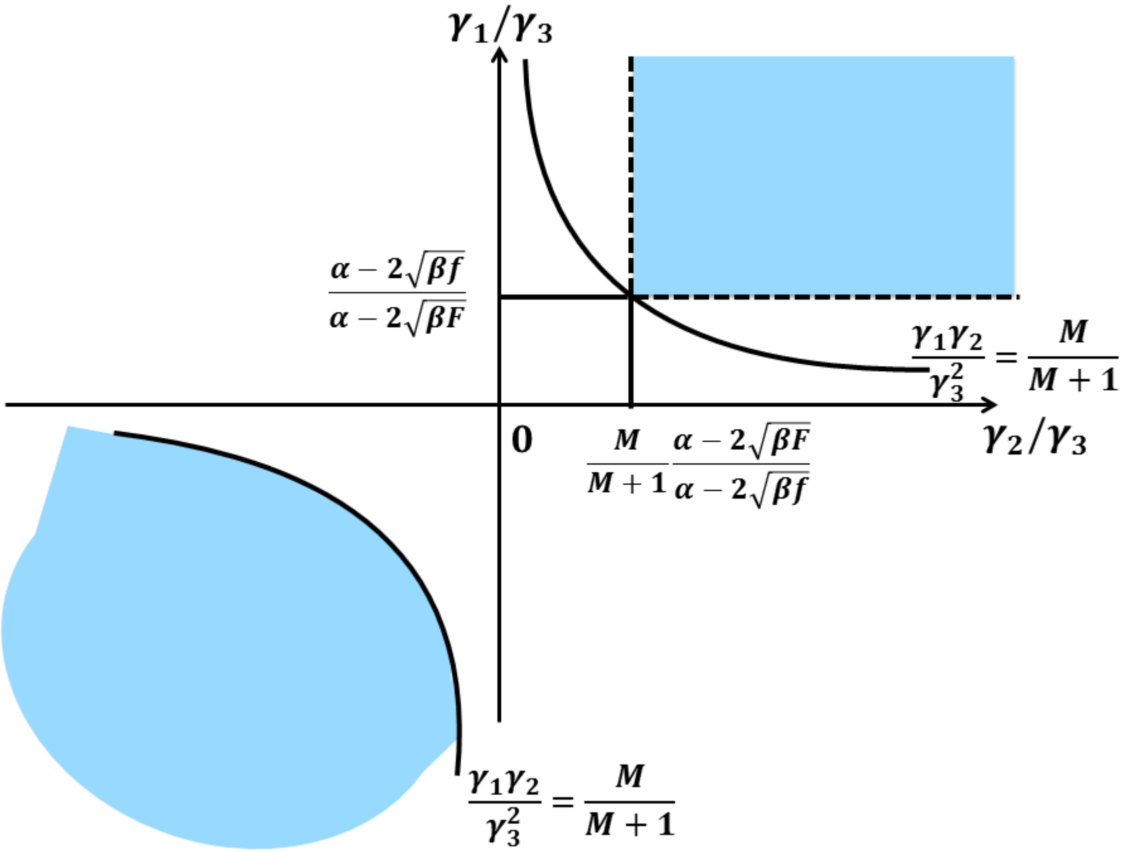


Figure 1. Conditions for a Unique Mixed Market Equilibrium

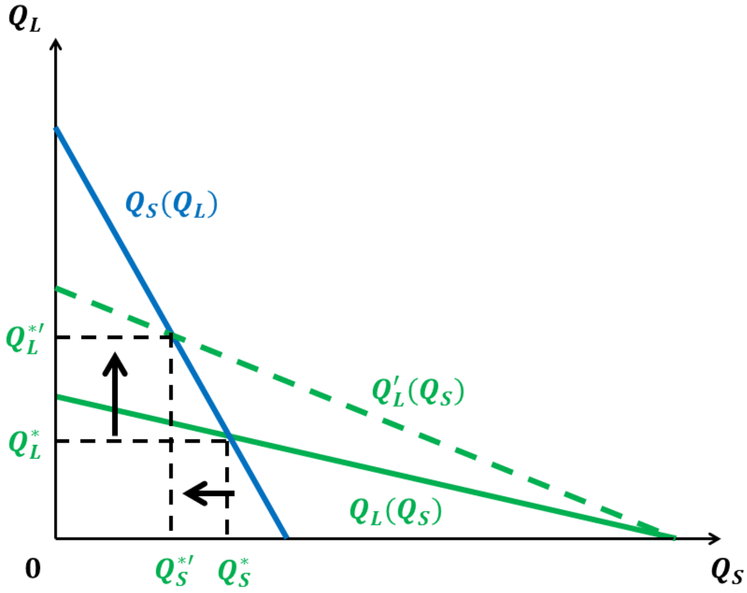


Figure 2a Goods are Substitutes across Large and Small Firms

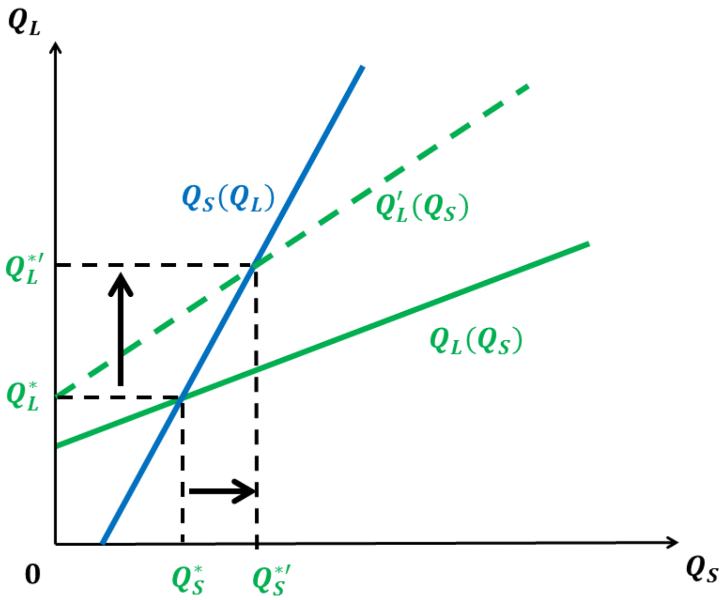


Figure 2b Goods are Complements across Large and Small Firms

Figure 2 Aggregate Reactions of Large and Small Firms

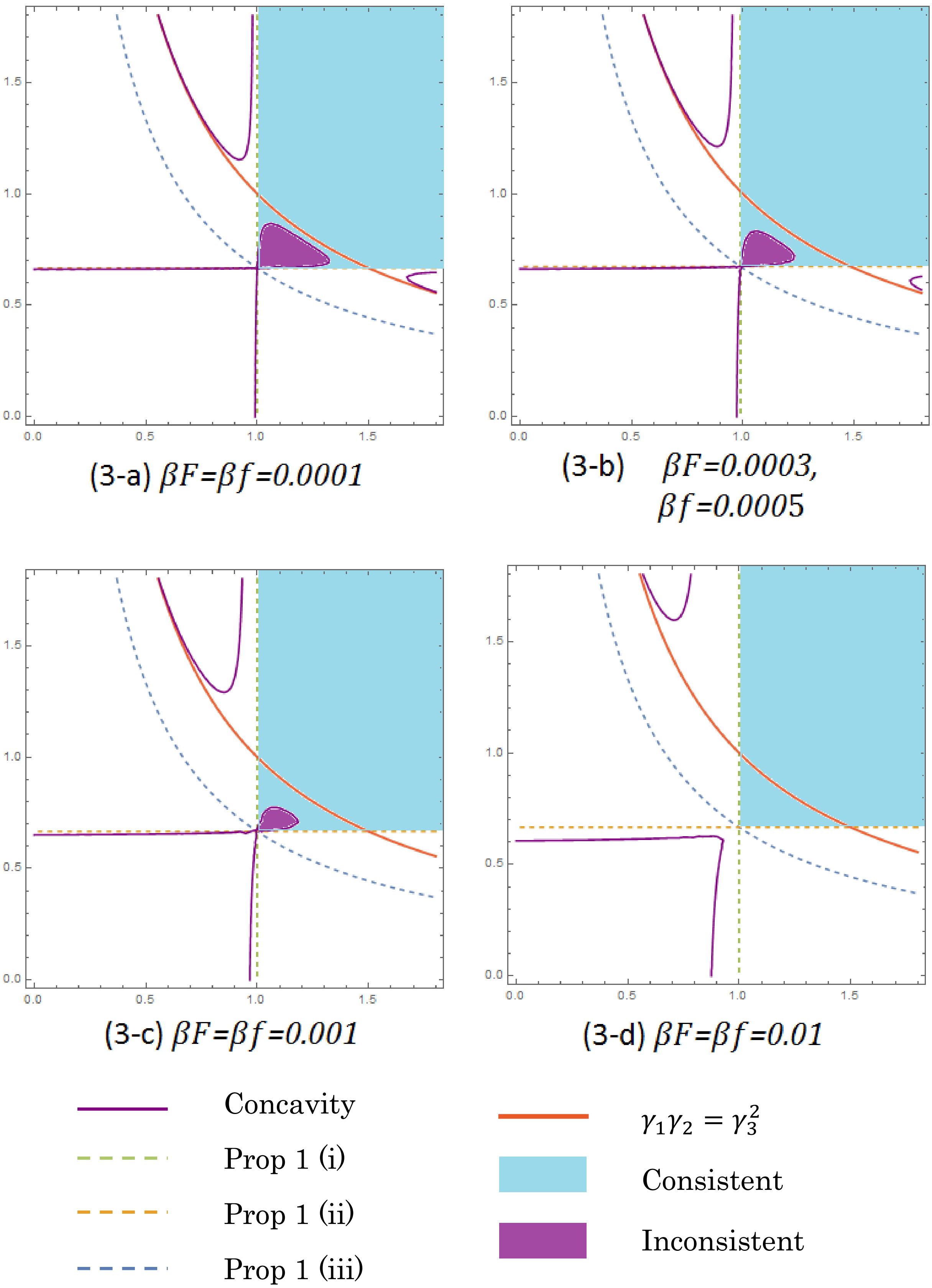


Figure 3 Consistency between Coexistence Condition and Concavity Condition When Goods are Substitutes across Large and Small Firms

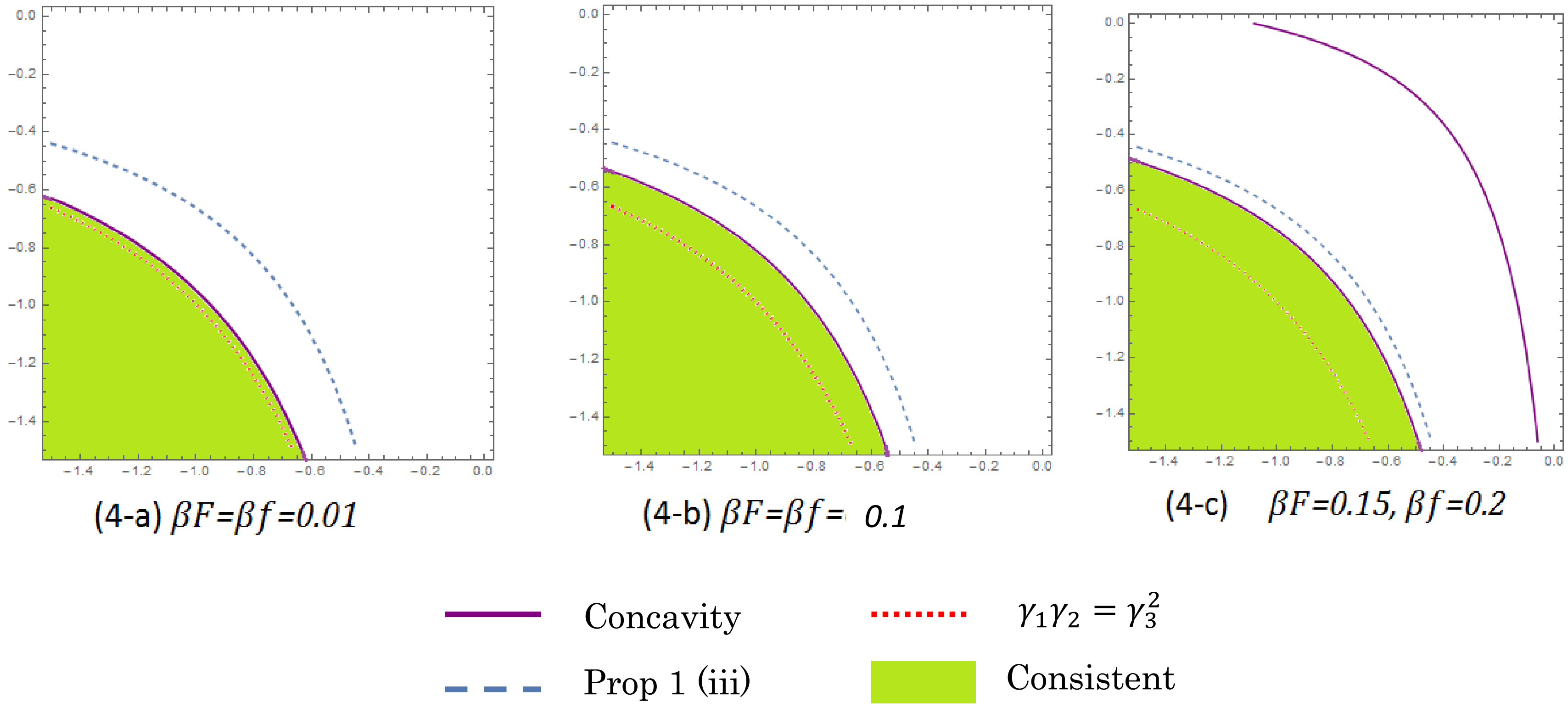


Figure 4 Consistency between Coexistence Condition and Concavity Condition When Goods are Complements across Large and Small Firms

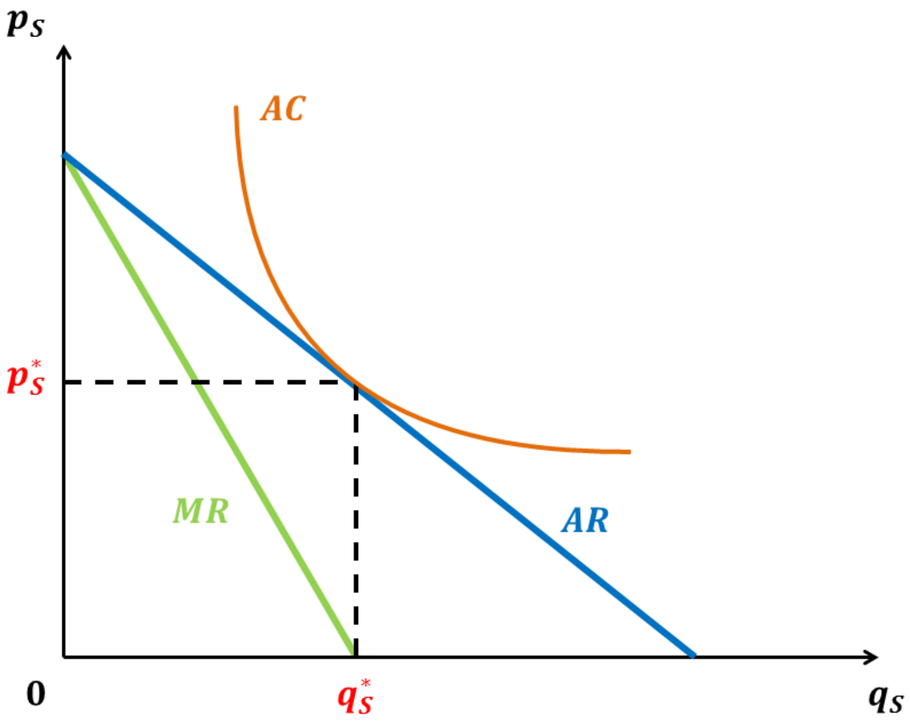
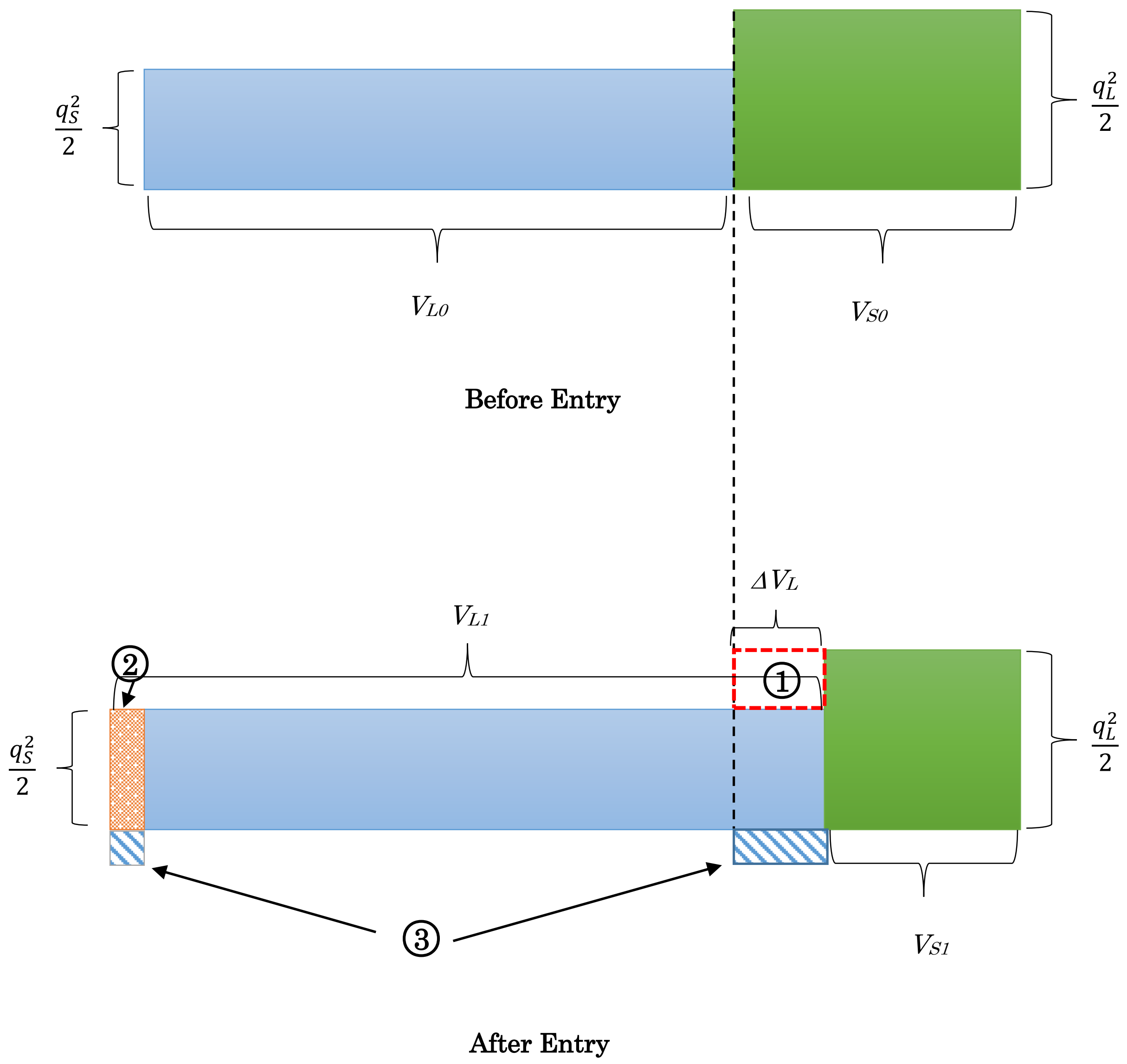


Figure 5 Small Firm's Production Behavior



① Replacement Effect
② Variety Change Effect
③ Marginal Increase in Producer Surplus

Figure 6 Social Welfare Change (Assume f > F)