# Media See-saws: Winners and Losers on Media Platforms[1]

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## Abstract

We customize the aggregative game approach to oligopoly to study asymmetric media markets. Advertiser, platform, and consumer surplus are tied together by a simple summary statistic. When media are ad-financed and ads are a nuisance to consumers we establish see-saws between consumers and advertisers. Entry of a lower-quality platform increases consumer surplus, but decreases advertiser surplus if industry platform profits decrease with entry. Merger decreases consumer surplus, but increases advertiser surplus if profits of each merged platform increase with the merger. By contrast, when platforms use two-sided pricing or consumers like advertising, advertiser and consumer interests are often aligned.

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# 1 Introduction

Standard imperfectly competitive markets tend to have consumer and producer interests diametrically opposed - what helps one side adversely affects the other side. For example, incumbent producers are hurt by new entrants, while consumers gain through lower prices and more variety. Contrarily, profitable mergers typically harm consumers (absent sufficient synergies), while benefitting firms. Our focus here is on two-sided markets – and media markets in particular – where there are three groups of protagonists which interact. In addition to firms (the media "platforms") and consumers, there are advertisers. Our research question is what happens to the advertisers when market structure changes. We show that consumer and platform interests are still opposed, so that we want to know with which side advertiser interests are aligned. In the base case where platforms are solely financed by advertising revenue, one might expect that it would be the advertisers who are hurt when platforms profits rise, for the advertisers are the customers paying the price. Surprisingly perhaps, it is in this case (when ads are a nuisance to consumers) that advertisers' interests are actually aligned with platforms. Because consumers are still hurt, we call this a *media see-saw* – advertiser and consumer interests are opposed.[1] Our objective in this paper is to determine the extent of such see-saws, by evaluating circumstances in which see-saws arise. When ads are actually valued by consumers, such see-saws tend not to arise; neither do they appear when platforms set prices to consumers in addition to advertisers. Instead, consumer and advertiser surpluses tend to both move together.

To establish when see-saws arise, and to investigate strategic interaction in media markets more generally, we develop a framework with asymmetric oligopoly media

[1] Rochet and Tirole (2006) refer to a seesaw principle when conducive to a low *price* on one side tend to lead to a high *price* on the other side. We use the term somewhat differently, as we ask whether factors conducive to a low *surplus* on one side lead to a high *surplus* on the other side.

platforms and deploy the toolkit of aggregative games (see Acemoglu and Jensen, 2013, for a summary). Through judicious use of an aggregator function, we can phrase the two-sided market models we consider as aggregative games. This approach delivers a full equilibrium characterization in various differing contexts. We can then engage these tools to describe the effects of entry, mergers, and ad caps in media markets. We show that in the standard media economics setting, all participants' surpluses can be tracked as a function only of the aggregate. Along the way, we also provide and engage a new result relating advertiser surplus to profits under the weak assumption of log-concavity of advertiser demand functions. This allows us to obtain clear-cut results on advertiser surplus to establish when see-saws arise.

In ad-financed media markets where ad is a nuisance, platform entry gives rise to a see-saw if platform industry profits decrease with entry. After entry consumers are better off, but advertisers are worse off. A merger between two media platforms gives rise to a see-saw if the profits of the merging platform with the higher quality increases with the merger. After the merger consumers are worse off, but advertisers are better off. The imposition of ad caps leads yet to another see-saw. An ad cap not only reduces ad levels of platforms exceeding the cap prior to its introduction, but also of the other (unconstrained) platforms. Consumers benefit from ad caps, while advertisers loose (even though additional consumers are attracted after the introduction of the cap).

Entry into some media markets (especially radio and broadcast TV) is controlled through licensing (e.g., by the FCC in the US), and media mergers are subject to stricter restrictions than other mergers. When analyzing the consequences in such contexts, it is important to recognize that there is an additional group impacted – the advertisers. When see-saws are present, consumer surplus standards may allow through policies that harm advertisers. Often such advertisers could be small businesses (rather than media monolith platforms) whose concerns might be highly valued socially. Our analysis underscores that ad-financed media markets require a dedicated welfare analysis.

Our paper builds on the literature of two-sided markets (Rochet and Tirole, 2003, 2006; Armstrong, 2006) and, in particular, media markets which feature one-sided pricing (Gabszewicz, Laussel and Sonnac, 2004; Anderson and Coate, 2005; Peitz and Valletti, 2008). All these analyses consider symmetric duopoly market where consumers are located on a Hotelling line. Crampes, Haritchabalet, and Jullien (2009) analyze symmetric oligopoly where platforms and consumers are located on the Salop circle (which, as with the above mentioned papers, features full market coverage). Anderson (2012) sketches a monopolistic competition media model with logit demand. We generalize the logit specification and analyze asymmetric platform oligopoly. To do so, we are the first in the context of two-sided markets to make use of aggregative game tools.

Our application to mergers connects to the classic merger literature (in particular, Deneckere and Davidson, 1985). It also connects to the recent empirical work on media mergers (e.g., Chandra and Collard-Wexler, 2009; Fan, 2013; Jeziorski, 2014a, 2014b; Ivaldi and Zhang, 2015).[2]

The plan of the paper is as follows. In section 2, we provide some relevant preliminaries on aggregative games. In section 3, we present the asymmetric oligopoly media platform model under ad-finance. In section 4, we characterize the equilibrium (when consumers like or dislike ads) with respect to equilibrium ad levels and provide comparative statics results for platform profits and consumer surplus. In section 5,

[2] An important empirical question is to identify how media platform characteristics change with a merger. In our theoretical analysis we presume that horizontal attributes of the media platform remain unchanged. Thus our analysis can be seen as a merger analysis under editorial independence. Such an analysis is relevant when the owner may deliberately decide not to intervene in the programming decisions by the editorial staff and maintain editorial independence of the two media platforms. Such independence may also be the result of a merger remedy imposed by the antitrust authority, as has happened in a number of newspaper merger cases. See also the counterfactual simulation by Ivaldi and Zhang (2015) for French free-to-air television.

we focus on advertiser surplus when ads are a nuisance and establish see-saws under entry, merger, and ad cap regulation. We also argue that see-saws are unlikely to emerge when consumers like ads. In section 6, we introduce two-sided pricing whereby platforms also make revenues from charging subscription fees to consumers. Even though each platform now has two instruments, we are able to construct an aggregator function and make use of aggregative game tools. We show that while see-saws may rarely emerge in such markets, they are not a general feature in such a context, as platforms do not use advertising levels strategically when they have a second instrument. In section 7 we conclude.

# 2 Preliminaries on aggregative games

The media market models in this paper have an aggregative game structure, which enables us to derive characterization and comparative static properties from the aggregative game approach. We next review the results we use from the aggregative game toolkit for Industrial Organization given in Anderson, Erkal, and Picinnin (2013).

Suppose that each firm's profit can be written as $\Pi_i(\psi_i, \Psi)$ where $\psi_i$ is firm $i$'s action variable, $i = 1, ..., n$, $\psi_0$ is a constant and $\Psi = \Sigma_{j=0}^{n} \psi_j$ is the aggregate (see, for instance, Acemoglu and Jensen, 2013). Prominent examples include oligopolies with price-setting firms and logit or CES demand,[3] and homogeneous product Cournot competition too. Each firm solves the problem $\arg\max_{\psi_i} \Pi_i(\psi_i, \psi_i + \sum_{j\neq i} \psi_j)$.

The first-order condition can be written as

$$\frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \psi_i} + \frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \Psi} = 0, \qquad i = 1, ..., n$$

and pins down a relationship between $\psi_i$ and $\Psi$. If $\Pi_i$ is strictly quasi-concave, this equation implicitly defines the *inclusive best reply* function, $r_i(\Psi)$, as $i$'s action

[3] For background references to such topics as the IIA property, logit and CES formulations, and differentiated product models of oligopoly, see Anderson, de Palma, and Thisse (1992).

that brings the total actions to $\Psi$. This follows Selten (1970) and differs from the standard way to write best replies as functions of the actions of all other players. However, the two concepts are quite related: in particular, $r_i$ is an increasing function if actions are strategic complements (see Anderson, Erkal, and Picinnin, 2013, for details). Suppose that the game is competitive in the sense that a higher $\Psi$ reduces profits (the simplest example is the homogeneous products Cournot model for which the aggregate is simply aggregate output), so that the second term in the first-order condition above is negative.

Equilibrium constitutes a fixed point, namely the equilibrium aggregate is given by $\Psi^* = \psi_0 + \Sigma_{i=1}^{n} r_i(\Psi^*)$ which is depicted simply graphically as the point where the sum of the inclusive best reply functions crosses the 45-degree line. Continuity of $r_i(\Psi)$ for all $i = 1, ..., n$ implies equilibrium existence. Absent continuity, strategic complementarity ensures equilibrium existence by Tarski's fixed point theorem. The equilibrium is unique if the $r_i(\Psi)$ are continuous and $\Sigma r_i'(\Psi^*) < 1$. Hence, for strategic complements, we need an upper bound for the slope of the inclusive best reply. A sufficient condition for uniqueness is that

**Condition 1** $r_i'(\Psi) < r_i(\Psi)/\Psi$ *for all* $i = 1, ..., n$.

Summing over all $i$, Condition 1 implies the desired slope property that $\Sigma r_i'(\Psi^*) < 1$.

Both the difference across agents in equilibrium and comparative statics can be depicted and derived simply with this device. In particular, "weaker" agents (in the sense of those with lower inclusive best reply functions) have lower equilibrium actions, and a change rendering an agent's behavior more aggressive (i.e., shifting up its best reply function) will increase its own equilibrium action, increase the aggregate and increase other players' actions when actions are strategic complements.

# 3 Ad-financed media: the actors and the model

We consider a market in which media deliver consumer attention to advertisers. Participants on both consumer and advertiser sides of the market are atomless. The platforms host ads and are attended by consumers. They set ad levels, which are observed by all players, and then consumers and advertisers choose which platform(s) to join.[4] We next describe the preferences of the advertisers, consumers, and platforms.

**Advertisers**

As we explain below, we assume that each consumer makes a discrete choice of which platform to attend, and therefore consumers "single-home". This means that the only way to reach a particular consumer is to place an ad on the channel she is watching.[5] Any ad on the channel is assumed to be seen by all the viewers there, and we assume there is no benefit to showing more than one ad per channel. Furthermore, advertisers' profits (gross of the costs of advertising) are assumed to be proportional to the number of consumers reached, and independent of the number or identities of other advertisers on the channels. Together, this means that each advertiser's decision on where to advertise is taken independently channel by channel, irrespective of whatever other channels are selected.

We rank advertisers in terms of decreasing per-viewer willingness to pay, $p$, to contact viewers and so $p(a_i)$ is the per-viewer willingness to pay of the marginal ($a_i$th) advertiser if there are $a_i$ ads on platform $i$. This willingness to pay is the expected surplus to the advertiser generated by an advertiser-viewer match.

[4] Recent surveys of the literature on such models are in the Handbook of Media Economics, in particular Anderson and Jullien (2015) on the two-sided ad-financed business model, Peitz and Reisinger (2014) on applications to the economics of the Internet, and Foros, Kind, and Sorgard (2015) for the anti-trust implications. The Handbook also includes surveys for particular industries (TV, radio, newspapers and magazines).

[5] This set-up gives rise to the "competitive bottleneck" of Armstrong (2006) that platforms control access to "their" consumers.

**Assumption 1** $p(a)$ is log-concave and twice continuously differentiable. There is an $\overline{a}$ such that $p(a) > 0$ for all $a < \overline{a}$ and $p(a) = 0$ for all $a > \overline{a}$.

The log-concavity assumption means that $p(a)$ is concave or not "too" convex, so the corresponding marginal revenue curve slopes downward at least as steeply as $p(a)$. The assumption that advertiser inverse demand drops to zero for the ad level sufficiently large is only needed when consumers like ads.

We define platform revenue per viewer as $R(a) = ap(a)$. We define the per viewer monopoly advertising level, $a^m$, as the solution to $R'(a) = 0$ (which is uniquely determined under Assumption 1).

Net advertiser surplus per viewer is

$$AS(a) = \int_0^a (p(x) - p(a))dx.$$

Gross advertiser surplus per viewer is $AS^G(a) = \int_0^a p(x)dx = AS(a) + R(a)$. Clearly, $dAS^G(a)/da = p(a)$, which is the per-viewer willingness to pay of the marginal advertiser, and $dAS(a)/da = -ap'(a)$. We establish an important property of $AS$ which will be used in the analysis below. In words, the ratio of advertiser surplus to ad revenues is non-decreasing in $a$.

**Lemma 1** *Under Assumption 1, $d(AS(a)/R(a))/da \geq 0$.*

The proof is relegated to Appendix 1, and relies on bounding inverse demand from above by a log-linear function in order to bound advertiser surplus. This lemma will play a key role in establishing see-saws in Section 5. Letting $\lambda_i$ denote the fraction of consumers on platform $i$, then the advertiser surplus on platform $i$ is $\lambda_i AS(a_i)$, and industry advertiser surplus is $IAS(\mathbf{a}) = \sum_{i=1}^n \lambda_i AS(a_i)$.

**Consumers**

We deploy a discrete-choice model of media consumption.[6] The attractiveness of a particular option depends upon its net quality, $v_i = s_i - \gamma a_i$, $i = 1, ..., n$, where $s_i$ is gross quality, and $\gamma$ denotes the net nuisance per ad, factoring in any expected consumer benefits from being exposed to the ad. We allow for this "nuisance" to be negative, so that $\gamma < 0$ corresponds to where consumers enjoy ads per se, or else benefit enough from ad exposure (e.g., learning about new consumption possibilities).

We assume that consumer demand for platform $i$ takes the fractional form associated with Luce (1959):

$$\lambda_i(\mathbf{a}) = \frac{h(v_i)}{\Sigma_{j=0}^{n} h(v_j)}, i = 1, ...n \tag{1}$$

where we assume

**Assumption 2** $h(v)$ is increasing, log-concave and twice continuously differentiable.

The borderline case is that $h$ is log-linear, in which case $h = \exp(v/\mu)$, which we shall refer to as the standard logit case, and where $\mu$ is a positive parameter reflecting platform heterogeneity. Notice that demand is higher for options delivering higher net quality, and let $h(v_0)$ denote the attractiveness of watching no channel. In the sequel, we shall use the denominator in (1) as a measure of consumer benefits from the media sector: the higher it is, the more the benefit. One justification is given in the next paragraph, although we do not need to espouse the particular model in order for our results to hold.

One possible consumer-theoretic underpinning for the form (1) is a familiar random utility model whereby each consumer chooses the platform (or outside option 0 with net quality $v_0$) to maximize

$$U_i = \ln h(v_i) + \varepsilon_i, \quad i = 0, 1, ..., n, \tag{2}$$

[6] Thus consumers single-home (as mentioned above). Recent work (Ambrus, Calvano, and Reisinger, 2014, and Anderson, Foros, and Kind, 2014) has included multi-homing consumers in two-sided media markets. We comment on this issue in the conclusion.

where the $\varepsilon_i$ are i.i.d. Type 1 Extreme Value (which delivers the logit model) with standard deviation $\mu > 0$.[7] A consumer with realization $\boldsymbol{\varepsilon}$ chooses option $i \in \{0, 1, ..., n\}$ if $U_i \geq U_j$ for all $j = 0, 1, ..., n$. This formulation yields the familiar log-sum form for consumer surplus associated to the Logit model:

$$CS = \mu \ln \left( \sum_{i=0}^{n} h(v_i) \right). \tag{3}$$

This form satisfies the claim above that consumer benefits increase in the value of the denominator of (1). It is also a useful module for extending the model to allow for subscription pricing.

**Platforms**

Platform $i$'s profit is

$$\Pi_i = a_i \lambda_i(\mathbf{a}) p(a_i) = R(a_i) \lambda_i(\mathbf{a}),$$

where Assumption 1 implies that the revenue per viewer, $R(a) = ap(a)$ is strictly log-concave in $a$.

**Actions and Aggregate**

We are now in a position to write each platform's objective as a function of an action $\psi_i$ and the corresponding aggregate $\Psi = \sum_{j=0}^{n} \psi_j$, where we define $\psi_0 = h(v_0)$ as the "constant action" of the outside option. Indeed, let $i$'s action be $\psi_i = h(v_i)$, where we recall that $v_i = s_i - \gamma a_i$. This defines the implicit relation between the action and the chosen ad level, with the property

$$a_i'(\psi_i) = -\frac{1}{\gamma h'(v_i)}. \tag{4}$$

Therefore the chosen ad level varies directly with the platform's action for $\gamma < 0$, and it varies inversely with it for $\gamma > 0$.

[7] See Anderson, de Palma, and Thisse (1992) for more details.

Demand for platform $i$ is $\lambda_i = \psi_i/\Psi$; i.e., demand of platform $i$ depends only on its own transformed action $\psi_i$ and the aggregate $\Psi$.[8] We can then write platform profit as

$$\Pi_i(\psi_i, \Psi) = R(a_i(\psi_i))\frac{\psi_i}{\Psi}, \qquad i = 0, 1, ..., n. \tag{5}$$

Clearly, this function satisfies the competitiveness property; i.e., that profits decrease in the aggregate $\Psi$.

# 4 Equilibrium analysis

## 4.1 Characterization

For $\gamma = 0$, viewer demand is independent of the advertising level. Hence, each platform acts as a monopolist on the advertiser side and market demand is exogenous. Therefore, platforms set the monopoly advertising level $a^m = \arg\max_a R(a)$. In the sequel all results exclude this case.

Strategic interaction arises when $\gamma \neq 0$. Each platform chooses its ad level, $a_i$, and because $\psi_i$ is a monotonic function of $a_i$ we can find the equilibrium action by differentiating (5) with respect to $\psi_i$. The first-order condition for platform $i$ is

$$\frac{\partial \Pi_i}{\partial \psi_i} = R'(a_i(\psi_i))a_i'(\psi_i)\frac{\psi_i}{\Psi} + R(a_i(\psi_i))\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) = 0, \quad i = 0, 1, ..., n.$$

The first-order conditions can be rewritten as

$$\begin{aligned} \frac{\psi_i}{\Psi} &= 1 + \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}a_i'(\psi_i)\psi_i & (6) \\ &= 1 - \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\frac{h(v_i)}{\gamma h'(v_i)}, & (7) \end{aligned}$$

[8]Demand of the form $\lambda_i = \psi_i/\Psi$ includes oligopoly models with logit and the Luce (1959) form of demand, and duopoly models based on Hotelling models which are predominant in the literature, such as the one presented in Anderson and Coate (2005). A logit specification is provided by Anderson (2012).

where the second line uses (4). Since the right-hand side is decreasing in $\psi_i$ (as shown in Lemma 2 below) while the left-hand side is increasing, first-order conditions define inclusive best reply functions $r_i(\Psi)$. In the Appendix we show that the inclusive best replies satisfy a key characterization property.

**Lemma 2** *Slopes of inclusive best replies obey* $0 < r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$*; i.e., actions are strategic complements and market shares decrease in* $\Psi$*.*

The lemma establishes that the slope Condition 1 holds.[9]

Whenever $\gamma \neq 0$, each platform chooses a larger action in response to an increase of the aggregate; however, their relative contribution to the aggregate declines. An increase in the aggregate means that competition is relaxed. For $\gamma > 0$, platform $i$ then chooses a larger advertising level closer to the monopoly level. For $\gamma < 0$, it chooses a smaller advertising level closer to the monopoly level. With respect to the viewer demand, competition in ad levels plays out similar to price competition in standard oligopoly models for $\gamma > 0$, whereas it is similar to quality competition for $\gamma < 0$; both cases exhibit strategic complementarities.

**Proposition 1** *There exists a unique equilibrium. In equilibrium, (7) holds for all platforms* $i$*.*

**Proof.** First note that we can restrict attention to ad levels $a_i \in [0, a^m]$ for $\gamma > 0$ because $a^m$ dominates any higher ad level.[10] Similarly, $a_i \in [a^m, \overline{a}]$ for $\gamma < 0$, where $\overline{a}$ solves $p(\overline{a}) = 0$ (see Assumption 3) because $a^m$ dominates any lower ad level, and the platform will never set a higher ad level than $\overline{a}$, as this would lead to zero revenues.

Under the monotone transformation $\psi_i = h(s_i - \gamma a_i)$, $\psi_i$ is chosen from $[h(s_i - \gamma a^m), h(s_i)]$ for $\gamma > 0$ and from $[h(s_i - \gamma a^m), h(s_i - \gamma \overline{a})]$ for $\gamma < 0$. Thus, the sum of

[9] The slope condition $r_i' < r_i/\Psi$ also implies that the second-order condition $d^2\Pi_i/d\psi_i^2 < 0$ holds.

[10] Both revenue per viewer and number of viewers would be lower for $a > a^m$.

inclusive best replies $\sum_{i=0}^{n} r_i(\Psi)$ is defined on $[\max_{i\in\{1,\ldots,n\}} h(s_i - \gamma a^m), \sum_{i=1}^{n} h(s_i) + \psi_0]$ for $\gamma > 0$, and on $[\max_{i\in\{1,\ldots,n\}} h(s_i - \gamma a^m), \sum_{i=1}^{n} h(s_i - \gamma \overline{a}) + \psi_0]$ for $\gamma < 0$.[11]

The sum of inclusive best replies function $\sum_{i=1}^{n} r_i(\Psi) + \psi_0$ maps from a compact interval into itself. Since $r_i(\Psi)$ for all $i$ is continuous in $\Psi$, there must exist a solution to $\psi_0 + \sum_{i=1}^{n} r_i(\Psi) = \Psi$ and, therefore, an equilibrium exists. Furthermore, since by Lemma 2 $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$(i.e., the slope Condition 1 holds), the sum of inclusive best replies has slope less than 1 in any equilibrium, and thus has to cross the diagonal from above. Hence, the equilibrium is unique. ■

In asymmetric markets, the pattern of platform characteristics $s_i$ matters for equilibrium levels. We characterize how the relative position of platforms with respect to their characteristic $s_i$ translates into their relative position with respect to market share $\lambda_i$ and advertising level $a_i$. The next (cross-section comparison) result describes economic outcomes when the only difference between media platforms is their content quality (in particular, no joint ownership or cross share-holdings). It shows that platforms' market shares follow the same ranking. Ad levels follow the same ranking for $\gamma > 0$,[12] and the opposite one for $\gamma < 0$.

**Proposition 2** *Consider any two platforms $i$ and $j$. For $\gamma > 0$, $s_i > s_j$ implies in equilibrium that $\lambda_i > \lambda_j$, $a_i > a_j$, and $\Pi_i > \Pi_j$. For $\gamma < 0$, $s_i > s_j$ implies in equilibrium that $\lambda_i > \lambda_j$, $a_i < a_j$, and $\Pi_i > \Pi_j$. ($s_i = s_j$ implies in equilibrium that $\lambda_i = \lambda_j$, $a_i = a_j$, and $\Pi_i = \Pi_j$.)*

**Proof.** We first show that $\lambda_i > \lambda_j$ if and only if $\gamma a_i > \gamma a_j$. The proof is by contradition. $\lambda_i > \lambda_j$ is equivalent to $\psi_i > \psi_j$, which, since $h$ is strictly increasing, is

[11] The max operator here ensures that $r_i(\Psi) < \Psi$ for all $i$ on the interior of the intervals.

[12] In the special case where $h$ is log-linear, Anderson (2012) has shown that higher quality implies higher ad levels.

equivalent to $v_i > v_j$. Using (7), the inequality $\psi_i > \psi_j$ is equivalent to

$$\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)} < \frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))}\frac{1}{\gamma}\frac{h(v_j)}{\gamma h'(v_j)} \tag{8}$$

Recall that along the best reply $R'/R$ has the same sign as $\gamma$ and that $h(v)/h'(v) > 0$. Thus, both sides are positive. The strict log-concavity of $R$ implies that $R'/R$ is strictly decreasing. Hence, when, by contradiction, $\gamma a_i < \gamma a_j$,

$$\frac{1}{\gamma}\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))} > \frac{1}{\gamma}\frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))}.$$

Thus, for (8) to be satisfied, we must have

$$\frac{h(v_i)}{h'(v_i)} < \frac{h(v_j)}{h'(v_j)}.$$

The log-concavity of $h$ in $v$ then implies that $h/h'$ non-decreasing and, hence, $v_i < v_j$, which is a contradiction.

Therefore, $v_i > v_j$ if and only if $a_i > a_j$ for $\gamma > 0$, whereas $v_i > v_j$ if and only if $a_i < a_j$ for $\gamma < 0$. Using the definition of $v_i$, since $v_i > v_j$ and $a_i > a_j$, we must have $s_i > s_j$ for $\gamma > 0$ and, since $v_i > v_j$ and $a_i < a_j$, we must again have $s_i > s_j$ for $\gamma < 0$. The result that $s_i = s_j$ implies that $\lambda_i = \lambda_j$ and $a_i = a_j$ is obvious.

Because each platform chooses its ad level in the increasing part of $R$ for $\gamma > 0$ and in the decreasing part of $R$ for $\gamma < 0$, $s_i > s_j$ implies that $R(a_i) > R(a_j)$. As a higher-quality platform also has more viewers, $s_i > s_j$ implies that $\Pi_i > \Pi_j$. ■

For $\gamma = 0$, each platform would set its ad level at the monopoly solution $a^m$ which solves $R'(a) = 0$. When ads are a nuisance ($\gamma > 0$), platforms set, in equilibrium, $a_i < a^m$ as they compete for viewers. The proposition establishes that in this case high-quality platforms carry more ads than lower-quality platforms, but are still more attractive such that they attract more viewers than lower-quality platforms despite the higher nuisance ($v_i > v_j$ if $s_i > s_j$). This finding is analogous to price competition models with horizontal product differentiation and quality differences between firms:

a high-quality firm sets a higher price and obtains a larger market share than a low-quality firm – this, for instance, holds in the Hotelling model (see Anderson and de Palma, 2001, for such a result for $n$-firm oligopoly). When viewers like ads, platforms choose ad levels that exceed optimal ad level for fixed viewer demand, $a^m$. Then a higher-quality platform chooses its ad level closer to the monopoly level than a lower-quality platform ($a_i < a_j$ if $s_i > s_j$).

Advertisers with a high willingness to pay advertise on all platforms. Advertisers with a rather low willingness to pay advertise on few platforms, if at all, and they advertise on high-quality platforms for $\gamma > 0$ and low quality ones for $\gamma < 0$.

## 4.2 Comparative statics

In this sub-section, we first deliver the analytical background needed to determine the comparative static results. Then we apply these methods to entry and mergers respectively.

**Consumer surplus and platform profits**

The key is that equilibrium values depend on the aggregate $\Psi$, and so we need to determine this relation for the various variables of interest. The first result is immediate from (3): $CS = \mu \ln \left(\sum_{i=0}^{n} h(v_i)\right) = \mu \ln \Psi$, where we used the definition of the aggregate, and hence

**Lemma 3** *Consumer surplus $CS$ is an increasing function of the aggregate $\Psi$.*

The monotonicity of $CS$ implies that comparative statics results on consumer surplus immediately follow from changes in the aggregate $\Psi$.

To evaluate the effect of policy interventions, we have to understand how market shares $\lambda_i = \psi_i/\Psi$ depend on the aggregate. Suppose that we compare two situations with two different aggregates. We call "outsiders" all those platforms whose inclusive best reply function are the same in both situations; i.e., the exogenous change or policy intervention has no effect on the outsiders' payoff function. By contrast, we call "insiders" those platforms whose inclusive best reply functions are shifted. A platform either belongs to the group of insiders, $i \in I$, or of outsiders, $i \in O$.

We first recall from Lemma 2 that outsiders' market shares decrease with $\Psi$. This effect is reinforced by higher equilibrium actions, and hence lower profits, as the next result establishes.

**Lemma 4** *A change that induces an increase in the aggregate $\Psi$ leads to lower platform profits for each outsider media platform, i.e., $d\Pi_i^*/d\Psi < 0$ for all $i \in O$.*

**Proof.** The profit change is $\frac{d\Pi_i^*}{d\Psi} = \frac{d\lambda_i}{d\Psi}R + \lambda_i \frac{dR}{d\Psi}$. By Lemma 2, the first term on the right-hand side is negative. Now write out the term $\frac{dR}{d\Psi} = R'(a)\, a'(\psi_i) r_i'(\Psi)$. Because $R'(a)$ and $a'(\psi_i)$ have opposite signs,[13] and $r_i'(\Psi) > 0$, then $\frac{dR}{d\Psi} < 0$ and the claim follows. ■

The effects on insiders depend on the particular exogenous variation or policy intervention. In what follows, we consider three such types: entry, media mergers, and advertising regulation and provide results on platform profits and consumer surplus. As results on industry advertiser surplus do not directly follow from changes of the aggregate we defer results on them to Section 5.

**Entry of Media Platforms**

We consider (exogenous) entry of a media platform; such exogenous entry may be the outcome of regulatory measures, e.g. by granting an additional license.[14] As illustrated by Figure 1 (where the superscript $N$ refers to the new situation, with entry), due to entry, the new firm's inclusive best reply shifts the sum of inclusive best replies upward. This implies that the equilibrium aggregate is larger after entry. The

[13] For $\gamma > 0$, $R' > 0$ in equilibrium because platforms carry less advertising than $a^m$, while $a'(\psi_i) < 0$ because ads are a nuisance. Contrarily, for $\gamma < 0$, $R' < 0$ in equilibrium, while $a'(\psi_i) > 0$.

[14] Our result with exogenous entry also translates into a setting with endogenous entry where, at a prior entry stage, firms decide whether to pay an entry cost to enter the market. A lower entry cost or an increase in the total mass of potential viewers then leads to entry.

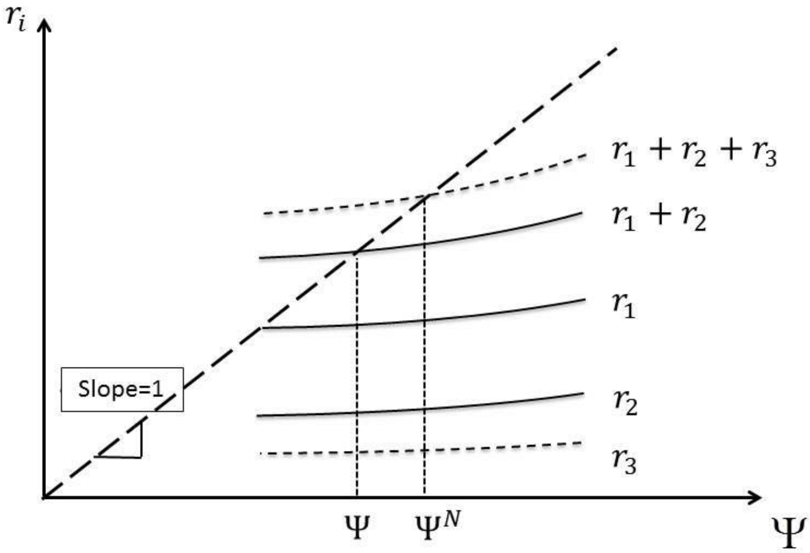


Figure 1: Entry and equilibrium aggregate

aggregative game framework delivers crisp results on the comparative static results for effects on consumer surplus and other platforms' profits.

**Proposition 3** *The entry of an additional platform*

1. *decreases other platforms' profits,*

2. *increases consumer surplus.*

**Proof.** The new platform $n+1$ has an inclusive best reply $r_{n+1}(\Psi) > 0$. Hence, the aggregate $\Psi$ goes up (for illustration, see Figure 1). Consumer surplus increases from Lemma 3. By strategic complementarity, $\psi_i$ increases for $i = 1, ..., n$: because all rivals' $\psi_j$ increase, platform $i$'s profit must decrease ($i \neq n+1$) and the first statement holds. ■

The opposite directions for profits of existing firms and consumer surplus are standard: what is new to the two-sided market case is what happens to the other platform participants, the advertisers. Entry of an additional platform decreases advertising on other platforms for $\gamma > 0$ and increases it for $\gamma < 0$: the effects on advertiser surplus are deferred to the next section.

Regarding platform industry profits, there is a tension between lower profits of the existing platforms $i = 1, ..., n$ and profits of the entering firm. Industry profits tend to increase with entry if platforms are poor substitutes and decrease if they are close substitutes. Whether or not industry platform profits increase with entry will turn out to be critical to evaluate the change of industry advertiser surplus in the following section.

**Media Mergers**

Media mergers have received quite some attention in the policy debate. Here, we explore the allocative effects of an exogenous media merger and its welfare implications in our model. Superscript $M$ refers to the new situation, after the merger.[15]

**Lemma 5** *The inclusive best reply of each merging platform is shifted downward by a merger. Hence a merger of two media platforms leads to a decrease in the aggregate.*

**Proof.** The merged entity of platforms $i$ and $j$ maximizes joint profits $\Pi_i(\psi_i, \Psi) + \Pi_j(\psi_j, \Psi)$.[16] The first-order condition regarding platform $i$ then becomes (see Anderson, Erkal, and Picinnin, 2013)

$$\frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \psi_i} + \frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \Psi} + \frac{\partial \Pi_j(\psi_j, \Psi)}{\partial \Psi} = 0.$$

[15] The Lemma also applies under two-sided pricing, as considered in Section 6.

[16] Or indeed, a cooperative venture or other coordination between agents.

The two first-order conditions can be solved simultaneously to find $\psi_i$ and $\psi_j$ as functions of the aggregate, giving $r_i^M(\Psi)$ and $r_j^M(\Psi)$ as the individual inclusive best reply functions under merger. The last term on the left-hand side of both first-order conditions is negative (by the competitiveness property). This implies that the inclusive reply function $r_i^M$ must take a lower value than before the merger; i.e., $r_i^M(\Psi) < r_i(\Psi)$ for all $\Psi$, and likewise for the other platform $j$. Therefore, the sum of the inclusive best replies falls, and the aggregate must be lower with the merger. ■

The next result delivers the effects of merger on platforms and consumers.

**Proposition 4** *The merger of two platforms*

1. *is profitable, and increases other platforms' profits too,*

2. *decreases consumer surplus,*

**Proof.** As per Lemma 5, the equilibrium aggregate goes down. Consumer surplus decreases, as per Lemma 3. Outsider platforms' actions decrease by strategic complementarity,[17] and their profits increase, as per Lemma 4. Profit of the merged platform increases because competitors' actions decrease and the merged platforms now jointly best reply to these. ■

Thus, similar to mergers in price competition models with differentiated products (see Deneckere and Davidson, 1985), a merger is always profitable and industry platform profits must increase. Merger increases advertising on all platforms for $\gamma > 0$, but decreases it for $\gamma < 0$.

# 5 Media See-saws

We have shown so far that consumer surplus and profits move in different directions in response to the changes we have considered. Our key question is which way advertiser

[17] So too do the merged parties' actions (by strategic complementarity and because their inclusive best replies moreover shift down).

surplus moves.

We recall that net advertiser surplus is $IAS = \sum_{i=1}^{n} \lambda_i AS(a_i)$. We first show that this can be written as a function of $\Psi$. The advertiser surplus on platform $i$ is

$$\lambda_i AS(a_i) = \lambda_i \int_0^{a_i} (p(x) - p(a_i))dx,$$

where $\lambda_i = \psi_i/\Psi = r_i(\Psi)/\Psi$ (because media platforms choose actions as functions of the aggregate $\Psi$). By inversion, $a_i$ can be written as a function of $\psi_i$. Hence net advertiser surplus is a function of the aggregate, and in the sequel we exploit this functional relationship to determine the consequences of changes.

For each platform, we know that a larger aggregate leads to a larger action $\psi_i = r_i(\Psi)$ and, thus, for $\gamma > 0$, a lower advertising level. Then, a larger $\Psi$ would always lead to a lower net advertiser surplus if each $\lambda_i$ did not change (or went down). However, the total market base (the sum of the $\lambda_i$'s) typically goes up with changes that raise $\Psi$. This argument suggests that a see-saw is at play when advertising is a nuisance and the market base expansion effect is weaker than the increased competition effect that decreases ad levels. In such cases, which, as we shall argue, constitute the norm for $\gamma > 0$, a larger value of the aggregate increases consumer surplus, but decreases advertiser surplus. We have to formally establish this see-saw by taking into account changing market shares $\lambda_i$ and, more subtly, changing ad levels on different platforms.

Some headway can be made for simple cases by evaluating changes of advertiser surplus per consumer and changes in the composition of consumers across platforms. However, we are able to obtain broader results by linking changes in advertiser surplus on a platform to changes in profits.

## 5.1 Entry of Media Platforms

We index the original $n$ platforms such that $s_i \geq s_{i+1}$, $i = 1, ..., n-1$. We suppose that entry is efficient in the sense that an increase in the number of platforms in the industry means that the most efficient among potential firms enters, while it is of lower quality than the incumbents.

To establish a see-saw, we have to show that industry advertiser surplus is decreasing with entry. If the market is fully covered this is easily argued. Here industry advertises surplus decreases with platform entry due to downshifting of consumers to the lowest quality platform, which generates a lower ad surplus per consumer.

However, with partial coverage, there is the countervailing benefit from market expansion. On the one hand, because $r_i' < r_i/\Psi$ (by Lemma 2), all original platforms lose market share to the new platform, as argued above, but now the overall market coverage increases. On the other hand, since competition among platforms becomes stronger with entry, ad levels decrease. While increased coverage is good for advertisers, lower ad levels are bad. Thus, it is *a priori* unclear whether advertisers benefit or suffer from entry. In asymmetric oligopoly it appears *a priori* even less clear what will happen, as platforms differ in the advertiser surplus per viewer they generate. Nonetheless, we are able to provide a simple and intuitive sufficient condition, the proof of which engages the characterization structure of the model (that higher quality platforms set higher ad levels, Proposition 2) and the regularity condition of Assumption 1 (log-concavity) on the advertiser demand function, which enables us to bound advertiser surplus changes from profit changes (by applying Lemma 1).

As the next proposition establishes, industry advertiser surplus decreases with entry if additional entry reduces industry platform profits; i.e., $\sum_{i=1}^{n} \lambda_i R(a_i) > \sum_{i=1}^{n+1} \lambda_i^N R(a_i^N)$, where the superscript $N$ refers to the new situation, with entry.

**Proposition 5** *For $\gamma > 0$, the entry of an additional platform $s_{n+1} \leq s_n$ decreases* *net advertiser surplus if entry reduces platform industry profits.*

**Proof.** We have to show that

$$\sum_{i=1}^{n} \lambda_i AS(a_i) > \sum_{i=1}^{n+1} \lambda_i^N AS(a_i^N). \tag{9}$$

The condition for entry to reduce platform industry profits can be written as

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] > \lambda_{n+1}^N R(a_{n+1}^N)$$

which says that the entrant's profit is smaller than the loss on other platforms. Equivalently,

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)} > \lambda_{n+1}^N R(a_{n+1}^N) \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)}. \tag{10}$$

From Proposition 2, we know that $a_i^N \geq a_j^N$ for all $i > j$. Applying Lemma 1,

$$\frac{AS(a_i^N)}{R(a_i^N)} \geq \frac{AS(a_{n+1}^N)}{R(a_{n+1}^N)}, \quad \text{for all } i = 1, ..., n.$$

In addition, platforms $i = 1, ..., n$ have lower profits after entry ($\lambda_i R(a_i) - \lambda_i^N R(a_i^N) > 0$ for all $i = 1, ..., n$). Thus inequality (10) implies

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) - \lambda_i^N R(a_i^N)\right] \frac{AS(a_i^N)}{R(a_i^N)} > \lambda_{n+1}^N AS(a_{n+1}^N). \tag{11}$$

From the analysis in Section 4.2 we also know that platforms $i = 1, ..., n$ choose lower advertising levels after entry; i.e., $a_i \geq a_i^N$. Thus, using Lemma 1, we must have

$$\frac{AS(a_i)}{R(a_i)} \geq \frac{AS(a_i^N)}{R(a_i^N)}.$$

Hence, inequality (11) implies

$$\sum_{i=1}^{n} \left[\lambda_i R(a_i) \frac{AS(a_i)}{R(a_i)} - \lambda_i^N R(a_i^N) \frac{AS(a_i^N)}{R(a_i^N)}\right] > \lambda_{n+1}^N AS(a_{n+1}^N). \tag{12}$$

Simplifying this expression, we obtain

$$\sum_{i=1}^{n}\left[\lambda_i AS(a_i) - \lambda_i^N AS(a_i^N)\right] > \lambda_{n+1}^N AS(a_{n+1}^N),$$

which is equivalent to inequality (9). ■

This proposition (combined with Proposition 3) establishes the see-saw under entry: consumers are better off, while advertisers are worse off as long as further entry reduces total platform industry profits.[18] Advertisers are then on the same "side" as the incumbent platforms, and the opposite side from consumers.

The see-saw holds under the sufficient condition that total platform industry profits should fall with entry. As background, one would usually expect total profits to be a hump-shaped function of the number of platforms. In a market with few firms and scarce market coverage, entrants are likely to have mild competitive and business-stealing effects. Conversely, if the market is close to fully covered already, the overall market expansion is very slight, and entry plays out in tougher competition in ad levels.[19] The latter case is when we should expect to see advertiser surplus go down – severe ad level reductions are not sufficiently offset by market expansion.

Proposition 5 clearly includes the case when platforms qualities are symmetric. In this case there are no cross-platform reallocations because of different ad levels across platforms to factor into the analysis. Proposition 5 allows for any pattern of platform asymmetries (modulo the proviso that the entrant is of no higher quality). A few words on the proof are in order. We express advertiser surplus per platform

[18]The condition is both necessary and sufficient when advertiser demand has constant elasticity. In that case, advertiser surplus per consumer is equal to a fixed fraction of revenue per consumer. Then, $IAS$ is a constant fraction of platform industry profit, so that $IAS$ rises (or falls) whenever total profits rise (or fall). While constant elasticity functions are not log-concave (they are "too convex"), the example illustrates the strong link between the two surpluses.

[19]These ideas can also be expressed in terms of model parameters. The lower is $\mu$, the more substitutable are platforms and the greater the reduction of competitor market share relative to market expansion, so that entry is likely to reduce industry profits. Coversely, the larger is $v_0$, the more attractive the outside option and the more likely it is that entry raises industry profit because the platforms are more strongly competing with the outside option and less so with each other.

summed over all platforms as platform profit times the ratio of advertiser surplus per platform to platform profits. The latter ratio is useful since Lemma 1 tells us that it is increasing in the ad level. We then use the result that higher-quality platforms have more ads, and that entry leads all platforms to reduce ad levels. This allows us to provide bounds on industry advertiser surplus. Then, the condition that industry profits decrease with entry implies that industry advertiser surplus also decreases.

The take-away is quite different if consumers like ads: for $\gamma < 0$, advertisers benefit from entry. Both of the impacts of entry bolster this conclusion. A larger consumer base, as the total consumer market expands, improves advertiser surplus. Moreover, the conflicting force in the $\gamma > 0$ case now works in the opposite direction: more competition increases ad levels for $\gamma < 0$, with concurrent increases in advertiser surplus per viewer, ceteris paribus. Thus, for $\gamma < 0$, consumer and advertiser welfare tend to be aligned: more advertisers tend to make more contacts. If the entrant is of lower quality it attracts some viewers from other platforms. For $\gamma < 0$ a lower quality is associated with a higher ad level. Hence, all effects work in the same direction: market coverage (weakly) rises, ad levels rise, and any diversion of demand to the new platform upshifts to higher ad-surplus per consumer. Therefore, advertiser surplus unambiguously increases with entry and there is no see-saw when consumers are ad-loving ($\gamma < 0$).

## 5.2 Media Mergers

Mergers induce two opposing effects on advertiser surplus when ads are a nuisance ($\gamma > 0$). While ad levels on platforms rise, market coverage falls (this holds since $\Psi$ is lower after the merger which boosts the market share of outside option $\psi_0/\Psi$). There are also shifts in platforms' relative market shares, which means that consumers may be shifted to platforms carrying more or fewer ads.

After a merger, each platform carries more ads and, thus, $AS(a_i)$ increases. Because the market share of each outsider media platform increases, as shown in Proposition 2, advertiser surplus associated to each outsider media platform must increase. However, the overall effect on the advertiser surplus associated with the merged entity is *a priori* unclear because $\lambda_i^M + \lambda_j^M < \lambda_i + \lambda_j$ after a merger between media platforms $i$ and $j$ (the merged platforms' combined base shrinks).

We can already give a preliminary analysis of the possibility of a see-saw by tracking how consumers switch platforms following a merger. Assume that the market is fully covered, in order to close down the effect of reduced overall market coverage. Suppose too that the merger involves the two lowest-quality media platforms and that their quality difference is small. (This latter stipulation ensures consumer reallocation goes towards platforms with more ads). Then merger increases advertiser surplus. To see this, first recall that the merged platforms $n$ and $n-1$ also feature more advertising after the merger than before. If both $\lambda_n$ and $\lambda_{n-1} + \lambda_n$ decrease after the merger, then all net shifts in consumers are shifts to platforms with more ads (since all other $\lambda$'s rise).[20] So it remains to show that $\lambda_n$ and $\lambda_{n-1} + \lambda_n$ decrease after the merger. The latter is a direct implication of Proposition 2 since the aggregate $\Psi$ is down and so all outsiders have a larger market share. The former necessarily holds if $s_n = s_{n-1}$ and, by continuity, for $s_n - s_{n-1}$ sufficiently small.

This see-saw result is of course very particular, but we cannot go much further by simply looking at the patterns of shifts, without drawing on some stronger restrictions that relate profit changes to advertiser surplus changes. Assumption 1 again provides just such a condition, and enables us to deploy Lemma 1 to bound advertiser surplus changes by insider profit changes. Recall that the merger is profitable, so total profit goes up on both insider platforms taken together. If profit goes up on each individually, then the Lemma tells us that advertiser surplus must go up. The possible

[20] Thus, market share needs to shift away from low-quality platforms towards high-quality platforms.

confound is when profit goes up on the weaker platform and down on the stronger one. But if it rises on the stronger one, the consumer reallocation effect works in the right direction. That is, we now get traction when $\lambda_j R(a_j) < \lambda_j^M R(a_j^M)$ for $s_j > s_i$, where $a_j^M$ denotes the advertising level after the merger. When this individual profitability condition does not hold, we recourse to a standard logit formulation (i.e., $h$ is log-linear) to show the result.

**Proposition 6** *For $\gamma > 0$, a merger of two platforms increases advertiser surplus if*

1. *the profit on the merged platform with higher quality increases, or*

2. *in the standard logit case.*

**Proof.** Proposition 4 shows that both insider and outsider platforms increase their profits. They also increase their ad levels. Because outsider market shares rise, advertiser surplus must increase on outsider platforms. The rest of the proof considers advertiser net surplus on insider platforms.

For part 1 we wish to show that a merger raises net advertiser surplus if profit on the merged platform with higher quality goes up. We distinguish two cases. First, suppose profit on each of the merged platforms goes up with the merger, so $\lambda_i^M R(a_i^M) > \lambda_i R(a_i)$, for $i \in I$. This inequality is equivalent to

$$\frac{AS(a_i^M)}{R(a_i^M)}\lambda_i^M R(a_i^M) > \frac{AS(a_i)}{R(a_i)}\lambda_i R(a_i).$$

Since by hypothesis $\lambda_i^M R(a_i^M) > \lambda_i R(a_i)$ for $i \in I$, this inequality is implied by $AS(a_i^M)/R(a_i^M) > AS(a_i)/R(a_i)$, for $i \in I$. Since $a_i^M > a_i$ and, by Lemma 1, $(d(AS(a)/R(a))/da \geq 0)$, advertiser surplus increases more strongly than revenue on each platform.

Second, suppose that profit of the platform $j$ of higher quality goes up, while profit of the platform $i$ of lower quality in the merger goes down; i.e., $\lambda_i R_i > \lambda_i^M R_i^M$

for $s_i < s_j$. We know that prior to the merger $a_j > a_i$. This ordering is preserved after the merger; i.e., $a_j^M > a_i^M$. In addition, we know that the merger increases advertising on each of the merging platforms; i.e., $a_j^M > a_j$ and $a_i^M > a_i$. Since the merger increases joint profits of the merged platforms, by rearranging and multiplying by $AS_j^M/R_j^M$, we obtain the inequality

$$(\lambda_j^M R_j^M - \lambda_j R_j)\frac{AS_j^M}{R_j^M} > (\lambda_i R_i - \lambda_i^M R_i^M)\frac{AS_j^M}{R_j^M}.$$

Since $a_j^M > a_i$, Lemma 1 implies that $AS_j^M/R_j^M > AS_i/R_i$, so that

$$(\lambda_j^M R_j^M - \lambda_j R_j)\frac{AS_j^M}{R_j^M} > (\lambda_i R_i - \lambda_i^M R_i^M)\frac{AS_i}{R_i}.$$

Since $a_j^M > a_j$ and $a_i^M > a_i$, Lemma 1 implies respectively that $AS_j^M/R_j^M > AS_j/R_j$ and $AS_i^M/R_i^M > AS_i/R_i$, thus implying:

$$\lambda_j^M R_j^M \frac{AS_j^M}{R_j^M} - \lambda_j R_j \frac{AS_j}{R_j} > \lambda_i R_i \frac{AS_i}{R_i} - \lambda_i^M R_i^M \frac{AS_i^M}{R_i^M}.$$

Simplifying and rearranging gives that $\lambda_i^M AS_i^M + \lambda_j^M AS_j^M > \lambda_i AS_i + \lambda_j AS_j$, as desired.

The proof of part 2 of the proposition where $h$ is log-linear is more involved and provided in Appendix 1. ■

The idea of the proof of part 1 is quite simple when profit of each of the merged platforms increases. We argued already at the start of this sub-section that advertiser surplus on outsider platforms necessarily rises. When profits also go up on *each* insider platform, then so does advertiser surplus.[21] When the merging parties have the same quality, this profit condition holds by symmetry.[22]

[21] For constant elasticity advertiser demands mentioned earlier, the result is even sharper. Because total advertiser surplus is proportional to profit, then advertiser surplus must increase because total platform profits rise with any merger.

[22] By continuity, it also holds when the qualities are not too different, so that Proposition 6(1) nests the analysis at the beginning of the present subsection.

While we might usually expect profits to go up for each party to a merger, this property may not be true if they are sufficiently asymmetric. However, our result continues to hold when the merger promotes the higher-quality platform at the expense of the other. Proposition 6(2) establishes a see-saw absent any profit condition under a specific functional form for $h$ (which corresponds to the standard logit model). The proof here exploits the property (which we establish) that the ad level for both parties to a merger is set at the same level in this case.

As we argue next, our results extend naturally to mergers with synergies as long as these synergies are not too pronounced. We consider synergies in the form of higher $s_i$ of the merged parties. As a borderline case, the merger is consumer surplus-neutral. Considering consumer-surplus neutral mergers simplifies the analysis because consumer surplus and advertiser surplus on outsider platforms is not affected. A particular case is where $\psi_i$ of each merged party is the same before and after the merger. For $\gamma > 0$, this implies that the merged parties increase their advertising $a_i$ so as to leave their action $\psi_i$ unchanged. Consequently, $\Psi$ is the same before and after the merger. However, since $a_i$ has increased on the merged platform, advertiser net surplus on this platform has gone up.

Considering only mergers which do not change the $\psi_i$ between the merged platforms is restrictive. For $\gamma > 0$, consider now a consumer surplus-neutral merger after which the content quality of both parties involved in the merger weakly increases. If, after the merger, $s_i^M > s_j^M$ of the merged platforms $i$ and $j$, then $a_i^M > a_j^M$ and $\lambda_i^M > \lambda_j^M$. Since the merger is consumer surplus-neutral, $\psi_i + \psi_j = \psi_i^M + \psi_j^M$. This implies that outside the special case considered above, after the merger either one of the platforms has a larger market share than before the merger. A sufficient condition for the merger to be advertiser surplus increasing is (i) each platform chooses a larger advertising level than the maximal advertising level among those two platforms prior to the merger – i.e., $\min\{a_i^M, a_j^M\} > \max\{a_i^M, a_j^M\}$ – or (ii) the higher-quality platform $i$ (with $s_i > s_j$) gains market share as a result of the merger – i.e., $\psi_i^M - \psi_i$ is positive whenever $s_i > s_j$ prior to the merger.

For mergers with lower synergies than those required for a consumer surplus neutral merger, there is a see-saw: consumers are worse off after the merger, while advertisers and platforms are better off.[23]

Finally, we consider the case where consumers like ads ($\gamma < 0$). A merger in this case decreases the aggregate and decreases ads on all platforms, with the total market shrinking. Both these indicators point to less advertiser surplus. For example, with symmetric platform qualities, there can be no possible advantageous reallocation of consumers toward platforms with higher advertiser surplus, so that there is no see-saw: platforms gain while consumers and advertisers lose.[24]

### 5.3 Advertising Regulation

Many countries limit the amount of advertising allowed on TV (e.g., EU Directive 97, with local ordinances in addition). Such regulation may benefit consumers when ads annoy consumers ($\gamma > 0$). However, it may negatively affect advertiser surplus. As we show, ad caps help consumers at the expense of advertisers and platforms through the see-saw effect.

The aggregative game approach, together with the log-concavity of advertiser demand, provides a clean way to analyze the effect on ad levels and consumer surplus of advertising regulation. Because $\psi_i = h\left(s_i - \gamma a_i\right)$, an ad cap constitutes a *floor* to the inclusive best reply function, and therefore renders the inclusive best reply flat for

[23] Clearly, the merged firm is better off after the merger. Since the aggregator is less than before the merger, also profits of outsider platforms are higher after the merger.

[24] To construct a case when advertiser surplus can rise with merger is a challenge, but consider the following. Suppose that the market were fully covered, and the merger involved two platforms of the highest quality. Then, it would have to be the case that sufficient numbers of consumers (sufficient to offset the lower ad levels everywhere) are diverted away from the merging parties and towards those other platforms, which carry more ads.

low levels of $\Psi$ up to the point where the cap no longer binds (i.e., at high enough $\Psi$, recalling that actions are strategic complements). Such a floor is depicted in Figure 2. The larger platforms (those with highest $s_i$) are the most affected because they are the ones that would otherwise choose the highest ad levels. In equilibrium, we may then have a mix of large, ad capped platforms and smaller, non-constrained ones (the reverse cannot happen). The floor induced by the ad-cap thus increases the inclusive best reply function. Consequently, if the cap is binding for at least one platform, the aggregate rises. Due to strategic complementarity, the equilibrium actions ($\psi_i$) of the non-constrained platforms must increase. This means that their ad levels decrease due to tougher competition for consumers. Because all platforms reduce advertising levels, consumers are necessarily better off whenever binding advertising caps are introduced (as is also seen by applying Lemma 3).

A priori, the effect of an advertising cap on advertiser surplus is far from clear. While advertisers are directly hurt by the cap (because it reduces ad levels and raises ad prices), a cap on the largest platform leads to an increase in that platform's consumer base in equilibrium.[25] The total consumer base also rises so that while advertiser surplus per viewer decreases on each platform there are more viewers in total and on the platform with the largest ad level in particular.

**Proposition 7** *The introduction of advertising caps*

1. *decreases all platforms' profits;*

2. *increases consumer surplus;*

3. *decreases advertiser surplus.*

[25] To see this, $\Psi$ rises and all unconstrained platforms' market shares decrease (by Lemma 2), as does the market share of the outside option. The capped platform's market therefore rises as does total viewership.

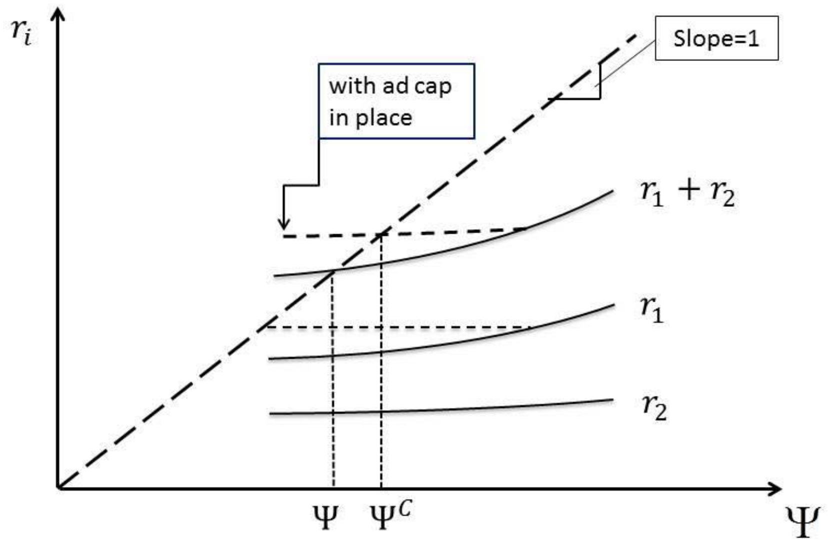


Figure 2: Ad cap regulation and equilibrium aggregate

**Proof.** Consider a cap that only binds on the highest-quality platform (for illustration, see Figure 2). The upward shift of its inclusive best reply leads to a larger equilibrium $\Psi$, and part 2 of the proposition follows. By the strategic complementarity result in Lemma 2, all unconstrained platforms' equilibrium actions $\psi_i$ rise and, therefore, their advertising levels fall in concert (hence ad levels decrease on all platforms). Moreover, by the slope result in Lemma 2, their market shares $\psi_i/\Psi$ fall.[26] Therefore, both profits and advertiser surplus on all uncapped platforms decreases. Moreover, the profit on the capped platform also decreases (despite the fact that its market share rises): the ad cap reduces its profit for given $\Psi$ and the rise in $\Psi$ further reduces its profit. This proves part 1.

Finally, consider advertiser surplus on the capped platform. Let superscript $C$ denote equilibrium values when advertising regulation is in place. For the ad-capped platform, we want to show that $\lambda^C AS(a^C) < \lambda AS(a)$.

This is equivalent to

$$\lambda^C R(a^C)\frac{AS(a^C)}{R(a^C)} < \lambda R(a)\frac{AS(a)}{R(a)}.$$

This is true because profit falls, $\lambda^C R(a^C) < \lambda R(a)$, and because $AS(a^C)/R(a^C) < AS(a)/R(a)$ by Lemma 1 given $a^C < a$. The argument extends to ad caps that affect multiple platforms. ■

While non-discriminatory ad caps necessarily affect the highest-quality platform, our proof applies for an ad cap imposed on any platform (or group of platforms). Thus, our result also holds for discriminatory ad caps on specific platforms. Such discriminatory ad caps often apply for public service broadcasters which are subject to more severe ad caps than their rivals. Advertising regulation delivers an unambiguous see-saw when ad caps apply to the public service broadcaster, but not to private

[26] Following the ad cap, outsider platforms take the hit in terms of reducing both "price" and "quantity" dimensions of profit: they reduce both ad levels and shares.

ones. Lowering the ad cap for public broadcasters (or imposing zero ads, such as on the BBC) leads to an increase of the aggregate and is, therefore, consumer-surplus increasing. Advertiser net surplus necessarily falls in the industry because all platforms reduce ad levels and more viewers watch public channels, which provide lower advertiser surplus after the cap is lowered. Hence, the see-saw holds for advertising regulation of public broadcasters.

# 6 Two-sided pricing

So far we analyzed ad-financed media platforms. Other media and trading platforms have revenues both from advertising and from subscription. Such platforms have two-sided pricing as their business model. Then, viewers are exposed to advertising and have to pay a subscription fee $f_i$ (which we allow to be negative) to subscribe to platform $i$.

We contend that see-saws have less currency in such an environment, the reason being that two-sided pricing uncouples the advertising decision from the equilibrium market share. In the following, we sketch the argument: full details are found in Appendix 2.

The viewer choice model is the same as in the previous setting except that we now include subscription pricing by writing market shares as $\lambda_i = \frac{h(v_i)\exp\{-f_i/\mu\}}{\sum_j h(v_j)\exp\{-f_j/\mu\}}$.[27] Each viewer generates revenues $R(a_i) + f_i$. Thus, the profit of platform $i$ is

$$\Pi_i = (R(a_i) + f_i) \frac{h(v_i)\exp\{-f_i/\mu\}}{\sum_j h(v_j)\exp\{-f_j/\mu\}} \tag{13}$$

We can treat the profit-maximization problem of each platform in two steps.[28] We define actions $\psi_i = h(v_i)\exp\{-f_i/\mu\}$, with the corresponding aggregate as $\Psi = \sum_{i=0}^{n} \psi_i$ and $\psi_0 = h(v_0)$. For any choice of action level $\psi_i$, platform $i$ determines the

[27] This can be derived by writing the utility (2) from choosing platform $i$ as $u_i = \ln h(v_i) - f_i + \varepsilon_i$.
[28] See also Anderson and Coate (2005) and Armstrong (2006).

price structure; i.e., the composition choice of $a_i$ and $f_i$, by maximizing $(R(a_i) + f_i)$ subject to $h(v_i)\exp(-f_i/\mu) = \psi_i$. Assuming that $R(.)$ is concave delivers a unique solution $\bar{a}_i$ such that

$$R'(\bar{a}_i) = \mu\gamma\frac{h'(\bar{v}_i)}{h(\bar{v}_i)}, \tag{14}$$

where $\bar{v}_i = s_i - \gamma\bar{a}_i$. The right-hand side increases in $a_i$ and the left-hand side decreases under the assumption that $R$ is concave, so that the unique solution is independent of the price $f_i$ and the decisions of other platforms. Moreover, with $h(.)$ strictly log-concave, ad levels increase (decrease) with $s_i$ for $\gamma > 0$ ($\gamma < 0$). We have therefore that $\psi_i = h(\bar{v}_i)\exp\{-f_i/\mu\}$, so that $\psi_i$ is a decreasing function of $f_i$ and so can be used as the action variable in the aggregate game.

The main plank for our contention that advertiser surplus and consumer surplus tend to be aligned starts from a couple of key properties. First, equilibrium ad levels are independent of market structure, as noted above. Second, the characterization results of Proposition 2 still hold, so that higher qualities garner higher equilibrium market shares (along with higher equilibrium ad levels for $\gamma > 0$, given the remark in the previous paragraph).

With induced changes in ad levels effectively off the table, the effects of market structure changes are now quite straightforward. Consumer surplus and profit changes are as before, which should not be too surprising. Advertiser surplus changes are now solely directed by changes in market shares, with the wrinkle again that consumers might be reallocated to platforms with higher advertiser surplus. Note that if $h$ is log-linear, by (14) all platforms carry the same ad levels (and this is true after mergers too), so that surplus simply follows total market coverage (this is true regardless of the sign of $\gamma$): in this case advertiser surplus and consumer surplus are fully aligned for entry and mergers.

Platform entry causes an overall expansion in market coverage, so the only possible offset (for $\gamma > 0$) to an increase in advertiser surplus is if the entering platform has lower quality.[29] We conclude that there is no see-saw for entry with symmetric qualities. Merger has the opposite effects and conclusions are analogous: with symmetric qualities, there is no see-saw.

Consumer and advertiser interests (perhaps surprisingly) also tend to be aligned under ad caps: both groups suffer from binding caps. The reason why consumers are worse off (despite aversion to ads) comes from platforms increasing their subscription prices. An ad cap makes the platform that is subject to this regulation become less aggressive for market share (a downward shift of the inclusive best reply – contrast the case of the ad-finance model), as each consumer becomes less valuable on the advertiser side. Hence, for given actions of non-constrained platforms, it offers a worse deal to consumers. By strategic complementarity, all other platforms increase their subscription fees too. Here the regulation of one "price" (the higher ad price that supports the lower ad level) affects the other price, namely the subscription fee: the higher ad price induces a higher consumer price. This is an instance of a "waterbed effect" where the utility loss from the induced higher subscription fee dominates the reduction of the ad nuisance.[30] This effect is so strong that consumers are actually worse off after the regulatory intervention. Advertisers tend to be worse off, as the capped platform delivers fewer ads and fewer consumers participate. If the non-capped platforms have weakly fewer ads than the capped platform advertisers are necessarily worse off. However, industry platform profits rise. Binding ad caps mean that at least some platforms are constrained in their use of instruments in extracting revenues. This is an instance where limiting the use of one strategic variable increases

[29] A decrease in advertiser surplus happens with a lower-quality entrant entering a fully covered market where $h(.)$ is strictly log-concave.

[30] The waterbed effect has been prominent in the debate on regulatory interventions in telecommunications markets. Genakos and Valletti (2011, 2015) find empirical evidence in support of the waterbed effect in mobile telecommunications markets.

industry profits to the detriment of both sides of the market.

# 7 Conclusion

Media platforms cater to two distinct audiences, advertisers and viewers-cum-consumers. Advertisers care about reaching viewers, while the utility of viewers is affected by the amount of advertising carried by the media platform of their choice. We present a multi-platform model in which consumers make discrete choices among media platforms and an outside option, and advertisers can advertise on multiple platforms. We find that markets with ad-financed media where advertising annoys viewers exhibit see-saws: market changes that increase consumer surplus reduce advertiser surplus and vice versa. In particular, entry benefits consumers, but tends to hurt advertisers, while a media merger reduces consumer surplus but tends to benefit advertisers. These see-saws mostly disappear when consumers are ad lovers or when platforms also charge viewers directly and so engage in two-sided pricing.

The results on advertiser surplus are the most intricate ones. Entry has two opposing effects. It increases total consumer participation, which is beneficial for advertisers, but it leads to less advertising on each platform, which hurts advertisers. The overall effect is necessarily negative for advertisers if platform industry profit decreases with entry (of a lower-quality platform). Media mergers necessarily increase advertiser surplus in the standard logit case. In the more general setting studied in this paper, media mergers increase advertiser surplus if the profit of the higher-quality platform that is part of the merger increases with the merger (which condition necessarily holds if the merging platforms are symmetric).

See-saws are also present when ad caps are used. In markets with ad-financed media platforms, advertisers suffer from lower advertising levels on all platforms. However, a mitigating effect is that the ad-capped platforms gain market share, which benefits advertisers because ad-capped firms carry more ads than firms for which the cap is not binding. We show that this latter effect is dominated, and so advertising regulation exhibits a see-saw. The see-saw disappears when platforms use two-sided pricing. While ad caps per se are in the interest of consumers, they reduce platforms' incentives to attract viewers. Then, they increase subscription fees with the overall effect that consumers suffer, as well as advertisers.

Our results immediately carry over to other two-sided markets. For instance, suppose that platforms decide on how many sellers to host and consumers obtain part of the gains from trade in the interaction with sellers. This setting corresponds to when consumers enjoy advertising. Our analysis then covers both business models in which only sellers pay, and those in which the platform charges consumers for participation. Competing shopping malls furnish one example; electronic market places which host shops in different product categories are another.

We have concentrated in this paper on situations where media consumers choose at most one media outlet to watch (or read, or listen to). This "single-homing" assumption gives rise to a "competitive bottleneck" situation (Armstrong, 2006) whereby each platform is the only conduit for reaching its consumers, while advertisers "multi-home," and therefore competition is primarily for viewers. See-saws seem unlikely to arise when there is a significant amount of consumer multi-homing. In such situations, competition for advertisers takes a stronger role. Ambrus, Calvano and Reisinger (2014) and Anderson, Foros, and Kind (2015) emphasize the ability of platforms to deliver exclusive viewers, and charge advertisers more for them than for viewers delivered by multiple platforms. Then, they argue that merger may raise prices to advertisers (and reduce their surplus) because merged platforms jointly control greater exclusive access. Entry, insofar as it offers more choice and hence more multi-homing, tends to reduce the numbers of exclusive viewers on platforms and reduce advertising prices while increasing total numbers of consumers accessed. Merger then reduces advertiser surplus, while entry raises it.

# 8 Appendix

## 8.1 Appendix 1: Relegated proofs

**Proof of Lemma 1.** We first show how the inverse demand is bounded above under the log-concavity assumption, and then use the bound to show the desired result.

Consider an advertiser inverse demand function $p(a)$ with slope $p'(a)$. We claim that

$$p(x) \leq p(a)\exp\left((x-a)p'(a)/p(a)\right) \quad \text{for } x \leq a.$$

Indeed, this condition can be rewritten as

$$\ln p(x) \leq \ln p(a) + (x-a)p'(a)/p(a) \quad \text{for } x \leq a.$$

This must hold because $f(x) \equiv \ln p(x)$ is concave ($f(x) \leq f(a)+(x-a)f'(a)$, which just states that the function lies below its tangent line).

Hence we can bound net advertiser surplus:

$$\begin{aligned}
\int_0^a p(x)dx &\leq p(a)\int_0^a \exp\left((x-a)\frac{p'(a)}{p(a)}\right)dx \\
&= p(a)\left[\frac{p(a)}{p'(a)}\exp\left((x-a)\frac{p'(a)}{p(a)}\right)\right]_0^a \\
&= \frac{p^2(a)}{p'(a)}\left(1-\exp\left(-\frac{ap'(a)}{p(a)}\right)\right).
\end{aligned} \tag{15}$$

We now use this bound to sign the derivative $(AS^G(a)/R(a))' \geq 0$, which is equivalent to

$$ap - (1+\frac{ap'}{p})\int_0^a p(x)dx \geq 0.$$

Define $\eta = -\frac{p}{ap'} > 0$, and so for $\eta \leq 1$ (which corresponds to $R'(a) \leq 0$, or, equivalently, $a \geq a^m$) the inequality necessarily holds. So consider $\eta > 1$.

To establish the desired inequality, given $(1+\frac{ap'}{p})>0$, it suffices to show that

$$ap-(1+\frac{ap'}{p})\left(-\frac{p^2}{p'}(\exp\left(-\frac{ap'(a)}{p(a)}\right)-1)\right)\geq 0,$$

which is equivalent to $g(\eta)=1-(\eta-1)(\exp(1/\eta)-1)\geq 0$. This is readily shown to be a decreasing function for $\eta>0$, which asymptotes to zero as $\eta$ tends to infinity, and hence the desired inequality holds.

For the corresponding advertiser net surplus, $d(AS(a)/R(a))/da = d((AS^G(a)-R(a))/R(a))/da = d([AS^G(a)/R(a)]-1)/da \geq 0$, as claimed. ■

**Proof of Lemma 2.** The right-hand side of (6) is denoted by

$$J(\psi_i)\equiv 1+\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}a_i'(\psi_i)\psi_i,$$

which is well-defined for $\gamma\neq 0$.

First, we show that (for any $\gamma\neq 0$) $J'(\psi_i)<0$. Using (6),

$$J'(\psi_i)=-\left(\frac{da_i}{d\psi_i}\right)\left\{\left(\frac{R'}{R}\right)'\frac{h}{h'\gamma}-\frac{R'}{R}\left(\frac{h}{h'}\right)'\right\}$$

The sign of $-\left(\frac{da_i}{d\psi_i}\right)$ has the sign of $\gamma$. Consider the term in curly brackets. Since $R$ is log-concave, $\left(\frac{R'(a_i)}{R(a_i)}\right)'$ is negative. Together with $h/h'>0$, this implies that the first term in the above expression has the sign of $-\gamma$. Since $h$ is log-concave $(h/h')'$ is positive. Together with the result that $a_i$ is chosen in the increasing part of $R$ for $\gamma>0$ and in the decreasing part of $R$ for $\gamma<0$, we have that $-\frac{R'}{R}\left(\frac{h}{h'}\right)'$ has the sign of $-\gamma$ too. Hence, as the term in curly brackets has the sign of $-\gamma$, $J'$ has the sign of $-\gamma^2<0$.

Thus, $\psi_i/\Psi=J(\psi_i)$ uniquely defines the inclusive best reply $r_i(\Psi)$ for all admissible $\Psi$.

Second, we show that (for $\gamma\neq 0$) inclusive best replies embody strategic complementarity, i.e., $r_i'(\Psi)>0$. Differentiating the inverse of the best reply, $\Psi=\psi_i/J(\psi_i)$

we obtain

$$\frac{d\Psi}{d\psi_i} = \frac{J(\psi_i) - \psi_i J'(\psi_i)}{J^2(\psi_i)}.$$

Since $J > 0$, it is sufficient that $J' < 0$, which has been established by the previous lemma.

Third, we show that (for $\gamma \neq 0$) slopes of inclusive best replies are below average actions, $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$. We can rewrite $r'(\Psi) < \frac{r_i(\Psi)}{\Psi}$ as $\frac{J^2}{J - \psi_i J'} < \frac{\psi_i}{\Psi}$. Using the first-order condition $\frac{\psi_i}{\Psi} = J$, this is equivalent to $\frac{J}{J - \psi_i J'} < 1$, which is satisfied as $J' < 0$. ■

**Proof of Proposition 6 (2), log-linear case.** First we show that if $h$ is log-linear, then after the merger advertising levels are the same on insider platforms $i$ and $j$, $\widetilde{a} \equiv \widetilde{a}_i = \widetilde{a}_j$. Note that profits of merged platforms $i$ and $j$ are

$$R(a_i)\frac{h(v_i)}{\Psi} + R(a_j)\frac{h(v_j)}{\Psi}.$$

The first-order condition with respect to $a_i$ can be written as

$$R'(a_i)\frac{h(v_i)}{\Psi} - \gamma R(a_i)\left(\frac{h'(v_i)}{\Psi} - \frac{h'(v_i)h(v_i)}{\Psi^2}\right) + \gamma R(a_j)\frac{h(v_j)h'(v_i)}{\Psi^2} = 0.$$

This is equivalent to

$$R'(a_i) - \gamma R(a_i)\left(\frac{h'(v_i)}{h(v_i)} - \frac{h'(v_i)}{h(v_i)}\frac{h(v_i)}{\Psi}\right) + \gamma R(a_j)\frac{h(v_j)}{\Psi}\frac{h'(v_i)}{h(v_i)} = 0$$

or

$$\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}R'(a_i) - R(a_i)\left(1 - \frac{h(v_i)}{\Psi}\right) + R(a_j)\frac{h(v_j)}{\Psi} = 0.$$

Rewriting this equation we have

$$\left(\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}\frac{R'(a_i)}{R(a_i)} - 1\right)R(a_i) = -R(a_i)\frac{h(v_i)}{\Psi} - R(a_j)\frac{h(v_j)}{\Psi}. \tag{16}$$

We obtain the corresponding equation for the first-order condition obtained from maximizing with respect to $a_j$. Since the right-hand side of these equations are the same, we must have

$$\left(1-\frac{1}{\gamma}\frac{h(v_i)}{h'(v_i)}\frac{R'(a_i)}{R(a_i)}\right)R(a_i)=\left(1-\frac{1}{\gamma}\frac{h(v_j)}{h'(v_j)}\frac{R'(a_j)}{R(a_j)}\right)R(a_j). \tag{17}$$

For $h$ log-linear, $h(v_i)/h'(v_i)$ is constant and $a_i = a_j$ must be a solution to this equation. It is the unique solution, as shown by contradiction. Suppose that there is a solution with $a_i > a_j$. Then, for $\gamma > 0$, $R(a_i) > R(a_j)$ and $R'(a_i)/R(a_i) < R'(a_j)/R(a_j)$. Since terms in brackets of (17) must be positive (by equation (16)), this is a contradiction. Similarly, for $\gamma < 0$, we have $R(a_i) < R(a_j)$ and $R'(a_i)/(\gamma R(a_i)) > R'(a_j)/(\gamma R(a_j))$, which also leads to a contradiction.

Second, since $a_i = a_j \equiv \widetilde{a}$, post merger, $R$ and $AS$ are the same on merging platforms $i$ and $j$ in this case.

Third, since a merger is profitable, there must exist artificial shares $\hat{\lambda}_i$, $\hat{\lambda}_j$ with $\hat{\lambda}_i + \hat{\lambda}_j = \widetilde{\lambda}_i + \widetilde{\lambda}_j$ such that, using these artificial shares, platform profits increase on each platform; i.e., $\hat{\lambda}_i R(\widetilde{a}) > \lambda_i R(a_i)$ and $\hat{\lambda}_j R(\widetilde{a}) > \lambda_j R(a_j)$.

Fourth, by Lemma 5 the merger leads to lower equilibrium actions $\psi_i$ and, thus, for $\gamma > 0$, higher advertising level on each of the merging platforms, $\widetilde{a} > a_i$, where $a_i$ denotes the equilibrium advertising level prior to the merger.

Fifth, we are now in a position to show that net advertiser surplus on insider platforms after the merger is larger than before the merger, $\sum_{i\in I}\widetilde{\lambda}_i AS(\widetilde{a}) = \sum_{i\in I}\hat{\lambda}_i AS(\widetilde{a}) > \sum_{i\in I}\lambda_i AS(a_i)$, where $I$ is the set of insiders. This inequality is equivalent to

$$\sum_{i\in I}\frac{AS(\widetilde{a})}{R(\widetilde{a})}\hat{\lambda}_i R(\widetilde{a}) > \sum_{i\in I}\frac{AS(a_i)}{R(a_i)}\lambda_i R(a_i).$$

Since, $\hat{\lambda}_i R(\widetilde{a}) > \lambda_i R(a_i)$ for $i \in I$, this inequality is implied by $AS(\widetilde{a})/R(\widetilde{a}) > AS(a^*)/R(a^*)$, for $i \in I$. For $\gamma > 0$, since $\widetilde{a} > a^*$ and, by Lemma 1, $d(AS(a)/R(a))/da \geq 0$, advertiser surplus increases more strongly than revenue on each platform. ■

## 8.2 Appendix 2: Supplementary analysis of two-sided pricing

Using the definition of $\psi_i$ from the main text in Section 6, we rewrite each platform's objective function

$$\begin{aligned}\Pi_i &= \left(R\left(\bar{a}_i\right)+f_i\right)\frac{\psi_i}{\Psi} \\ &= \left(k_i-\mu\ln\psi_i\right)\frac{\psi_i}{\Psi}\end{aligned}$$

where $k_i = R\left(\bar{a}_i\right)+\mu\ln h\left(\bar{v}_i\right)$.

**Proposition 8** *Suppose that $R$ is strictly concave. There exists a unique equilibrium.*

**Proof.** The inclusive best reply $r_i(\Psi) = \arg\max_{\psi_i}\Pi_i(\psi_i,\Psi)$ satisfies the first-order condition of profit maximization with respect to $\psi_i$,

$$\frac{-\mu}{\Psi}+\left(k_i-\mu\ln\psi_i\right)\left(\frac{1}{\Psi}-\frac{\psi_i}{\Psi^2}\right)=0$$

or, equivalently,

$$-\mu+\left(k_i-\mu\ln\psi_i\right)\left(1-\frac{\psi_i}{\Psi}\right)=0.$$

This can be rewritten as

$$1-\frac{\mu}{\left(k_i-\mu\ln\psi_i\right)}=\frac{\psi_i}{\Psi}. \tag{18}$$

We define

$$J^P\left(\psi_i\right)\equiv 1-\frac{\mu}{k_i-\mu\ln\psi_i}$$

and write the first-order condition as $\psi_i/J^P(\psi_i)=\Psi$. Since we immediately observe that

$$\left(J^P\right)'=-\frac{\mu}{\psi_i}\frac{\mu}{\left(k_i-\mu\ln\psi_i\right)^2}<0,$$

the slope of the inclusive best reply with two-sided pricing lies between 0 and $\lambda_i$.

Notice that a profit-maximizing platform sets $f_i \in [-R(\overline{a}_i), \infty)$. Actions $\psi_i$ must exceed the monopoly action $\psi_i^{\min}$ defined as the solution to

$$1 - \frac{\mu}{(k_i - \mu \ln \psi_i)} = \frac{\psi_i}{\psi_i + h(v_0)}.$$

Thus $\psi_i$ is chosen in $[\psi_i^{\min}, h(\overline{v}_i) \exp\{R(\overline{a}_i)/\mu\}]$. Thus, the sum of inclusive best replies $\sum_{i=1}^n r_i(\Psi) + \psi_0$ is defined on $[\max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}, \sum_{i=1}^n h(\overline{v}_i) \exp\{R(\overline{a}_i)/\mu\} + h(v_0)]$. Consider $\sum_{i=1}^n r_i(\Psi) + \psi_0$ which maps from a compact interval into itself. Clearly, $\sum_{i=1}^n r_i(\max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}) + h(v_0) > \max_{i\in\{1,\ldots,n\}}\{\psi_i^{\min}\}$ and $\sum_{i=1}^n r_i(\sum_{i=1}^n h(\overline{v}_i) \exp\{R(\overline{a}_i)/\mu\} + h(v_0)) + h(v_0) < \sum_{i=1}^n h(\overline{v}_i) \exp\{R(\overline{a}_i)/\mu\} + h(v_0)$. Since $\sum_{i=1}^n r_i(\Psi)$ is continuous in $\Psi$, there must exist an interior solution to $\psi_0 + \sum_{i=1}^n r_i(\Psi) = \Psi$ and, therefore, an equilibrium exists. Furthermore, since $r_i'(\Psi) < \frac{r_i(\Psi)}{\Psi}$ in any equilibrium the sum of inclusive best replies crosses the diagonal from above and the equilibrium is unique.

■

The equilibrium is characterized by equations (18). As the following proposition establishes, the cross-section characterization of Proposition 2 also holds with two-sided pricing when $h$ is strictly log-concave.

**Proposition 9** *Suppose that $R$ is strictly concave. Consider any two platforms $i$ and $j$. Whenever $s_i > s_j$, in equilibrium $\lambda_i > \lambda_j$ and $R(a_i) \geq R(a_j)$. For $\gamma > 0$ and $h$ strictly log-concave, $s_i > s_j$ implies that $a_i > a_j$. For $\gamma < 0$ and $h$ strictly log-concave, $s_i > s_j$ implies that $a_i < a_j$. For $h$ log-linear, all platforms choose the same ad level.*

**Proof.** As $k_i = R(\bar{a}_i) + \mu \ln h(\bar{v}_i)$, we have $\frac{dk_i}{ds_i} = R'(\bar{a}_i) \frac{d\bar{a}_i}{ds_i} + \mu \frac{h'(\bar{v}_i)}{h(\bar{v}_i)} \left(1 - \gamma \frac{d\bar{a}_i}{ds_i}\right)$. Denote $\rho = \mu (\ln h(\bar{v}_i))''$. Now, using the definition of $\bar{a}_i$, $R'(\bar{a}_i) - \mu\gamma \frac{h'(\bar{v}_i)}{h(\bar{v}_i)} = 0$, we have that $\frac{d\bar{a}_i}{ds_i} = \frac{\gamma\rho}{R" + \gamma^2\rho}$ which has the sign of $\gamma$ (under the assumption that $R$ is concave). So now both terms in the expression $\frac{dk_i}{ds_i}$ above are positive: the first because $R'(\bar{a}_i)$ and $\frac{d\bar{a}_i}{ds_i}$ have the same sign; the second because $1 - \gamma \frac{d\bar{a}_i}{ds_i} = \frac{R''}{R" + \gamma^2\rho} > 0$. Therefore, $k_i$ is increasing in $s_i$ (regardless of the sign of $\gamma$).

Consider now the inclusive best reply. Recall that the inclusive best reply satisfies (the LHS is the function $J^{MP}$ used above):

$$1-\frac{\mu}{(k_i-\mu\ln\psi_i)}=\frac{\psi_i}{\Psi};$$

or inverse inclusive best reply is

$$\Psi=\frac{\psi_i}{1-\frac{\mu}{(k_i-\mu\ln\psi_i)}}.$$

This shows the property that higher $k_i$ (from higher $s_i$) shifts the inclusive best reply up; i.e., for larger $k_i$, a given $\Psi$ is associated with a higher $\psi_i$. To summarize, when $h$ strictly log-concave, $s_i > s_j$ implies that $a_i > a_j$ and $\lambda_i > \lambda_j$ for $\gamma > 0$, while $s_i > s_j$ implies that $a_i < a_j$ and $\lambda_i > \lambda_j$ for $\gamma < 0$. When $h$ is log-linear, $s_i > s_j$ implies that $a_i = a_j$ and $\lambda_i > \lambda_j$. ■

Of course, subscription fees depend on quality. Using the first-order condition, we obtain by implicit differentiation that $\frac{df_i}{ds_i}$ has the sign of

$$-R'(\bar{a}_i)\frac{d\bar{a}_i}{ds_i}\frac{\mu}{(R(\bar{a}_i)+f_i)^2}+\frac{h'(\bar{v}_i)\exp\{-\frac{f_i}{\mu}\}(1-\gamma\frac{da_i}{ds_i})}{\Psi}.$$

Clearly, in the log-linear case, ad level are independent of quality and, therefore, $f_i$ is increasing in $s_i$.

As in the previous section we consider three exogenous changes of market structure. First, we consider entry of an additional platform.

**Proposition 10** *The entry of an additional platform*

1. *leaves advertising on other platforms unchanged,*

2. *decreases other platforms' profits,*

3. *increases consumer surplus,*

4. *increases advertiser surplus if platforms are symmetric or $h$ is log-linear; for $\gamma > 0$ and $h$ strictly log-concave, it decreases advertiser surplus if $s_{n+1} < \min\{s_1, ..., s_n\}$ and $v_0 = -\infty$; for $\gamma < 0$ and $h$ strictly log-concave, it unambiguously increases advertiser surplus if $s_{n+1} < \min\{s_1, ..., s_n\}$.*

**Proof.** As shown in the main text, ad levels are independent of market share and, thus, unaffected by entry. In line with the proof of Proposition 3, after entry, in equilibrium, the aggregate $\Psi$ goes up. Because all rivals' $\psi_j$ increase, platform $i$'s profit must decrease, $i = 1, ..., n$. Because $\Psi$ goes up, from Lemma 3, consumer surplus increases.

In the log-linear case, since ad levels are the same for all platforms and there are more viewers in total, advertiser surplus must increase. For $\gamma > 0$ and $h$ strictly log-concave, for asymmetric platforms there is a reshuffling of viewers toward the lowest-quality platform, which carry fewer ads. Thus, advertiser surplus necessarily decreases if platform $n+1$ has lower quality than all other platforms and all consumers participate. For $\gamma < 0$ and $h$ strictly log-concave, the lowest-quality platform has more ads; entry then leads to a reshuffling of viewers toward the lowest-quality platform. Furthermore, additional consumers may participate after entry. For both reasons, advertiser surplus increases with entry in this case. ■

Different from markets with ad financed platforms, entry does not affect the advertising decision of other platform. Hence, changes in advertiser surplus are purely due to reshuffling viewers. By contrast, under ad finance, additional entry has the effect that platform offer less annoying programs by reducing the advertising level. Under symmetry and full coverage, this does not lead to a see-saw effect. By contrast, in the two-sided pricing model, under symmetry and full coverage, advertisers are unaffected. Proposition 10 adds that there may be a see-saw effect for $\gamma > 0$, in line with what we found for ad-financed media platforms. With two-sided pricing results consumer and advertiser surplus are aligned for $\gamma < 0$.

Second, we consider a merger of two platforms.

**Proposition 11** *The merger of two platforms*

1. *leaves advertising on all platforms unchanged,*
2. *is profitable and increases other platforms' profits,*
3. *decreases consumer surplus,*
4. *decreases advertiser surplus if platforms are symmetric or $h$ is log-linear; for $\gamma > 0$ and $h$ strictly log-concave, it increases advertiser surplus if the two lowest-quality platforms merge and $v_0 = -\infty$; for $\gamma < 0$ and $h$ strictly log-concave, it increases advertiser surplus if the two highest-quality platforms merge and $v_0 = -\infty$.*

**Proof.** Again, ad levels are independent of market share; they are also unaffected by the merger. The merger shifts the inclusive best response of the merged platforms down. Hence, the merger decreases the aggregate $\Psi$ and consumer surplus is down. The second claim follows from the same argument as made in Proposition 4.

If platforms are symmetric or $h$ is log-linear all platforms choose the same ad level. Hence, advertiser surplus is monotone in the number of consumers that are served. Since the merger leads to less consumer participation ($\psi_0/\Psi$ is decreasing in $\Psi$), advertiser surplus is lower after the merger. However, for $\gamma > 0$, under full participation, a merger among the lowest-quality platforms has the effect that these platforms loose viewers who instead join higher-quality platforms. Therefore, advertiser surplus increases in this case. Analogously, for $\gamma < 0$, under full participation, a merger among the highest-quality platforms has the effect that these platforms loose viewers who instead join lower-quality platforms. Since, for $\gamma < 0$, lower-quality platforms carry viewer ads, advertiser surplus increases also in this case. ■

Outside the above special cases, for $\gamma > 0$, a merger under two-sided pricing decreases advertiser surplus if the two merging platforms are the highest-quality platforms.

The merger result with two-sided pricing is in stark contrast to the results with ad financing. We observe that with two-sided pricing advertiser and consumer surplus tend to be aligned: if $h$ is logconcave or platforms offer the same quality, then both sides of the market suffer from a merger. This result can only be offset if the number of active viewers does not depend strongly on the merger and if platforms with low ad levels merge, as the merger then leads to a reshuffling of viewers to platforms with higher ad levels.

Third, we consider an ad cap on highest-quality platform ($h$ strictly log-concave, $\gamma > 0$).

**Proposition 12** *The introduction of symmetric advertising caps that becomes binding for one platform*

1. *decreases consumer surplus,*
2. *decreases advertiser surplus.*

**Proof.** As shown above, the introduction of an ad cap that is binding for the highest-quality firm reduces $k_i$. This shift the inclusive best reply downward, as is seen by implicitly differentiating (18) with respect to $k_i$.

$$\frac{d\psi_i}{dk_i} = \frac{\frac{\mu}{(k-\mu \ln \psi_i)^2}}{\frac{1}{\Psi} + \frac{\mu}{(k-\mu \ln \psi_i)^2}\frac{\mu}{\psi_i}} > 0$$

The downward shift of platform $i$'s inclusive best reply leads to a lower aggregate $\Psi$ after the introduction of the ad cap. As shown in the main text, all non-capped platforms do not change the ad level. Consumer surplus decreases as the aggregate has gone down.

Since $\Psi$ decreases, the market share of the uncapped platforms must increase. Competition becomes less intense with an ad cap. As uncapped platforms do not adjust ad levels, a higher market share implies that advertiser surplus on those platforms is up (also profit is up).

Market share of the capped platform is down and market share of outside option is up. Regarding advertiser surplus per viewer we note the following: For all consumers who stay with the outside option or one of the uncapped platforms advertiser surplus per viewer remains the same after the introduction of an ad cap. Some consumers move from the capped platform to one of the other platforms (which contain less advertising) or the outside option. Thus, advertiser surplus per viewer is down for those consumers. The last group of consumers consists of those consumers which stay with the platform that is subject to a binding cap after its introduction. By design, this platform hosts fewer ads and thus advertiser surplus per viewer declines also for these consumers. Combining all these changes, advertiser surplus must be decreasing.
$\blacksquare$

# References


[1] Acemoglu, D. and M. K. Jensen. 2013. Aggregate Comparative Statics. *Games and Economic Behavior*, 81, 27-49.

[2] Ambrus, A., E. Calvano, and M. Reisinger. 2014. Either or Both Competition: A "Two-Sided" Theory of Advertising with Overlapping Viewerships. Mimeo.

[3] Anderson, S. 2012. Advertising on the Internet. In: Peitz, M. and J. Waldfogel (eds.), Oxford Handbook of the Digital Economy, Oxford: Oxford University Press.

[4] Anderson, S. and S. Coate. 2005. Market Provision of Broadcasting: A Welfare Analysis. *Review of Economic Studies*, 72, 947-972.

[5] Anderson, S., N. Erkal, and D. Picinnin. 2013. Aggregative Oligopoly Games with Entry. CEPR Discussion Papers 9511.

[6] Anderson, S., Ø. Foros and H. J. Kind. 2015. Competition for Advertisers and for Viewers in Media Markets. CEPR Discussion Paper 10608.

[7] Anderson, S. and B. Jullien. 2015. Advertising Finance and Two Sided Business Models. Forthcoming in: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics*. Amsterdam: Elsevier.

[8] Anderson, S. and A. de Palma. 2001. Product Diversity in Asymmetric Oligopoly: Is the Quality of Consumer Goods Too Low? *Journal of Industrial Economics* 49, 113-135.

[9] Anderson, S., A. de Palma and J. F. Thisse. 1992. *Discrete Choice Theory of Product Differentiation*. Cambridge, MA: MIT Press.

[10] Armstrong, M. 2006. Competition in Two-Sided Markets. *Rand Journal of Economics*, 37, 668-691.

[11] Chandra, A. and A. Collard-Wexler. 2009. Mergers in Two-Sided Markets: An Application to the Canadian Newspaper Industry. *Journal of Economics and Management Strategy,* 18, 1045-1070.

[12] Crampes, C., C. Haritchabalet, and B. Jullien. 2009. Advertising, Competition, and Entry in Media Industries. *Journal of Industrial Economics*, 57, 7-31.

[13] Deneckere, R. and C. Davidson. 1985. Incentives to Form Coalitions with Bertrand Competition. *Rand Journal of Economics*, 16, 473-486.

[14] Fan, Y. 2013. Ownership Consolidation and Product Characteristics: A Study of the US Daily Newspaper Market. *American Economic Review*, 103, 1598-1628.

[15] Foros, Ø., H. J. Kind and L. Sorgard. 2015. Merger Policy and Regulation in Media Industries. Forthcoming in: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics.* Amsterdam: Elsevier.

[16] Gabszewicz, J. J., D. Laussel and N. Sonnac. 2004. Programming and Advertising Competition in the Broadcasting Industry. *Journal of Economics & Management Strategy,* 13, 657-669.

[17] Genakos, C. and T. Valletti. 2011. Testing the 'Waterbed' Effect in Mobile Telecommunications. *Journal of the European Economic Association* 9, 1114-1142.

[18] Genakos, C. and T. Valletti. 2015. Evaluating a Decade of Mobile Termination Rate Regulation. *Economic Journal*, forthcoming.

[19] Ivaldi, M. and J. Zhang. 2015. Advertising Competition in the French Free-To-Air Television Broadcasting Industry. CEPR Discussion Paper 10726.

[20] Jeziorski, P. 2014a. Estimation of Cost Synergies from Mergers: Application to the U.S. Radio. *Rand Journal of Economics*, 45, 816-846.

[21] Jeziorsky, P. 2014b. Effects of Mergers in Two-Sided Markets: The U.S. Radio Industry. *American Economic Journal: Microeconomics*, 6, 35-73.

[22] Luce, R. D. 1959. *Individual Choice Behavior: A Theoretical Analysis.* New York: Wiley.

[23] Peitz, M. and M. Reisinger. 2014. The Economics of Internet Media. University of Mannheim Working Paper 14-23. Forthcoming in: Anderson, S., Stromberg, D., and J. Waldfogel (eds.). *Handbook of Media Economics.* Amsterdam: Elsevier.

[24] Peitz, M. and T. Valletti. 2008. Advertising and Content in Media: Pay-TV versus Free-to-Air. *International Journal of Industrial Organization*, 26, 949-965.

[25] Rochet, J.-C. and J. Tirole. 2003. Platform Competition in Two-Sided Markets. *Journal of the European Economic Association*, 1, 990-1029.

[26] Rochet, J.-C. and J. Tirole. 2006. Two-Sided Markets: A Progress Report. *Rand Journal of Economics*, 37, 645-667.

[27] Selten, R. 1970. *Preispolitik der Mehrproduktenunternehmung in der Statischen Theorie*, Berlin: Springer Verlag.