Secular Fertility Declines, Baby Booms and Economic Growth: International Evidence

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Abstract

We present a model capable of explaining 200 years of declining fertility, 200 years of rising educational achievement and a significant Baby Boom for the United States and twenty other industrialized market countries. We highlight the importance of secularly declining young adult mortality risk for producing secularly declining fertility and a sudden decline in housing costs after the end of the Second World War, but ending by 1970. In addition we introduce a new puzzle to the profession. Given the magnitude of the Baby Boom, roughly equal to fertility in 1900 for many of these countries, why did schooling of the Baby Boom cohorts not fall to the 1900 level of their predecessors? In fact, not only do they not fall, but their schooling levels are higher than previous cohorts. Using a quantitative model we are able to identify the magnitude of the reduction in costs of education necessary to explain this paradoxical increase in schooling. We produce a novel data set on historical education expenditures with over 1500 observations. We find empirical support for these cost reductions.

In this paper we present a model of secular fertility decline and baby boom, which we view as an identification exercise. Principally we are interested in identifying the forcing variables for both secularly declining fertility in the United States, as well as the developed world, throughout the past 200 years, as well as each country's Baby Boom. The Baby Boom was a dramatic deviation from the secular trend in children ever born by women. In the United States, fertility declined from 7 children per woman in 1800 to 2.4 children prior to the Baby Boom. At the peak of the Baby Boom, the typical American woman had 3.2 children over her reproductive life. At the end of the Baby Boom the typical American woman had 2 children over her reproductive life. A less know fact is that schooling rose in every country that experienced a baby boom, and in fact the schooling levels of baby boom children are either on trend, or deviant from trend because they are higher, not lower, than trend! Again for the US, the typical child in 1800 had .8 years of schooling, by 1930 this had risen to 10.6 years of schooling. At the peak of the US baby boom in the late 1950s, these children had 13 years of schooling! At the end of the century, when fertility is 2.0 instead of 3.2, these children are predicted to receive 14.3 years of schooling.

Recent work by Greenwood, et. al. (2005) provides one explanation for the secular decline in fertility and the Baby Boom. They identify rising market productivity as the cause for the secular decline

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in fertility. A once and for all increase in household productivity caused the Baby Boom to occur in the United States and in other western countries. In particular they argue that the introduction of modern appliances in the household dramatically lowered the cost of fertility. Adoption of mechanical washing machines, refrigerators, stoves and ovens would lower the cost of household production.[1] However a recent paper by Bailey and Collins (2011) shows that the Greenwood, et.al. hypothesis has other difficulties. Much of the effect identified by Greenwood, et. al. requires that the household be wired for electricity. Using a data set on electricity usage in households, they find little correlation between electrification and fertility. Additionally they show that the Amish had the same Baby Boom as others in the United States, even though they do not use modern appliances or electricity.

Doepke, Hazan and Maoz (2007) show that differential mobilization rates of women during World War II can produce differential baby booms across countries. High mobilization rates of women in the US, Australia, Canada, New Zealand and the UK contrast with much lower rates in continental Europe. These high mobilization rate countries had much larger baby booms than the low mobilization rate countries. The logic is that more women in the high mobilization countries continued to work after the war, blocking the entry level jobs of younger women in the post war period. These younger women then chose to marry earlier and start their families earlier.[2]

Albanesi and Olivetti (2010) illustrate that differences in the reduction of maternal mortality across the US states, arising from the differential adoption of antibiotics and blood transfusion are associated with differential baby booms across the US states. In Albanesi (2011) a model of fertility with maternal mortality is developed. She shows that the differential declines in maternal mortality across countries, as well as differential timing of the decline is consistent with the differential magnitudes and timing of international baby booms.[3]

In the Easterlin (1961, 1966) works, children raised during the Great Depression became accustomed to a level of consumption. After World War II, the expected return to the Depression living standards did not arise, but rather a post war boom occurred. As a result of this, the children of the Depression consumed some of this unexpected wealth in larger families. With the large cohorts of the baby boom entering into the labor market with higher expectations of living standards, the slowing of labor productivity growth produced smaller families. Easterlin does not attempt to explain the changes in schooling that are identified here.

In this paper we identify secularly declining young adult mortality, as in Tamura (2006), as the cause for declining fertility. However unlike Greenwood, et. al., Albanesi and Olivetti (2010), Albanesi (2011), Doepke, et. al. and Easterlin, we identify falling price for space as the principal cause of the Baby Boom.[4] We have in mind the dramatic expansion in the suburban land area available for housing as a result of the interstate highway system in the United States as the cause of a reduction in housing costs, c.f. Baum-Snow (2007). In addition we require time varying educational efficiency in order to explain the secular

[1] One would also have to believe that the further improvements on these appliances are generally neutral towards fertility, or at least too small to overcome the rising return to labor market participation of women. The introduction of radio, television and particularly color television appear to serve as substitutes for children, in terms of their timing.

[2] One requirement would be that these younger women married men that had higher earnings to support these larger families. One possibility is that the labor of their husbands was complementary to the labor of the older women in the labor force.

[3] Albanesi (2011) provides a model that can produce rising births and rising human capital investments during the baby boom as a result of declining maternal mortality. To our knowledge this is the only model that can produce this result. In this paper we use a declining cost of schooling during the baby boom to produce rising schooling.

[4] Murphy, Simon and Tamura (2008) provides the US fertility and schooling experience by state from 1840-2000 using the model of this paper.

trend in completed years of schooling. The model does an excellent job of fitting the data for fertility as well as years of schooling, and serves to identify the magnitude of shocks to housing costs and educational efficiencies. The model is capable of fitting real output per worker. Finally we show that the model's measures of educational efficiency, and space are consistent with the historical international data.

1 Model

In this section we present a model with parental choice of fertility, x, human capital of their children, h', a composite consumption good, c, and space, S.[5] Parents choose the number of children to have in an environment of young adult mortality. Parental preferences are:

$$\alpha \left(c_t^{\psi} S_t^{1-\psi}\right)^{\varphi} \left[(1-\delta_t)x_t - a\right]^{1-\varphi} + \Lambda(Zh_{t+1})^{\varphi} - \frac{Z^{\varphi}\beta\delta_t^{\nu_t}}{\left[(1-\delta_t)x_t - a\right](1-\delta_t)^{\varepsilon}}, \tag{1}$$

where ν is a time varying preference parameter that becomes constant by 1950.[6] We assume that the young adult mortality rate is δ. Further we assume that expected net fertility is what parents care about, $(1-\delta)x - a$, $a \geq 0$. Thus we model the parental fertility choice similar to Jones (2001), where elasticity of substitution of net expected fertility with human capital investments is greater than 1. This in turn exceeds the elasticity of substitution between net expected fertility and space, 1. The final term, with $\varepsilon > 0$, in the preferences captures something like a precautionary demand for fertility as in Kalemli-Ozcan (2002, 2003) and Tamura (2006).[7] Notice that it also depends on the level of total factor productivity. The more productive the economy, the more costly young adult mortality is from the perpsective of utility. With falling young adult mortality rates, which in the limit reach 0, the final term in preferences disappears.

The budget constraint facing the typical parent is given by:

$$c_t + r_t x_t S_t = Zh_t \left[1 - x_t\left(\theta + \kappa_t \tau_t\right)\right] \tag{2}$$

where Z is the constant total factor productivity in production, θ is the time cost of rearing children, τ is the time spent educating children, κ is the time efficiency of education time, p is the price of consumption and r is the price per unit of space.[8] Finally we assume that the human capital accumulation technology

[5]The results of the numerical solution for the United States were used in Murphy, Simon and Tamura (2008) in order to identify the relevant price of space. That paper compared the model solution for the required time series on price of space with measures of population density. Population density was computed for each state in each census year by computing the population per square mile in each county, and then aggregating the state counties weighting by population. In this manner what we compute is how many people reside within a square mile of a randomly chosen persion.

[6]In the appendix we indicate via the country specific graphs what the time series for ν is. For almost all countries the stationary value of ν is the same, .40. In an earlier version, as well as in Murphy, Simon and Tamura (2008) we had preferences that included an endogenous time varying total factor productivity in each of the three terms. However we have been able to simplify the model and eliminate the need for endogenous total factor productivity, hence Z is constant.

[7]The preferences are similar to those contained in Tamura, SImon and Murphy (2011). In that paper the precautionary portion of preferences is merged with the utility from child human capital as $\Lambda(Zh_{t+1})^{\varphi}[1 - \frac{\beta_t\delta_t^{\nu_t}}{[(1-\delta_t)x_t - a](1-\delta_t)}]$. The advantage of this specification is that it allows the price of space to fit the population density data. It does come with a tradeoff of a time varying β. Since we do not have any comparable estimates of population density outside of the United States, we chose to stay with our earlier preferences.

[8]Alternatively we could have specified the first term in preferences as depending on a composite of space, S, and all other consumption goods, c, and net expected fertility: $\alpha X^{\varphi}\left[(1-\delta)x - a\right]^{\lambda-\varphi}$, where $X = \left\{\sigma c^{\frac{1}{\rho}} + (1-\sigma)S^{\frac{1}{\rho}}\right\}^{\rho}$. If ρ were negative, so that goods were stronger complements than the Cobb-Douglas case examined here.

is given by:[9]

$$h_{t+1} = A\overline{h}_t^{\rho} h_t^{1-\rho} \tau_t^{\mu} \tag{3}$$

This accumulation technology is from Tamura (1991, 2006).[10] We assume that the US is the frontier human capital country, so that $h_{US,t} = \overline{h}_t$ for all countries.[11] Substituting (3) and (2) into (1) and differentiating produces the three Euler conditions determining optimal choices of human capital investments, fertility and space:

$$\frac{\partial}{\partial \tau} : \psi\varphi\alpha c_t^{\psi\varphi-1} S_t^{(1-\psi)\varphi} \left[(1-\delta_t) x_t - a\right]^{1-\varphi} Z h_t x_t \kappa_t = \mu\varphi Z^{\varphi} A^{\varphi} (\overline{h}_t^{\rho} h_t^{1-\rho})^{\varphi} \tau_t^{\mu\varphi-1} \tag{4}$$

$$\frac{\partial}{\partial x} : \psi\varphi\alpha c_t^{\psi\varphi-1} S_t^{(1-\psi)\varphi} \left[(1-\delta_t) x_t - a\right]^{1-\varphi} \left[Z h_t \left[\theta + \kappa_t \tau_t\right] + r_t S_t\right]$$

$$= (1-\varphi)\alpha c_t^{\psi\varphi} S_t^{(1-\psi)\varphi} \left[(1-\delta_t) x_t - a\right]^{-\varphi} (1-\delta_t) + \frac{Z^{\varphi}\beta\delta_t^{\nu_t}}{x_t^2 (1-\delta_t)^{\varepsilon}} \tag{5}$$

$$\frac{\partial}{\partial S} : \psi\varphi\alpha c_t^{\psi\varphi-1} S_t^{(1-\psi)\varphi} \left[(1-\delta_t) x_t - a\right]^{1-\varphi} r_t x_t$$

$$= \alpha (1-\psi) \varphi c_t^{\psi\varphi} S_t^{(1-\psi)\varphi-1} \left[(1-\delta_t) x_t - a\right]^{1-\varphi} \tag{6}$$

We can solve for c_t as a function of S_t and x_t. This produces:

$$c_t = \left(\frac{\psi}{1-\psi}\right) r_t x_t S_t \tag{7}$$

Substituting this into the budget constraint produces:

$$r_t x_t S_t = (1-\psi) Z h_t \left[1 - x_t (\theta + \kappa_t \tau_t)\right]$$

Substituting this back into the objective function produces the following problem facing the household:

$$\max_{x_t, \tau_t} \left\{ \begin{array}{c} \alpha (\psi)^{\psi\varphi} \left(\frac{1-\psi}{r_t x_t}\right)^{(1-\psi)\varphi} \left(Z h_t \left[1 - x_t (\theta + \kappa_t \tau_t)\right]\right)^{\varphi} \left[(1-\delta_t) x_t - a\right]^{1-\varphi} \\ + \Lambda (Z h_{t+1})^{\varphi} - \frac{Z^{\varphi}\beta\delta_t^{\nu_t}}{(1-\delta_t)^{\varepsilon}} \left[(1-\delta_t) x_t - a\right]^{-1} \end{array} \right\} \tag{8}$$

What is most interesting to us is the decline in the fertility rate with the decline in young adult mortality, δ, as well as the relationship between the price of space, r, and fertility. Due to the interaction of fertility with both space as well as human capital investments, the budget constraint facing the typical parent is not convex. As a consequence, the comparative static exercise does not lead to any nice analytical results. We thus utilize numerical solution methods to examine the interaction of the precautionary demand for fertility and human capital investments in the long term. Our method takes note of the fact that for a given level of fertility, x, the problem is concave in (c, S, τ). Thus we use a grid over possible values of fertility, and solve the household's problem, and then we choose the fertility that maximizes utility. Although the problem is not standard, the parameters chosen for preferences, $(\alpha, \psi, \varphi, a)$, and technology, (ρ, μ), still

[9] In the numerical solutions we allow μ to vary across countries. Within a country μ is constant. Holding κ constant, changes in μ imply differential efficiency in converting study time to human capital. Since the proportion of the first period of life spent in school is a fraction less than 1, increasing μ implies lower productivity in schooling holding time in school fixed.

[10] In Tamura, Dwyer, Devereux and Baier (2011) human capital accumulation is a decreasing returns adventure. That is to say, $h_{t+1} = A\overline{h}_t^{\rho} h_t^{\beta} exp(.10E + .0495x - .0007x^2)$, where $\rho + \beta < 1$, and E is education, and x is experience.

[11] For the UK, we backsolved the growth rate of the US human capital stock and used that as the spillover value for the period 1600-1800.

produces interior solutions for (x, c, S, τ).

The numerical solutions presented below indicate the requisite decline in the price of space in order to induce a baby boom. Thus in the numerical solutions, we produce the secular decline in fertility arising from the rising survival rate, or falling mortality rate, as well as the rising levels of human capital investment. Furthermore one possible mechanism of the baby boom is the falling price of space. We are able to replicate the broad pattern of fertility as well as human capital investment.

We also use the parameter κ to produce the appropriate secular rise in human capital investment time.[12] We use information on years of schooling in the labor force for the US from Turner, Tamura, Mulholland and Baier, (2007), as a measure of τ. We assume that a period length is 40 years, so that $40\tau_t$ is the years of schooling for the typical individual born in year t.[13] In the international comparisons, we use data from Baier, Dwyer and Tamura (2006) as well as Baier, Devereux, Dwyer and Tamura (2008) to fit years of schooling, fertility, young schooling, output per worker.There is a strong quality and quantity tradeoff in the model. The solutions show that the current low rate of fertility in these countries are often so low that they imply counterfactual schooling attainment. Furthermore the baby booms in the countries typically imply dramatic reductions in schooling for that cohort as well as schooling in the population. Thus we used the efficiency of time for schooling, κ, as a means to control for these counterfactual schooling levels.

1.1 Stationary Values and Numerical Solutions

In this section we analyze the stationary solution as well as present numerical solutions. We assume that the stationary fertility rate is 1. Examining the Euler equation with respect to fertility when mortality risk is 0, we can produce the parameter restriction on a as a function of parameters and the stationary human capital investment rate, $\overline{\tau}$:

$$a = 1 - \frac{(1-\varphi)\left(1-[\theta+\overline{\tau}]\right)}{\varphi\left(1-\psi\left(1-[\theta+\overline{\tau}]\right)\right)}$$

We can also find the implicit function determining the stationary human capital investment rate, τ:

$$1 = \frac{\Lambda\mu\left[Ar^{1-\psi}\right]^{\varphi}\left(1-\theta-\overline{\tau}\right)^{1-\varphi}}{\alpha\left[\psi^{\psi}\left(1-\psi\right)^{1-\psi}\right]^{\varphi}\left(1-a\right)^{1-\varphi}\overline{\tau}^{1-\mu\varphi}},$$

where under the balanced growth path, $\overline{h}_t = h_t$, and the right hand side is constant under a balanced growth path. Under these parameter restrictions and convergence of mortality risk to 0, perhaps due to human capital accumulation as in Tamura (2006), the long run fertility rate and human capital investment rate will be $x = 1$, $\tau = \overline{\tau}$.

What follows is the generation of time series on total fertility rates, years of schooling, consumption and space. We present these results in comparison with the actual data, under varying assumptions. The model solutions presented result from numerically solving the Euler equation for τ_t for a range of possible

[12]In Tamura, Simon and Murphy (2011) we use variations in fertility and schooling across races, black and white, across states of the US from 1800 (white) 1820 (black) to 2000 to identify κ_{ijt}, where i refers to state, j refers to black or white, and t refers to birth cohort. This allows us to compute compensating and equilibrating variations, for blacks and whites, to measure the value of equal opportunity in education.

[13]In the numerical solutions we assume that the nonlinear budget constraint provides the possibility that fertility may be at a corner, as in Ehrlich and Lui (1991). Thus our algorithm allows for this, although in practice all choice variables are interior solutions.

values of fertility, x_t. The resulting solutions are compared and the one generating the highest utility is selected as optimal.[14]

Before continuing we discuss how we calculated our measure of young adult mortality risk, δ. We used the actual time series history on probability of dying between the ages of 1 and 35, $d_{1,35}$ to produce our measure of δ. Specifically we used data from Tamura (2006) on infant mortality, m, and the probability of dying between the ages of 1 and 35, $d_{1,35}$, to construct a forecast probability of dying between the ages of 1 and 35. We do not assume that individuals have perfect foresight, instead we assume that they made rational forecasts of the probability of dying between the ages of 1 and 35 prior to 1900. We regressed the log of the probability of dying between the ages of 1 and 35 for years prior to 1900 on a time trend. We then used the forecast values, $p_{1,35}$ as our measure of the perceived risk of dying between 1 and 35. We then ran the regression of log of the probability of dying between the ages of 1 and 35 for 1900 - 1949 on a time trend. We then used the forecast values as our measure of the preceived risk of dying between 1 and 35 for years 1900 - 2000. We did this because the introduction of penicillin in medical use was a dramatic change in the probability of dying of infectious diseases. We do not assume that the typical individual knew about penicillin at all. By 2000 the deviation of the forecast from the actual probability is quite small. We did the same thing for infant mortality to produce our forecast of infant mortality, $\widehat{m}$. Thus our measure of the relevant young adult mortality risk is δ:

$$\delta = p_{1,35} + \frac{\widehat{m}}{3} \tag{9}$$

As can be seen, we downweight the effect of infant mortality by $\frac{1}{3}$. This is due to the fact that an infant death is less costly to replace than a death of a child say at age 12. This for the obvious reason that after 12 years a substantial amount of human capital investment could have been made, a substantial level of support has been delivered, and perhaps even more importantly, a large proportion of a woman's child bearing potential has disappeared. For if a child born when a woman was 15 dies at age 12, she is now 27, with only 17 years of reproductive life remaining. [15]

figures 1-21 below contain the data and the model solutions for children ever born, years of schooling in the labor force, output per worker, and schooling of the youngest cohort. We also plot the model solutions for the base case, and the three time varying cases contained in Table 1, taste: ν_t, taste & rental price of space: (ν_t,r_t), and the full model (ν_t, r_t, κ_t). In Figure 1, the case of the United States, there are three major features of the children ever born data: (1) the secular decline from 7 children born over a woman's reproductive life in 1800 to 2 children born by the end of the period, (2) the baby boom for women in the years 1946-1964, (3) the mini bulge in fertility for women born between 1840-1860. The base model, picks up on the secular decline in fertility arising from the secular decline in δ, but completely misses the baby boom. Adding time varying taste parameters, ν_t allows the model to pick up a portion of the baby boom. Adding time varying price of space, r_t fully captures the United States baby boom. The falling cost of education during the baby boom, κ_t is required in order to fit the data on schooling as well as young schooling. For the other 20 countries time varying taste parameter, ν_t, generally does not pick up

[14] We use this method in order to allow for the possibility of corner solutions. Since the budget constraint is not linear, the budget set is not convex, and hence it is plausible that fertility might head either towards a maximum level, i.e. $x = \frac{1}{\theta}$, or a minimal level, i.e. $x = a + \zeta$, for arbitrarily small ζ.

[15] Ideally one would consider the relative price of mortality at 2 years of age versus say 12 years of age to construct an economically relevant young adult mortality in a sequential fertility-human capital investment model.

the baby boom, but rather does a better job fitting the secular trend in fertility.[16] For all of the remaining 20 countries, time varying price of space, r_t, is required to produce the baby boom. Furthermore a time varying cost of schooling parameter, κ_t, is necessary to fit schooling levels of the young baby boom cohort.

In Figures 1-21, we only present the data on and the solutions to fertility, schooling, real output per worker and young cohort schooling. We present all four cases, from the base case (no time varying parameters), preferences (time varying ν_t), preferences and time varying rent (ν_t , r_t) and the full time varying parameter model, ν_t, (r_t, κ_t). By and large the results reproduce the Baby Boom in each country's data, but do not fit the years of schooling data at all. Notice as well that the base model clearly picks up the secular increase in schooling in the labor force. Clearly however the model dramatically over predicts years of schooling. This arises due to the below replacement rate of fertility in the base model. The addition of time varying rental rates for space, r_t produces the standard quality-quantity tradeoff of Becker and Lewis (1973).

After Figure 1, the entries are alphabetic by country name: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.[17] In all of the cases, the base model generally can pick up the time series of fertility with the exception of the Baby Boom experiences. As with the case for the United States, the introduction of time varying price per unit of space to produce a Baby Boom, dramatically alters the years of schooling in the labor force. Thus the need for ν_t, (r_t, κ_t) in order to produce the secular rise in schooling in each of these countries. Notice that for almost all of the countries there is if anything, an acceleration in accumulation of years of schooling that accompanies their Baby Boom. It is fair to say that the Baby Boom experiences in the United States as well as these other industrialized market countries arose from shocks to the total demand for children. This increase in demand was for both quantity and quality, so that the typical quality-quantity tradeoff did not occur during this cycle.

The data for fertility come from Tamura (2006) and additional sources contained in the data appendix. The remaining data for schooling, both average and young cohort, and real output per worker come from Tamura, Dwyer, Devereux, and Baier (2012). In the column labeled base, the model assumes only a time varying mortality rate. Thus the typical regression run was:

$$y_t = \alpha + \beta x_t$$

where y_t is the year t observation on either children ever born or average years of schooling in the labor force, etc.; x_t is the year t observation from the model on children ever born, or the average years of schooling in the labor force between the ages of 20 to 65, or output per worker, or schooling of the youngest cohort. Under the null hypothesis that the model fits the data, $\alpha = 0$, and $\beta = 1$. The row marked with p provides the p value on the joint test of these hypotheses.

Table 1 below presents the fit of the model with various assumptions on parameters, ν, preferences, r, the rental price of space, and κ the cost of teaching time. There are four panels in Table 1. Starting in

[16] There is one country, Canada, in which there is no need for time varying tastes, $\nu_t = \nu, \forall t$.

[17] These are the same rich countries that were modeled in Tamura (2006). The only exception is the exclusion of Korea. Korean data on fertility and mortality only goes back to 1950 and the data is one of monotone decline in fertility. South Korea does not have a Baby Boom in their post war period. The other countries in Tamura (2006), India, China, and the regions of Latin America, Central and Eastern Europe and the rest of Asia do not have any Baby Boom cycles either. The data to fit these series comes from both Tamura (2006) and Tamura, Devereux, Dwyer and Baier, (2011).

the upper left hand corner of the table, we present the regressions results for fertility. The top right hand corner presents the regression results for schooling. The bottom left hand corner presents the results for log income per worker, and the bottom right hand corner presents the results for schooling of the young cohort. Within each panel we present the fit of the model with the data for four specifications. The base specification has a constant price of space, r, and constant preferences. The second column of each panel, labeled ν_t, contains the fit with the time varying preference parameter on the precautionary demand for fertility utility component. The column marked (ν, r_t) allows for time varying preferences and prices of space. The final column within each panel is the full model specification with time varying $(\nu, \mathrm{r}_t, \kappa_t)$. In all four panels of Table 1, the full model provides the best fit with the data for 21 countries. This is true based on improvements in the goodness of fit, $\bar{R}^2$, and the closeness of the slope coefficient to 1, and the constant to 0. In two of the four regressions, the p value on the null of $\beta = 1$ and $\alpha = 0$ is greater than .05. Thus in half of the cases we accept the null that that the model solutions and the data are statistically identical.

The base United States case that is assumed is that there is a constant price for $S : r = 1.285531$.[18] We present the time series of price of space needed in the model to reproduce the Baby Boom in the US. We compare it with the average population density of the US.[19] This is contained in Figure 32. Curiously the population density of the US has been constant since 1980, at values lower than those that produced the Baby Boom, and yet US fertility did not remain high. In Murphy, Simon and Tamura (2008) we show that the dramatic decline in the price of space required here is consistent with an example of suburbanization. In that paper an example with a central urban county is surrounded by four rural-suburban counties. We computed the marginal population density, in which all new households are responsible for the growth in population. Thus we weighted the county population densities by the relative population growth of each county.[20]

1.2 Cost of Schooling: κ_t

In the Appendix, we present the graphs of the time series of preference parameters, ν_t, the rental price of space, r_t, and the cost of schooling κ_t. In all cases the cost of schooling must fall dramatically during the Baby Boom in order to fit the observed time series of schooling of the Baby Boom cohort and the average schooling in the population. This is the first time any paper has identified this phenomena.

Recall that the total expenditures on schooling of the next generation, E_t are given by:

$$E_t = Zh_t x_t \kappa_t \tau_t$$

Total income is given by Zh_t, so expressed as expenditures per student, e_t, we have:

$$e_t = Zh_t \kappa_t \tau_t$$

[18]Technically we used a price for consumption, c, of $p = 1.002461$.

[19]We computed this by calculating the population density by county, and then weighting each county by the population in the county. Thus we are measuring the average number of people an individual is surrounded by, and not the average number of people per mile.

[20]Thus in periods where the urban core county population declines, its population density receives a negative weight, and the rest of the counties receive relative weights that aggregate to more than unity.

Dividing by total income produces the share of output spent on education per student:

$$s_{e_t} = \kappa_t \tau_t$$

This provides us with a way to test the model empirically. We can compute an estimate of $\kappa_t \tau_t$ in the data if we take the share of GDP spent on education, both public and private, and divide by the number of students and multiply by population (over the age of 4 not enrolled in school). This is given below:

$$S_{e_t}^{data} = \frac{[\text{ public + private education expenditures}]_t}{Y_t} \frac{[\text{population over 4 not enrolled in school}]_t}{[\text{public + private student population}]_t}$$

Furthermore if we divide s_{e_t} by the model's predicted length of time of schooling, τ_t, we can identify κ_t. In the data we can divide S_{e_t} by estimates of the length of schooling, τ_t^{data}, schooling of the young cohort. In order to express this as a share of lifetime, which in the model is two periods of 40 years each, we divided both the model value of schooling length, τ_t, and the data value of schooling length, τ_t^{data}, by 80 years of expected life. Luckily we have exactly this measure of expected schooling by cohort, because it is contained in Tamura, Dwyer, Devereaux and Baier (2012). Thus we have:

$$\frac{s_{e_t}}{\tau_t} = \kappa_t \tag{10}$$

$$\frac{S_{e_t}^{data}}{\tau_t^{data}} = \kappa_t^{data} \tag{11}$$

In Table 2 we present evidence on the decline of schooling cost during the Baby Boom for both measures of education expenditures. We further provide evidence for the US separately, as we have the most data for the US, covering 1850-2008, and annual data from 1885 to 2008.[21] As with the previous goodness of fit regressions, we run either

$$S_{e_t}^{data} = \alpha + \beta s_{e_t} \tag{12}$$

$$\kappa_t^{data} = \alpha + \beta \kappa_t \tag{13}$$

The evidence from Table 3 is broadly supportive. For the US, the results are very strongly positive. In every instance the coefficient on the model solution is positive and significant. We use dummy variables to control for the 20th century, for World War II, and for each country's Baby Boom years, as given by Albanesi (2011).[22] While in every regression we reject the null that the slope is 1 and the intercept is 0, we do see that the model solution is strongly positively correlated with the US data. Our model solutions for both $\kappa_t \tau_t$ expenditures per pupil, and cost of schooling κ_t are quite closely correlated to the observed data. In all eight specifications the coefficient is positive and significant at the 1% level. The coefficients are robust to inclusion of the time dummies for the 20th century, World War II years, and the US Baby Boom years. In fact the magnitude of the coefficients in both regressions increase with the inclusion of the

[21]We used multiple sources to get education expenditures as a share of GDP, Tanzi and Schuknecht (2000), Lindert (2004), *Digest of Education Statistics* (2011), *One Hundred Years of Economic Statistics* by Liesner (1989), *Historical Statistics of the United States: Millennial Edition*, and various issues of the *Human Development Report*. For cohort schooling length we used Tamura, Dwyer, Devereux, and Baier (2012), for enrollments and population we used B.R. Mitchell (2003abc) and *Historical Statistics of the United States: Millennial Edition* (2006).

[22]See the data appendix for years of the Baby Boom for each country. The only exception we made was for the United States.

Baby Boom dummy variable.

For all 21 countries the model's solutions are positively correlated with the data. In all eight specifications, the model solutions are significantly positively correlated with the actual data. The magnitude of the coefficient on the model solutions is similar to the coefficient obtained only on the US data. Unlike in the US subsample, the Baby Boom dummy variable has no affect on the coefficient of the model solution using all data.

In Table 4 we use fixed effects panel regressions to control for possible country effects. We find that in the $\kappa_t \tau_t$ regressions, the panel regressions reinforce the results from the Prais-Winsten regressions. The results also hold for the κ_t regressions.

Broadly speaking we find that the model is identifying something novel. That is concurrent with the Baby Boom, there was a sharp decline in the cost of schooling that enabled Baby Boom children to receive education comparable to or better than their parents. In the most cases, the schooling of the Baby Boom cohort was either on the long term trend or actually shifted higher to a higher trend than earlier generations! This has never before been identified to our knowledge, and the model shows just how large the deviations in schooling costs must have been in order to observe the schooling of Baby Boomers. Recall that the model was selected to fit fertility and schooling, but not anything to do with the cost of schooling. Thus our regression results represent something like an out of sample test of the model. We feel the model is strongly confirmed.

2 School efficiency

School efficiency in the model is separated from the diffusion of knowledge arising from the spillover parameter, ρ. In the solutions we assumed a value of ρ=.40. However countries can differ in their efficiency in transforming school time into human capital, via μ. In order to fit the growth rate of income we used μ as a major source of the cross country differences. Since the United States had the smoothest income process by far, and since it was generally the most productive country in the 1800 start of the data, most countries would have had a μ smaller than the United States, except for the spillover effect of human capital. This is contained in Table 3. This is very close to the work of Schoellman (2010). While Table 3 contains both a taste parameter, β, the crucial parameter for us is μ.[23] The smaller the μ the more efficient a society is at transforming school time into human capital. Notice that there is a large cluster of countries at the US value of $\mu = .0850$. Only France and Norway have schooling efficiency better than the US. The countries with inferior schooling efficiency than the US are Belgium, Canada, Denmark, Germany, Greece, Japan, Netherlands, New Zealand, Portugal, Spain, Sweden, Switzerland and the UK. The countries that are comparable in efficiency are Australia, Austria, Finland, Ireland,and Italy.

[23]In the precautionary demand for fertility term in preferences, the stationary value of ν differs from the baseline case of .400 in only three cases, France, Netherlands and New Zealand. There is more heterogeneity in preference parameter β. The possible values are .18 (nine countries), .27 (three countries), .30 (one country), .36 (two countries), .45 (one country), .54 (two countries), .63 (one country), .72 (one country) and .75 (one country). In the limit, the young adult mortality converges to 0, and all of these differences vanish.

3 Conclusion and future work

This paper has presented a model capable of capturing the secular decline in fertility of the United States and 20 other western countries. In addition it captures the significant Baby Booms that occurred in these countries. It is able to fit the dramatic increase in schooling for all of these countries over the last 200 years, and more importantly to capture the increase in schooling the occurred even during or in some countries accelerated during the Baby Boom. The model assumes that the secular decline in fertility that occurred in these countries is mainly driven by falling precautionary demand for children as young adult mortality declines. The model assumes that the price of space is the driving variable for the Baby Booms in these countries. One weakness in this model is that we must have an offsetting change in the price of schooling during the declining price of space in order to produce the observed rise in schooling.[24] We consider the numerical solutions to be an exercise in quantitative identification. That is to say we force the model to fit the data by allowing the crucial variables to vary over time. Thus the model solutions provide economists with the time series of the price of space and cost of schooling that must have occurred in order to produce the observed phenomena. We provide comprehensive information on the cost of schooling. The evidence is supportive of the model.

In terms of the price of space, in other work, we, along with Kevin M. Murphy, have produced new data for the states of the United States. We show that a the same model used here for cross country differences can be applied to the US states separately. The data is consistent with a declining price of space inducing a Baby Boom in the 1950s. We construct new measures of population density that are correlated with the model's price of space. That state population density is constructed by producing population per square mile for each county in a state, and then weighting the counties by their share of the state population. In this manner we show that population density has been declining since the mid 20th century, coincident with the Baby Boom.

We document in our joint paper (2008) that the states of the United States have the similar declines in fertility secularly, but differential magnitudes of Baby Booms. Using data from censuses of the US population, we produce time series of fertility, mortality risk from 1850 - 2000. We combine this data with data from Turner, Tamura, Mulholland and Baier (2007) on schooling at the state level over this period. Typically the states that had below the national average fertility, the New England, Middle Atlantic, East North Central West North Central, and Pacific census regions, had larger than average Baby Booms. The remaining states, the South Atlantic, East South Central, West South Central and Mountain census regions, had smaller than average Baby Booms. We also show that the states with larger than average Baby Booms had higher schooling at the start, but added fewer years of schooling over time, than their smaller than average Baby Boom counterparts. The states with larger than average Baby Booms are also the states that had higher initial population density, and would have had the greatest reduction in the price of space arising from suburbanization.

[24]Ideally it would have been preferred to have the price of space and the cost of schooling be relatively uncorrelated. However many school systems are tied to the property tax on housing. So at least in terms of the United States we would expect to see a correlation between the two. The dramatic decline in the price of space along with the decline in the cost of schooling suggest that the suburbanization mechanism must be studied in greater detail. For the model to be correct, it must be the case that along with the falling price of space, the newer suburban schools must have been of dramatically higher quality than the urban schools that existed before them.

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4 Data Appendix

Mortality data come from Tamura (2006), see data appendix for that paper. Fertility data prior to 1980 come from Tamura (2006). Fertility data for the European countries for years 1980, 1985, 1990, 1995, 2000 and 2003 come from Table 1.7 of *Housing Statistics in the European Union: 2004*. Annual population and school enrollments come from B.R. Mitchell (2003abc). Real income per worker, education, education of young cohort come from Tamura, Dwyer, Devereux and Baier (2012). Education expenditures come from a variety of sources. We detail these sources by country below.

4.1 Australia

We assumed 1.0 percent for 1870, as we had information on all the other categories. Years 1900 and 1910 come from Table C3. of Lindert (2004), and only includes public expenditures for all levels of education. Table 11.5 of Tanzi and Schuknecht (2000) provide datum for 1937, which again is only for public education. For years 1949-1986, inclusive, the data come from Liesner (1989), *One Hundred Years of Economic Statistics* Tables A.1 (nominal GDP) and A.9 (public finance), which only provides public education expenditures. To convert public education GDP share into total of public education and private education expenditure share of GDP, we used the 1960, 1985 and 1986 years in which both public expenditure shares and total education shares are observed. We averaged the ratios of total education GDP shares to public education GDP shares in these three years and multiplied the observed public shares by this average. For years from 1990 to 2007, public education GDP data shares for 1990-2005 come from various issues of the *Human Development Report*, except for 1992, which was interpolated. The 2007 and 2008 public education GDP shares come from WDI. The 2006 value was interpolated from the 2005 and 2007 values. We converted these into total education GDP shares by using the 1991 observation from the 1994 *Human Development Report* which reports both public and total education shares.

Baby Boom years are 1937-1965, inclusive.

4.2 Austria

For years 1870 and 1913 we used the average value for France and Germany. Tanzi and Schuknecht provide data for 1937, which again is only for public education, Table 11.5 of Tanzi and Schuknecht (2000), which again is only for public education. The 1953-1984 observations of public education shares of GDP come from various issues of *UNESCO Statistical Yearbooks*. Unfortunately we do not have any measure of private education shares. The 1957 value was interpolated from 1956 and 1958. We interpolate between the 1937 and 1953 values to produce 1950-1952 observations. Data for 1985-2005 come from various issues of the *Human Development Report*, with the 1989 value interpolated from 1988 and 1990 values. Values for 2007 and 2008 come from WDI, and the 2006 value was interpolated from 2005 and 2007 observations.

Baby Boom years are 1938-1965, inclusive.

4.3 Belgium

Data for 1850, 1870 and 1880 come from Table C.3 Lindert (2004). For 1860 we interpolated between the 1850 and 1870 data. Our 1913 value comes from Table 11.5 of Tanzi and Schuknecht (2000). We interpolated our 1890 and 1900 values from our 1870 and 1913 observations. Our values for 1951-1984 come from various issues of the *UNESCO Statistical Yearbooks*. For 1955, 1961, 1971, 1974 we interpolated from surrounding years. Our values from 1985-2005 come from various issues of the *Human Development Report*. In this period we interpolated 1987, 1996 and 1997 from surrounding years. Our 2007 and 2008 values are from OECD, and we interpolate the 2006 value. These values provide both public and total (public and private) education expenditures. We use this overlapping data to inflate the values for 1985-2006. Those values are originally only for public expenditures as a share of GDP.

Baby Boom years are 1944-1965, inclusive.

4.4 Canada

Data for 1870, 1880, 1890, 1900 and 1910 come from Table C2. of Lindert (2004). These values only contain public expenditures on primary and secondary schools. Values for 1933-1959, 1960-1984 and 1986 come from Liesner (1989), *One Hundred Years of Economic Statistics* Table C.1 (nominal GDP) and C9. (public finance), which only provides public education expenditures. We interpolated in order to produce our 1913 value. Our 1985-2005 values are from various issues of *Human Development Report*. Our 2007 and 2008 values come from the OECD. We interpolated for our 2006 value. The 1960, 1991, 2007 and 2008 years provide both public expenditure share and total expenditure share. For years between 1991 and 2007, we interpolated the changing share of total expenditures that are private in order to produce our estimates of total public expenditure shares in 1992-2006. For 1985-1990 we used the 1991 ratio of total expenditure shares relative to public education expenditure shares to produce total expenditure shares. For 1933-1984 we used the 1960 observation to produce total expenditure shares from public expenditure shares. We used the

Baby Boom years are 1941-1960, inclusive.

4.5 Denmark

The 1890 value comes from Table C3. of Lindert (2004) for Norway. For 1953, 1956-1958, 1961, 1963-1975, 1977-1980, 1982-1984 our data come from various issues of *UNESCO Statistical Yearbooks*. We interpolated for years 1950-1952, 1959, 1962, 1976 and 1981. Our values for 1960, 1985-2005 come from various issues of *Human Development Report*. We interpolated our 1989 value. Our 2007-2008 values come from OECD. We interpolated our 2006 value. All values are for public education, at no time do we observe private or total expenditure shares.

Baby Boom years are 1938-1964, inclusive.

4.6 Finland

The 1950 value is an average value of the 1950 values for Norway and Sweden. For 1954, 1955, 1957, 1958, 1961-1984 our values come from various issues of *UNESCO Statistical Yearbooks*. We interpolated for 1951-1953, 1956, 1959 values. Our 1960, 1985-2005 values come from various issues of *Human Development Report*. We interpolated our 1989 value. For 2007 and 2008 the information comes from OECD. We interpolated our 2006 value. The 1991 observation provides both total expenditure share and public expenditure share. We used the ratio of total to public expenditure shares to produce total expenditure shares from 1951-1990, 1992-2008.

Baby Boom years are 1937-1959, inclusive.

4.7 France

Values for 1850, 1860, 1870, 1880, 1890, 1900 and 1910 come from Table C4. of Lindert (2004). These are total primary, secondary and higher education expenditures, public and private. The 1913 value comes from Table 11.5 of Tanzi and Schuknecht (2000). For the years 1931-1984 the data come from Tables F.1 (nominal GDP) and F.9 (public finance) of Liesner (1989), which is only for public education expenditures. For 1985-2005, the data comes from various issues of *Human Development Report*. The 2007 and 2008 values come from the OECD. We interpolated 1987, 1989, 1990, and 2006. The 1960, 1986 1991, 2007, 2008 years provide both total expenditure shares and public education expenditure shares. For 1931-1959 we used the 1960 relative public expenditure share to total expenditure share to produce total expenditure share. For 1961-1985 we interpolated the relative public expenditure shares to total expenditure shares to produce total expenditure shares. For 1988 we used the 1986 value of relative public expenditure shares to total expenditure share to produce total expenditure shares. For 1990 we used the average of the 1991 and 1986 relative public expenditure shares to total expenditure shares to produce total expenditure shares. For years 1992-2006 we used the 1991 relative share of public expenditures to total expenditure shares to produce total expenditure shares.

Baby Boom years are 1941-1965, inclusive.

4.8 Germany

Values for 1860, 1870, 1880, 1890, 1900, 1910 and 1914 come from Table C4. of Lindert (2004). These are total primary, secondary and higher education expenditures, public and private. For all other years 1885-1985, the values come from the Tables G.1 (nominal GDP) and G.9 (public finance) of Liesner (1989), which is only for public education expenditures. We interpolated the 1937 value using the 1932 and 1950 observations. The 1986, 1988,1991-2005 observations come from various issues of *Human Development Reports*. The 1991 observation provides both public expenditure shares and total expenditures shares on education. We used this observation to convert all observations since 1914 to total expenditure shares. The 2007 value comes from *World Development Indicators*. We used the 2007 value for 2008. The 1987, 1989, 1990 and 2006 values are interpolated.

Baby Boom years are 1951-1969, inclusive.

4.9 Greece

Values for 1952-1984 come from various issues of *UNESCO Statistical Yearbooks*. We interpolated for 1956, 1958, 1959, 1972, 1976 values. All of these values are for public expenditures on education. Our 1985-2005 values come from various issues of *Human Development Reports*. For 2006-2008 we assumed the 2005 value held constant.

Baby Boom years are 1961-1975, inclusive.

4.10 Ireland

We assumed 1870 and 1913 values equal to those of the UK. The 1937 value comes from Table 11.5 of Tanzi and Schuknecht (2000). The 1954-1984 values come from various issues of *UNESCO Statistical Yearbooks*. The 1960, 1985-2005 values come from various issues of *Human Development Reports*. The 2007 and 2008 values are from *World Development Indicators*. We interpolated for the 1950, 1951, 1952, 1953, 1957, 1968 and 2006 observations. The 1991 observation from the *Human Development Report* provides both total expenditure shares and public expenditure shares. We used this to produce total expenditure shares for all other years.

Baby Boom years are 1953-1971, inclusive.

4.11 Italy

Values for 1880, 1890, 1900 come from Table C4. of Lindert (2004). These are total primary, secondary and higher education expenditures, public and private. For 1860, 1870 and 1910 we use Table C3. of Lindert (2004) and adjust the public expenditures to total expenditures using the same average ratio of public to total from 1880-1900. For 1914-1964 we used Tables I.1 (nominal GDP) and I.9 (public finance) of Liesner (1989), which is only for public education expenditures. For 1968-1984 we used various issues of *UNESCO Statistical Yearbooks*. For 1985-2005 we used various issues of *Human Development Reports*. Our 2007 and 2008 values come from *World Development Report*. We interpolated our 1965, 1967, 1968, 1987, 1989, 1990, 1994 and 2006 observations. Our 1988 observation contains both total expenditure shares and public expenditure shares. We also observe the average ratio of public to total expenditure shares in 1880, 1890 and 1900. We interpolated this ratio between 1913 and 1987, inclusive to adjust our public expenditure shares to total expenditure shares. For years after 1988 we used the 1988 ratio of total expenditure share to public expenditure share to produce total expenditure share.

Baby Boom years are 1955-1969, inclusive.

4.12 Japan

Table C3. of Lindert (2004) provides public expenditure shares for 1880, which we adjust using the ratio of total expenditure share to public expenditure share from 1890. For 1870 we assumed the same value as 1880. Our 1890 value comes from Table C4. of Lindert (2004). These are total primary, secondary and higher education expenditures, public and private. Values for 1913 and 1937 come from Table 11.5 of Tanzi and Schuknecht (2000). Our 1954-1984 values come from various issues of *UNESCO Statistical Yearbooks*. The exceptions are 1960 (*Human Development Report 1993*), 1968, 1969, 1974-1979, and 1981 (Tables J.1 (nominal GDP) and J.9 (public finance) of Liesner (1989)). We have overlapping information for years 1954-1959 with Tables J.1 (nominal GDP) and J.9 (public finance) of Liesner (1989), and we use the average ratio of *UNESCO Statistical Yearbooks* with Liesner to convert 1947-1953 Liesner observations to public expenditure shares. We interpolated for 1938-1946 observations. Our 1960 and 1985-2005 values come from various issues of *Human Development Reports*. The 2007 and 2008 observations are from *World Development Indicators*. We interpolated our 2006 observation. Our 1991and 1890 observations have both total expenditure share and public expenditure share. We interpolate between these two ratios and apply these to produce total expenditure share for years 1913-2008.

Baby Boom years are 1965-1974, inclusive.

4.13 Netherlands

Our values for 1850, 1860, 1870, 1880, 1890, 1900 and 1910 come from Table C3 of Lindert (2004). They are total public expenditures on all levels of education as share of GDP. Values for 1937 come from Table 11.5 of

Tanzi and Schuknecht (2000). We interpolate for 1913. Our observations for 1955-1984 come from various issues of *UNESCO Statistical Yearbooks*. We interpolated for years 1950-1954 and 1962. Our values for 1985-2004 come from various issues of *Human Development Reports*. We interpolated our observations for 1987 and1989. Our 2007 and 2008 observations are from the *World Development Indicators*. We interpolated our 2005 and 2006 observations. Our 1991 observation has both information on total expenditure shares and public expenditure shares. We use this to produce total expenditure shares for all years.

Baby Boom years are 1940-1962, inclusive.

4.14 New Zealand

Our 1890 and 1900 values are from Table C3 of Lindert (2004). They are total public expenditures on all levels of education as share of GDP. We assumed an 1870 value equal to the US value, which is total expenditures on education as a share of GDP. Our 1937 value comes from Table 11.5 of Tanzi and Schuknecht (2000). We interpolate for 1913. Our 1952-1984 values come from various issues of *UNESCO Statistical Yearbooks*. We interpolated our 1950, 1951 and 1956 values. Our 1960, 1985-2005 observations come from various issues of *Human Development Reports*. We interpolated our 1989 observation. Our 2007 and 2008 observations come from *World Development Indicators*. We interpolated our 2006 observation. All observations are for public expenditure shares.

Baby Boom years are 1937-1961, inclusive.

4.15 Norway

Our 1880, 1890, 1900 and 1910 observations are from Table C3. of Lindert (2004). They are total public expenditures on all levels of education as share of GDP. We assumed an 1870 value equal to the 1880 value. Our 1913 and 1937 observations come from Table 11.5 of Tanzi and Schuknecht (2000). Observations for years 1952-1984 come from various issues of *UNESCO Statistical Yearbooks*. We interpolated our 1950, 1951, 1956 and 1958 observations. Our observations for 1985-2005 come from various issues of *Human Development Reports*. We interpolated our 1987 and 1989 observations. For 2007 and 2008 we used *World Development Indicators*. We interpolated our 2006 observation. In 1991 we observed both total expenditure shares and public expenditure shares. We used the ratio of total to public to construct total expenditure shares for all years.

Baby Boom years are 1938-1966, inclusive.

4.16 Portugal

Our 1952-1984 observations come from various issues of *UNESCO Statistical Yearbooks*. Our 1950 value is assumed to be 90% of Spain's 1950 value. We interpolated our 1951, 1956, 1958, 1968, 1969 observations. We used various issues of *Human Development Reports* for years 1985-2005. Our 2008 value comes from *World Development Indicators*. We interpolated for our 1992, 2006 and 2007 observations. Our 1990 observation provides both total expenditure share and public expenditure share. We used the ratio of these to produce total expenditure shares for all other years.

Baby Boom years are 1955-1964, inclusive.

4.17 Spain

Our 1890 value comes from Table C1. of Lindert (2004). They are total public expenditures on primary education as share of GDP. We assumed an 1870 value equal to the 1890 value. Our 1913 and 1937 observations come from Table 11.5 of Tanzi and Schuknecht (2000). Our 1953-1984 observations come from various issues of *UNESCO Statistical Yearbooks*. We interpolated for our 1950, 1951, 1952, 1956, 1965, 1977 and 1978 observations. For years 1985-2005 we used various issues of *Human Development Reports*. Our 2007 and 2008 observations come from *World Development Indicators*. We interpolated for years 1989 and 2006. Our 1991 observation comes with both total expenditure share and public expenditure share. We use the ratio of these to produce total expenditure share for all other years.

Baby Boom years are 1955-1970, inclusive.

4.18 Sweden

Our 1870, 1880, 1890, 1900 and 1910 values come from Table C1. of Lindert (2004). They are total public expenditures on primary education as share of GDP. Our 1903, 1936-1964 values come from Tables S.1 (nominal GDP) and S.9 (public finance) of Liesner (1989), which is only for public education expenditures. We use various issues of *UNESCO Statistical Yearbooks* for 1965-1984. Our 1986-2005 observations come from various issues of *Human Development Reports*. We used overlapping years of coverage by *UNESCO Statistical Yearbooks* and Liesner (1989) in order to convert Liesner data into total public expenditure shares. Our 2007 and 2008 observations come from *World Development Indicators*. We interpolated our 1913, 1937, 1989 and 2006 values.

Baby Boom years are 1934-1965, inclusive.

4.19 Switzerland

Our 1870, 1880, 1890 values come from Table C1. of Lindert (2004). They are total public expenditures on primary education as share of GDP. For years 1948-1984 we used various issues of *UNESCO Statistical Yearbooks*. Our 1985-2005 observations come from various issues of *Human Development Reports*. Our 2007 and 2008 values come from *World Development Indicators*. We interpolated for years 1900, 1913, 1937, 1950, 1951, 1952, 1953, 1955, 1961, 1963, 1966, 1989, and 2006. We observe both total expenditure shares and public expenditure shares in 1986, which we use to construct total expenditure shares for all other years.

Baby Boom years are 1939-1964, inclusive.

4.20 United Kingdom

The web address http://www.ukpublicspending.co.uk/download_multi_year_1770_2015UKp_12s1li011mcn_20t Christopher Chantrill provides data for 1802-1959. For 1960-1987 we use Tables UK.1(nominal GDP) and UK.18 (public finance)of Liesner (1989), which is only for public education expenditures. For 1988-2005 we use various issues of *Human Development Reports*. For 2007 and 2008 we use *World Development Indicators*. We interpolated for 1989 and 2006. Our 1880 and 1900 observations provide information on total expenditure shares and public shares. We use the ratio of these to construct total education shares from 1802-1900. For years prior to 1880 we use the the 1880 ratio. For years between 1880 and 1900 we geometrically interpolated the average ratios and applied these to our public expenditure shares. In 1986 we observe both from *Human Development Report* and from Liesner (1989) observations. We use the ratio of these two observations to adjust the *Human Development Report* data.

Baby Boom years are 1937-1964, inclusive.

4.21 United States

Our 1850 and 1860 values come from Table C4. of Lindert (2004).[25] It is for total expenditures, public and private, on all levels of education as share of GDP. For 1869-1973 we use information from *Historical Statistics of the United States: Millennial Edition*. For nominal GDP we use Table Ca184-191, Volume 3, on nominal gross national product for years 1869-1929. We averaged the values from Kuznets Variant I and Variant III for 1869-1888, and Kuznets Variant I and Variant III as well as Kendrick for 1889-1929. For years 1930-1973 we used Table Ca1-8, Volume 3, on nominal Gross Domestic Product. For expenditures we use total expenditures on public elementary and secondary schools from Table Bc909-925, Volume 2 for years 1869-1973. We also have total current revenue 1909, biennially from 1919-1965, annually from 1966-1973, and total revenue for 1889 and 1899. Also we have total private elementary and secondary receipts biennially from 1948-1970. We interpolated for missing years. Prior to 1950 we used 12% of total public elementary and secondary expenditures as an estimate of private spending. This is the average value over the 1948-1970 period. Also going back before 1889 we used a rule that 20% of total public and elementary and secondary schooling expenditure was spent on all higher education. For years after 1973 we used the Table 28 of *2011 Digest of Education Statistics*.

For public primary and secondary enrollments we used Table Bc7-18, Volume 2, for years 1869-1995. We also have private primary and secondary enrollments from this table for years 1888-1995. For years prior to 1888 we assumed 10% of total public primary and secondary enrollments were enrolled in private primary and secondary schools. We used Table Bc523-536, Volume 2, for years 1869, 1879, 1889, 1904, 1909-1995 for total higher education enrollments. For years not listed we geometrically interpolated. For 1850 and 1860 we used Turner, Tamura, Mulholland and Baier (2007).

Baby Boom years are 1946-1964, inclusive.

[25] Lindert also supplies values for 1870, 1880, 1890, 1900 and 1910. However as they agree essentially with our other sources, we chose to use the other source for all of these years.

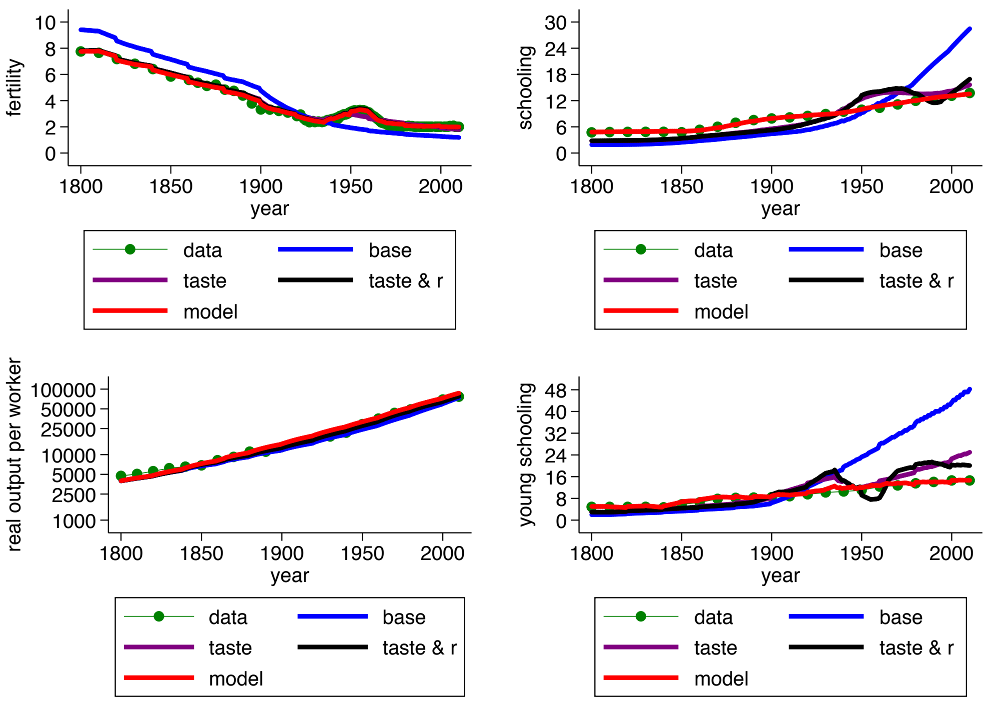

Figure 1: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): USA

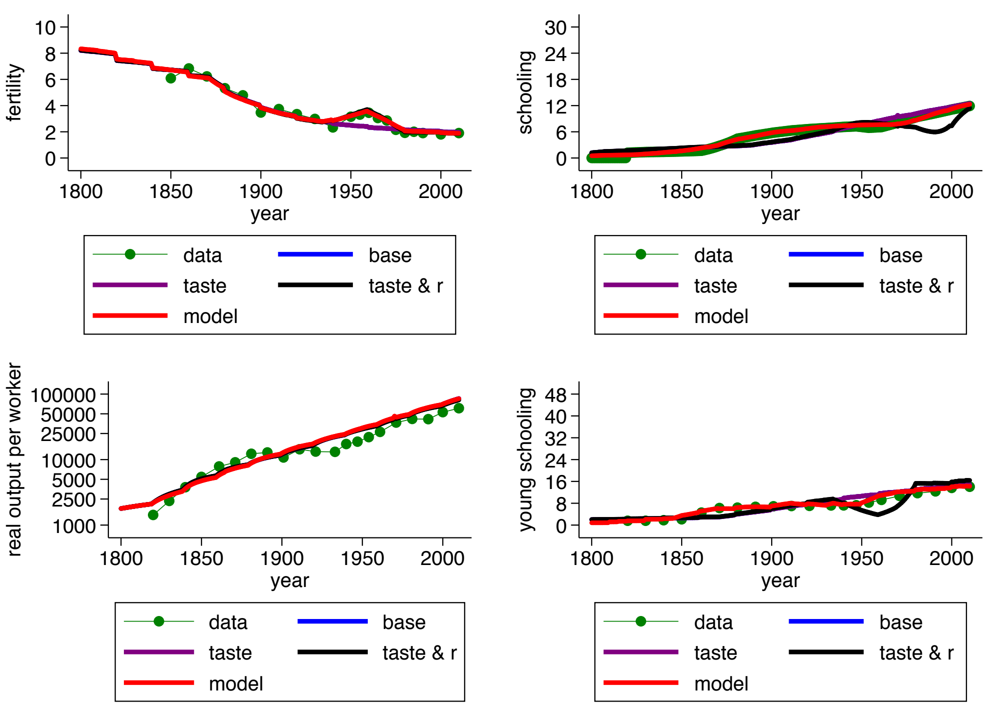

Figure 2: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Australia

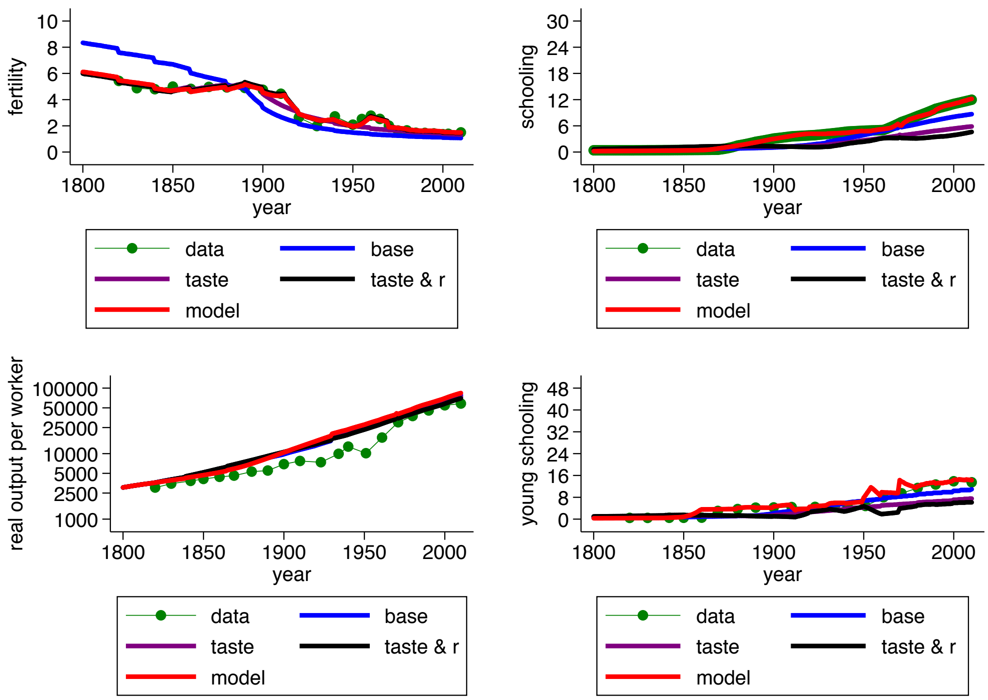

Figure 3: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Austria

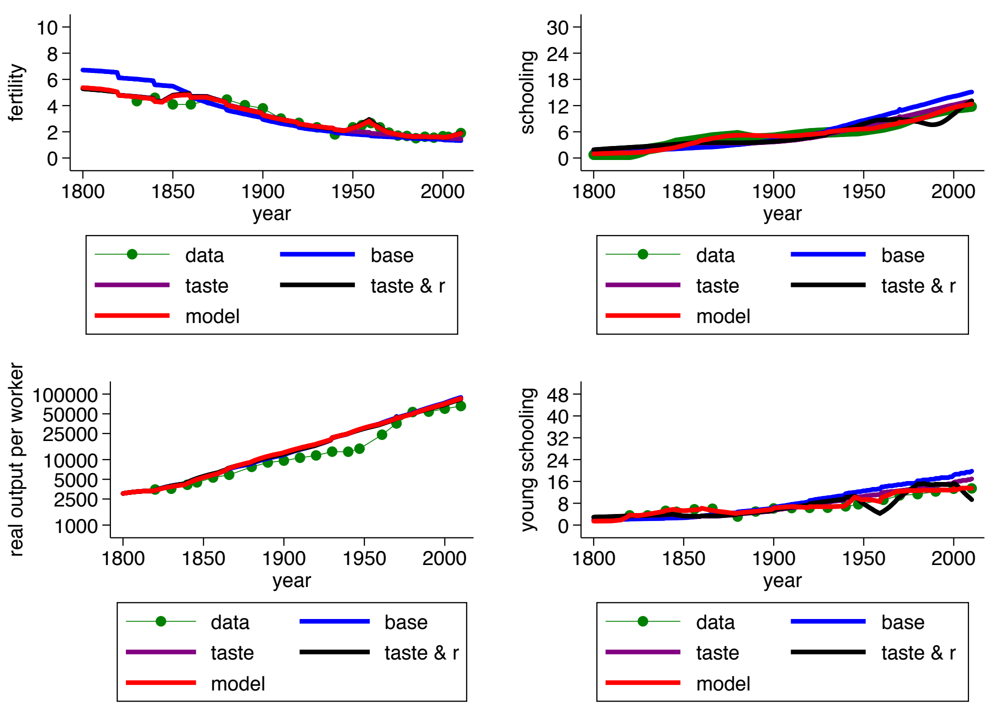

Figure 4: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Belgium

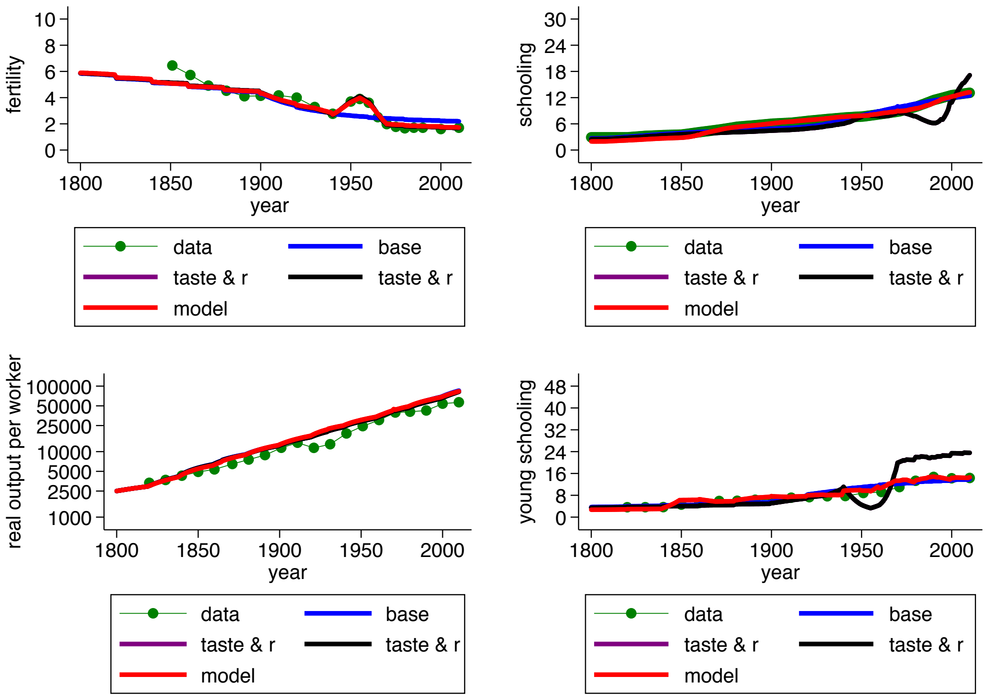

Figure 5: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Canada

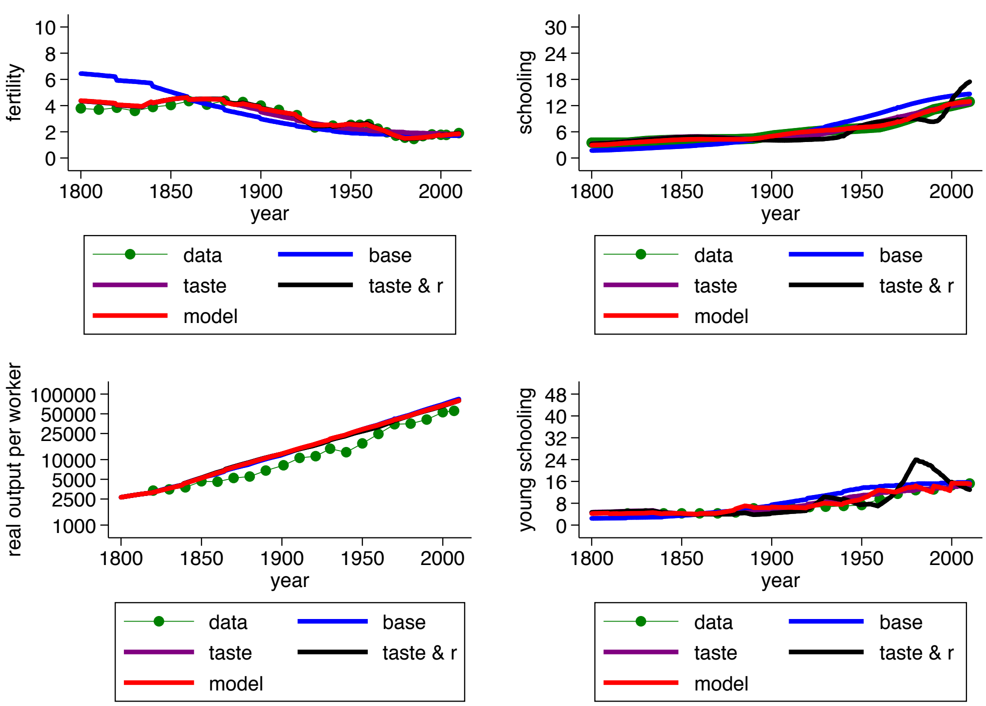

Figure 6: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Denmark

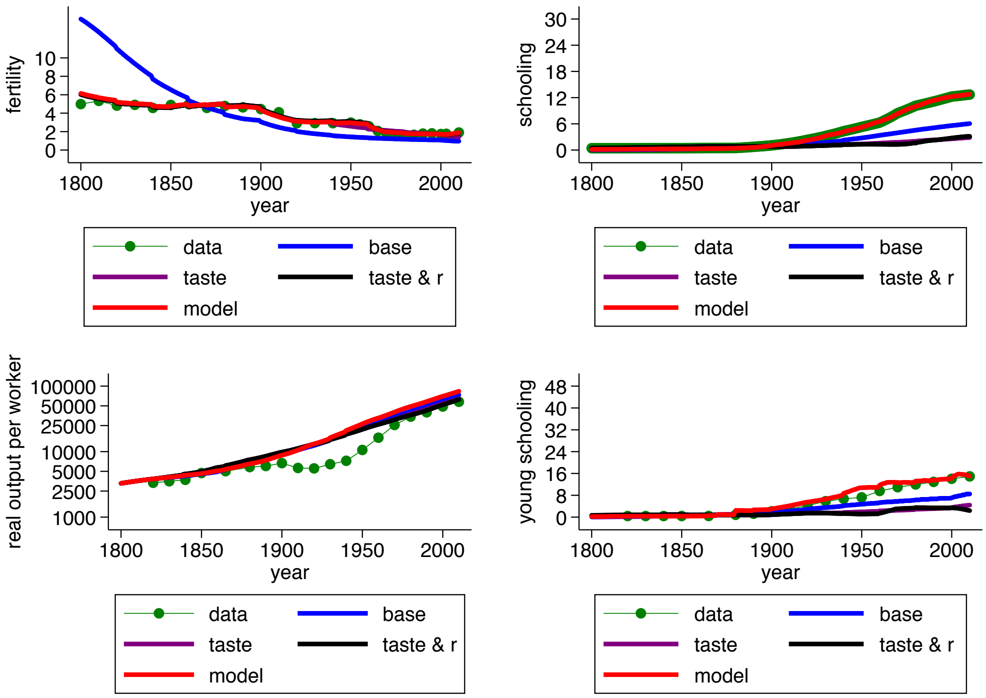

Figure 7: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Finland

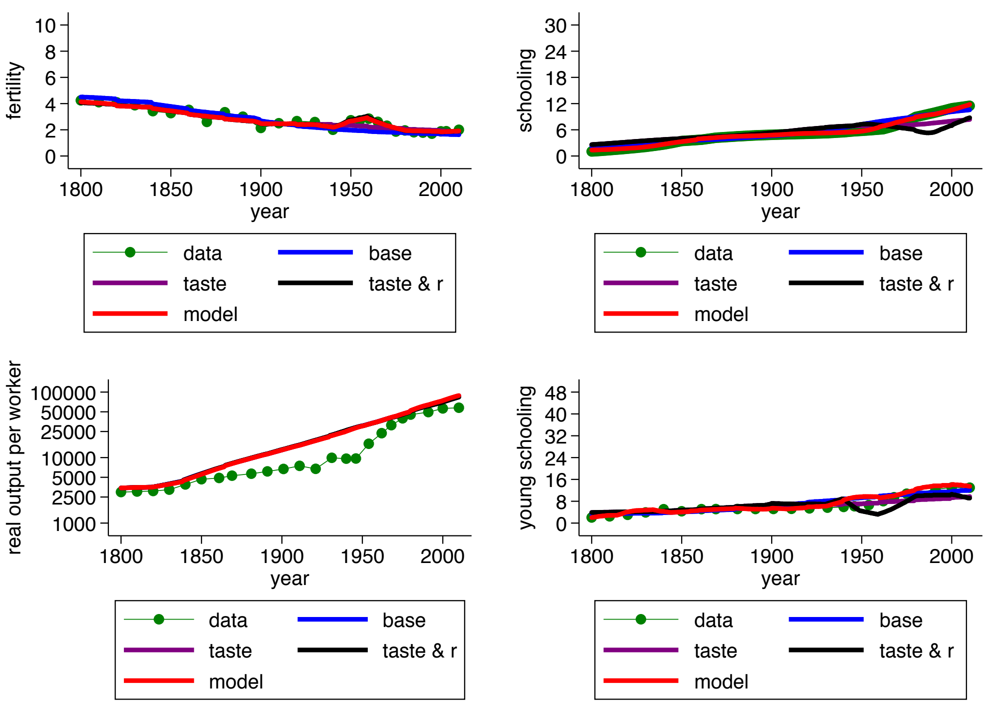

Figure 8: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): France

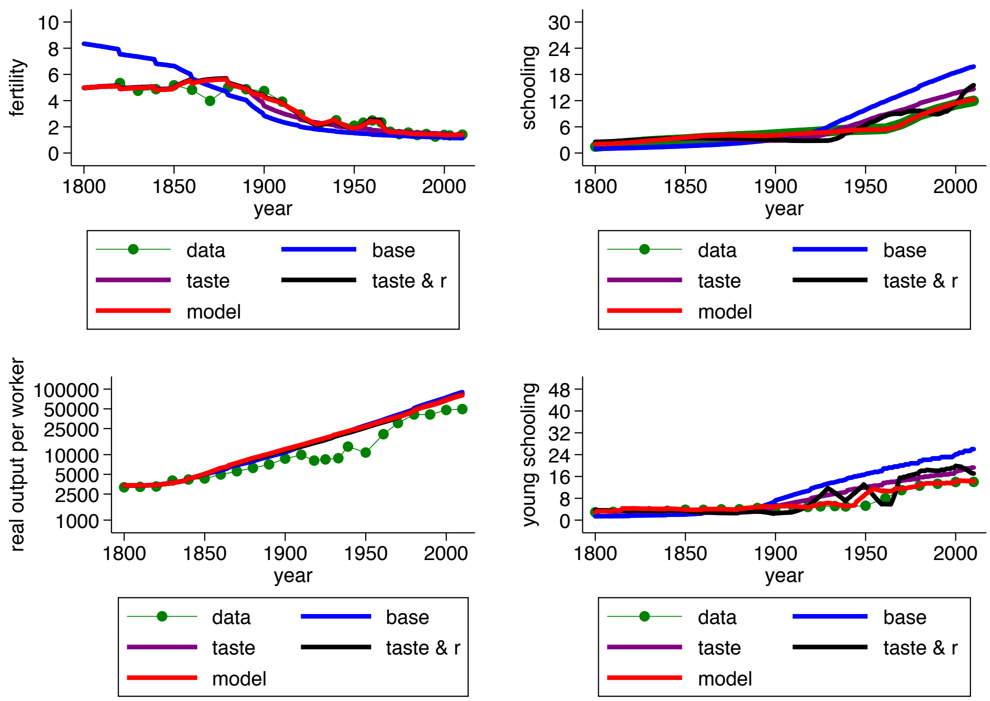

Figure 9: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Germany

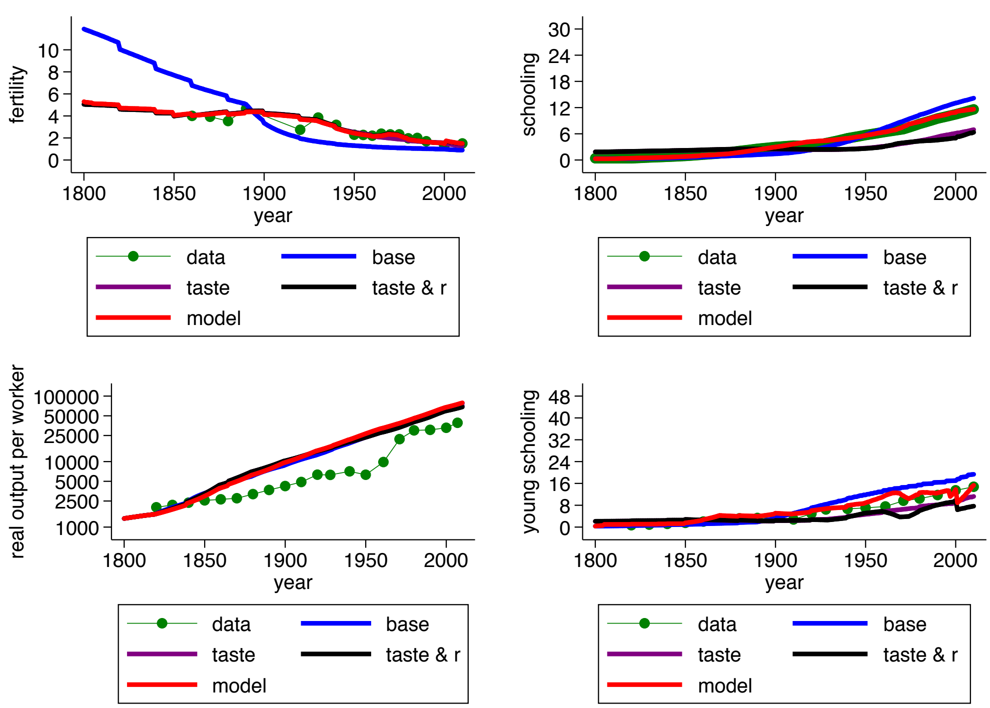

Figure 10: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Greece

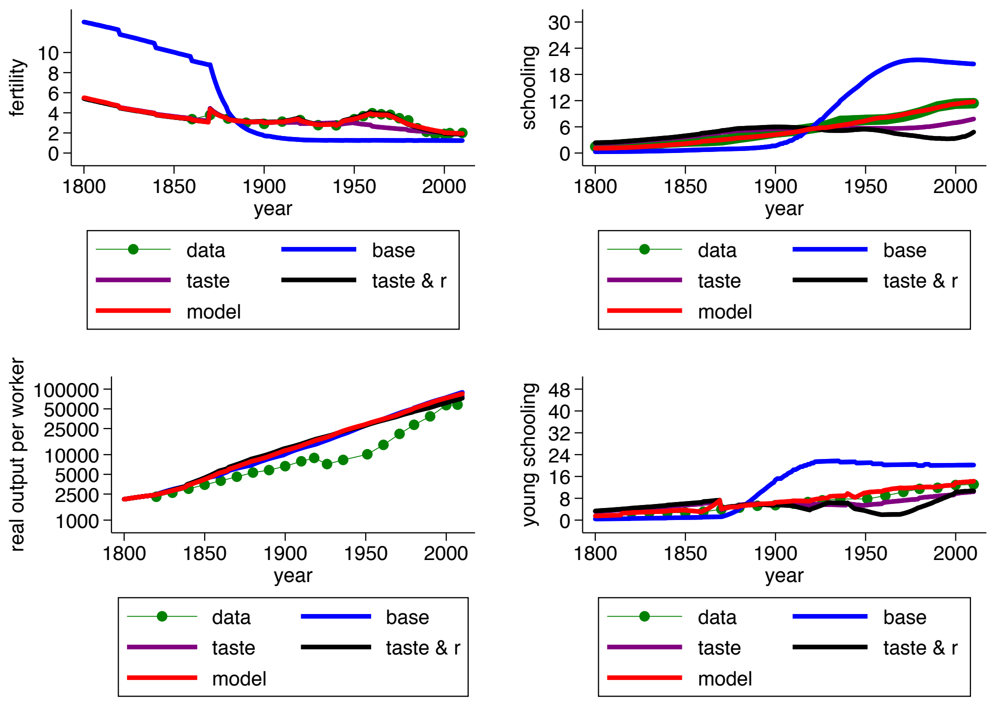

Figure 11: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Ireland

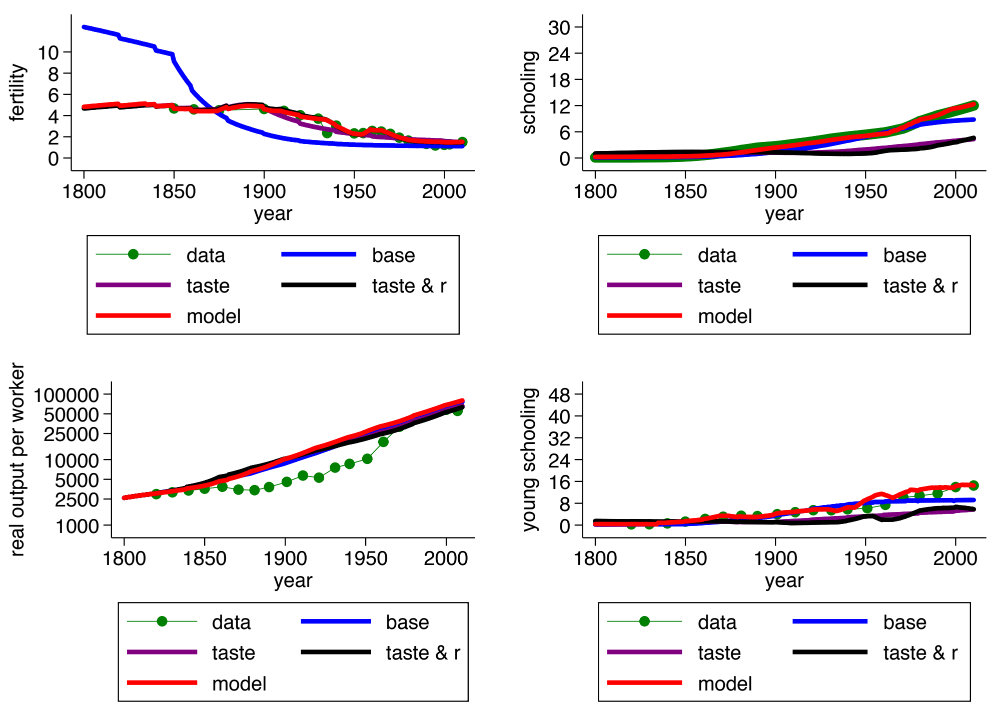

Figure 12: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Italy

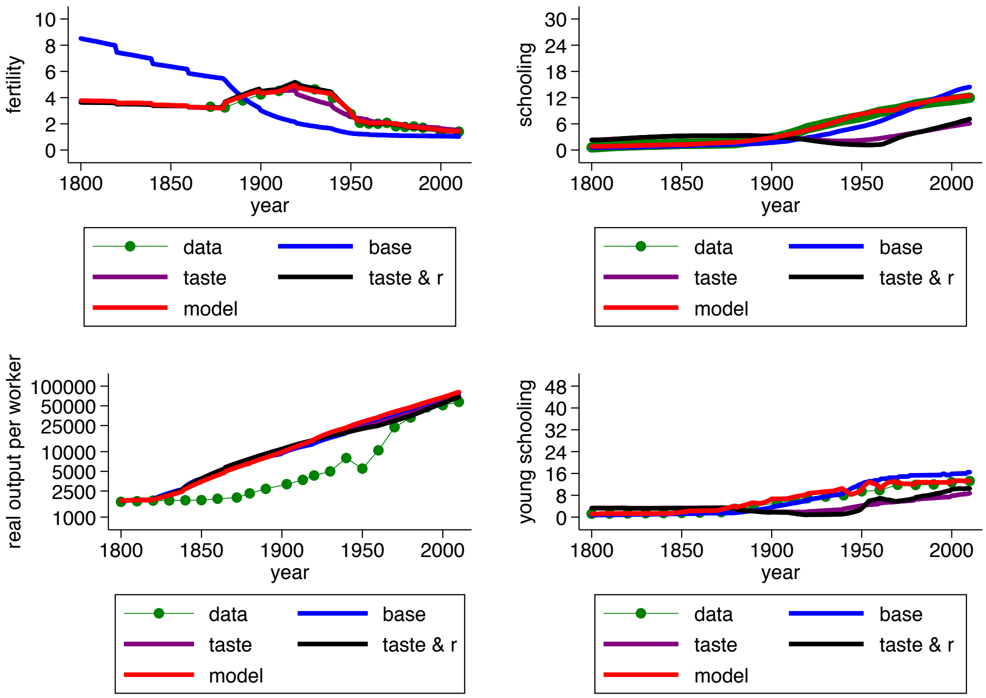

Figure 13: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Japan

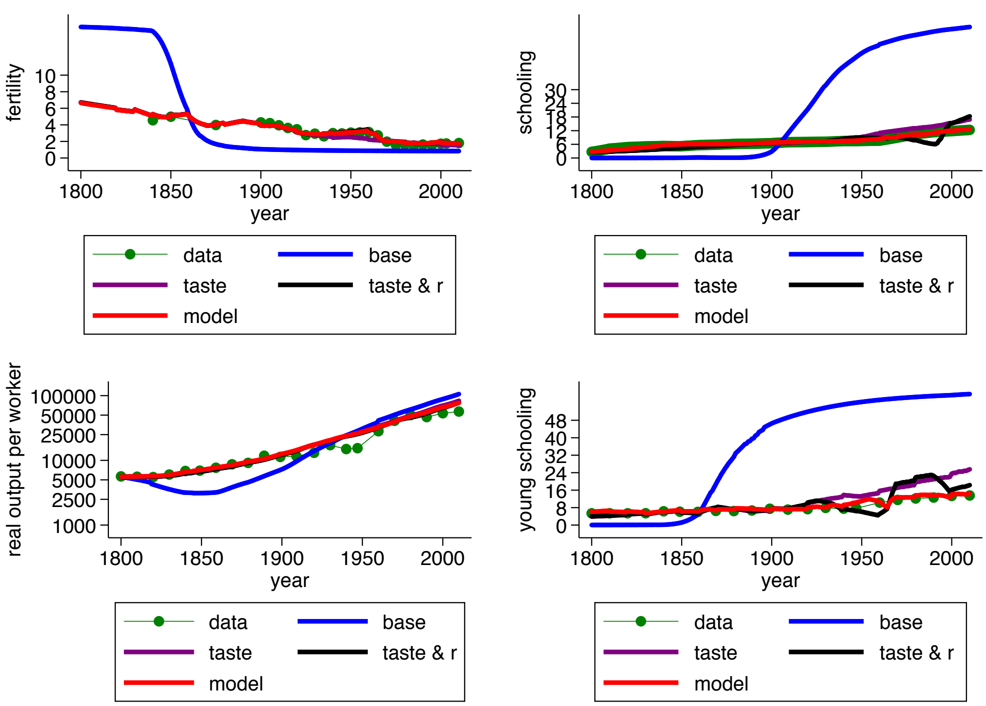

Figure 14: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Netherlands

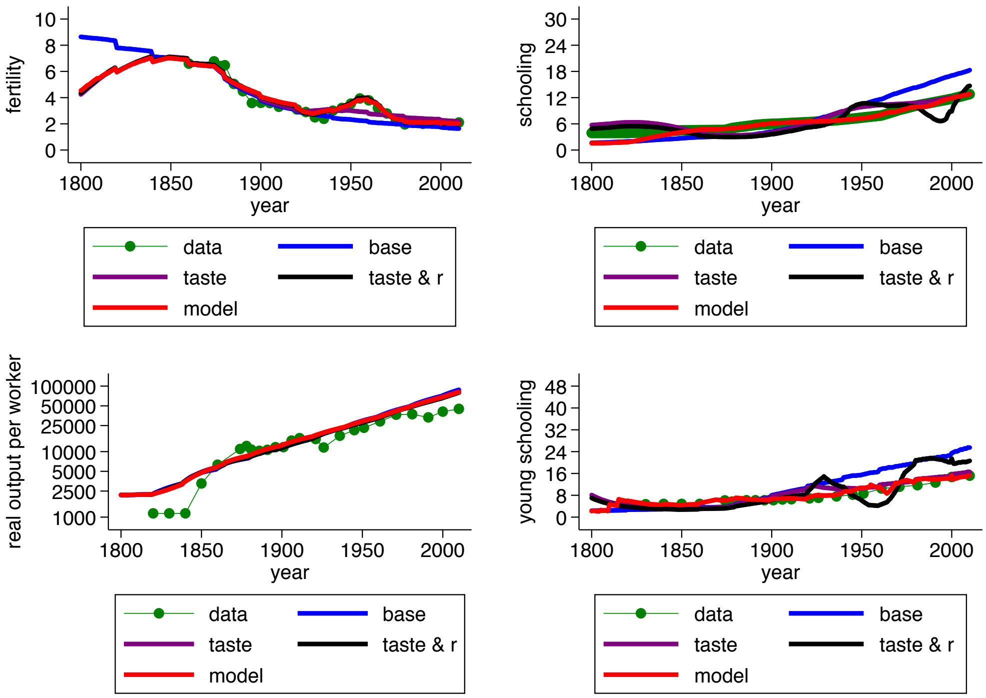

Figure 15: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): New Zealand

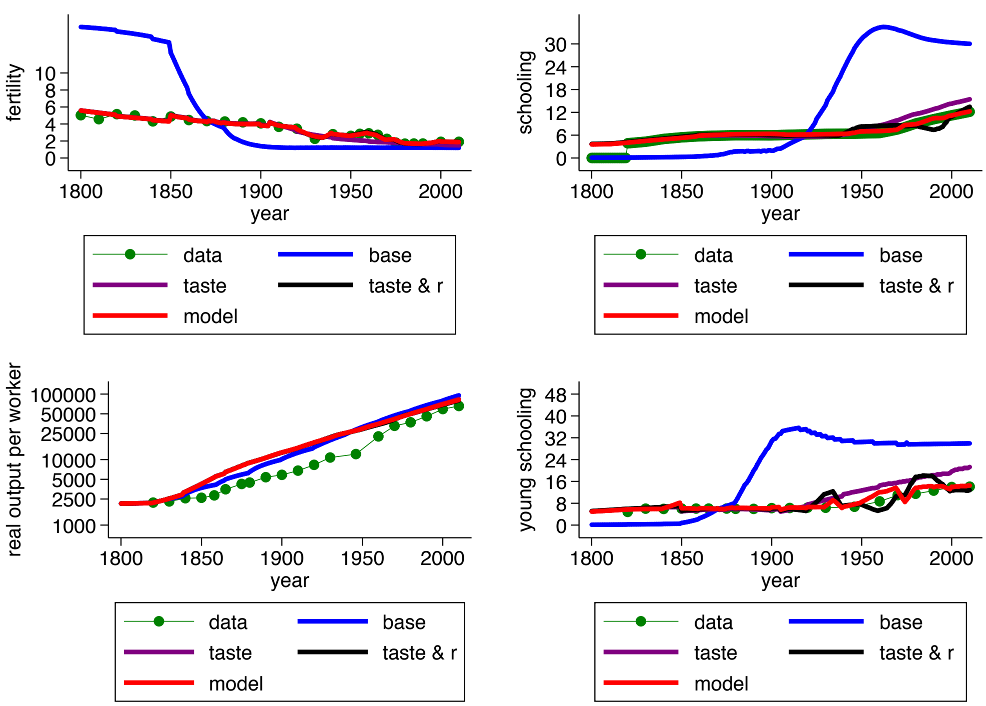

Figure 16: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Norway

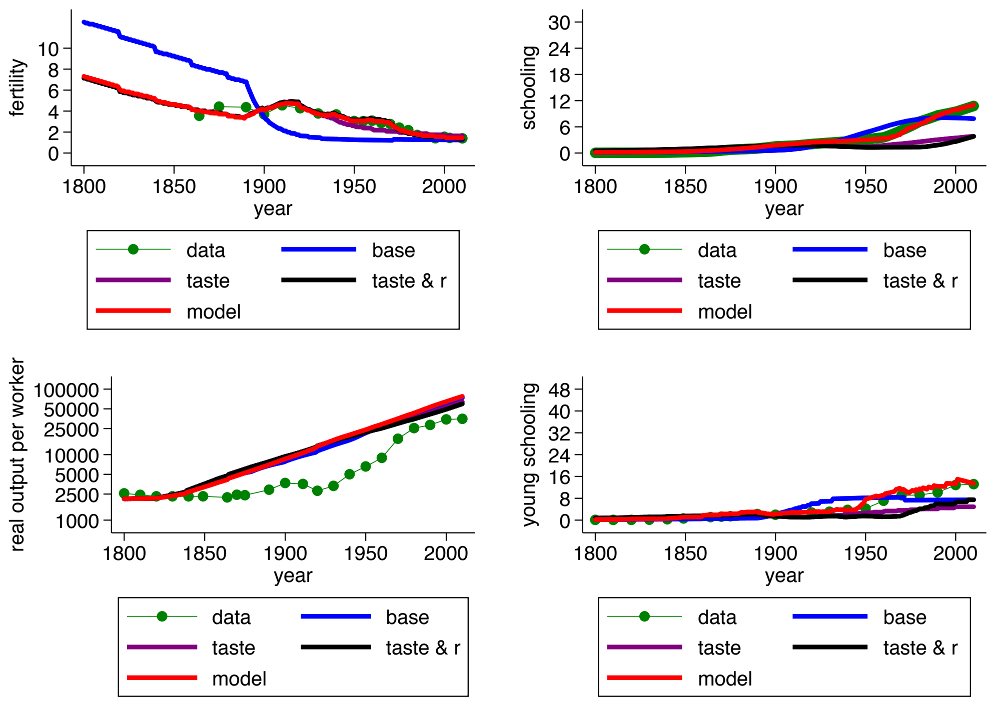

Figure 17: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Portugal

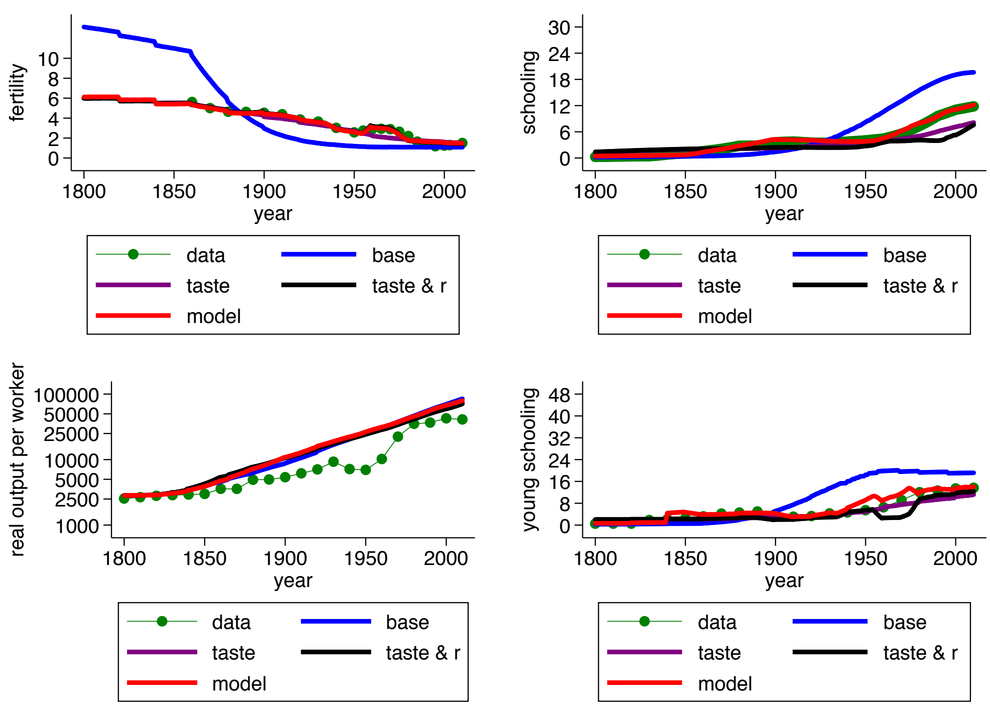

Figure 18: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Spain

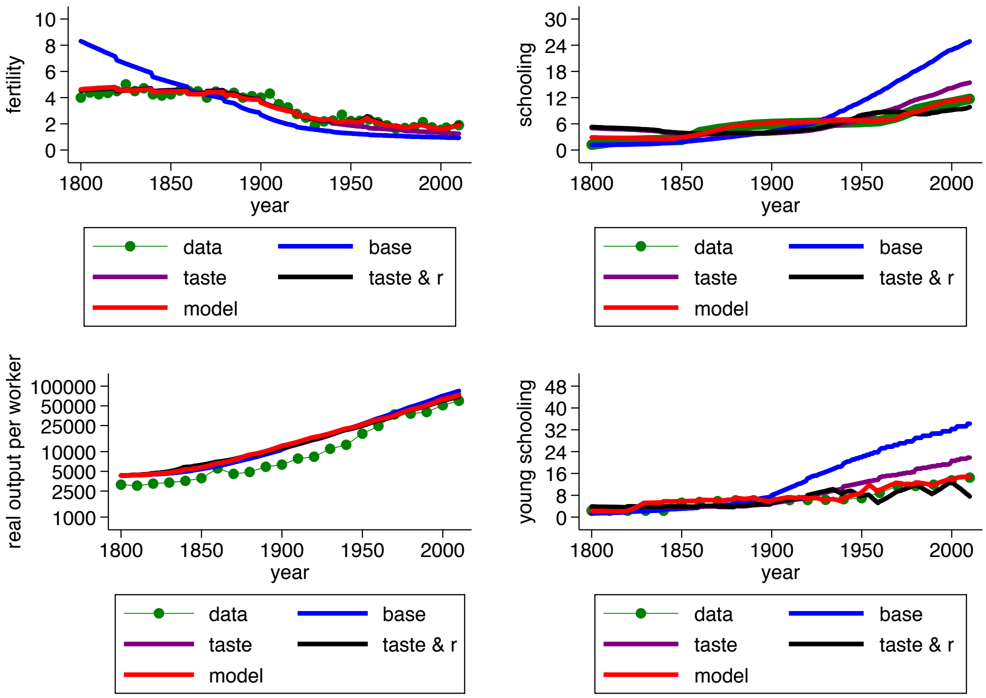

Figure 19: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Sweden

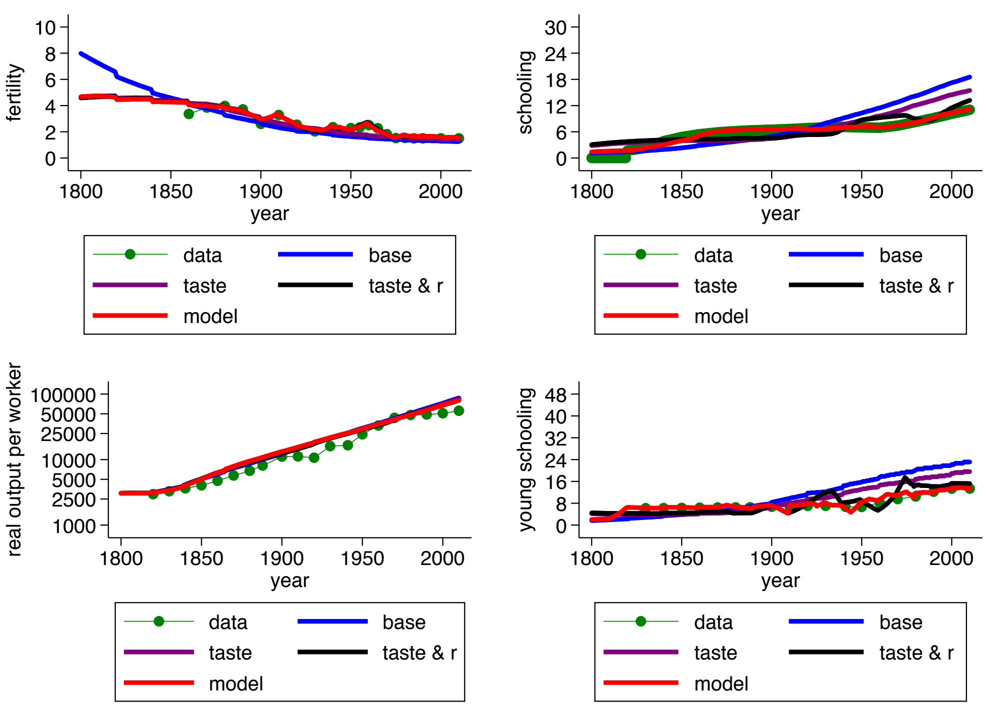

Figure 20: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): Switzerland

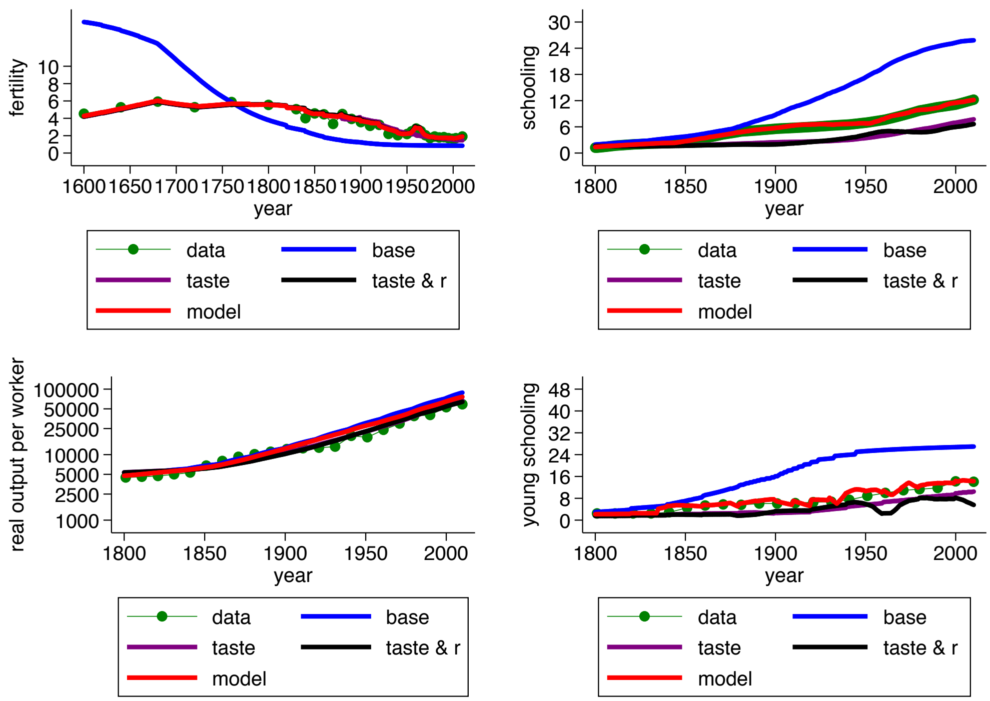

Figure 21: Fertility, Schooling, Real Output Per Worker, Young Schooling (clockwise): United Kingdom

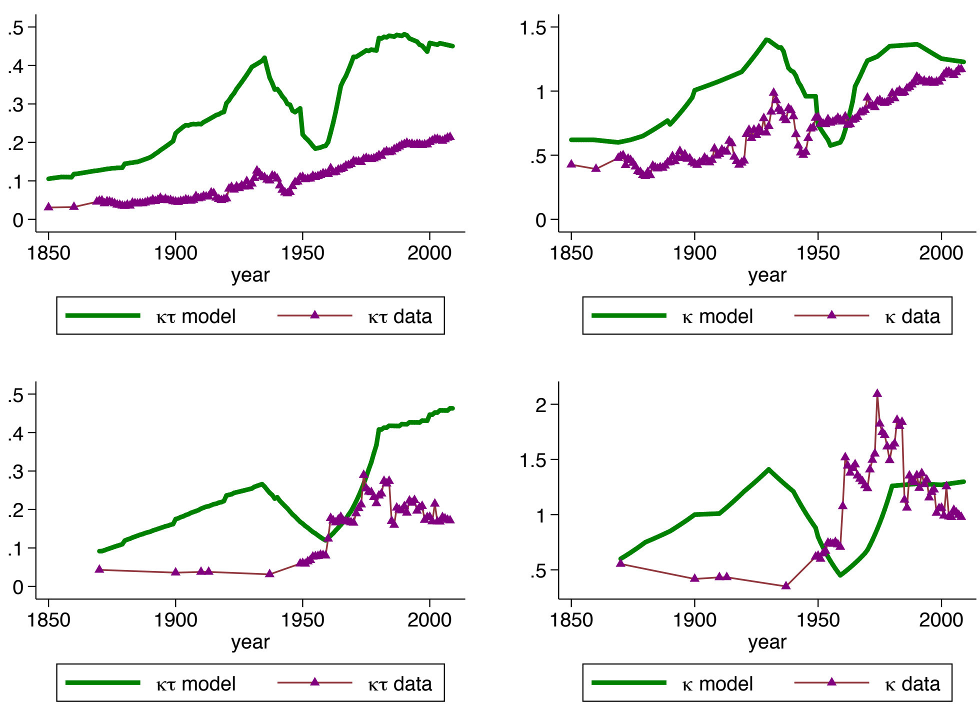

Figure 22: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: USA top, Australia bottom

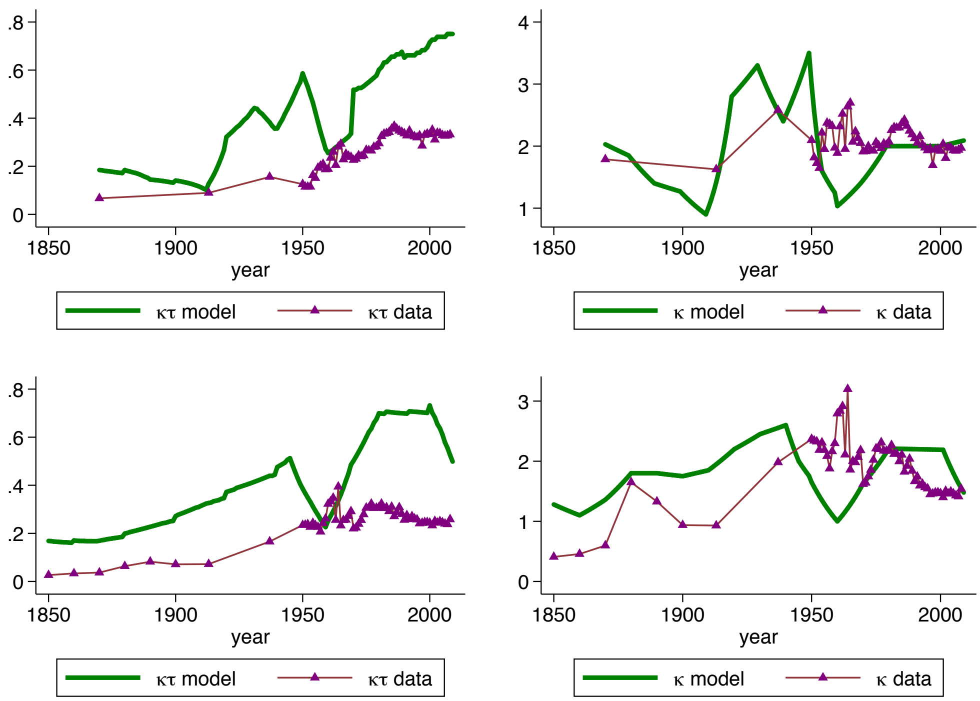

Figure 23: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Austria top, Belgium bottom

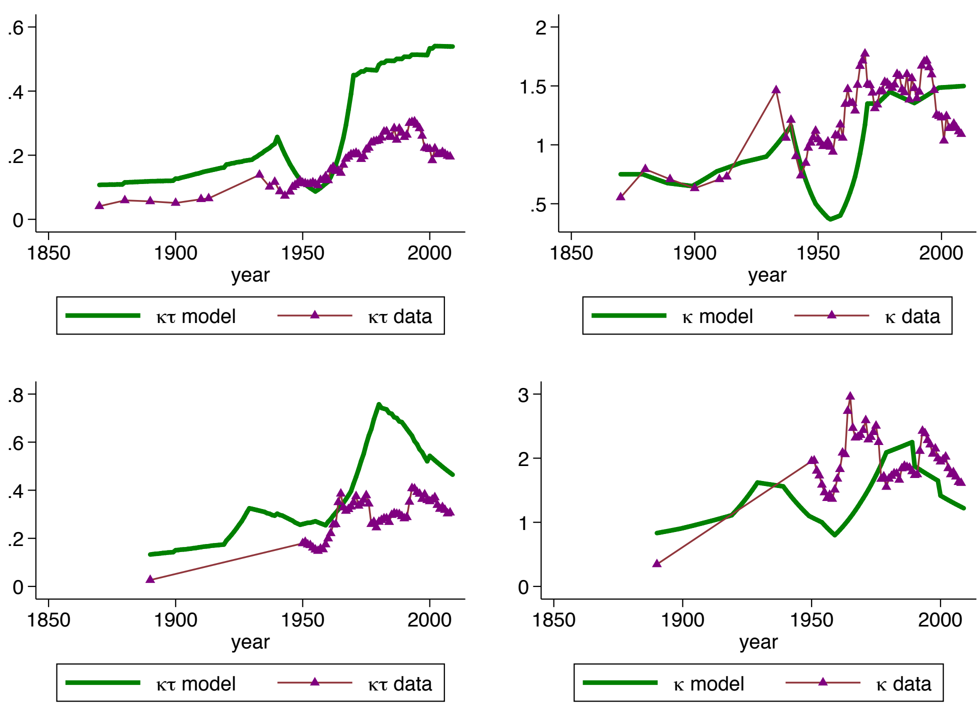

Figure 24: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Canada top, Denmark bottom

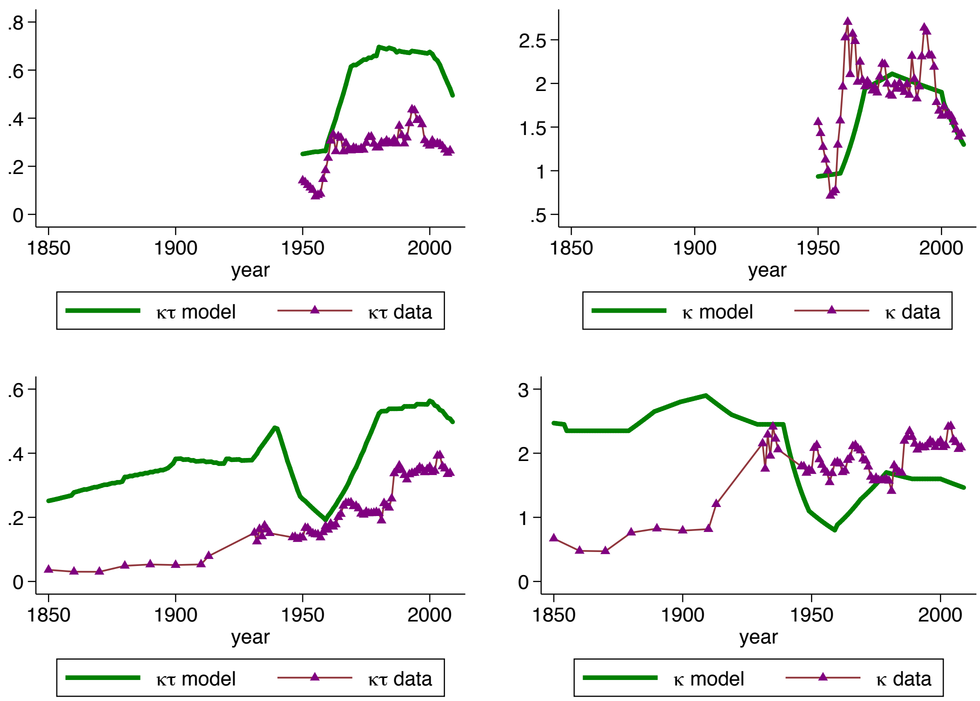

Figure 25: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Finland top, France bottom

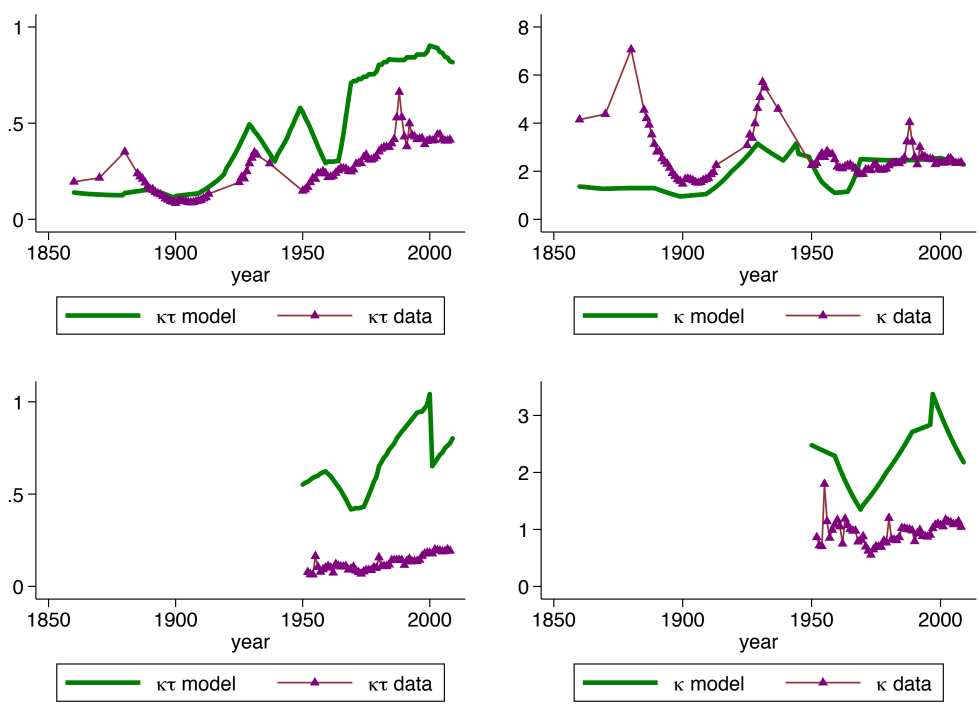

Figure 26: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Germany top, Greece bottom

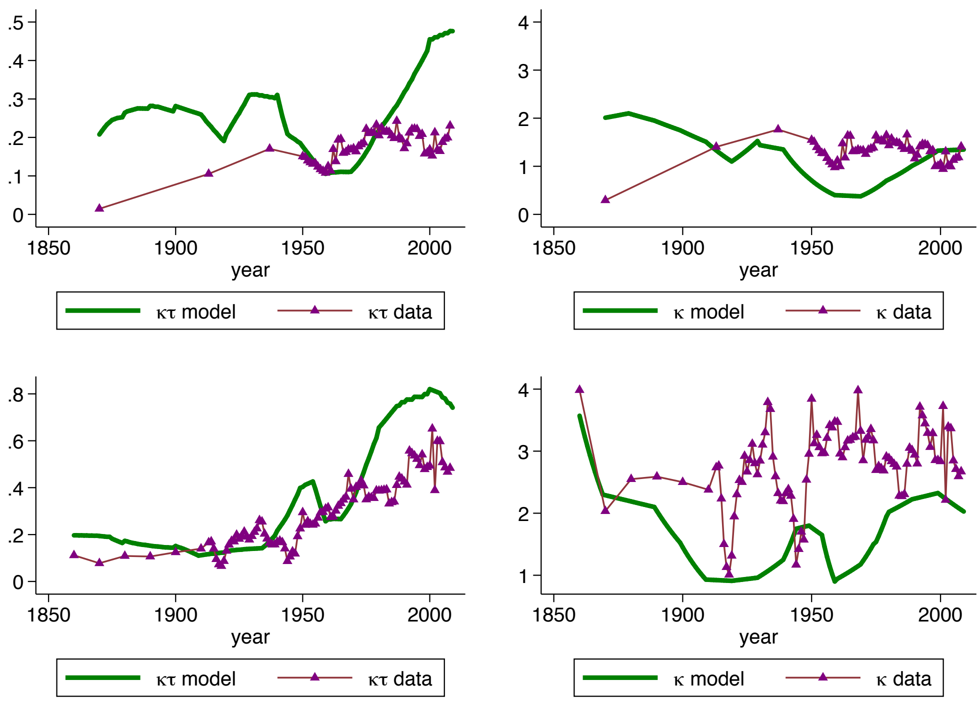

Figure 27: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Ireland top, Italy bottom

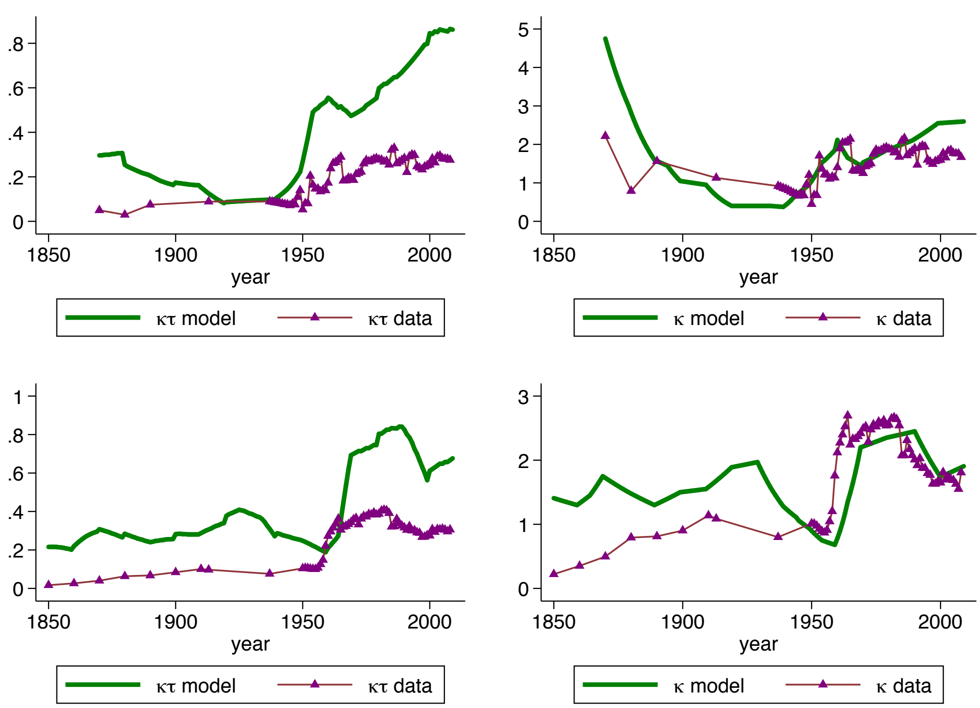

Figure 28: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Japan top, Netherlands bottom

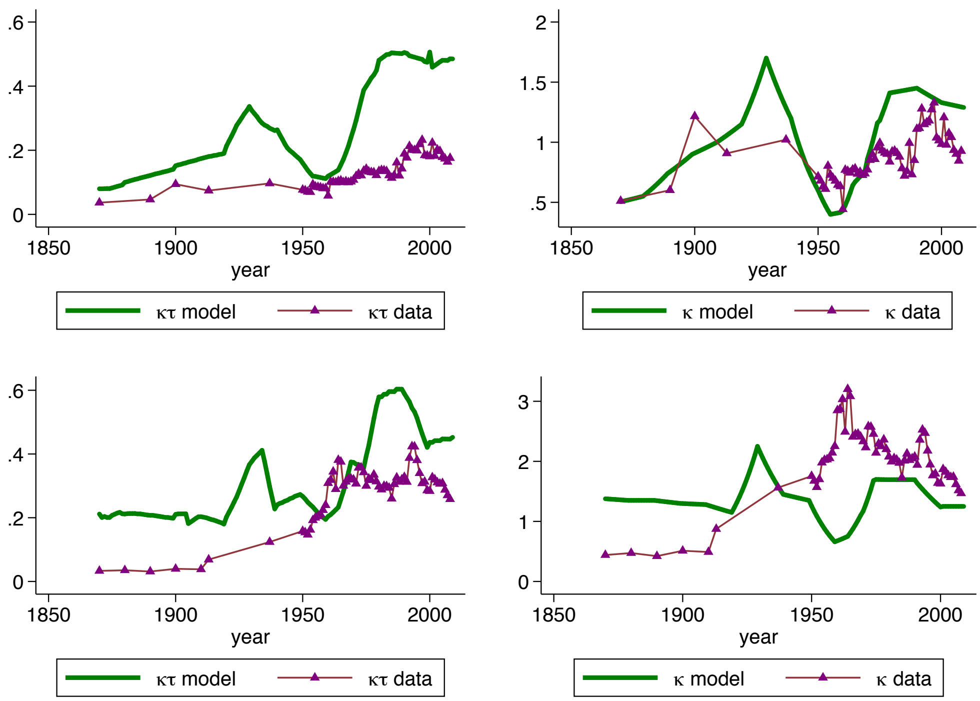

Figure 29: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: New Zealand top, Norway bottom

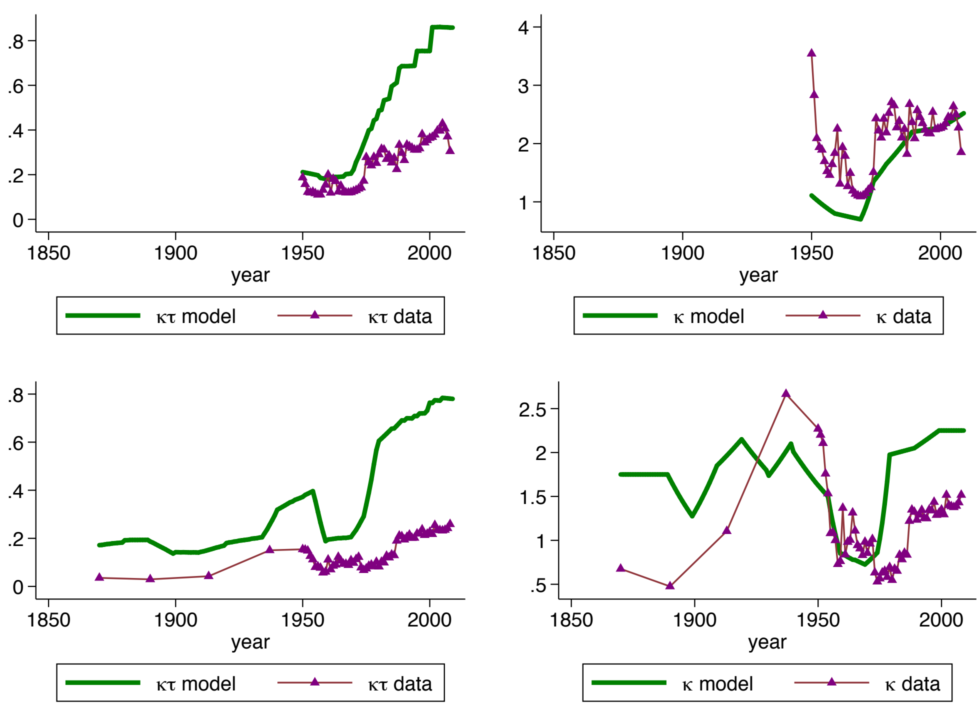

Figure 30: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Portugal top, Spain bottom

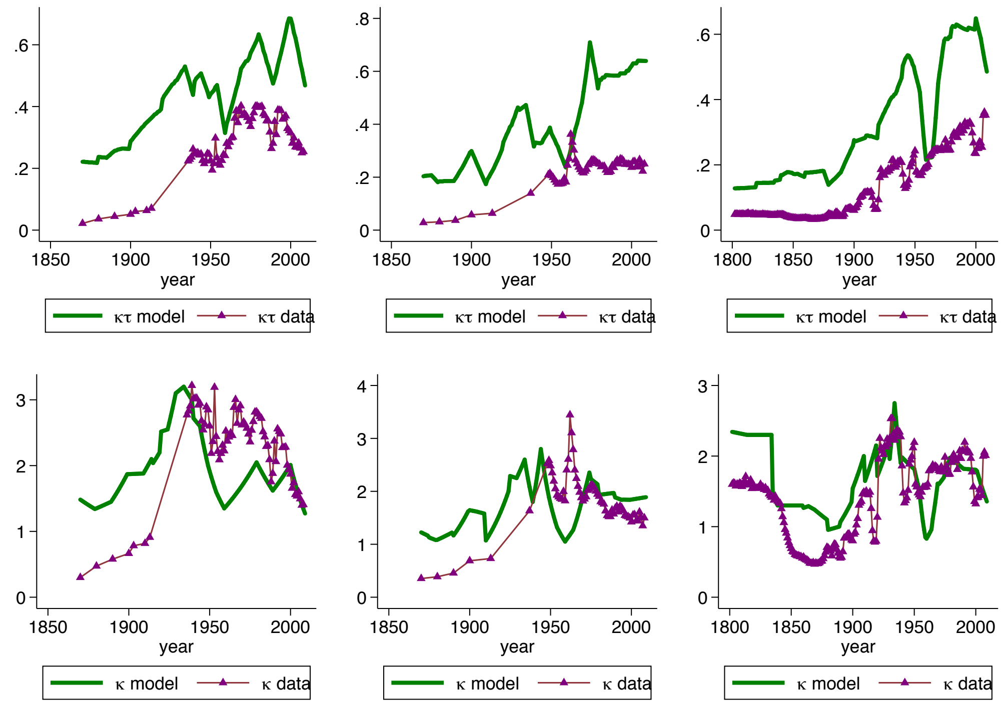

Figure 31: $(\kappa\tau)_{model}$ & $(\kappa\tau)_{data}$, κ_{model} & κ_{data}: Sweden left, Switzerland middle, United Kingdom right

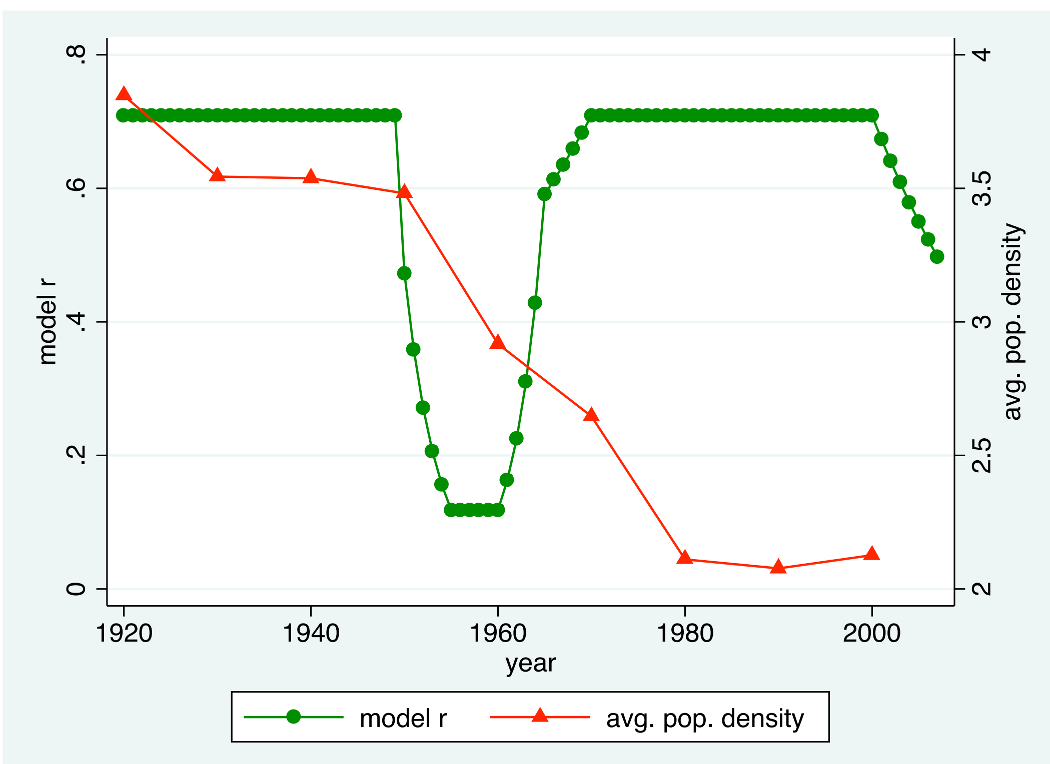

Figure 32: Model Rental Price of Space and US Population Density

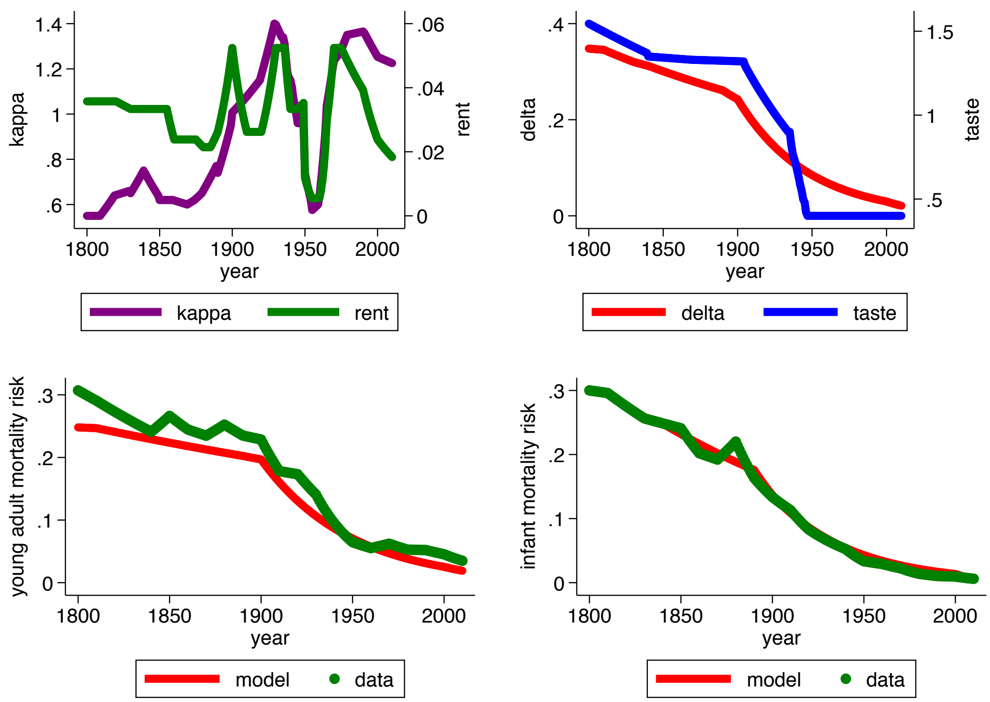

Figure 33: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: USA

5 Appendix A: Rents, κ, ν and Mortality

In this section we present the data for rents, r, preference parameter ν, the cost of schooling per child, κ and mortality. We present both the model mortality, based on splined regressions, both for mortality between the ages of 1 and 35, and infant mortality as well as the data for mortality. In the splined regressions we split the data into two regimes, pre 1900 and 1900-1950. We shut off mortality from 1950 onward in the projections in order to capture the idea that the mass introduction of penicillin was unexpected. The top left panel graph contains the price of schooling, κ_t, and rent series. The top right panel graph contains the model young adult mortality, δ_t, and taste parameter, ν_t. The bottom two panels contain components of young adult mortality. The bottom left graph contains the model probability of dying between the ages of 1 and 35, and the actual probability of dying before 40. The bottom right graph contains the model infant mortality, m_t, and the data on infant mortality. Recall that $\delta_t = \frac{m_t}{3} + prob(\text{dying between 1 and 35})$.

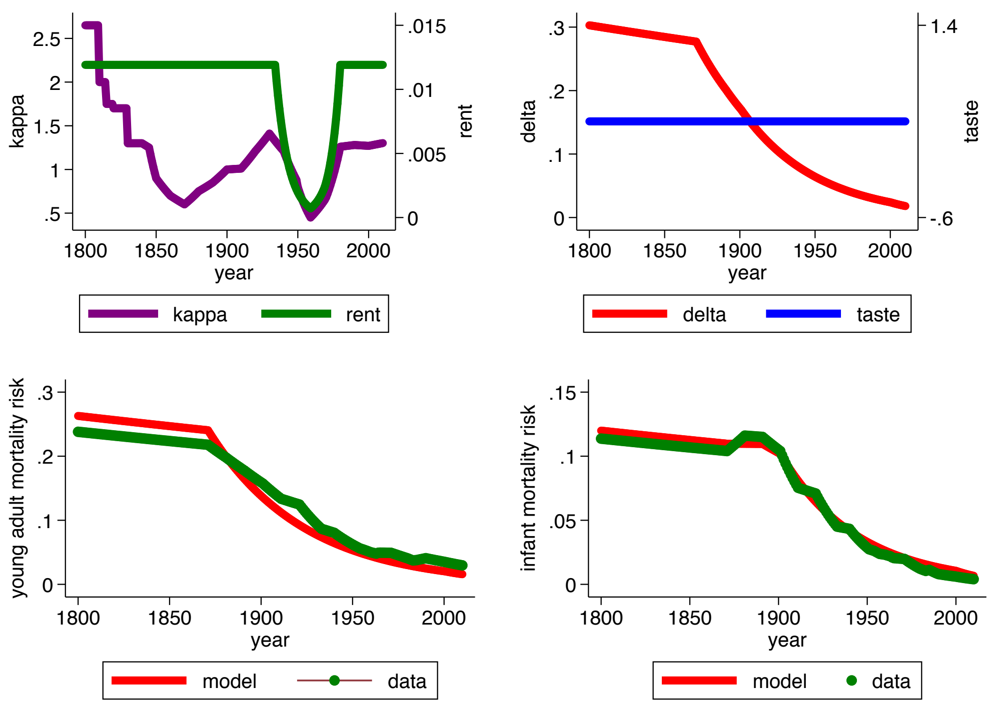

Figure 34: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Australia

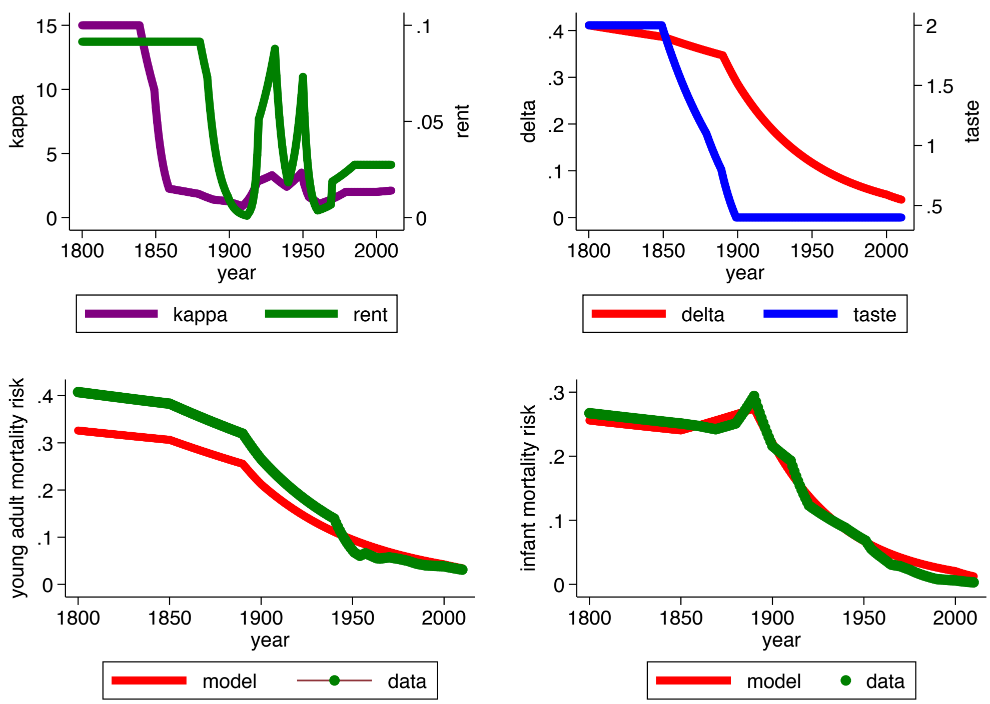

Figure 35: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Austria

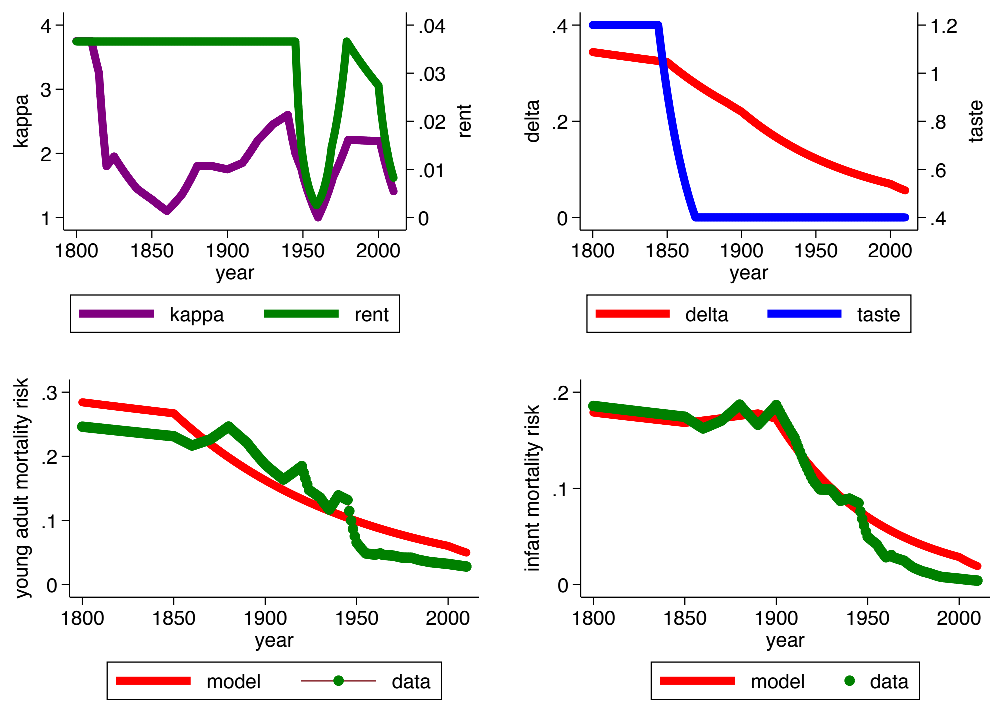

Figure 36: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Belgium

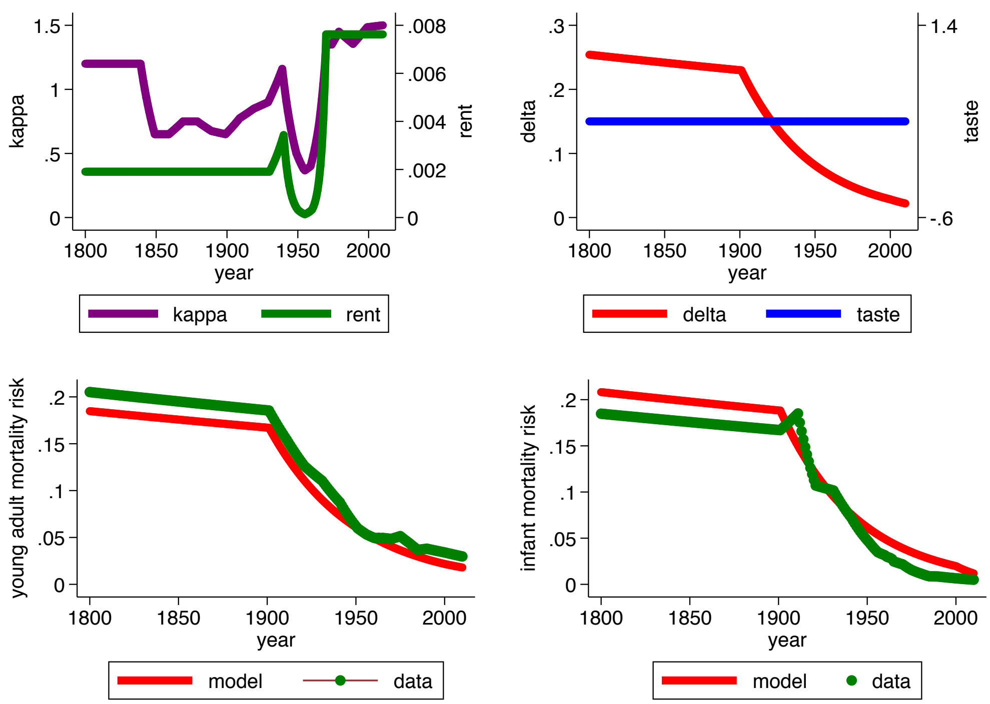

Figure 37: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Canada

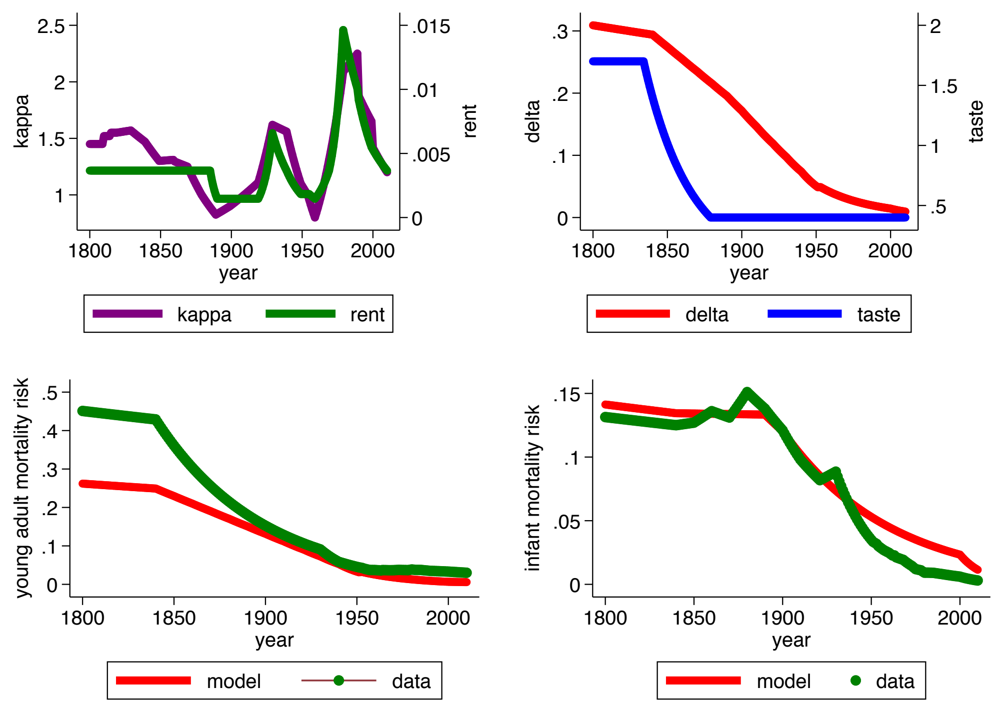

Figure 38: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Denmark

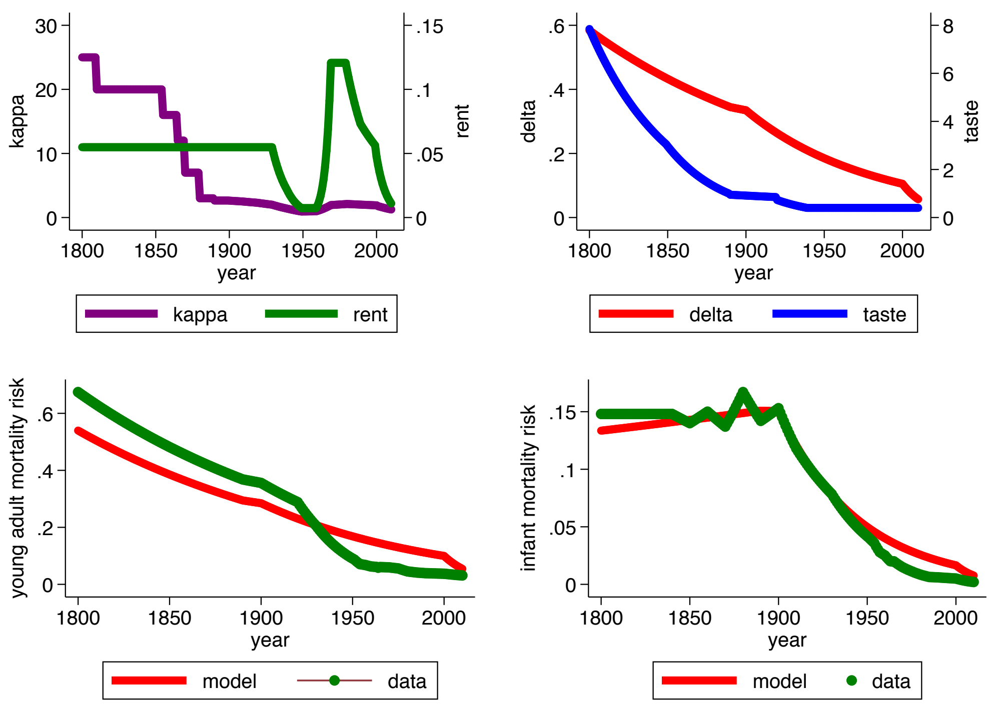

Figure 39: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Finland

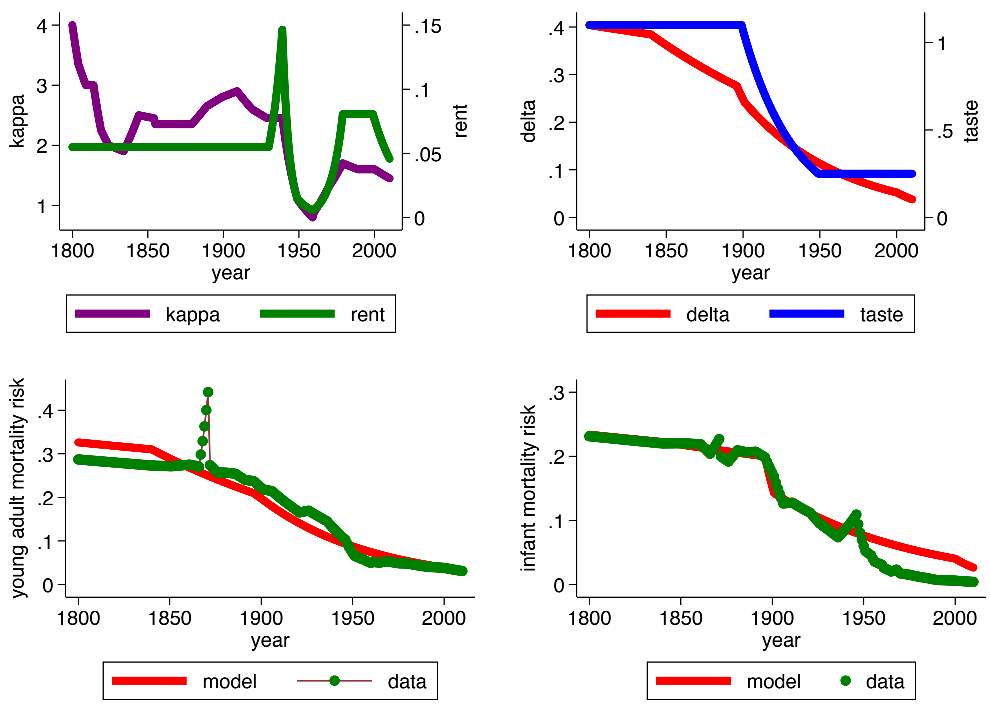

Figure 40: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: France

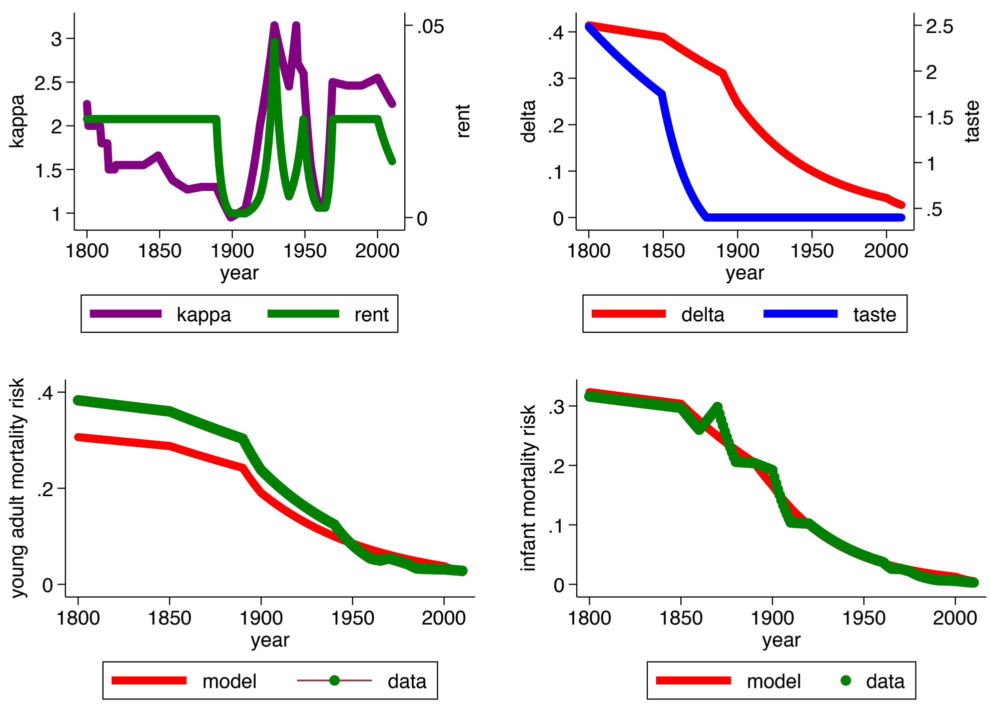

Figure 41: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Germany

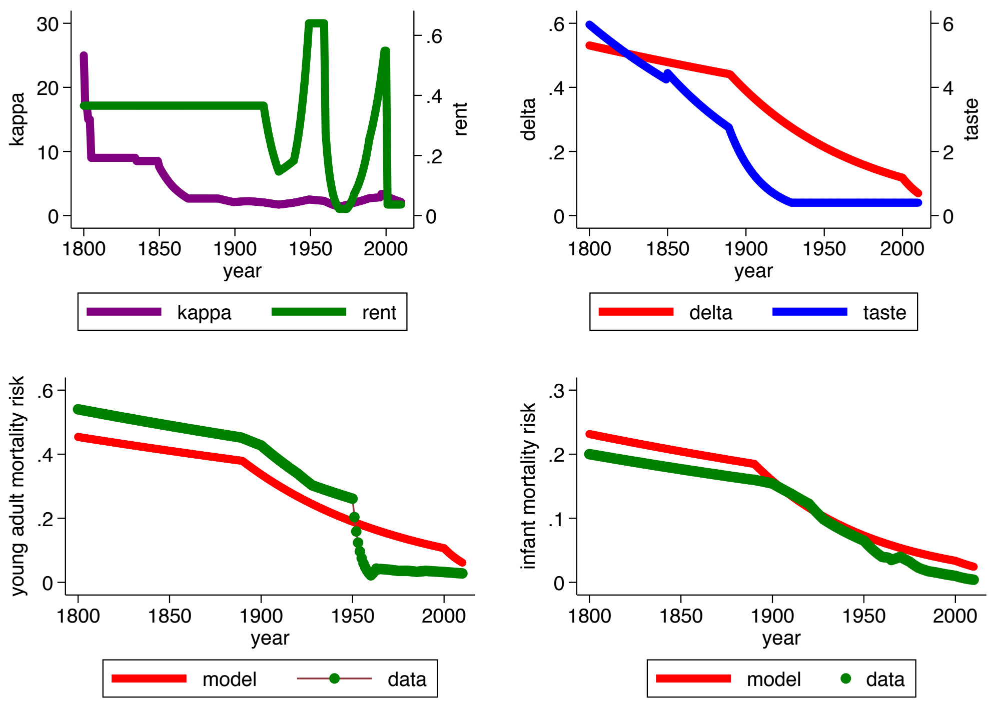

Figure 42: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Greece

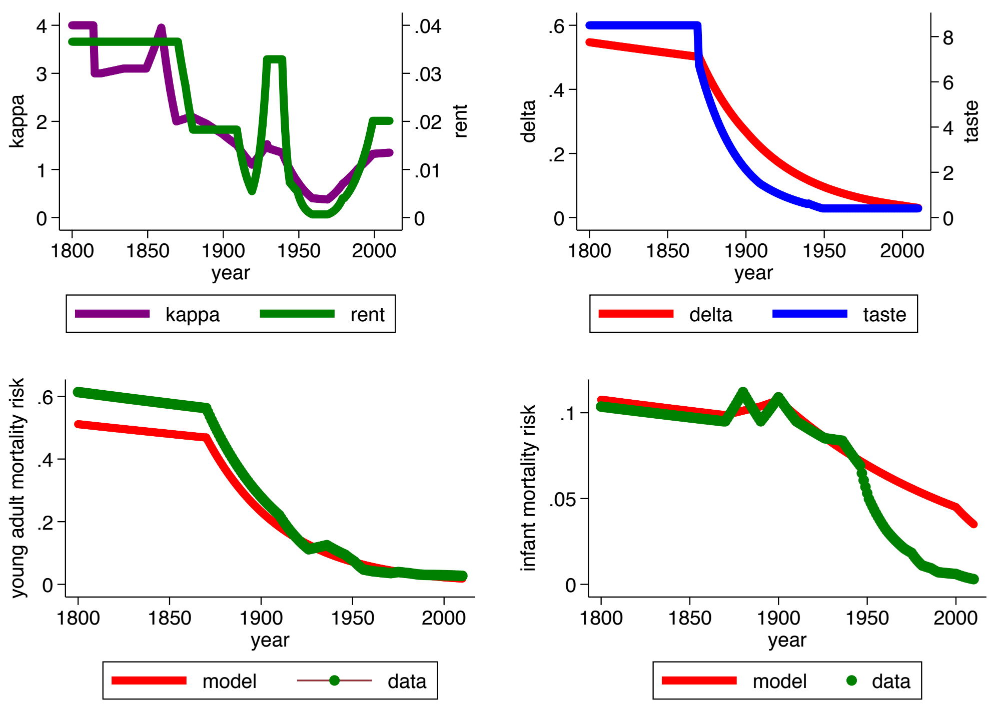

Figure 43: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Ireland

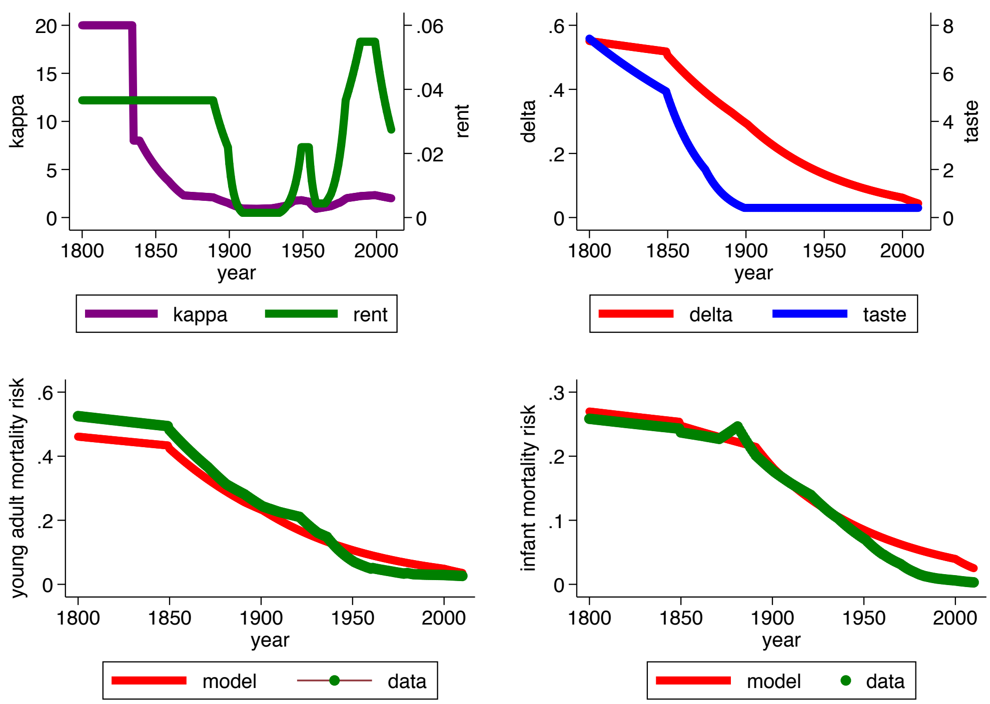

Figure 44: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Italy

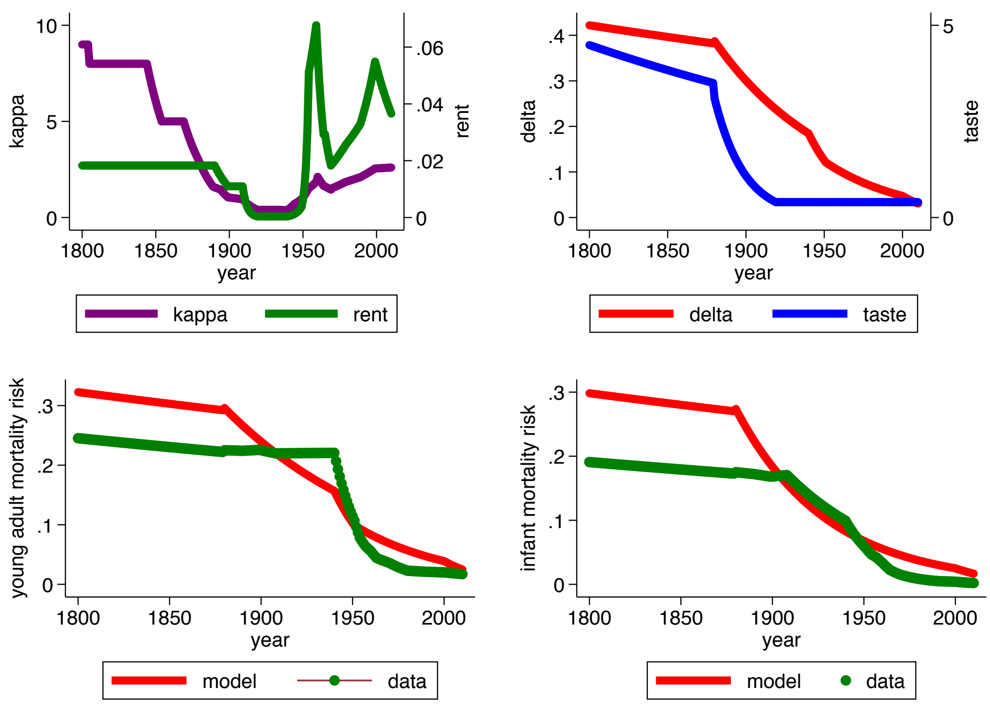

Figure 45: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Japan

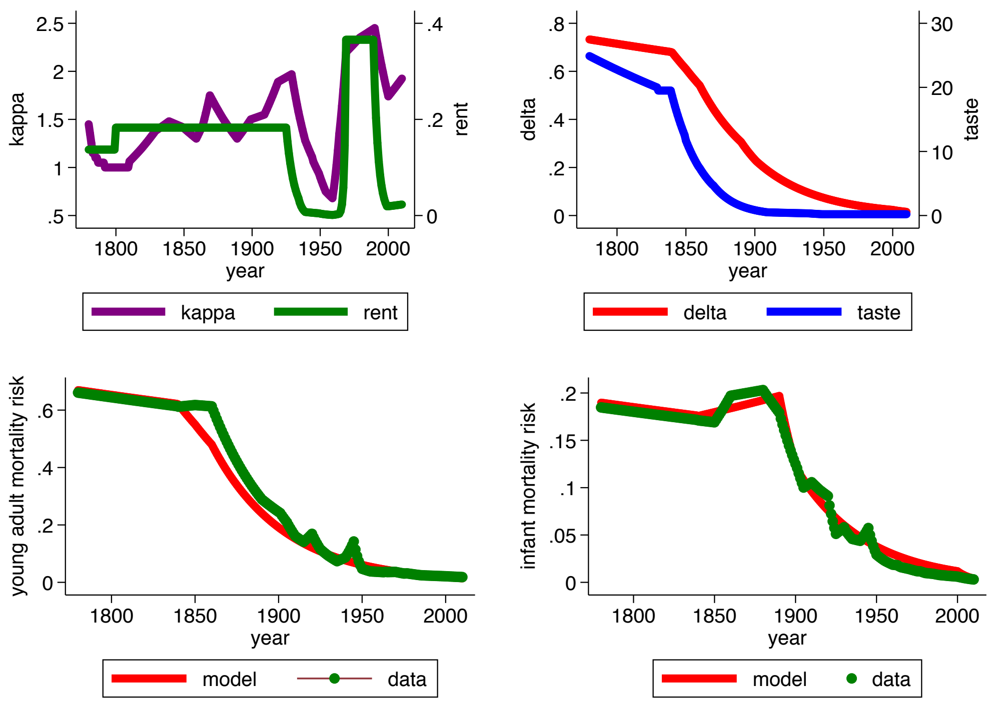

Figure 46: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Netherlands

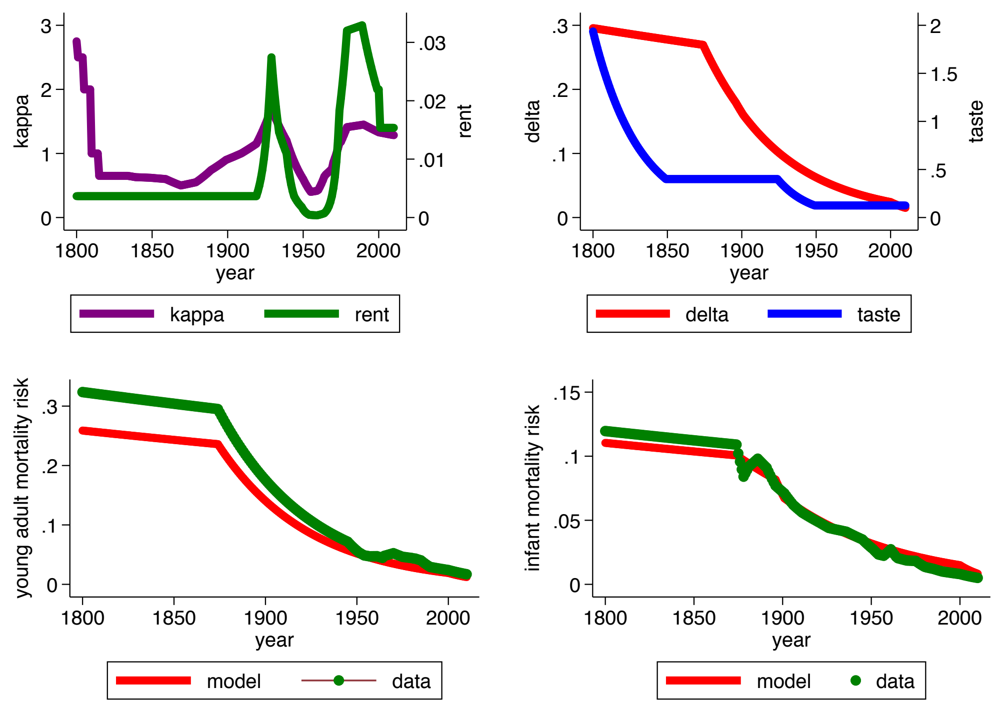

Figure 47: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: New Zealand

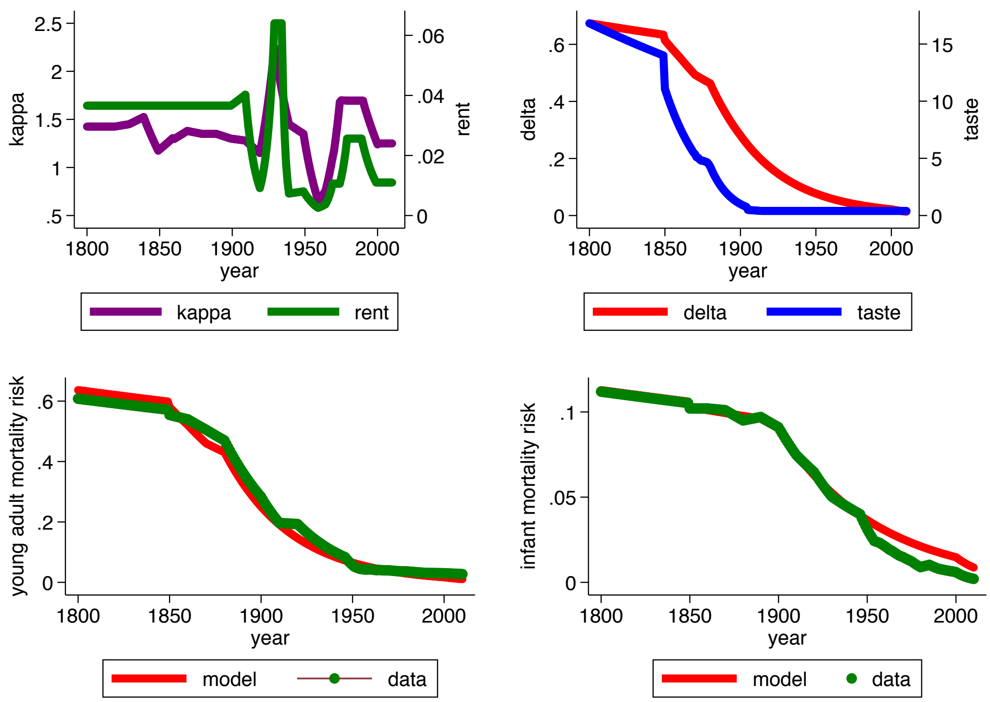

Figure 48: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Norway

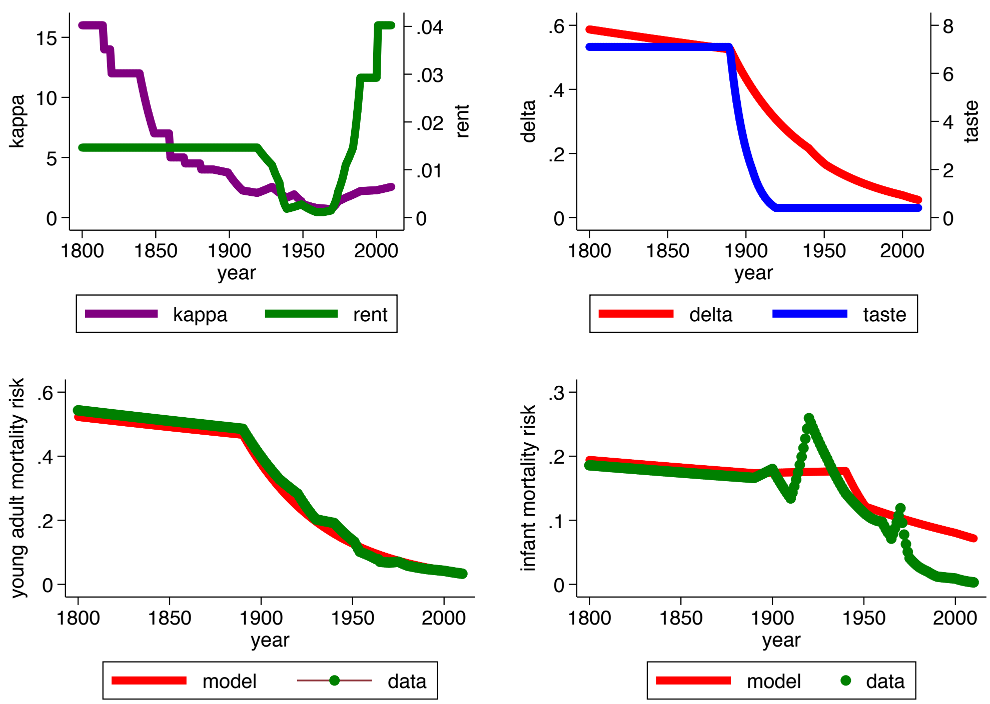

Figure 49: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Portugal

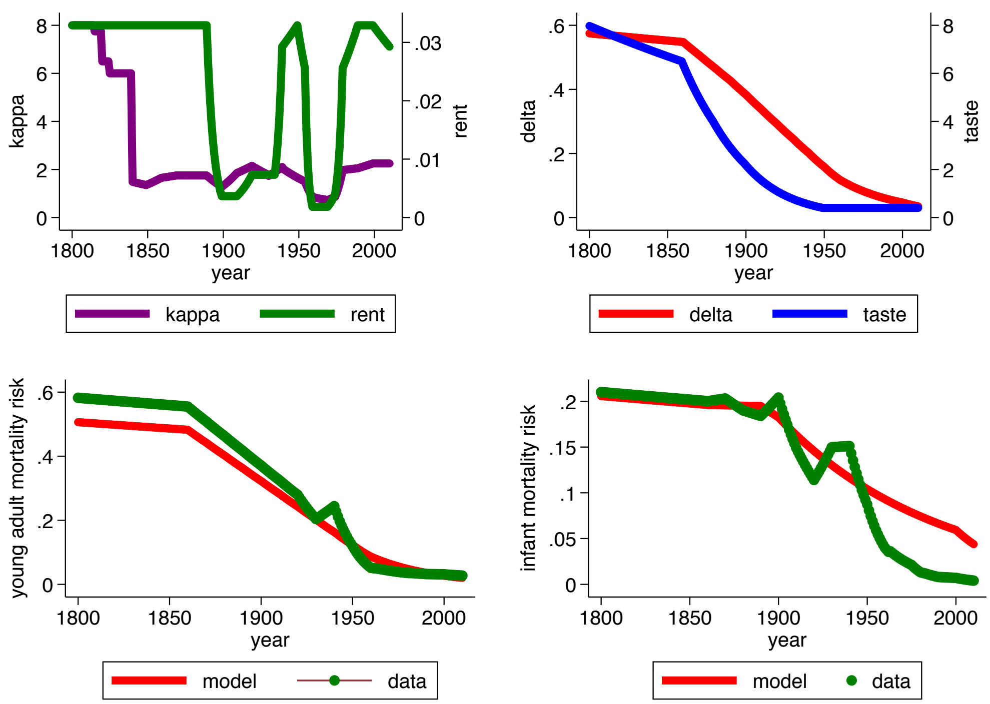

Figure 50: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Spain

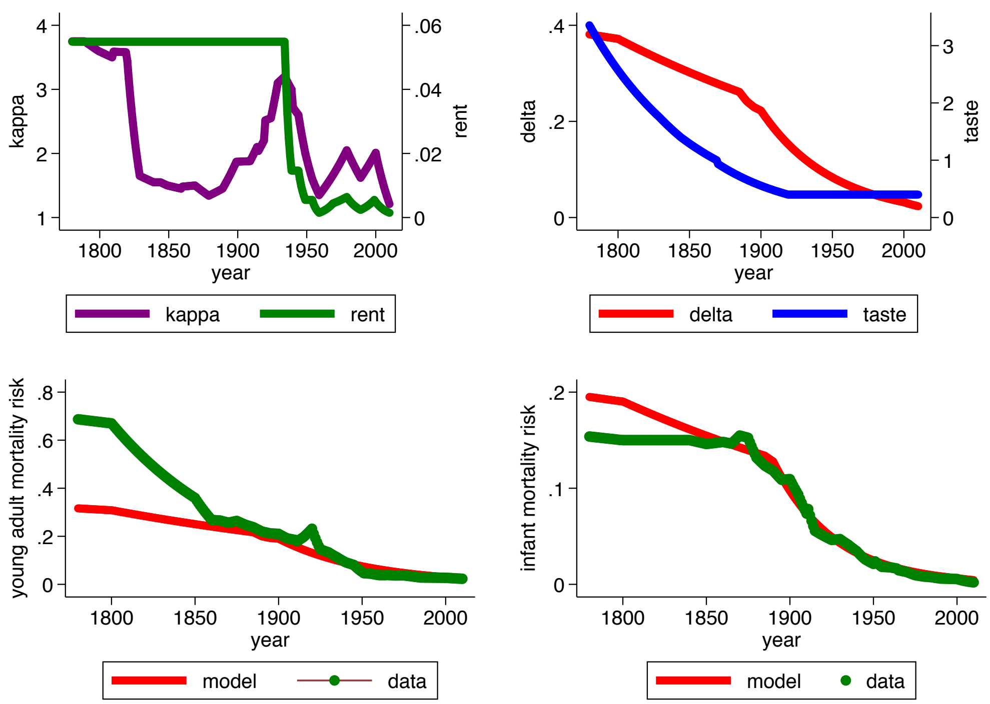

Figure 51: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Sweden

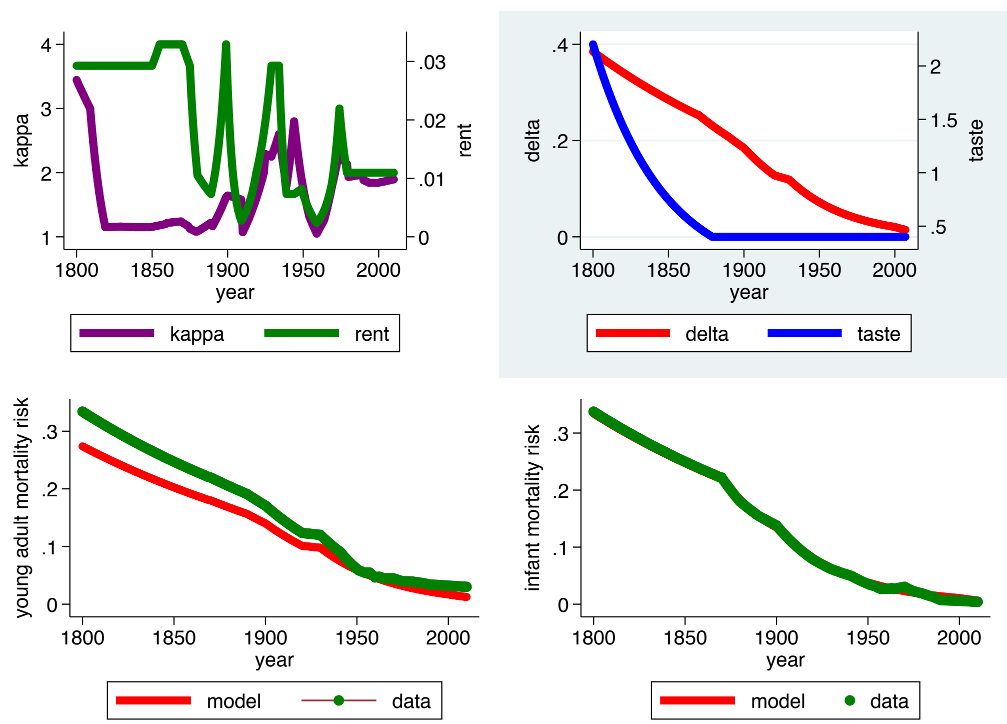

Figure 52: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: Switzerland

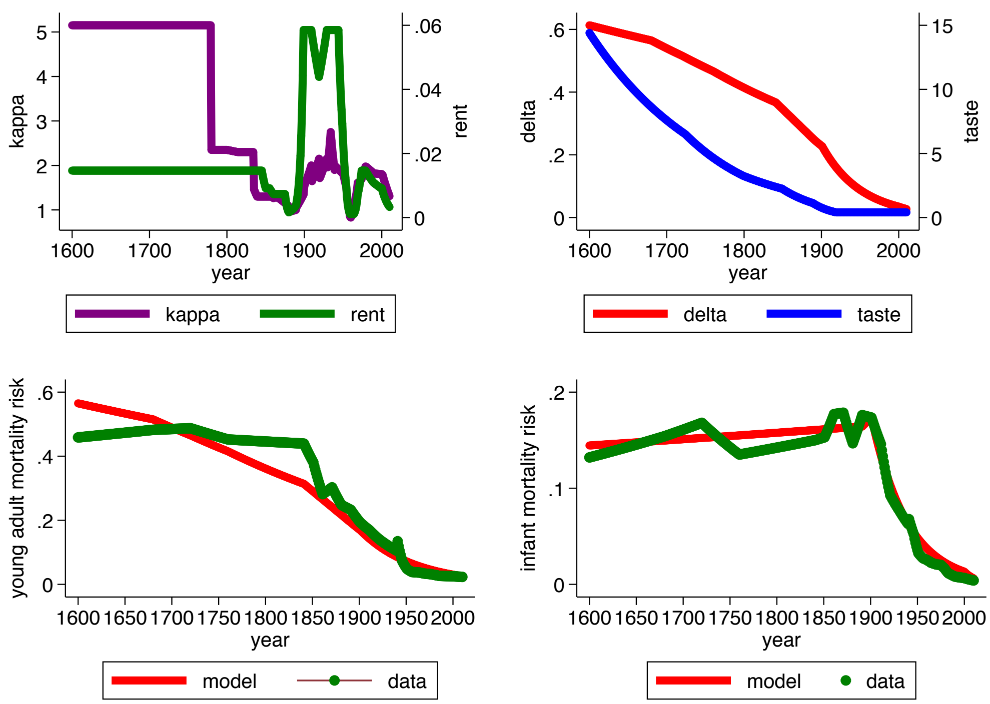

Figure 53: Rents, κ_t, δ_t, ν_t, model prob(dying between 1 & 35) & data prob(dying before 40), infant mortality: United Kingdom

Table 1: Parameter Values & Calibration

parameter	value	parameter	value	parameter	value
α	0.275	μ	0.085	A	1.55
ψ	0.660	$\overline{\tau}$	0.38125	p	1.000
φ	0.550	a	0.40073833	r	1.529679
θ	0.125	Λ	2.014672872		
			Calibration		
variable	model	minimum	maximum	average	notes
fertility	2.00	1.4	2.1	1.8	*Human Development Report 2014*
schooling	15.25	13.0	15.2	14.0	Tamura, Dwyer, Devereux & Baier
		15.2	19.8	16.4	*Human Development Report 2014*
		15.1	19.4	16.0	males only *Human Development Report 2014*
annualized growth rate	1.80%			1.80%	US 1840 - 2000[1]
housing share	0.19			0.19	US value[2]
				0.21	rest of OECD[2]
next generation share	0.44			0.48	US
				0.21	avg 1950-2011 US investment rate[3]
				0.08	US public & private education rate[4]
				0.05	US 0-44 pop health expenditures[5]
				0.02	US R&D net of university[6]
				0.12	foregone earnings: schooling beyond 12 years[7]
				0.45	non US
				0.23	avg 1950-2010 non US investment rate[3]
				0.05	avg non US education rate
				0.03	health share for 0-44 pop health expenditures[8]
				0.02	non US R&D share[9]
				0.12	foregone earnings: schooling beyond 12 years[10]

Parameter values that are constant throughout the solution. The value of a and Λ are determined by the other parameters and are given by (13) and (14), respectively. We assume that consumption is the numeraire. The value of r is the average white state density for 2000, where we weighted by the 2000 white population. [1]For the model we assumed that the growth rate is computed as $\ln(A\overline{\tau}^{\mu})/20$. Annualized growth of real output per worker from 1840-2000, Turner, Tamura and Mulholland (2010). [2]*OECD Better Life Index.* [3]*Penn World Tables.* [4]*Digest of Education Statistics* [5]Lassman, et al. [6]Figure comes from WDI less R&D expenditures by universities, the latter figure from *Chronicle of Higher Education Almanac issue 2013-14*. [7]Authors' calculations using a 4.5% discount rate, \$31,700 median full time male worker earnings of high school graduates (25-34 years old), and \$41,700 median full time male worker earnings of workers with an Associate Degree (25-34 years old). [8]$\frac{1}{3}$ of *HDR* health share of GDP for 0-44 age population. [9]*World Development Indicators.* [10]We assumed that outside of the US tuition is 0, and in fact workers receive a student stipend equal to one third of the high school wage to attend 4 years of schooling after high school.

Table 2: Pooled Regressions of Actual Observations on Model Solutions

	fertility				schooling			
	base	ν_t	ν_t r_t	ν_t r_t κ_t	base	ν_t	ν_t r_t	ν_t r_t κ_t
β	0.3390***	0.9345***	0.9799***	0.9954***	0.2606***	0.7853***	0.7778***	0.9767***
	(0.0573)	(0.0183)	(0.0096)	(0.0115)	(0.0845)	(0.0534)	(0.0445)	(0.0061)
α	2.1140***	0.2822***	0.0127	-0.0153	3.7037***	1.6323***	1.9600***	0.0942*
	(0.1664)	(0.0665)	(0.0281)	(0.0334)	(0.5404)	(0.2639)	(0.2566)	(0.0573)
N	576	576	576	576	446	446	446	446
$\bar{R}^2$	.5074	.9010	.9580	.9574	.4619	.6001	.4853	.9897
p	.0000	0.0000	.0002	.0188	.0000	.0000	.0000	.0000

	young schooling				ln(income)			
	base	ν_t	ν_t r_t	ν_t r_t κ_t	base	ν_t	ν_t r_t	ν_t r_t κ_t
β	0.2408***	0.6217***	0.5715***	0.9281***	0.9135***	0.9587***	0.9710***	0.9321***
	(0.0711)	(0.0520)	(0.0396)	(0.0084)	(0.0231)	(0.0157)	(0.0162)	(0.0160)
α	4.1610***	2.6244***	3.2526***	-0.2116**	0.5587**	0.1143	0.0089	0.3527**
	-	(0.5876)	(0.3005)	(0.2675)	(0.0645)	(0.1503)	(0.1537)	(0.1469)
N	461	461	461	461	446	446	446	446
$\bar{R}^2$	.4503	.6123	.5212	.9531	.9394	.9335	.9311	.9401
p	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Notes: Table reports results from fixed effects regressions with clustered errors on the country. *** 1%, *** 5%, * 10%. The final row, marked p, is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$. The regressions on ln(income) contain dummy variables for World War I, for years 1914-1918 inclusive, the Great Depression, for years 1930-1939, and for World War II, for years 1939-1945 inclusive.

Table 3: Pooled Regressions of Actual Observations on Model Solutions: Education Expenditures and κ

	$\kappa_t \tau_t$				κ_t			
	US	US	US	US	US	US	US	US
β	0.1529***	0.1966***	0.2746***	0.4169***	0.2239*	0.2294*	0.3827***	0.6159***
	(0.0424)	(0.0397)	(0.0359)	(0.0239)	(0.1200)	(0.1198)	(0.1085)	(0.0860)
α	0.0447***	0.0381***	0.0184	-0.0157**	0.4209***	0.4233***	0.2938***	0.0886
	(0.0169)	(0.0137)	(0.0115)	(0.0071)	(0.1424)	(0.1406)	(0.1129)	(0.0839)
20^{th} century	no	yes	no	yes	no	yes	no	yes
world war ii	no	yes	yes	yes	no	yes	yes	yes
baby boom	no	no	yes	yes	no	no	yes	yes
N	142	142	142	142	142	142	142	142
$\bar{R}^2$	.	0260	.1466	.6315	.0324	.0572	.1080	.2539
p	.0000	0.0000	.0000	.0000	.0000	.0000	.0000	.0000

	$\kappa_t \tau_t$				κ_t			
	all 21 countries				all 21 countries			
β	0.2810***	0.2948***	0.2908***	0.2977***	0.3347***	0.3622***	0.3527***	0.3710***
	(0.0223)	(0.0213)	(0.0218)	(0.0213)	(0.0568)	(0.0534)	(0.0555)	(0.0535)
α	0.0495***	0.0397**	0.0472***	0.0386***	0.9101***	0.8389***	0.8937***	0.8233***
	(0.0087)	(0.0089)	(0.0086)	(0.0089)	(0.0918)	(0.0890)	(0.0900)	(0.0896)
20^{th} century	no	yes	no	yes	no	yes	no	yes
world war ii	no	yes	yes	yes	no	yes	yes	yes
baby boom	no	no	yes	yes	no	no	yes	yes
N	1679	1679	1679	1679	1679	1679	1679	1679
$\bar{R}^2$	.0094	.0188	.0134	.0200	.1592	.1730	.1664	.1748
p	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Notes: Table reports results from pooled regressions with errors corrected for panel autocorrelation and Prais-Winsten heteroskedastic error correction. *** 1%, *** 5%, * 10%. The final row, marked p, is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$.

Table 4: Panel Regressions of Actual Observations on Model Solutions: Education Expenditures and κ

	$\kappa_t \tau_t$				κ_t			
	fixed effects				fixed effects			
β	0.3692***	0.3666***	0.3911***	0.3852***	0.3272***	0.3440***	0.4315***	0.4146***
	(0.0263)	(0.0236)	(0.0669)	(0.0261)	(0.0765)	(0.0703)	(0.0843)	(0.0731)
α	0.0494***	0.0311**	0.0352**	0.0233*	1.1538***	0.8746***	0.9402***	0.7617***
	(0.0111)	(0.0130)	(0.0131)	(0.0135)	(0.1174)	(0.1567)	(0.1435)	(0.1685)
20^{th} century	no	yes	no	yes	no	yes	no	yes
world war ii	no	yes	yes	yes	no	yes	yes	yes
baby boom	no	no	yes	yes	no	no	yes	yes
N	1679	1679	1679	1679	1679	1679	1679	1679
$\bar{R}^2$	.6020	.6207	.6203	.6282	.0936	.1637	.1336	.1808
p	.0000	0.0000	.0000	.0000	.0000	.0000	.0000	.0000

Notes: Table reports results from pooled fixed effects regressions with errors clustered about the country. *** 1%, *** 5%, * 10%. The final row, marked p, is the p-value on the null hypothesis that $\beta = 1$ and $\alpha = 0$.

Table 5: Preference Parameters, β, ν, and Education Efficiency, μ

country	β	ν	μ
Australia	.36	.400	.0850
Austria	.18	.400	.0850
Belgium	.18	.400	.0900
Canada	.27	.400	.0950
Denmark	.30	.400	.1000
Finland	.18	.400	.0850
France	.18	.250	.0750
Germany	.18	.400	.0950
Greece	.18	.400	.0900
Ireland	.63	.400	.0850
Italy	.18	.400	.0850
Japan	.36	.400	.0950
Netherlands	.75	.200	.1150
New Zealand	.54	.125	.1000
Norway	.27	.400	.0825
Portugal	.18	.400	.0900
Spain	.27	.400	.0900
Sweden	.45	.400	.1200
Switzerand	.18	.400	.0900
United Kingdom	.54	.400	.1100
United States	.72	.400	.0850

Notes: Table reports stationary values of β, and ν. It also reports the efficiency of schooling time, μ.