An Efficient Numerical Procedure for Financial Engineering Using Quasi-Monte Carlo Method

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Abstract

In this research, we propose an efficient numerical procedure that consists of a general random number generation and a use of enhanced quasi-Monte Carlo (QMC) method. We first introduce a simple yet practical scheme to generate random variates from any continuous distribution. The inverse of the cumulative distribution function is derived from its characteristic function via the fast Fourier transform. We conduct several numerical experiments to assess the accuracy and efficiency of the scheme.

Next, we discuss possible acceleration of QMC method. It is known that the dimension and the smoothness of the integrands are the two key factors that affect the efficiency of the QMC method. We first review a couple of dimension reduction methods that have been proposed to decrease the effective dimension of typical financial problems. The orthogonal transformation method, on the other hand, was proposed to handle the smoothness of the problem. However, both of these methods are unsatisfactory in that they only achieve optimality single sidedly in the sense that the dimension reduction methods enhance QMC by effective dimension reduction while the orthogonal transformation method accomplishes the same objective by discontinuity realignment. It should be noticed that there are many problems in finance that are both of high dimensionality and discontinuity. We propose the integrated linear transformation method that has advantage of addressing both dimensionality and discontinuity concurrently. The numerical examples indicate that relative to the standard QMC, the proposed method is extremely effective and could attain a variance reduction as high as several thousand times. We further discuss a possible extension of the proposed method to the case under a Lévy process and demonstrate its competitive efficiency using some additional numerical examples.

Keywords: Quasi-Monte Carlo method, Integrated Linear Transformation, Lévy process, Option Pricing

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