Innovation, Delegation, and Asset Price Swings*

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Abstract

We propose a dynamic asset-market equilibrium model in which (1) an "innovative" asset with as-yet-unknown average payoff is traded, and (2) investors delegate investment to experts. Experts secretly renege on investors' orders and take on leveraged positions in the asset to manipulate investors' beliefs, thereby attracting more orders and fees. Despite agents' full rationality, the combination of experts' moral hazard and investors' learning generates bubble-like price dynamics: gradual upswing, overshoot, and reversal. Consistent with empirical observations, hedge funds "ride" price swings, adjusting holdings counter-cyclically to other financial intermediaries. Capping fees may lower fund leverage, dampen price swings, and improve welfare.

JEL classification: D80, G10, G23

Key words: asset price swings, delegated investment, innovation, learning, moral hazard, signal jamming.

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1 Introduction

Bubble-like price movements have recurred in financial markets throughout history. Many of them followed a common pattern: upswings, triggered by technological innovation, are eventually followed by dramatic downswings, leading to persistent economic downturns.[1] The 2007–08 financial crisis is no exception: the rise in the U.S. housing prices, fueled by financial innovation (i.e., securitization), and the subsequent reversal are the key factors behind the global turmoil. Given their serious impacts on the real economy, it is important to understand the mechanism of asset price swings. Especially, studying the entire cycle—emergence of upswing, its overshoot, and eventual reversal—coherently in a unified framework appears to be a critical task.

To tackle this problem, we develop a fully rational, dynamic asset-market equilibrium model with delegated investment. We consider a market for a new and "innovative" asset, whose average payoff is as-yet-unknown and subject to learning. Investors delegate their investment to financial experts. We highlight the roles of (1) moral hazard in delegated investment, and (2) investors' learning about the asset's average payoff. Despite full rationality of long-lived agents, the combination of these two elements generates endogenous bubble-like price dynamics: gradual upswing, overshoot, and eventual downswing.

Our model builds on the signal-jamming framework developed in Sato (2014). Unlike Sato (2014), which does not provide implications for the dynamics of asset prices and investors' trading activities, our paper explores why bubble-like swings in security prices arise endogenously and how trades evolve over time behind such price swings. We consider a discrete time model with finite horizon. There are one risky asset and one riskless asset. Initially, the agents have large uncertainty about the risky asset's average payoff. Over time, they learn about it based on the asset's payoff history. This asset is interpreted as a financial asset backed by an unprecedented and/or hard-to-understand technology—such

[1] See Brunnermeier and Oehmke (2013) for a historical overview of bubbles and crises.

as Internet stocks, biotech stocks, or sophisticated structured products—whose underlying profitability is initially unknown to most investors due to the lack of track record and background knowledge. There is a continuum of investment funds, each with a financial expert and an investor. The investor can invest directly in the riskless asset. However, investing in the risky asset requires that she submits to the expert a purchase order that specifies the number of shares of the asset to be purchased on her behalf. Each period, the expert earns a delegation fee proportional to the order.

There are two items the investor cannot directly observe. First, the expert's actual purchase of the asset is unobservable. The expert can secretly renege on the investor's purchase order (at a cost) and boost the asset purchase by using leverage. An example of such a fund can be a hedge fund adopting a flexible trading strategy that is not communicated to investors. Second, although fund returns are (obviously) observable, the periodic payoffs of the risky asset in the fund portfolio are unobservable.[2] These two layers of unobservability create a signal-jamming problem akin to Sato (2014). The investor tries to learn about the risky asset's average payoff from the observed fund returns: the higher the fund returns, the better the investor's assessment of the risky asset's average payoff, hence the larger her purchase orders (and thus fees). So the expert is inclined to boost the expected fund return by secretly levering up and increasing the purchase of the risky asset, inflicting excessive risk on the investor. The investor is not fooled in equilibrium because she is rational; nevertheless, the expert still reneges and levers up since otherwise his fund's future prospect would be underestimated by the investor who believes that the expert does renege secretly.

In equilibrium, the risky asset's price path exhibits a bubble-like pattern on average: it rises gradually, surpasses the benchmark level that would be obtained in the case in

[2] An alternative—and perhaps more realistic—assumption yielding exactly the same results is that each investor can directly observe the asset's payoff by incurring a small effort cost $\epsilon > 0$. Even if ϵ is very close to 0, it would be optimal for the investors to *not* observe the payoff directly because, in equilibrium, they learn it perfectly and costlessly from the fund return anyway.

which the asset's average payoff is fully known, and eventually falls and converges to the benchmark level over time. Intuitively, these swings are caused by the combination of the following two effects that have opposing pressures on the asset's aggregate demand and thus on its market-clearing price.

1. *Learning effect.* Initially, the investors' estimate of the asset's average payoff has low precision. So, being risk averse, they hesitate to purchase the asset. The associated demand for the asset is weak; thus, ceteris paribus, the initial price is low. But, as the investors' learning progresses over time, the precision of their estimate increases. This leads them to increase purchase orders over time, having an upward pressure on the asset's demand and thus its price.

2. *Leverage effect.* Initially, the investors' estimate of the asset's average payoff has low precision and hence is susceptible to the experts' manipulation. This leads the experts to renege on purchase orders and choose high leverage. The associated aggregate demand is high; so, ceteris paribus, the initial price is high. But, as the investors' learning progresses, their estimate becomes precise and less subject to manipulation. Accordingly, the experts deleverage over time, having a downward pressure on the asset's demand and hence its price.

In early periods, where the investors still have large uncertainty about the asset, the learning effect dominates the leverage effect, initiating upswing in the price. On average, the price overshoots the level of the benchmark case, in which neither of the above effects is at work, because the experts' use of leverage pushes up the asset's aggregate demand and its market-clearing price. As investors' learning progresses, the learning effect weakens and is dominated by the leverage effect, leading to downswing of the price. At some point, the investors' estimate becomes so precise that it is no longer worthwhile for the experts to attempt to manipulate it. The leverage effect disappears, and the price converges to the benchmark level over time.

The up-and-down swings are pronounced if the investors' initial estimate has low precision. This is because for these swings to occur we need both the learning and leverage effects to be strong, which is the case when the investors' estimate has very low precision. Thus, the model predicts that swings and overshooting of prices are more pronounced for new and innovative assets with highly uncertain payoff characteristics than for old-economy assets already familiar in the market. This prediction is consistent with the historical observation that bubble-like price movements tend to arise in times of technological change (e.g., railroads or the Internet) or financial innovation (e.g., securitization), as noted by Brunnermeier and Oehmke (2013).

If we allow the experts to choose the delegation fee rate, their choices may be too high from the social perspective. This is because a high fee rate incentives themselves to take on excessive leverage, ultimately deterring the investors from providing capital. Although the experts understand that they would all be better off if they collectively charge a low fee, it is individually optimal for each expert to exploit the investor by charging a high fee. As a policy implication, we show that imposing a cap on the fee rate effectively alleviates the experts' coordination failure, thereby achieves a Pareto improvement and also dampens swings and overshooting of the price.

Last, to study the evolution of funds' holdings and trading volume over time, we extend the model to accommodate two types of funds: hedge funds (HFs), which can secretly renege on the investors' orders, and other funds (OFs), which cannot do so due to statutory disclosure requirements. Despite the same preferences and the same asset valuation, these funds trade the asset over time: the OFs serve as trading counterparties to the HFs who adjust holdings according to the evolution of the learning and leverage effects. We show that the HFs tend to increase their holdings in the price upturn and then decrease them in the downturn, consistent with the empirical finding of Brunnermeier and

Nagel (2004) that hedge funds were "riding" the 1998–2000 dot-com bubble. In contrast, the OFs' holdings are negatively related to those of the HFs over time because they are counterparties to each other. This is consistent with Ang, Gorovyy, and van Inwegen (2011) who document that hedge funds' leverage was counter-cyclical to that of other financial intermediaries during the 2007–2009 crisis.

Our paper is related to the theoretical literature providing rational explanations to price anomalies such as bubbles or momentum and reversal. Vayanos and Woolley (2013) also study momentum and reversal in a model with delegated investment. As noted in their paper, however, delegated investment is not essential for momentum and reversal to arise in Vayanos and Woolley (2013): the driving force is delay in the reaction of fund flows to returns. We do not assume delays in agents' reactions; in our model, delegation and the associated moral hazard problem are critical. The possibility that experts attempt to manipulate investors' beliefs by deviating from their equilibrium strategies—which Vayanos and Woolley (2013) exclude by assumption—is the key driver of price swings in our model. Like our paper, Pastor and Veronesi (2009) develop a fully rational model and obtain bubble-like patterns in the prices of "new-economy" assets. The key ingredients of their model are a time lag between the introduction and adoption of a new technology and investors' learning during that lag. While investors' learning plays a central role also in our model, we do not distinguish the introduction and adoption of a new technology; our results are driven by agency relationship in delegated investment in which investors' learning is potentially influenced by experts. DeMarzo, Kaniel, and Kremer (2007) also study bubbles caused by technological innovation. While their model is static and focuses on bubbles in real investment, our model studies dynamic bubble-like pattens—both upswings and reversals—in security prices.

Our paper is also related to the literature on opacity/complexity and obfuscation in financial markets. Sato (2014) is most closely related to ours: both papers feature signal jamming associated with delegated investment. While Sato (2014) studies a stationary equilibrium and therefore has no implications for the dynamics of asset prices and holdings, our primary purpose is to study time-varying, bubble-like patterns of asset prices and the evolution of funds' holdings behind such price swings.[3] Carlin (2009) and Carlin and Manso (2011) argue that financial professionals may deliberately obfuscate their products to extract informational rents from unsophisticated investors. Financial experts in our model also try to exploit less-informed investors strategically. But unlike those papers, our focus is on the dynamics of security prices and trading activities in competitive markets.

In recent years, a theoretical literature studying the equilibrium implications of delegated portfolio management has been growing (Allen and Gorton 1993; Shleifer and Vishny 1997; Berk and Green 2004; Vayanos 2004; Cuoco and Kaniel 2011; Guerrieri and Kondor 2012; He and Krishnamurthy 2012, 2013; Malliaris and Yan 2012; Kaniel and Kondor 2013). To our knowledge, bubble-like price dynamics—upswing, overshoot, and reversal—are not yet discussed in this literature. Our paper contributes to this literature by showing that, despite agents' full rationality, portfolio delegation and the associated moral hazard generate such patterns of equilibrium asset prices.[4]

The paper proceeds as follows. Section 2 presents the model. Section 3 characterizes the equilibrium. Section 4 studies the price dynamics. Section 5 discusses policy implications. Section 6 studies the evolution of holdings and trading volume. Section 7 concludes. All proofs are in the Appendix.

[3]Moreover, our modeling approach is more standard than that of Sato (2014): while Sato (2014) assumes overlapping generations of short-lived risk-neutral investors whose decisions stem from a decreasing-returns-to-scale assumption à la Berk and Green (2004), we consider a standard dynamic optimization problem of long-lived risk-averse investors.

[4]Our paper is also related to the literature on career concerns and asset prices (Dasgupta and Prat 2008; Guerrieri and Kondor 2012; Dasgupta, Prat, and Verardo 2011). In these papers, fund managers seek to influence investor beliefs about their ability. In our paper, experts try to influence investor expectations about the innovative asset's future prospects.

2 Model

Time t is discrete and finite: $t = 0, ..., T + 1$. Period T is the last period in which the market is open and agents make decisions. In period $T + 1$, the agents just consume their entire wealth. There is a single risky asset and a riskless asset. The riskless asset has an infinitely elastic supply at an exogenous rate of return $r > 0$ and is freely accessible to all agents. There are two classes of agents: investors and experts. The investors can purchase the risky asset only through investment funds run by the experts. Each investor provides capital to a fund, specifying the number of shares of the risky asset to be purchased on her behalf. The expert can secretly lever up the investor capital and buy a larger number of shares of the asset than asked by the investor. The experts' leverage, the investors' demand for the risky asset, and the asset's price are determined in equilibrium.

2.1 Risky asset

In period $t = 1, 2, ..., T + 1$, the risky asset yields a per-share payoff $\delta_t = \bar{\delta} + u_t$. The transitory component u_t is i.i.d. across time, normally distributed with mean 0 and variance $1/\eta_u$, and unobservable to anyone. The average payoff $\bar{\delta}$ is a constant drawn by nature in period 0 from a normal distribution with mean $\hat{\delta}_0$ and variance $1/\eta_0$. The agents do not observe $\bar{\delta}$ directly, and learn about it over time based on the payoff history $\mathcal{H}_t \equiv \{\delta_\tau\}_{\tau=1}^{t}$. We denote the period-$t$ estimate of $\bar{\delta}$ given $\mathcal{H}_t$ by $\hat{\delta}_t \equiv \mathrm{E}[\bar{\delta}|\mathcal{H}_t]$. Parameter η_0 measures (the inverse of) the risky asset's "innovativeness." The asset with small η_0 is interpreted as an innovative asset whose cash flow is backed by an unprecedented and/or hard-to-understand technology—such as Internet stocks, biotech stocks, or sophisticated structured products—because most market participants initially have large uncertainty about such an asset's average payoff due to the lack of track record and background knowledge. The investors cannot directly observe the realized payoff δ_t for all t, whereas the experts can do so. Although it is unobservable, as shown later, in equilibrium the investors learn δ_t perfectly from the observed fund return.[5]

In period $t = 0, ..., T$, the asset is traded in the market at a publicly observable market-clearing price, P_t. The asset's supply $S > 0$ is constant over time. Let $R_{t+1} \equiv \delta_{t+1} + P_{t+1} - (1 + r)P_t$ denote the excess return on the risky asset per share. We denote the expected excess return conditional on $\mathcal{H}_t$ by $\hat{R}_{t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{H}_t]$.

2.2 Delegated investment

There is a continuum with mass one of investment funds, each indexed by $i \in [0, 1]$. Fund i consists of a risk-neutral expert and a risk-averse investor, who both live from $t = 0$ to $t = T + 1$. For simplicity, we assume that investor i can neither invest in nor observe activities in the other funds. This assumption eliminates the possibility that an expert attracts an infinitely large amount of capital to have price impact in the asset market.

In each period $t = 0, ..., T$, investor i submits to expert i a *purchase order*, which specifies the number of shares of the risky asset, $y_{i,t} \in [0, \infty)$, that the expert is supposed to purchase on behalf of the investor. On top of the capital needed for this purchase ($P_t y_{i,t}$ dollars), the investor pays $\phi y_{i,t}$ dollars to the expert as the *delegation fee*, where the fee rate $\phi > 0$ is an exogenous parameter.

After receiving capital and fee from the investor, the expert buys the asset. Despite being asked to purchase only $y_{i,t}$ shares, the expert can renege and buy $(1 + \xi_{i,t})y_{i,t}$ shares of the asset, where $\xi_{i,t} \in [0, \infty)$ is the expert's choice variable. We assume $\xi_{i,t}$ is unobservable to the investor and the expert cannot commit to his choice of $\xi_{i,t}$. Since the expert has zero personal wealth, choosing $\xi_{i,t} > 0$ requires borrowing funds from outside lenders. Thus, $\xi_{i,t}$ is also a measure of the fund's *leverage*. To choose $\xi_{i,t}$, the expert

[5] As noted in footnote 2, we could alternatively assume that each investor can observe δ_t with a very small effort cost $\epsilon > 0$. However small ϵ is, the investors would choose not to observe δ_t because they learn it perfectly and costlessly from the fund return.

incurs a nonpecuniary cost of reneging, $\kappa\xi_{i,t}$ with $\kappa > 0$. This represents the cost of effort to "cook the books" and/or manipulate the disclosure documents to make them look like the expert adhered to the investor's purchase order.[6]

In period $t+1$, the fund's period-t investment yields the total proceeds of $Q_{i,t+1} \equiv R_{t+1}(1+\xi_{i,t})y_{i,t} + (1+r)P_t y_{i,t}$ dollars.[7] The investor can directly observe $Q_{i,t+1}$.

2.3 Maximization problems

Each expert maximizes the discounted sum of his lifetime expected utilities, where his within-period utility is the difference between the fee and the cost of reneging. That is, expert i's problem in period $t = 0, ..., T$, denoted by $\mathcal{P}^E_{i,t}$, is to choose $\xi_{i,t} \in [0, \infty)$ to maximize

$$\mathrm{E}\left[\sum_{\tau=0}^{T-t} \beta^\tau \left(\phi y_{i,t+\tau} - \kappa\xi_{i,t+\tau}\right) \middle| \mathcal{F}^E_{i,t}\right], \tag{2.1}$$

where $\beta \in (0,1)$ is a discount factor common for all agents, and $\mathcal{F}^E_{i,t} = \{Q_{i,\tau}, y_{i,\tau}, \xi_{i,\tau}, P_\tau, \delta_\tau : \tau \leq t\}$ is his information set in period t.

Investor i's problem in period $t = 0, ..., T$, denoted by $\mathcal{P}^I_{i,t}$, is to choose purchase order $y_{i,t}$ and consumption $c_{i,t}$ to maximize the discounted sum of her lifetime expected utilities

$$-\mathrm{E}\left[\sum_{\tau=0}^{T-t} \beta^\tau \exp\left(-\nu c_{i,t+\tau}\right) \middle| \mathcal{F}^I_{i,t}\right], \tag{2.2}$$

[6] It is not important that we allow only nonnegative $\xi_{i,t}$. The reason for assuming $\xi_{i,t} \geq 0$ is that it is compatible with the proportional reneging cost, $\kappa\xi_{i,t}$. If we allow $\xi_{i,t} < 0$, we would need a cost function of the form $\kappa|\xi_{i,t}|$. This would reduce tractability of analysis while the results remain unchanged because, in equilibrium, the experts may choose $\xi_{i,t} > 0$ but never $\xi_{i,t} < 0$. An alternative cost function accommodating $\xi_{i,t} < 0$ is a quadratic form, $\kappa\xi^2_{i,t}/2$. With this specification, the experts still do not choose $\xi_{i,t} < 0$ in equilibrium and the model yields very similar results.

[7] $Q_{i,t+1}$ is computed as follows. In period t, the investor invests $P_t y_{i,t}$ dollars in the fund. The expert borrows $P_t\xi_{i,t}y_{i,t}$ dollars from outside lenders and invests $P_t(1+\xi_{i,t})y_{i,t}$ dollars in the risky asset (i.e., buys $(1+\xi_{i,t})y_{i,t}$ shares). In period $t+1$, the fund receives $\delta_{t+1}(1+\xi_{i,t})y_{i,t}$ dollars of payoff from the asset, and obtains $P_{t+1}(1+\xi_{i,t})y_{i,t}$ dollars from selling the asset in the market. The expert pays back $(1+r)P_t\xi_{i,t}y_{i,t}$ dollars to the lenders. Thus, the total proceeds from the fund's investment is $Q_{i,t+1} = \delta_{t+1}(1+\xi_{i,t})y_{i,t} + P_{t+1}(1+\xi_{i,t})y_{i,t} - (1+r)P_t\xi_{i,t}y_{i,t} = R_{t+1}(1+\xi_{i,t})y_{i,t} + (1+r)P_t y_{i,t}$.

where $\nu > 0$ is the coefficient of constant absolute risk aversion and $\mathcal{F}^I_{i,t} = \{Q_{i,\tau}, y_{i,\tau}, P_\tau : \tau \leq t\}$ is her information set in period t, subject to her dynamic budget constraint

$$W_{i,t+1} = Q_{i,t+1} - \phi y_{i,t} + (1+r)(W_{i,t} - c_{i,t} - P_t y_{i,t}), \tag{2.3}$$

where $W_{i,t}$ is her wealth in period t. Constraint (2.3) states that the investor's next-period wealth is the sum of the proceeds from the delegated investment net of fee and her own investment in the riskless asset. In the final period $t = T+1$, in which there is no market for the asset, she simply consumes her entire wealth: $W_{i,T+1} = c_{i,T+1}$.

2.4 Definition of equilibrium

The equilibrium consists of the price P_t, the investor's purchase order $y_{i,t}$, and the expert's leverage $\xi_{i,t}$ for $i \in [0,1]$ such that, for all $t = 0, ..., T$,

1. given P_t and the others' actions, investor i solves $\mathcal{P}^I_{i,t}$,
2. given P_t and the others' actions, expert i solves $\mathcal{P}^E_{i,t}$, and
3. the risky asset's market clears:

$$\int_0^1 (1+\xi_{i,t}) y_{i,t} \,\mathrm{d}i = S. \tag{2.4}$$

3 Analysis

This section characterizes the equilibrium of the model. We look for a linear equilibrium such that P_t is linear in $\hat{\delta}_t$ on the equilibrium path. We follow the following steps to solve the model.

1. Characterize the evolution of the agents' estimates of $\bar{\delta}$ (Section 3.1).

2. Conjecture a linear form of the equilibrium price. Also conjecture that the sequence of each expert's optimal leverage $\{\xi_\tau^*\}_{\tau=0}^T \geq 0$ is deterministic (Section 3.2).

3. Specify the investors' out-of-equilibrium beliefs (Section 3.3).

4. Solve each investor's problem (Section 3.4) for her optimal purchase order $y_{i,t}$.

5. Solve each expert's problem (Section 3.5). Verify that $\{\xi_\tau^*\}_{\tau=0}^T$ is indeed deterministic as conjectured in step 2, and obtain ξ_t^* as a function of $\hat{R}_{t+1}$ (Eq.(3.16)).

6. Impose market clearing (Section 3.6) and obtain $\hat{R}_{t+1}$ as a function of ξ_t^* (Eq.(3.17)).

7. Solve Eqs. (3.16) and (3.17) for two unknowns ξ_t^* and $\hat{R}_{t+1}$ (Section 3.6; Figure 1). Verify that the resulting equilibrium price is indeed linear as conjectured in step 2.

3.1 Evolution of estimates

Since $\bar{\delta}$ is unobservable to anyone, all agents learn about it over time by Kalman filtering. The experts, who observe $\mathcal{H}_t$ directly, update their period-t estimate $\hat{\delta}_t$ by

$$\hat{\delta}_t = \lambda_t \hat{\delta}_{t-1} + (1 - \lambda_t)\delta_t, \tag{3.1}$$

with the updating factor $\lambda_t \in (0, 1)$ that increases over time deterministically according to $\lambda_{t+1} = 1/(2 - \lambda_t)$ from its initial value $\lambda_1 = \eta_0/(\eta_0 + \eta_u)$ (see Appendix A).

The investors, who cannot observe $\mathcal{H}_t$ directly, estimate $\bar{\delta}$ based on the $\mathcal{H}_t$ that they *infer* from the available information. Let $\mathcal{H}_{i,t}^I \equiv \{\delta_{i,\tau}^I\}_{\tau=1}^t$ denote the payoff history inferred by investor i, where $\delta_{i,t}^I$ is the value of δ_t inferred by her. Her period-t estimate of $\bar{\delta}$ is denoted by $\hat{\delta}_{i,t}^I \equiv \mathrm{E}[\bar{\delta}|\mathcal{F}_{i,t}^I]$, and her conditional expectation of the excess return is denoted by $\hat{R}_{i,t+1}^I \equiv \mathrm{E}[R_{t+1}|\mathcal{F}_{i,t}^I]$. If she infers that δ_t is $\delta_{i,t}^I$, she updates $\hat{\delta}_{i,t}^I$ as

$$\hat{\delta}_{i,t}^I = \lambda_t \hat{\delta}_{i,t-1}^I + (1 - \lambda_t)\delta_{i,t}^I, \tag{3.2}$$

where $\hat{\delta}_{i,0}^I = \hat{\delta}_0$ is exogenously given, and the updating factor λ_t is the same as that of (3.1) (see Appendix A).

Importantly, the investor's learning (3.2) potentially departs from (3.1) on off-the-equilibrium paths because the experts can manipulate the investors' inference by secretly deviating from their equilibrium strategy (i.e., choosing $\xi_{i,t}$ that is not anticipated by the investors). However, on the equilibrium path the investors correctly anticipate $\xi_{i,t}$; consequently $\mathcal{H}_{i,t}^I = \mathcal{H}_t$ holds, and thus (3.2) coincides with (3.1).

3.2 Conjectures

First, we propose and later verify the following conjecture about the equilibrium price.

Conjecture 1 (Price)**.** *The equilibrium price for $t = 0, ..., T$ is of the form*

$$P_t = a_t \hat{\delta}_t^I - b_t \quad \textit{with} \; \hat{\delta}_t^I \equiv \int_0^1 \hat{\delta}_{i,t}^I \, \mathrm{d}i, \tag{3.3}$$

where

$$a_t \equiv \sum_{\tau=1}^{T+1-t} \left(\frac{1}{1+r} \right)^{\tau} \tag{3.4}$$

is a riskless discount factor and $\{b_\tau\}_{\tau=0}^T > 0$ is a deterministic sequence.

The first term of (3.3), $a_t \hat{\delta}_t^I$, is the present value of the *average* of all the investors' expectations about the asset's future payoffs discounted at the riskless rate. The second term, b_t, involves risk premium. This term is time-varying because the premium demanded by the investors changes together with their learning about $\bar{\delta}$. Since there is no market in period $T+1$, we set $P_{T+1} \equiv 0$ (i.e., $a_{T+1} \equiv 0$ and $b_{T+1} \equiv 0$).

Second, we conjecture and later verify the expert's equilibrium strategy as follows.

Conjecture 2 (Expert's optimal strategy)**.** *There exists a deterministic sequence $\{\xi_\tau^*\}_{\tau=0}^T \geq 0$ such that, for all $i \in [0, 1]$, expert i optimally chooses $\xi_{i,t} = \xi_t^*$ in period $t = 0, ..., T$ on* *the equilibrium path and also on off-the-equilibrium paths where* $\hat{\delta}^I_{i,t} \neq \hat{\delta}_t$.

Conjecture 2 states that the expert's optimal leverage is deterministic, irrespective of his potential deviations in the past, $\{\xi_{i,\tau}\}_{\tau=0}^{t-1}$.

3.3 Out-of-equilibrium beliefs

As shown later, all investors infer $\mathcal{H}_t$ correctly on the equilibrium path and therefore have the same estimate of $\bar{\delta}$. So, on the equilibrium path, each investor observes P_t and confirms that the other investors' average estimate $\hat{\delta}^I_t = \frac{P_t+b_t}{a_t}$ is the same as her estimate based on her inferred payoff history $\mathcal{H}^I_{i,t}$. That is, $\mathrm{E}[\bar{\delta}|\mathcal{H}^I_{i,t}] = \frac{P_t+b_t}{a_t}$ for all i and t on the equilibrium path. However, on off-the-equilibrium paths where some agents deviate from their equilibrium strategies, an investor may observe P_t and realize that her estimate $\mathrm{E}[\bar{\delta}|\mathcal{H}^I_{i,t}]$ does not equal $\frac{P_t+b_t}{a_t}$. For such a case, we specify the following out-of-equilibrium belief of investors.

$$\text{If } \mathrm{E}[\bar{\delta}|\mathcal{H}^I_{i,t}] \neq \frac{P_t+b_t}{a_t} \text{ then } \hat{\delta}^I_{i,t} = \mathrm{E}[\bar{\delta}|\mathcal{H}^I_{i,t}]. \tag{3.5}$$

That is, an investor whose estimate (based on her inferred payoff history $\mathcal{H}^I_{i,t}$) disagrees with the observed price P_t would stick with her own estimate. We have two remarks on (3.5).

Remark 1 (Comparison with REE models). One may argue that (3.5) is implausible because, based on the rational-expectations equilibrium (REE) logic, each investor should revise her belief in favor of the price. Such an argument does not apply to our model, as the model setting and the nature of analysis are fundamentally different from those of the standard REE models. In asymmetric information models à la Grossman and Stiglitz (1980), uninformed investors should indeed revise their estimates in favor of the price because it reflects informed investors' superior signals. Also, in differential information models à la Grossman (1976), investors should revise their estimates in favor of the price because it aggregates all investors' signals and smooths out their idiosyncratic noises. In contrast, in our model, the price need not convey information superior to each investor's because no investor has information superior to other investors' and no one has private information that would be collectively useful. That is, each investor has no reason to believe that the others' average estimate is more informative than her own. On the equilibrium path, all investors infer the same payoff history $\mathcal{H}_t$ and thus do not *learn* new information from P_t: each of them just *confirms* that her own estimate $\mathrm{E}[\bar{\delta}|\mathcal{H}_{i,t}^I]$ is equal to $\frac{P_t+b_t}{a_t}$ (which equals $\hat{\delta}_t^I$ *on* the equilibrium path). If $\mathrm{E}[\bar{\delta}|\mathcal{H}_{i,t}^I] \neq \frac{P_t+b_t}{a_t}$ on an off-the-equilibrium path, she may potentially attribute the discrepancy to the following events: (1) her estimate $\mathrm{E}[\bar{\delta}|\mathcal{H}_{i,t}^I]$ is biased because expert i has deviated from the equilibrium strategy, (2) the others' average estimate $\hat{\delta}_t^I$ is biased because some other experts have deviated, (3) P_t does not reflect $\hat{\delta}_t^I$ in the form of (3.3) because some other investors have deviated, or a combination of these three. The investor cannot conduct a statistical inference as to which of these three events is more likely than the others, because all of them are supposed to occur with probability zero. The out-of-equilibrium belief (3.5) states that each investor attributes the discrepancy to (2) or (3) instead of (1).

Remark 2 (Unique linear price). There is another, important reason why the out-of-equilibrium belief of the form (3.5) is plausible: it ensures that P_t reflects the unbiased estimate $\hat{\delta}_t$ on the equilibrium path. Indeed, somewhat paradoxically, it is precisely because each investor would prioritize her own estimate over P_t when there were a discrepancy between them that P_t reflects the unbiased $\hat{\delta}_t$ on the equilibrium path. To see this point, suppose to the contrary that each investor would prioritize P_t over her own estimate (that is, consider the out-of-equilibrium belief of the form: if $\mathrm{E}[\bar{\delta}|\mathcal{H}_{i,t}^I] \neq \frac{P_t+b_t}{a_t}$ then $\hat{\delta}_{i,t}^I = \frac{P_t+b_t}{a_t}$). Under such a belief, there would be infinitely many equilibrium prices of the form (3.3). Specifically, $P_t = a_t z - b_t$ for an arbitrary number z would support an equilibrium because each investor observes P_t, revises her estimate to $\hat{\delta}_{i,t}^I = z$ and forms demand on that basis, which then consistently translates into the market-clearing price $P_t = a_t z - b_t$ even on the equilibrium path. Such a multiplicity of equilibria would significantly lower the model's predictive power. The out-of-equilibrium belief (3.5) eliminates this multiplicity and guarantees that P_t reflects only $\hat{\hat{\delta}}_t$ on the equilibrium path.

3.4 Investor's optimization

We conjecture and later verify that investor i's value function in period $t = 0, ..., T+1$ is, for all i,

$$V_t(W_{i,t}) = -\exp\left(-A_t W_{i,t} - B_t\right), \tag{3.6}$$

where $\{A_\tau\}_{\tau=0}^{T+1}$ and $\{B_\tau\}_{\tau=0}^{T+1}$ are deterministic sequences obtained later. Function $V_t(\cdot)$ is time-varying because the investor's maximized expected utility changes over time as their learning about $\bar{\delta}$ progresses. The Bellman equation is

$$V_t(W_{i,t}) = \max_{c_{i,t}, y_{i,t}} \left\{-\exp(-\nu c_{i,t}) + \beta \mathrm{E}\left[V_{t+1}(W_{i,t+1}) | \mathcal{F}_{i,t}^I\right]\right\}. \tag{3.7}$$

The following two lemmas characterize the investor's optimization.

Lemma 1 (Investor's value function)**.** *The investor's value function* (3.6) *satisfies the Bellman equation* (3.7) *if*

$$A_t = \frac{\nu}{1 + a_t} \quad \textit{for } t = 0, ..., T \tag{3.8}$$

and $A_{T+1} = \nu$*, and*

$$B_t = \sum_{s=t}^{T} \left(\prod_{k=t}^{s} \frac{a_k}{1 + a_k}\right) \left\{ \begin{array}{c} -\ln\beta + \frac{1}{2\chi_s}\left(\frac{\nu S}{(1+r)a_s}\right)^2 \\ +\frac{1}{a_s}\ln\left(\frac{1}{a_s}\right) - \frac{1+a_s}{a_s}\ln\left(\frac{1+a_s}{a_s}\right) \end{array} \right\} \quad \textit{for } t = 0, ..., T \tag{3.9}$$

and $B_{T+1} = 0$, where

$$\chi_t \equiv \frac{1}{\mathrm{Var}_t[R_{t+1}]} = \frac{\eta_u \lambda_{t+1}}{\left(1 + a_{t+1}(1 - \lambda_{t+1})\right)^2} \tag{3.10}$$

is the period-t precision of the risky asset's excess return.

Lemma 2 (Investor's purchase order)**.** *Given P_t, investor i asks the expert to buy*

$$y_{i,t} = \frac{\chi_t \left(\hat{R}^I_{i,t+1}(1 + \xi^*_t) - \phi\right)}{A_{t+1}(1 + \xi^*_t)^2} \tag{3.11}$$

shares of the risky asset.

The investor's optimal order (3.11) can be viewed as a mean-variance solution, standard in CARA-normal setting. It is increasing in the after-fee expected fund return, $\hat{R}^I_{i,t+1}(1 + \xi^*_t) - \phi$, and is decreasing in the volatility of fund return, $(1 + \xi^*_t)^2/\chi_t$, and the time-adjusted risk aversion, A_{t+1}. There are three points worth noting. First, $y_{i,t}$ is increasing in the asset return precision χ_t, which measures how much the investor's learning has progressed. Over time, χ_t increases as the uncertainty about $\bar{\delta}$ is unraveled, encouraging the risk-averse investor to increase $y_{i,t}$. As shown later, this upward pressure on $y_{i,t}$ generates upswing in the equilibrium price P_t. Second, $y_{i,t}$ depends on the term $(1+\xi^*_t)$ because the investor anticipates that the expert will renege on her purchase order and buy $(1 + \xi^*_t)y_{i,t}$ shares. Importantly, it is the investor's *belief* (ξ^*_t) about the expert's choice and not the choice itself $(\xi_{i,t})$ that affects $y_{i,t}$, because $\xi_{i,t}$ is unobservable. This is the source of the expert's moral hazard that is central to the following analyses. Third, $y_{i,t}$ depends on the expected excess return from investor i's point of view, $\hat{R}^I_{i,t+1}$, which is not necessarily equal to $\hat{R}_{t+1}$ on some off-the-equilibrium paths. On the equilibrium path, of course, $\hat{R}^I_{i,t+1} = \hat{R}_{t+1}$ for all i and t because all the investors infer $\mathcal{H}_t$ correctly.

3.5 Expert's optimization

The expert's problem is solved in a similar fashion to Sato (2014). To verify that it is indeed optimal for expert i to choose $\{\xi^*_\tau\}_{\tau=0}^T$ deterministically, we consider what would happen if he deviated from his equilibrium play and instead chose an arbitrary sequence of leverage $\{\xi_{i,\tau}\}_{\tau=0}^T \neq \{\xi^*_\tau\}_{\tau=0}^T$ even as investor i still believes that $\{\xi^*_\tau\}_{\tau=0}^T$ is played.

What would be investor i's inference $\delta^I_{i,t+1}$ of the payoff δ_{t+1} that is unobservable to her? The value of $\delta^I_{i,t+1}$ solves

$$\big(\delta^I_{i,t+1} + P_{t+1} - (1+r)P_t\big)\left(1+\xi^*_t\right) = \big(\delta_{t+1} + P_{t+1} - (1+r)P_t\big)(1+\xi_{i,t}). \tag{3.12}$$

The right-hand side (RHS) is $R_{t+1}(1+\xi_{i,t})$, whose value is known to investor i who observes $Q_{i,t+1}$. It depends on the expert's actual choice, $\xi_{i,t}$, and the true payoff, δ_{t+1}. The left-hand side (LHS) is the decomposition of the RHS as inferred by investor i. It depends on her incorrect belief about the expert's action, ξ^*_t, and her erroneous inferred payoff, $\delta^I_{i,t+1}$. Rearranging (3.12), we have

$$\delta^I_{i,t+1} = \delta_{t+1} + \left(\frac{\xi_{i,t} - \xi^*_t}{1+\xi^*_t}\right) R_{t+1}. \tag{3.13}$$

This implies that if the expert plays $\xi_{i,t} > \xi^*_t$ and if $R_{t+1} > 0$, then the investor will overshoot her inference, i.e., $\delta^I_{i,t+1} > \delta_{t+1}$.

The investor's erroneous perception about δ_{t+1} biases her learning subsequently. Specifically, if $\delta^I_{i,t+1} > \delta_{t+1}$ then her estimates of $\bar{\delta}$ and asset return will be both biased upward in future periods, i.e., $\hat{\delta}^I_{i,t+\tau} > \hat{\delta}_{t+\tau}$ and $\hat{R}^I_{i,t+\tau+1} > \hat{R}_{t+\tau+1}$ for $\tau = 1, 2, ..., T-t$ (see Appendix C). When choosing $\xi_{i,t}$ in period t, the expert takes into account the fact that he can potentially inflate the investor's perceived expected returns, $\hat{R}^I_{i,t+\tau+1}$ $(\tau \geq 1)$, and therefore her purchase orders, $y_{i,t+\tau}$ $(\tau \geq 1)$, by choosing $\xi_{i,t} > \xi^*_t$. Lemma 3 characterizes his optimal choice in period t, both on and off the equilibrium path.

Lemma 3 (Expert's leverage)**.** *Taking $\hat{R}_{t+1}$ and ξ_t^* as given, the expert's optimal choice of leverage $\xi_{i,t}$ is as follows.*

$$\text{If} \quad \frac{\phi \Omega_t \hat{R}_{t+1}}{1+\xi_t^*} \begin{cases} > \kappa & \text{then} \quad \xi_{i,t} \to \infty, \\ = \kappa & \text{then} \quad \xi_{i,t} \in [0,\infty) \ \text{(indifferent)}, \\ < \kappa & \text{then} \quad \xi_{i,t} = 0, \end{cases} \tag{3.14}$$

where

$$\Omega_t \equiv (1-\lambda_{t+1}) \sum_{\tau=1}^{T-t} \beta^{\tau} \frac{\chi_{t+\tau}\left(1+a_{t+\tau+1}(1-\lambda_{t+\tau+1})\right)}{A_{t+\tau+1}(1+\xi_{t+\tau}^*)} \left(\prod_{k=t+2}^{t+\tau} \lambda_k \right) \ \text{for}\ t = 0,...,T-1 \tag{3.15}$$

and $\Omega_T = 0$.

Lemma 3 states that the expert chooses $\xi_{i,t}$ by weighing the marginal gain from influencing the investor beliefs in future periods, $\phi \Omega_t \hat{R}_{t+1}/(1+\xi_t^*)$, against the marginal cost, κ. The deterministic variable Ω_t measures the sensitivity of the expert's expected future gain to an increase in $\xi_{i,t}$. In $t = 0, ..., T-1$, Ω_t is positive because the expert can potentially gain from influencing the investor's future beliefs. However, Ω_T is zero because there is no "future" in period T: since the investor makes no decisions in period $T+1$, the expert has no benefit from influencing her belief in period T.

Note that Lemma 3 only characterizes the choice of $\xi_{i,t}$ optimal from expert i's personal perspective, taking the equilibrium level ξ_t^* as fixed. To determine ξ_t^*, we need to ensure that investor i's belief about $\xi_{i,t}$ is consistent with the expert's optimal choice. As will be shown in Section 3.6, $\hat{R}_{t+1}$ is a deterministic variable; thus, the investor's belief is consistent (that is, Conjecture 2 is correct) if and only if $\xi_{i,t} = \xi_t^*$ in (3.14), or

$$\xi_t^* = \max\left\{0, \frac{\phi \Omega_t \hat{R}_{t+1}}{\kappa} - 1\right\}. \tag{3.16}$$

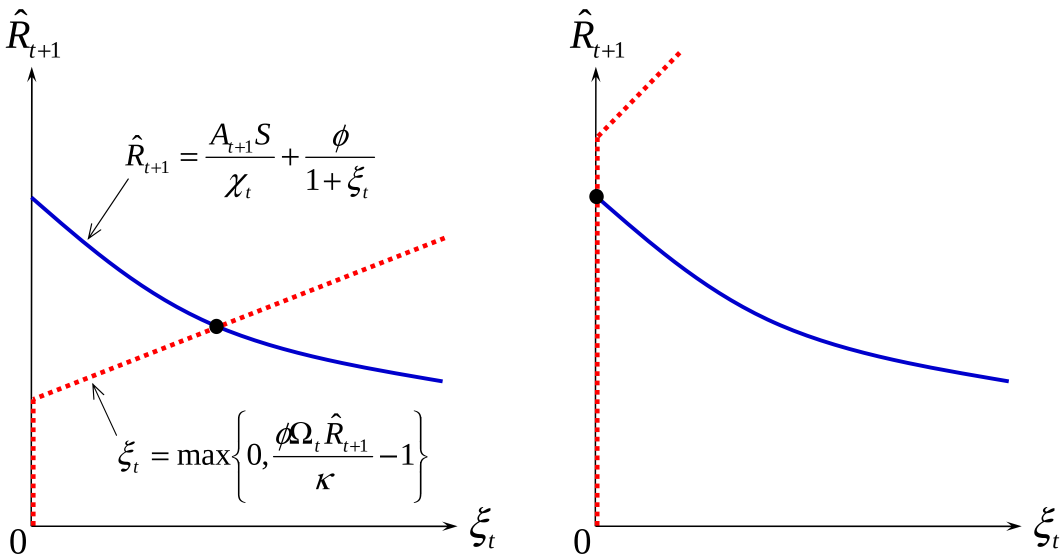
Note that $\xi_t^* = 0$ if Ω_t is small enough. That is, the experts do not renege on the purchase orders if the benefit of manipulating investor beliefs is small. Indeed, in period T they will surely choose $\xi_T^* = 0$ because $\Omega_T = 0$. If Ω_t is large enough, ξ_t^* is positive and increases with $\hat{R}_{t+1}$. This make sense: a large $\hat{R}_{t+1}$ means a large marginal benefit for the experts from influencing the future investor beliefs by reneging (i.e., the LHS of (3.14) is large), inducing them to increase leverage ξ_t^*. We will pin down the equilibrium values of ξ_t^* and $\hat{R}_{t+1}$ explicitly in Section 3.6 by imposing market clearing.

3.6 Equilibrium

The market-clearing condition (2.4) determines the asset's expected return and the agents' actions. Plugging the investor's optimal policy (3.11) into (2.4), and noting that $\hat{R}_{i,t+1}^I = \hat{R}_{t+1}$ holds for all i in equilibrium, we obtain $\hat{R}_{t+1}$ as a function of ξ_t^*:

$$\hat{R}_{t+1} = \frac{A_{t+1}S}{\chi_t} + \frac{\phi}{1+\xi_t^*}. \tag{3.17}$$

The first term on the RHS is the risk premium demanded by investors, which increases with the degree of their risk aversion ν and decreases with the precision χ_t of asset return. The second term $\phi/(1+\xi_t^*)$ is the "fee premium," i.e., the return that compensates investors for the delegation fees they pay. Importantly, (3.17) implies that $\hat{R}_{t+1}$ decreases with the expert's leverage ξ_t^* because the fee premium is decreasing in ξ_t^*. Why does the fee premium decrease with ξ_t^*? The reason is that the fee effectively serves as a fixed cost of investment in the risky asset from the investors' perspective. An increase in ξ_t^* lowers the average cost per share of the asset purchased, leading to a lower fee premium demanded by investors. Note that the risk premium does not depend on ξ_t^* despite the fact that a rise in ξ_t^* increases the risk borne by investors, all else equal. This is because each investor responds to the higher ξ_t^* by decreasing her purchase order $y_{i,t}$ so that the



(a) High Ω_t (b) Low Ω_t

Figure 1: Determination of the risky asset's expected excess return $\hat{R}_{t+1}$ and the fund leverage ξ_t^*

total risk she bears remains the same.

Given Ω_t, the equilibrium levels of ξ_t^* and $\hat{R}_{t+1}$ are obtained by solving the system of equations (3.16) and (3.17). The solutions are

$$\xi_t^* = \max\left\{0, \frac{\phi\Omega_t}{2\kappa}\left(\frac{A_{t+1}S}{\chi_t} + \sqrt{\frac{A_{t+1}^2 S^2}{\chi_t^2} + \frac{4\kappa}{\Omega_t}}\right) - 1\right\} \quad \text{and} \tag{3.18}$$

$$\hat{R}_{t+1} = \min\left\{\frac{A_{t+1}S}{\chi_t} + \phi, \frac{1}{2}\left(\frac{A_{t+1}S}{\chi_t} + \sqrt{\frac{A_{t+1}^2 S^2}{\chi_t^2} + \frac{4\kappa}{\Omega_t}}\right)\right\}. \tag{3.19}$$

Figure 1 illustrates the determination of (3.18) and (3.19). Panel (a) shows the case with $\Omega_t > \kappa/(\phi(A_{t+1}S/\chi_t + \phi))$. Since the experts have strong desire to influence investors' future beliefs (i.e., Ω_t is large), they choose high leverage given $\hat{R}_{t+1}$ (see (3.16)), leading to $\xi_t^* > 0$ in equilibrium. The equilibrium level of $\hat{R}_{t+1}$ decreases with Ω_t for the following reason. For a given $\hat{R}_{t+1}$, a rise in Ω_t induces the experts to increase ξ_t^*, which leads to a higher aggregate demand $(1 + \xi_t^*)y_t$ for the risky asset. Thus, $\hat{R}_{t+1}$ decreases (i.e., P_t

increases) to clear the market. Panel (b) is the case with $\Omega_t \leq \kappa/(\phi(A_{t+1}S/\chi_t + \phi))$. Here, Ω_t is so small that the equilibrium leverage is $\xi_t^* = 0$. The resulting small aggregate demand for the asset is accompanied by a low market clearing price and a high $\hat{R}_{t+1}$.

Once $\hat{R}_{t+1}$ is identified, the equilibrium price is readily determined. Conjecture 1 implies that P_t can be written as $P_t = a_t\hat{\delta}_t^I - (b_{t+1} + \hat{R}_{t+1})/(1+r)$.[8] This implies that Conjecture 1 is correct if and only if $b_t = (b_{t+1} + \hat{R}_{t+1})/(1+r)$, or

$$b_t = \sum_{\tau=1}^{T+1-t} \left(\frac{1}{1+r}\right)^{\tau} \hat{R}_{t+\tau}. \tag{3.20}$$

That is, b_t is the present value of the future expected excess returns.

Proposition 1. *There is a linear equilibrium in which*

1. *the risky asset's excess return R_{t+1} is, conditional on t, normally distributed with mean $\hat{R}_{t+1}$ and precision χ_t, which are given by* (3.19) *and* (3.10), *respectively;*

2. *the risky asset's price is $P_t = a_t\hat{\delta}_t - b_t$, where a_t and b_t are given by* (3.4) *and* (3.20), *respectively;*

3. *each expert's leverage is $\xi_{i,t} = \xi_t^*$, given by* (3.18)*;*

4. *each investor asks the expert to buy $y_{i,t} = S/(1+\xi_t^*)$ shares of the asset;*

5. *investor's consumption in $t = 0, ..., T$ is an affine function of wealth,*

$$c_{i,t} = \frac{W_{i,t}}{1+a_t} + \frac{1}{\nu}\left(\frac{a_t}{1+a_t}\right)\left(-\ln\beta + \frac{1}{2\chi_t}\left(\frac{\nu S}{(1+r)a_t}\right)^2 + B_{t+1} + \ln a_t\right),$$

[8] This is shown as follows. From (C.7) in Appendix C, all investors' average expected excess return is $\hat{R}_{t+1}^I \equiv \int_0^1 \hat{R}_{i,t+1}^I \mathrm{d}i = (1 + a_{t+1}(1-\lambda_{t+1})) \int_0^1 \hat{\delta}_{i,t}^I \mathrm{d}i + a_{t+1}\lambda_{t+1}\hat{\delta}_t^I - b_{t+1} - (1+r)P_t = (1+a_{t+1})\hat{\delta}_t^I - b_{t+1} - (1+r)P_t$. Rearranging this and noting that $a_t = (1+a_{t+1})/(1+r)$, we have $P_t = a_t\hat{\delta}_t^I - (b_{t+1} + \hat{R}_{t+1}^I)/(1+r)$. Since $\hat{R}_{t+1}^I = \hat{R}_{t+1}$ in equilibrium for all t, the result follows.

where B_t is given by (3.9)*; in the final period $t = T + 1$, she consumes her entire wealth, i.e., $c_{i,T+1} = W_{i,T+1}$.*

Proposition 1 characterizes the equilibrium in closed form. Although it is costly to renege on the investor's order, each expert chooses $\xi_t^* > 0$ if it gives him sufficiently large marginal benefit through manipulating the investor's future beliefs (i.e., if Ω_t is large enough). The investors understand the experts' desire to fool them and hence their beliefs are not manipulated on the equilibrium path; nonetheless, the experts still renege on the investors' orders and lever up. This is because, given the investors' beliefs that the experts will lever up, the experts indeed lever up optimally since otherwise the funds' future prospects would be underestimated by the investors. By itself, this over-leverage result may not be surprising: it is in line with other signal-jamming modes (Holmström 1999; Stein 1989; Sato 2014). The primary purpose of this paper, which distinguishes ours from existing works, is to examine the implication of the experts' signal-jamming behavior for the dynamics of security prices. We explore this issue in Section 4.

4 Dynamics: Asset Price Swings

This section studies the dynamics of P_t determined in Proposition 1. First, we show that P_t on average exhibits a bubble-like pattern: gradual upswing, overshoot, and eventual reversal. Second, we show that such a pattern is pronounced when the asset is innovative in that its average payoff is highly uncertain when introduced to the market (i.e., low η_0).

4.1 Price path

To obtain the sequence of P_t, first we need to obtain the sequences of Ω_t, ξ_t^*, and $\hat{R}_{t+1}$. As shown in Appendix E, the deterministic sequence $\{\Omega_\tau\}_{\tau=0}^{T-1}$ is obtained by solving the following difference equation of Ω_t, backward from the terminal values $\Omega_T = 0$ and $\xi_T^* = 0$:

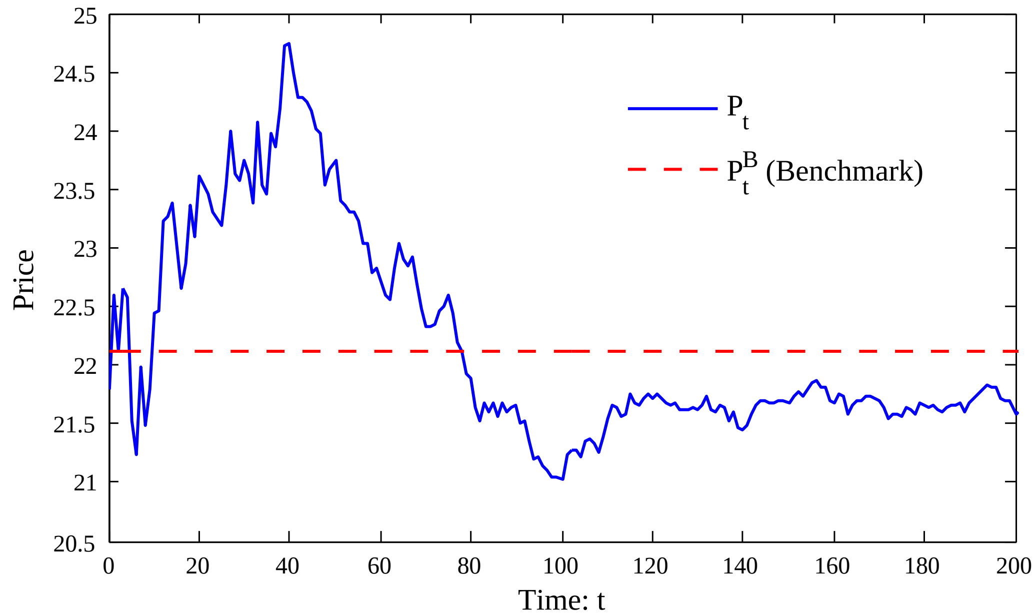
$$\Omega_t = \beta \left(\Omega_{t+1} + \left(\frac{1-\lambda_{t+2}}{\lambda_{t+2}} \right) \frac{\chi_{t+1} \left(1 + a_{t+2}(1-\lambda_{t+2})\right)}{A_{t+2}(1+\xi_{t+1}^*)} \right). \tag{4.1}$$

We already know the deterministic values of $\{\lambda_\tau\}_{\tau=1}^{T+1}$, $\{a_\tau\}_{\tau=0}^{T+1}$, $\{A_\tau\}_{\tau=0}^{T+1}$, and $\{\chi_\tau\}_{\tau=0}^{T}$ from (A.5), (3.4), (3.8), and (3.10), respectively. Thus, together with (4.1), we can identify the deterministic values of $\{\xi_\tau^*\}_{\tau=0}^{T}$ and $\{\hat{R}_\tau\}_{\tau=1}^{T+1}$ backward by (3.18) and (3.19), respectively, which then determine the deterministic values of $\{b_\tau\}_{\tau=0}^{T+1}$ by (3.20). Generating a sequence of normal random payoffs $\{\delta_\tau\}_{\tau=1}^{T+1}$, we compute the associated estimates $\{\hat{\delta}_\tau\}_{\tau=1}^{T+1}$ by (3.1). Then we can simulate a path of the price, $\{P_\tau\}_{\tau=0}^{T+1}$, by $P_t = a_t\hat{\delta}_t + b_t$.

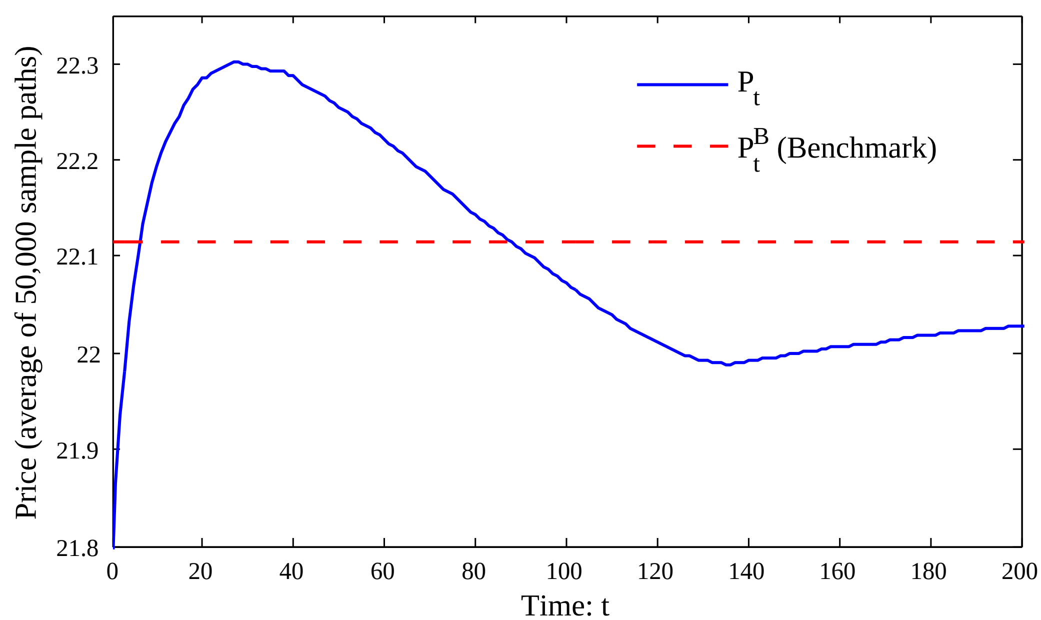
Panel (a) of Figure 2 plots a sample path of P_t (blue solid line). Here we assume that the agents have the correct prior expectation in $t = 0$, i.e., $\hat{\delta}_0 = \bar{\delta}$. So the obtained price pattern is not driven by the agents' incorrect prior about $\bar{\delta}$. We also plot the benchmark price path, denoted by P_t^B (red dashed line), which corresponds to a "noninnovative" asset whose average payoff $\bar{\delta}$ is fully known in period 0 (i.e., $\hat{\delta}_0 = \bar{\delta}$ and $\eta_0 = \infty$). Although we set $T = 1,000$, we present only the first 200 periods in the figure because our focus is on the price dynamics of an innovative asset, whose $\bar{\delta}$ is highly uncertain to the investors. For very large t, the asset is no longer "innovative" since the agents have already learned $\bar{\delta}$ with high precision. Also, the price paths in later periods are not interesting economically: P_t simply converges to P_t^B.[9]

The innovative asset's price P_t is highly volatile in early periods. This makes sense. Since the precision of the agents' estimate $\hat{\delta}_t$ is low in the early phase of learning, they update $\hat{\delta}_t$ drastically when having a new realization of stochastic payoff δ_t (that is, λ_t is

[9]After the periods shown in the figure, the path of P_t approaches that of P_t^B and stays almost flat for most of the remaining periods. When the final period approaches, both of these price paths start to fall (around $t = 850$) and reach zero in the final period ($t = 1,001$). This price fall occurs because there is no market in the very last period $t = T + 1$—which is an inevitable assumption in this finite-period setting—and thus the investors cannot sell the asset in that period (i.e., $P_{T+1} = 0$). We do not view this price fall in last periods as an economically relevant result because it is merely an artifact of the finite-horizon assumption. Thus, we do not report it in the figure and focus on early periods.



(a) Simulated price path.



(b) Average of 50,000 simulated price paths.

Figure 2: Price dynamics. We plot the time paths of P_t obtained in Proposition 1 and the benchmark price P_t^B that would be obtained in the case in which $\bar{\delta}$ is fully known in $t = 0$. Panel (a) plots a simulated path. Panel (b) plots the average of 50,000 simulated paths. The parameter values are $r = 0.04$, $\phi = 0.1$, $\kappa = 1$, $\nu = 0.2$, $\beta = 1/(1+r)$, $S = 10$, $\eta_u = 5$, $\bar{\delta} = 1$, $\hat{\delta}_0 = 1$, and $T = 1{,}000$. We set $\eta_0 = 50$ for P_t and $\eta_0 = \infty$ for P_t^B.

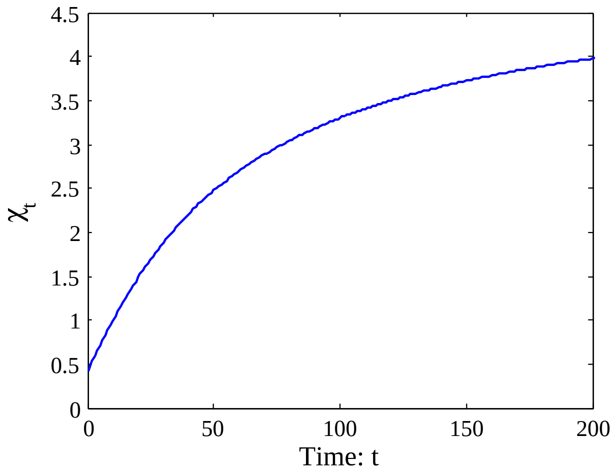
small in early periods), making the price volatile. But the price becomes less volatile over time because, as learning progresses, the agents become more "confident" about their estimate: they do not change $\hat{\delta}_t$ so much with a new realization of δ_t (that is, λ_t increases over time). Indeed, the conditional price volatility $\mathrm{Var}_t[P_{t+1}] = a_{t+1}^2(1-\lambda_{t+1})^2/(\eta_u \lambda_{t+1})$ decreases with t. By contrast, if the asset is noninnovative the agents do not update their estimate (i.e., $\hat{\delta}_t = \bar{\delta}$ for all t), and hence P_t^B is deterministic and almost flat in early periods.

To see the overall trend of the price dynamics more clearly, we plot the average of 50,000 simulated price paths in panel (b) of Figure 2. The innovative asset's price P_t exhibits bubble-like dynamics on average: it rises gradually ("upswing"), surpasses the noninnovative-asset benchmark P_t^B ("overshoot"), and then falls gradually ("downswing"). Around $t = 140$ in the figure, it starts increasing again and converges to P_t^B over time.

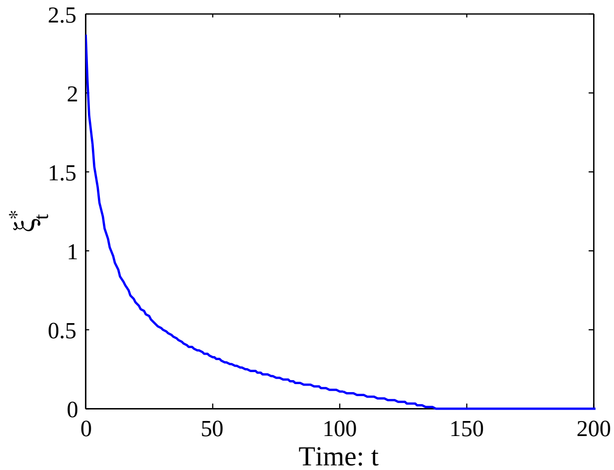
Intuitively, the initial up-and-down swings in P_t are caused by the combination of the following two effects that have opposing pressures on the asset's aggregate demand and therefore its market-clearing price.

1. *Learning effect.* Initially, the investors' estimate of the asset return has low precision; i.e., χ_t is small for small t. So, being risk averse, they hesitate to purchase the asset. Thus, ceteris paribus, the associated aggregate demand is weak and the price is low. But, as time goes on, the investors learn about $\bar{\delta}$ and thus χ_t increases (Figure 3(a)). This encourages them to increase $y_{i,t}$ over time, having an upward pressure on the aggregate demand and thus the price.

2. *Leverage effect.* Initially, the investors' estimate $\hat{\delta}_{i,t}^I$ has low precision and is susceptible to manipulation; i.e., λ_t is small for small t. So Ω_t is large, leading the experts to choose high leverage ξ_t^*. Thus, ceteris paribus, the associated aggregate demand is high and the price is high. But, as time goes on, the investors learn about $\bar{\delta}$



(a) Return precision χ_t



(b) Expert's leverage ξ^*_t

Figure 3: Time paths of the risky asset's return precision $\chi_t \equiv 1/\mathrm{Var_t[R_{t+1}]}$ and the expert's equilibrium leverage ξ^*_t. The parameter values are the same as those of Figure 2.

> and thus $\hat{\delta}^I_{i,t}$ becomes precise, lowering the experts' desire to manipulate it (i.e., Ω_t decreases). Accordingly, the experts deleverage over time (Figure 3(b)), having a downward pressure on the aggregate demand and thus the price.

In sum, due to the learning (leverage) effect, the price tends to be low (high) initially and then increases (decreases) over time; the combination of these two effects generates the inverse-U pattern of Figure 2(b). In the noninnovative-asset benchmark, neither of these effects exists because the agents do not conduct learning (i.e., $\chi_t = \eta_u \ \forall t$) and the experts do not use leverage (i.e., $\xi^*_t = 0 \ \forall t$). For the parameter values used in Figures 2 and 3, the learning effect dominates the leverage effect in early periods, initiating the upswing in P_t. It even surpasses P^B_t. This overshoot is caused by the experts' use of leverage: higher ξ^*_t is associated with larger aggregate demand for the risky asset, pushing up the market-clearing price P_t. As learning unravels $\bar{\delta}$ over time, the learning effect fades out because the incremental increase in χ_t diminishes over time towards 0; that is, χ_t approaches η_u asymptotically (Figure 3(a)). At some point, the leverage effect dominates the weakened learning effect, leading to downswing in the average P_t (around $t = 30$ in Figure 2(b)).

Eventually, the investors' estimate becomes so accurate that it is no longer beneficial for the experts to use leverage to influence investor beliefs. That is, ξ_t^* reaches 0 and the leverage effect disappears (around $t = 140$ in Figures 2(b) and 3(b)). Afterwards, the average P_t increases over time due to the learning effect (which is weakened but still at work), and converges to P_t^B as χ_t converges to η_u.

Remark (Is this a bubble?). Figure 2 resembles bubble-like price movements observed in reality. Is it a "bubble"? The answer is no, if we define a bubble as a situation in which an asset is overpriced *even though investors are contemporaneously aware that the price is too high* (Allen, Morris, and Postlewaite 1993; Abreu and Brunnermeier 2003). In our model, the fact that P_t overshoots P_t^B *on average* means that all agents know at $t = 0$ that $P_t > P_t^B$ is *likely* to occur in the near future periods. Also, the agents are likely to realize *ex post*, after they have learned $\bar{\delta}$ with a high precision, that P_t was higher than P_t^B in early periods. However, when they are *actually* making investment decisions in early periods, they are not sure whether $P_t > P_t^B$ or $P_t < P_t^B$ because they do not know the benchmark level P_t^B that depends on $\bar{\delta}$ they are still learning about. Indeed, while Figure 2(a) reports a typical sample path in which overshoot occurs, it is possible to obtain (rare) simulated paths such that $P_t < P_t^B$ for all t.

4.2 Effect of asset's innovativeness

Under what circumstances are price swings pronounced? This section examines how the price dynamics change in response to a change in η_0 (inverse of the risky asset's innovativeness). Figure 4 plots the average of 50,000 simulated paths of P_t for different levels of η_0. The average price exhibits up-and-down swings only if η_0 is small enough ($\eta_0 \leq 50$ in the figure), i.e., only if the agents have sufficiently large uncertainty about $\bar{\delta}$ initially. For large η_0 ($\eta_0 \geq 300$ in the figure), the learning effect is so weak that it is

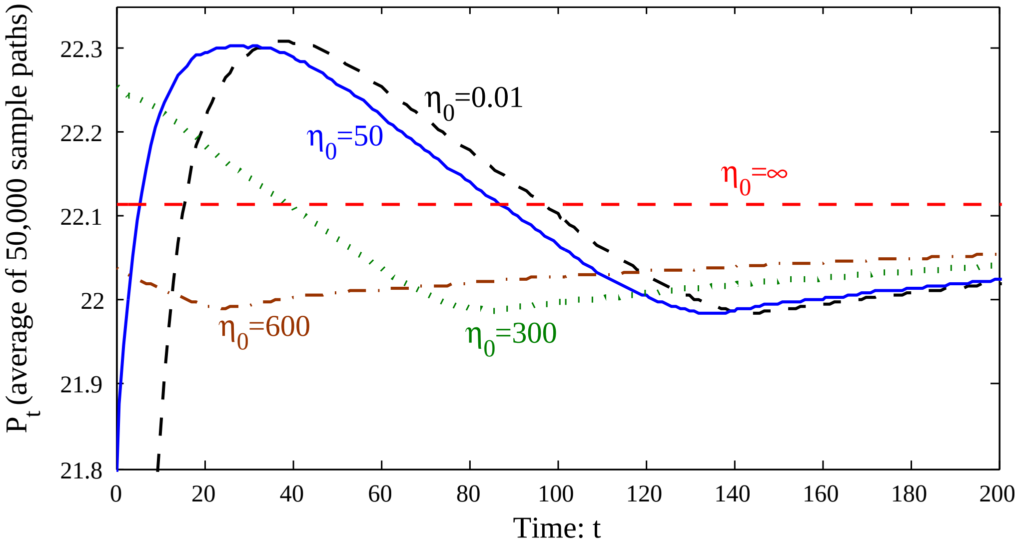


Figure 4: The average of 50,000 sample price paths for different levels of η_0 (inverse of the risky asset's innovativeness). The parameter values other than η_0 are the same as those of Figure 2.

already dominated by the leverage effect in $t = 0$, and thus the initial upswing does not occur. Moreover, if η_0 is very large ($\eta_0 \geq 600$), the experts' leverage ξ_t^* is so low that even overshoot does not occur on average. As η_0 increases further, the price swings are toned down and the path converges to that of the benchmark P_t^B as $\eta_0 \to \infty$.

Thus, the model predicts that swings and overshooting of prices are more pronounced for new and innovative assets with highly uncertain payoff characteristics than for old-economy assets already familiar in the market. This prediction is consistent with the historical observation that bubble-like price movements tend to arise in times of technological change (e.g., railroads or the Internet) or financial innovation (e.g., securitization), as noted by Brunnermeier and Oehmke (2013).

Note that price swings are pronounced with small η_0 because *both* the learning and leverage effects are large when η_0 is small. The intuition is as follows. Suppose that a financial asset backed by an unprecedented and/or hard-to-understand technology—such as Internet stocks, biotech stocks, or structured products—is newly introduced to the market. The investors have large uncertainty about such an asset's average payoff due to the lack of track record and background knowledge (i.e., η_0 is small). On the one hand, the investors, being risk averse, hesitate to purchase such an asset initially; but they increase demand gradually as learning progresses, generating a gradual upswing in the price (the learning effect). On the other hand, the experts, being motivated by career concerns, try to exploit the as-yet-unknown nature of the asset. Initially, they take on high leverage and invest in the asset aggressively in an effort to trick investors into believing that the asset is more profitable than it really is, causing the price overshoot; however, as the asset's true nature becomes known to investors, experts lose their desire to influence the investor beliefs and thus deleverage, causing a downswing in the price (the leverage effect).

5 Policy Implication: Capping Fees

So far, we have taken the fee rate ϕ as exogenous. What happens if we allow the experts to choose ϕ? In this section, we show that they may choose an inefficiently high ϕ, and imposing a cap on ϕ achieves a Pareto improvement and also dampens asset price swings.

Consider a simple modification to the model of Section 2. Each period t is split into two stages. In stage 1, investment decisions are made: each investor submits $y_{i,t}$ and each expert chooses $\xi_{i,t}$. In stage 2, each expert offers ϕ to the investor. To keep the analysis simple, we restrict his choice to $\phi \in [0, \phi^{\max}]$, where $\phi^{\max}$ represents the level above which the investor would decline the offer and walk away from the fund (though we do not model her outside option explicitly). The expert's choice of ϕ is observable to the investor.

The timing assumption that the expert chooses ϕ *after* the investor determines $y_{i,t}$ captures the observation in the real-world investment management industry: a significant part of fund costs ultimately borne by investors are unspecified at the time they put their capital in the funds. Of course, funds do charge some prespecified fees such as management fees and broker fees. But their charges may not end there; anecdotal evidence suggests that there are various other charges, in the name of trading fees, transaction costs, or administrative costs, which are not explicitly declared when investors allocate their capital.[10] Such non-prespecified charges seem to be of significance in terms of size. Indeed, Blake (2014) estimates that 80-85% of fund costs ultimately incurred by investors are not communicated to them in advance.

Lemma 4. *If each expert can choose fee rate $\phi \in [0, \phi^{\max}]$, he chooses $\phi = \phi^{\max}$ every period. All the equilibrium variables are identical to those of Section 3 with $\phi = \phi^{\max}$.*

This is shown as follows. Due to the lack of commitment, the expert cannot influence the investor's $y_{i,t}$ by choosing ϕ. Thus, in stage 2 where $y_{i,t}$ is already fixed, it is optimal for him to charge the highest possible fee $\phi^{\max}$ to maximize his fee revenue. In stage 1, the investor anticipates that the expert will charge $\phi^{\max}$, and thus her choice of $y_{i,t}$ depends on $\phi^{\max}$. Hence, the resulting equilibrium is as if $\phi = \phi^{\max}$ was imposed exogenously.

Now, consider a regulator who chooses in period 0 a constant, ϕ^{cap}, such that all the experts' choices are restricted to $\phi \in [0, \phi^{\text{cap}}]$ for all t, to maximize the social welfare. Our goal is to show that each expert's choice $\phi = \phi^{\max}$ can be inefficiently high, i.e., $\phi^{\text{cap}} < \phi^{\max}$. Note that the regulator effectively ignores the impact of her choice on each investor's expected utility because, as shown in Lemma 1, the investor's value function $V_t(\cdot)$ is independent of ϕ for all t. The intuition is as follows. Each investor is worse off with a higher ϕ, holding the expected asset return $\hat{R}_{t+1}$ constant. But, in equilibrium, a high ϕ translates into a high $\hat{R}_{t+1}$ through the high fee premium (the second term of (3.17)), compensating exactly the increase in her fee payment. Thus, her overall expected

[10] David Pitt-Watson writes about such undeclared charges in *The Financial Times* (January 16, 2015): "Those costs are paid directly from our account, but never made explicit. No one is required to collect this data, or to add up the many other hidden charges. [...] It is as if we had hired an agent to rent out our home while we were away, and on our return had asked them how much their service had cost. They tell us how much they have charged, but not how much the repair to the roof had cost, or the property taxes they had paid. They just give us a cheque and say: "Trust me. Here is what is left over from the rent that was received.""

utility is invariant to changes in ϕ in equilibrium. Thus, effectively, the regulator chooses ϕ^{cap} to maximize each expert's expected utility.

On the equilibrium path, each expert's maximized utility is deterministic because $y_{i,t+\tau}$ and $\xi_{i,t+\tau}$ in (2.1) are equal to $S/(1+\xi^*_{t+\tau})$ and $\xi^*_{t+\tau}$, respectively. Thus, his period-0 discounted lifetime utility, which is the regulator's objective function, is

$$U^*(\phi) \equiv \sum_{\tau=0}^{T} \beta^\tau \left(\frac{\phi S}{1+\xi^*_\tau} - \kappa \xi^*_\tau \right). \tag{5.1}$$

The impact of ϕ on U^* is ambiguous: while a rise in ϕ increases the fee revenue $\phi S/(1+\xi^*_t)$ ceteris paribus, it also increases ξ^*_t (see (3.18)), lowering U^*. Here, ξ^*_t increases with ϕ for the following reason. A rise in ϕ allows each expert to collect higher fees for a given rise in the investor's estimate $\hat{\delta}^I_{i,t}$. That is, the marginal gain from increasing $\xi_{i,t}$ (the LHS of (3.14)) increases with ϕ, leading him to take on higher leverage. This accompanies a higher cost $\kappa\xi^*_t$ and a lower purchase order $S/(1+\xi^*_t)$, leading to a lower U^*.

Lemma 5. *If T is large enough, the expert's lifetime utility $U^*(\phi)$ is inverse-U shaped in ϕ. We have $U^*(0) = 0$, $U^*(\phi) > 0$ for small $\phi > 0$, and $U^*(\phi) < 0$ for large $\phi > 0$.*

For $\phi = 0$, obviously, $U^* = 0$ because the expert's fee revenue is zero and he chooses $\xi^*_t = 0$ for all t. For small ϕ, $U^* > 0$ because his fee revenue is positive and yet ξ^*_t is so small (even zero) that U^* remains positive. But if ϕ is large, he chooses large ξ^*_t for a long period time. This entails large renege costs and also decreases the fee revenues, leading to a negative U^*. Lemma 5 implies that there exists a positive and finite value of ϕ that maximizes U^*. If the regulator sets ϕ^{cap} to such a level, the social welfare improves.

Proposition 2. *Imposing a cap $\phi^{\text{cap}} = \arg\max U^*(\phi)$ on the delegation fee ϕ achieves a Pareto improvement. It does not affect each investor's expected utility. It makes each expert better off if $\phi^{\text{cap}} < \phi^{\max}$, whereas it does not affect his utility if $\phi^{\text{cap}} \geq \phi^{\max}$.*

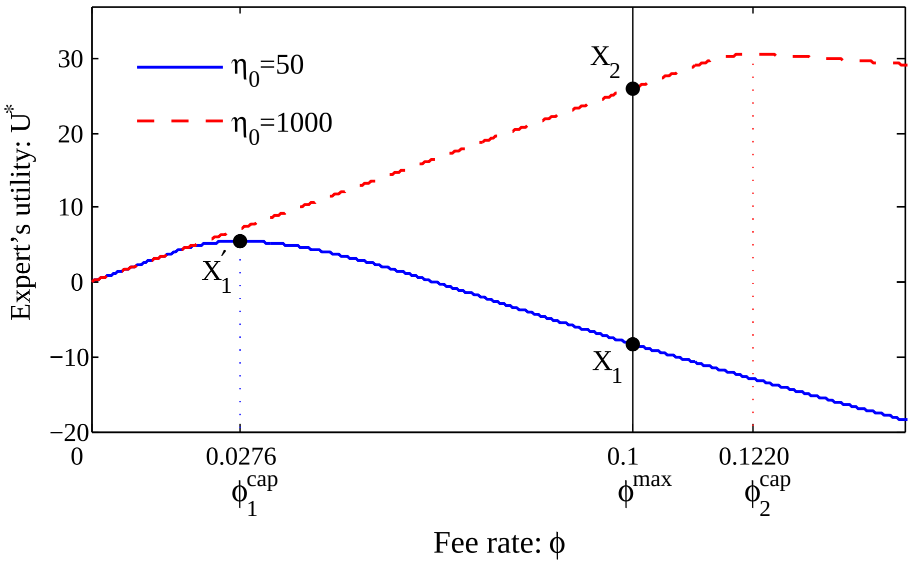


Figure 5: Expert's lifetime utility U^* versus fee rate ϕ for different values of η_0. The parameter values other than η_0 are the same as those of Figure 2.

Proposition 2 is illustrated in Figure 5, which plots U^* as a function of ϕ for two values of η_0. The blue solid line is the case with low η_0 ($\eta_0 = 50$), and the red dashed line is the case with high η_0 ($\eta_0 = 1,000$). For both cases, we set $\phi^{\max} = 0.1$. In the low-η_0 case, if there were no cap the expert would choose $\phi = \phi^{\max}$ (Lemma 4) and achieve lifetime utility corresponding to point X_1. If the regulator sets a cap of $\phi_1^{\text{cap}} = 0.0276$, it is binding because $\phi_1^{\text{cap}} < \phi^{\max}$. The expert chooses the highest possible fee $\phi = \phi_1^{\text{cap}}$ and achieves a higher utility at point X_1'. By contrast, in the high-η_0 case, the cap set by the regulator, $\phi_2^{\text{cap}} = 0.1220$, is not binding as it is larger than $\phi^{\max}$. Note that the expert cannot choose $\phi = \phi_2^{\text{cap}}$ because the investor would walk away from the fund for $\phi > \phi^{\max}$. So the expert charges $\phi = \phi^{\max}$, as if there is no cap. That is, regardless of the cap, the expert achieves the same level of utility corresponding to point X_2.

In the low-η_0 case, capping ϕ restores efficiency because it effectively allows the experts to coordinate their actions and choose low ϕ, lowering their leverage ξ_t^* in equilibrium. Of course, even without such a cap, the experts understand that if all of them choose $\phi < \phi^{\max}$ together, they would all be better off. However, such a behavior is not in line

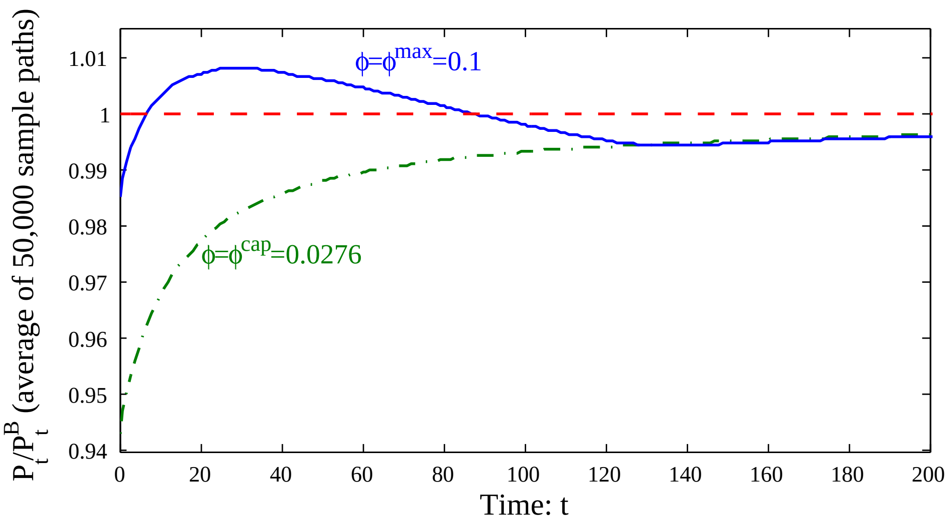


Figure 6: Impact of capping fees on the dynamics of P_t (average of 50,000 sample paths). The parameter values other than ϕ are the same as those of Figure 2.

with each expert's personal incentive. Having received purchase order $y_{i,t}$, it is individually optimal for her to charge a highest possible fee $\phi^{\max}$ to maximize her fee revenue. The cap alleviates this coordination failure and improves social welfare.

Note that the level of ϕ^{cap} varies with underlying parameter values, and the policy is effective only under specific conditions. That is, the model's important policy implication is that the regulator should adjust the cap flexibly depending on the market condition, rather than fixing it at a certain level at all times. In times when investors have very high uncertainty about the asset (the blue line in Figure 5), capping the fee is effective as it deters the experts from taking on excessive leverage. However, in "normal" times when η_0 is not so low (the red line in Figure 5), the cap is not needed because the experts would not use excessive leverage anyway. Keeping $\phi^{\text{cap}} < \phi^{\max}$ in such times would rather destroy social welfare by lowering the experts' fee revenue unnecessarily.

What is the impact of the cap on the price dynamics? Figure 6 plots the average of 50,000 simulated paths of P_t/P_t^B, assuming $\eta_0 = 50$ as in the blue-line case of Figure 5. We present the case without a cap ($\phi = \phi^{\max} = 0.1$; the blue solid line) and the case with a cap ($\phi = \phi^{\text{cap}} = 0.0276$; the green dashed-dotted line). Instead of presenting both P_t and the benchmark P_t^B in the figure, we normalize P_t by P_t^B because P_t^B itself changes with ϕ. While up-and-down swings and overshoot ($P_t/P_t^B > 1$) occur without a cap, they are eliminated by the cap. The intuition is simple. Capping ϕ prevents the experts from using excessive leverage, lowering the asset's aggregate demand and thus its market-clearing price P_t. Note that capping ϕ does not affect the learning effect because the Kalman filter (3.1) is independent of ϕ. Solely because of the toned-down leverage effect, the price swings and overshoot are eliminated.

6 Extension: Holdings and Trading Volume

The model of Section 2 provides an explanation of bubble-like price swings by shedding light on the role of leveraged, opaque financial companies such as hedge funds. However, the model is silent about how such funds' stock holdings evolve over time behind the price swings. Indeed, since all the funds are identical and the asset's supply is fixed at S shares, every fund's asset holding in equilibrium is constant at S for all t and thus the trading volume is zero for $t \geq 1$. This is clearly counterfactual. The empirical literature documents that hedge funds actively adjust their holdings in times of large price swings, and their trading activities in such times are different from other players in the market. Brunnermeier and Nagel (2004) find that hedge funds increased their holdings of the technology stocks during the upturn of the 1998–2000 dot-com bubble, but they decreased them before the bubble collapsed. Ang, Gorovyy, and van Inwegen (2011) report that hedge funds' leverage was counter-cyclical to that of other market participants during the 2007–2009 crisis. This section attempts to explain these empirical observations by rationalizing time-varying holdings of the asset.

6.1 Setup

We make a single modification to the model of Section 2. There are two types of funds: *hedge funds* (HFs), indexed by $i \in [0, \gamma]$, and *other funds* (OFs), indexed by $i \in (\gamma, 1]$. The proportion γ of HFs is exogenous. The HFs are the same as the funds of Section 2, whereas the OFs' experts cannot renege on the investors' purchase orders. That is, the only difference between these types is that HF experts can choose $\xi_{i,t} \geq 0$ while OF experts cannot.[11] The OFs can be viewed as representing various financial institutions subject to statutory disclosure requirements, such as mutual funds, banks, or investment banks. Alternatively, since the OF experts just take the investors' orders passively with no agency frictions stemming from $\xi_{i,t}$, the OFs can also be interpreted as individual investors, each of whom incurs a cost (such as a brokerage fee) of ϕ per share she purchases on her own.[12] For notational clarity, we append a tilde to the variables related to the OFs. We look for an equilibrium in which, for all t, every agent is optimizing and the risky asset's market clears, i.e., $\int_0^\gamma (1 + \xi_{i,t}) y_{i,t} \mathrm{d}i + \int_\gamma^1 \tilde{y}_{i,t} \mathrm{d}i = S$, where $\tilde{y}_{i,t}$ is OF investor i's purchase order.

6.2 Equilibrium

The equilibrium of this economy is derived following steps similar to those in Section 3 (see Appendix G for details).

Proposition 3. *There is a linear equilibrium in which*

1. *the risky asset's excess return R_{t+1} is, conditional on t, normal with mean*

$$\hat{R}_{t+1} = \min\left\{ \frac{A_{t+1}S}{\chi_t} + \phi, \frac{1}{2}\left(\frac{A_{t+1}S}{\chi_t} + (1-\gamma)\phi + \sqrt{\left(\frac{A_{t+1}S}{\chi_t} + (1-\gamma)\phi \right)^2 + \frac{4\gamma\kappa}{\Omega_t}} \right) \right\}$$

[11]An alternative assumption yielding the same results is that each OF's expert can choose $\xi_{i,t} \geq 0$ but his choice is observable to the investor. In such a case, the OFs' experts choose $\xi_{i,t} = 0$ for all t because $\xi_{i,t} > 0$ would not influence the investors' behavior and yet is costly to choose.

[12]It is not important that the cost ϕ is the same for both types of funds. Allowing for different values of ϕ does not change the main results.

and precision χ_t*, where* A_t*,* χ_t*, and* Ω_t *are given by* (3.8), (3.10), *and* (3.15), *respectively;*

2. *the risky asset's price is* $P_t = a_t\hat{\delta}_t - b_t$*, where* a_t *and* b_t *are given by* (3.4) *and* (3.20), *respectively;*

3. *each HF expert's leverage is, for all* $i \in [0,\gamma]$, $\xi_{i,t} = \xi_t^*$ *with*

$$\xi_t^* = \max\left\{0, \frac{\phi\Omega_t}{2\kappa}\left(\frac{A_{t+1}S}{\chi_t} + (1-\gamma)\phi + \sqrt{\left(\frac{A_{t+1}S}{\chi_t} + (1-\gamma)\phi\right)^2 + \frac{4\gamma\kappa}{\Omega_t}}\right) - 1\right\};$$

4. *each HF investor asks the expert to buy* $y_{i,t} = \frac{S}{1+\xi_t^*} + \frac{\phi(1-\gamma)\chi_t\xi_t^*}{A_{t+1}(1+\xi_t^*)^2}$ *shares of the asset, and each OF investor asks the expert to buy* $\tilde{y}_{i,t} = S - \frac{\phi\gamma\chi_t\xi_t^*}{A_{t+1}(1+\xi_t^*)}$ *shares of the asset;*

5. *HF investor's value function is* $V_t(W_{i,t}) = -\exp(-A_tW_{i,t} - B_t)$ *with*

$$B_t = \sum_{s=t}^{T}\left(\prod_{k=t}^{s}\frac{a_k}{1+a_k}\right)\left\{\begin{array}{l} -\ln\beta + \frac{1}{2\chi_s}\left(\frac{\nu S}{(1+r)a_s} + \frac{(1-\gamma)\phi\chi_s\xi_s^*}{1+\xi_s^*}\right)^2 \\ +\frac{1}{a_s}\ln\left(\frac{1}{a_s}\right) - \frac{1+a_s}{a_s}\ln\left(\frac{1+a_s}{a_s}\right)\end{array}\right\} \quad \textit{for } t = 0,...,T$$

and $B_{T+1} = 0$*, and OF investor's value function is* $\tilde{V}_t(\tilde{W}_{i,t}) = -\exp(-A_t\tilde{W}_{i,t} - \tilde{B}_t)$ *with*

$$\tilde{B}_t = \sum_{s=t}^{T}\left(\prod_{k=t}^{s}\frac{a_k}{1+a_k}\right)\left\{\begin{array}{l} -\ln\beta + \frac{1}{2\chi_s}\left(\frac{\nu S}{(1+r)a_s} - \frac{\gamma\phi\chi_s\xi_s^*}{1+\xi_s^*}\right)^2 \\ +\frac{1}{a_s}\ln\left(\frac{1}{a_s}\right) - \frac{1+a_s}{a_s}\ln\left(\frac{1+a_s}{a_s}\right)\end{array}\right\} \quad \textit{for } t = 0,...,T$$

and $\tilde{B}_{T+1} = 0$*;*

6. *each HF investor's consumption is*

$$c_{i,t} = \frac{W_{i,t}}{1+a_t} + \frac{1}{\nu}\left(\frac{a_t}{1+a_t}\right)\left(-\ln\beta + \frac{1}{2\chi_t}\left(\frac{\nu S}{(1+r)a_t} + \frac{(1-\gamma)\phi\chi_t\xi_t^*}{1+\xi_t^*}\right)^2 + B_{t+1} + \ln a_t\right)$$

for $t = 0, ..., T$ *and* $c_{i,T+1} = W_{i,T+1}$*, and each OF investor's consumption is*

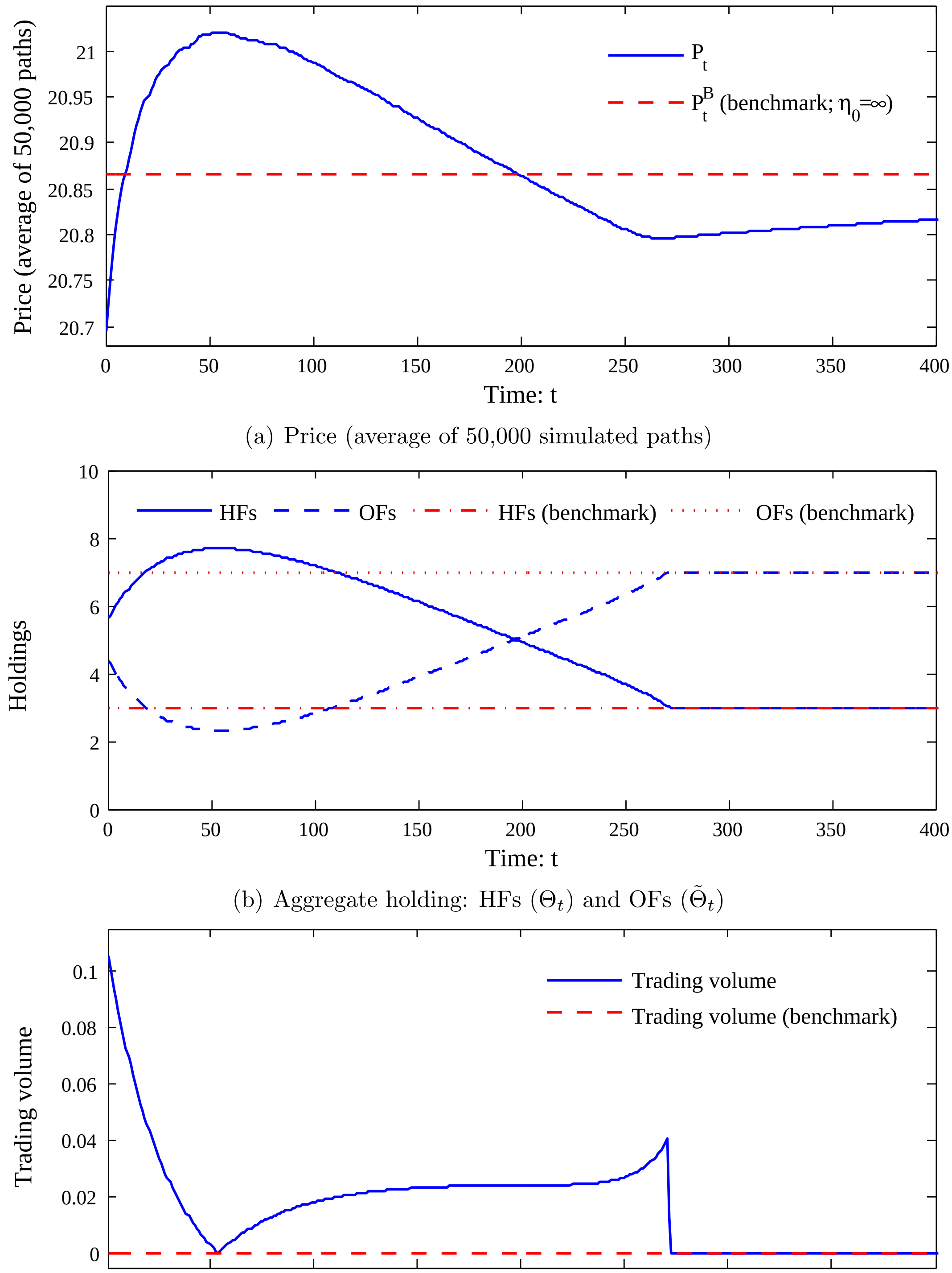
$$\tilde{c}_{i,t} = \frac{\tilde{W}_{i,t}}{1+a_t} + \frac{1}{\nu}\left(\frac{a_t}{1+a_t}\right)\left(-\ln\beta + \frac{1}{2\chi_t}\left(\frac{\nu S}{(1+r)a_t} - \frac{\gamma\phi\chi_t\xi_t^*}{1+\xi_t^*}\right)^2 + \tilde{B}_{t+1} + \ln a_t\right)$$

for $t = 0, ..., T$ *and* $\tilde{c}_{i,T+1} = \tilde{W}_{i,T+1}$.

Proposition 3 provides a characterization of the equilibrium in closed form, nesting Proposition 1 as a special case with $\gamma = 1$. Panel (a) of Figure 7 plots the average of 50,000 paths of P_t. It exhibits a bubble-like pattern similar to that of Figure 4 of Section 4. This is not surprising, as there is a significant fraction ($\gamma = 0.3$) of HFs, whose learning effect and leverage effect jointly shape the inverse-U dynamics of P_t as in Section 4. The primary purpose of this section is to study how the funds' holdings—and the associated trading volume—evolve over time behind such swings in P_t. The key to understanding it is the relationship between the HF leverage ξ_t^* and the purchase orders, $y_{i,t}$ and $\tilde{y}_{i,t}$. Statement 4 of Proposition 3 implies that not only the HF investors' $y_{i,t}$ but also the OF investors' $\tilde{y}_{i,t}$ depend on ξ_t^* because these investors' decisions depend on P_t that reflects ξ_t^* in equilibrium. Thus, both funds' holdings evolve over time depending on ξ_t^*, as shown in the following corollary.

Corollary 1. *The HFs' aggregate holding is* $\Theta_t \equiv \int_0^\gamma (1+\xi_{i,t})y_{i,t}\mathrm{d}i = \gamma S + \Pi_t$*, and the OFs' aggregate holding is* $\tilde{\Theta}_t \equiv \int_\gamma^1 \tilde{y}_{i,t}\mathrm{d}i = (1-\gamma)S - \Pi_t$*, where* $\Pi_t \equiv \frac{\gamma(1-\gamma)\phi\chi_t\xi_t^*}{A_{t+1}(1+\xi_t^*)} \geq 0$.

Corollary 1 states that the HFs' holding Θ_t is the sum of the "stationary" level γS and a time-varying component Π_t, whereas the OFs' holding $\tilde{\Theta}_t$ is the stationary level $(1-\gamma)S$ subtracted by Π_t. Here, Π_t increases with χ_t and ξ_t^*, reflecting the HFs' learning effect and leverage effect, respectively. Panel (b) of Figure 7 presents the dynamics of Θ_t and $\tilde{\Theta}_t$ for the same parameter values as panel (a). For comparison, we also plot the benchmark case with a noninnovative asset (i.e., $\eta_0 = \infty$), where $\Pi_t = 0$ because $\xi_t^* = 0$ for all t. The following two points are worth noting.



(a) Price (average of 50,000 simulated paths)

(b) Aggregate holding: HFs (Θ_t) and OFs ($\tilde{\Theta}_t$)

(c) Trading volume ($|\Delta\Theta_t| = |\Delta\tilde{\Theta}_t|$)

Figure 7: Dynamics of price, holdings, and trading volume. The parameter values are $\gamma = 0.3$, $r = 0.04$, $\phi = 0.15$, $\kappa = 1.2$, $\nu = 0.2$, $\beta = 1/(1+r)$, $S = 10$, $\eta_u = 5$, $\eta_0 = 100$, $\bar{\delta} = 1$, $\hat{\delta}_0 = 1$, and $T = 1,000$.

First, the evolution of the HFs' holding Θ_t is positively related to that of the average price P_t of panel (a). That is, the HFs tend to increase their asset holdings together with the price that is growing beyond its benchmark level, and reduce them with the price downturn. This result is consistent with Brunnermeier and Nagel (2004), who find that hedge funds were "riding" the 1998–2000 technology bubble: their stock holdings were heavily tilted toward the technology stocks when their prices were rising, but they cut back their holdings before the prices collapsed. Brunnermeier and Nagel (2004) argue that their empirical finding is consistent with the model of Abreu and Brunnermeier (2003), in which rational arbitragers such as hedge funds ride a bubble that emerges and grows exogenously due to "irrationally exuberant behavioral traders." Our result complements their argument and offers a further insight: hedge funds may not only ride/avoid the upturn/downturn of asset prices but also *generate* these price swings. Indeed, in our model, the up-and-down dynamics of P_t reflect the evolution of the HFs' demand. In early periods, the HFs increase holdings as the investors increase $y_{i,t}$ together with the return precision χ_t (the learning effect). But, as the investor learning progresses, they lower ξ_t^* and decrease their holdings. In Section 4.1 where there are only HFs, the learning and leverage effects are entirely absorbed by the movement of P_t and are not reflected on their holdings, as market clearing requires the equilibrium holdings to equal S for all t. In contrast, in this section, those two effects are also reflected on the HFs' holdings because they are partly absorbed by the holdings of the OFs who act as the HFs' trading counterparties.

Second, the evolution of the HFs' holding Θ_t is negatively related to that of the OFs' holding $\tilde{\Theta}_t$. While the HFs adjust their holdings to the same direction of the average P_t, the OFs alter them to the opposite direction. The OFs' holding is largest when the HFs have finished unloading the asset and ξ_t^* hits zero (around $t = 270$). This result is consistent with Ang, Gorovyy, and van Inwegen (2011), who report that hedge funds' leverage was counter-cyclical to that of other financial intermediaries during the 2007–2009 crisis: hedge fund leverage decreased before the crisis and was lowest in early 2009 when financial sector leverage was highest. Mathematically, this result is trivial because market clearing requires $\Theta_t + \tilde{\Theta}_t = S$ and thus $\Delta\Theta_t = -\Delta\tilde{\Theta}_t$. Economically, the result follows because the OFs serve as trading counterparties to the HFs that alter holdings over time due to their agency frictions. In early periods, each HF expert is bidding up the price to cater to growing purchase orders $y_{i,t}$ as well as the leveraged purchase $\xi_t^* y_{i,t}$. For such a high price, the OFs—which are not subject to agency frictions—optimally sell the asset to the HFs, reducing their holdings over time. At some point, the HFs' leverage effect surpasses the learning effect and the HFs start to unload the asset. They deleverage and push down the price over time, to which the OFs optimally respond by increasing their holdings. After the HF leverage ξ_t^* reaches zero, their holdings stay at the stationary levels.

Panel (c) of Figure 7 shows the dynamics of trading volume, defined as the number of shares traded in the market, i.e., $|\Delta\Theta_t|$ (which equals $|\Delta\tilde{\Theta}_t|$). In the benchmark case with an noninnovative asset (the red dashed line), the volume is zero for all $t \geq 1$ because all the funds trade only in $t = 0$ and keep the stationary levels of holdings for the rest of the time horizon. In the innovative asset case (the blue solid line), there is a "trading frenzy" right after the asset is introduced to the market: the trading volume is large when t is very small where the investors are highly uncertain about the asset, as the HFs aggressively buy it from the OFs. Intuitively, the volume is largest in the beginning because the speed of investor learning is fastest in the beginning in the sense that the return precision χ_t is concave in t (see Figure 3(a)), and thus the learning effect is strongest in the beginning. The HFs continue increasing the holdings (at a diminishing speed) until around $t = 50$, where they switch to selling the asset to the OFs.[13] Afterwards, the

[13]In the figure, it looks like the volume hits zero at the turning point around $t = 50$; but it actually stays positive at a low level.

trading volume increases over time because the HFs unload the asset more and more aggressively, reflecting the fact that the learning effect is fading out over time and hence the leverage effect becomes pronounced relatively. After the HF leverage ξ_t^* reaches zero, the trading volume is zero since the funds keep their stationary levels of holdings.

7 Conclusion

This paper develops a fully rational, dynamic asset-market equilibrium model in which (1) a new and innovative asset with as-yet-unknown average payoff is traded (e.g., Internet stocks, biotech stocks, or sophisticated structured products), and (2) investors delegate investment to experts. Over time, investors learn about the asset's average payoff from fund returns. Experts can secretly renege on investors' purchase orders and take on leveraged positions in the asset in an attempt to manipulate investors' beliefs, thereby attracting more orders and thus more fees. Despite full rationality of long-lived agents, the asset's equilibrium price exhibits bubble-like dynamics on average: gradual upswing, overshoot, and eventual reversal. The up-and-down swings are caused by the combination of (1) the learning effect—an upward pressure on the price as the investors' learning unravels the asset's uncertainty over time, and (2) the leverage effect—a downward pressure on the price as the experts deleverage over time. The price tends to overshoot because the experts' use of leverage pushes up the asset's aggregate demand and thus its market-clearing price. The model predicts that swings and overshooting of prices are more pronounced for new and innovative assets with highly uncertain payoff characteristics than for old-economy assets already familiar in the market. Capping delegation fees deters the experts from taking on excessive leverage, thereby achieves a Pareto improvement and also dampens swings and overshooting of asset prices. Consistent with empirical evidence, hedge funds increase holdings during the bubble-like price upturn but decrease them in the downturn, counter-cyclically to other market participants' holdings.

For future research, it would be interesting to explore this paper's idea in a model of imperfectly competitive and/or illiquid financial markets, because in reality a significant amount of innovative financial assets are traded in over-the-counter markets (where asset prices are not made public) or thin markets (where investors have price impact). Studying the relation between delegated investment and dynamics of such assets' prices in an OTC market model (e.g., Duffie, Gârleanu, and Pedersen 2005) or in a double share auction model (e.g., Kyle 1989) may yield further economic insights and policy implications.

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Appendix

A Evolution of Estimates

Let $\eta_t \equiv 1/\text{Var}[\bar{\delta}|\mathcal{H}_t]$ be the precision of the experts' period-t estimate of $\bar{\delta}$. By standard Kalman filtering, a new observation of δ_t will update the estimate of $\bar{\delta}$ and its precision as follows:

$$\hat{\delta}_t = \lambda_t \hat{\delta}_{t-1} + (1-\lambda_t)\delta_t \quad \text{for} \;\; \lambda_t \equiv \frac{\eta_{t-1}}{\eta_t}, \tag{A.1}$$

where

$$\eta_t = \eta_{t-1} + \eta_u. \tag{A.2}$$

The initial value of $\hat{\delta}_t$, $\hat{\delta}_0 > 0$, and the initial value of η_t, $\eta_0 > 0$, are exogenously given. The initial value of λ_t is $\lambda_1 = \eta_0/\eta_1 = \eta_0/(\eta_0 + \eta_u)$. From (A.1) and (A.2), we have

$$\lambda_{t+1} = \frac{\eta_t}{\eta_{t+1}} = \frac{\eta_t}{\eta_t + \eta_u} \tag{A.3}$$

and

$$\lambda_t = \frac{\eta_{t-1}}{\eta_t} = \frac{\eta_t - \eta_u}{\eta_t} \quad \Longleftrightarrow \quad \eta_t = \frac{\eta_u}{1-\lambda_t}. \tag{A.4}$$

Plugging (A.4) into (A.3) yields

$$\lambda_{t+1} = \frac{\frac{\eta_u}{1-\lambda_t}}{\frac{\eta_u}{1-\lambda_t} + \eta_u} = \frac{1}{2-\lambda_t}. \tag{A.5}$$

The updating factor λ_t is the same for (3.1) and (3.2). This is shown as follows. Let $\eta_{i,t} \equiv 1/\text{Var}[\bar{\delta}|\mathcal{H}^I_{i,t}]$ be the precision of investor i's estimate of $\bar{\delta}$. As in (A.2), $\eta_{i,t}$ evolves as

$$\eta_{i,t} = \eta_{i,t-1} + \eta_u. \tag{A.6}$$

Since $\mathcal{H}_0$ and $\mathcal{H}^I_{i,0}$ are both empty sets, $\eta_{i,0} = \eta_0$ for all i. Thus, (A.2) and (A.6) imply that $\eta_{i,t} = \eta_t$ for all i and t. So we have $\eta_{i,t-1}/\eta_{i,t} = \eta_{t-1}/\eta_t = \lambda_t$, as required.

B Proof of Lemmas 1 and 2

First, we derive the period-t precision of asset return, $\chi_t \equiv 1/\mathrm{Var}_t[R_{t+1}]$. To do so, it is useful to compute the conditional volatility of δ_{t+1}:

$$\mathrm{Var}_t[\delta_{t+1}] = \mathrm{Var}_t[\bar{\delta} + u_{t+1}] = \frac{1}{\eta_t} + \frac{1}{\eta_u} = \frac{1}{\eta_u}\left(\frac{1}{\lambda_{t+1}} - 1\right) + \frac{1}{\eta_u} = \frac{1}{\eta_u \lambda_{t+1}}. \tag{B.1}$$

Plugging the price conjecture (3.3) into the definition of R_{t+1} and using the investor's learning rule (3.2), the excess asset return is

$$\begin{aligned}
R_{t+1} &\equiv \delta_{t+1} + P_{t+1} - (1+r)P_t \\
&= \delta_{t+1} + a_{t+1}\hat{\delta}^I_{t+1} - b_{t+1} - (1+r)P_t \\
&= \delta_{t+1} + a_{t+1}\int_0^1 \hat{\delta}^I_{i,t+1}\mathrm{d}i - b_{t+1} - (1+r)P_t \\
&= \delta_{t+1} + a_{t+1}\int_0^1 \left(\lambda_{t+1}\hat{\delta}_{i,t} + (1-\lambda_{t+1})\delta_{t+1}\right)\mathrm{d}i - b_{t+1} - (1+r)P_t \\
&= \delta_{t+1} + a_{t+1}\lambda_{t+1}\hat{\delta}^I_t + a_{t+1}(1-\lambda_{t+1})\delta_{t+1} - b_{t+1} - (1+r)P_t \\
&= \left(1 + a_{t+1}(1-\lambda_{t+1})\right)\delta_{t+1} + a_{t+1}\lambda_{t+1}\hat{\delta}^I_t - b_{t+1} - (1+r)P_t.
\end{aligned} \tag{B.2}$$

From (B.2) and (B.1), we have

$$\begin{aligned}
\mathrm{Var}_t[R_{t+1}] &= \left(1 + a_{t+1}(1-\lambda_{t+1})\right)^2 \mathrm{Var}_t[\delta_{t+1}] \\
&= \frac{\left(1 + a_{t+1}(1-\lambda_{t+1})\right)^2}{\eta_u \lambda_{t+1}},
\end{aligned} \tag{B.3}$$

which yields χ_t as in (3.10).

Now, we derive the investor's value function and investment policy. In the final period $t = T+1$, the investors do not have optimization problems. Each of them just consumes her entire wealth, i.e., $c_{i,T+1} = W_{i,T+1}$. Thus, $A_{T+1} = \nu$ and $B_{T+1} = 0$. The investor's problem in period $t = 0, ..., T$ is solved as follows. Using dynamic budget constraint (2.3) and conjectured value function (3.6), we have

$$\mathrm{E}\left[V_{t+1}(W_{i,t+1})|\mathcal{F}^I_{i,t}\right] = -\exp\begin{pmatrix} -A_{t+1}\Big(\hat{R}^I_{i,t+1}(1+\xi^*_t)y_{i,t} - \phi y_{i,t} + (1+r)(W_{i,t} - c_{i,t}) \\ -\frac{1}{2}A_{t+1}(1+\xi^*_t)^2 y^2_{i,t}\frac{1}{\chi_t}\Big) - B_{t+1} \end{pmatrix}. \tag{B.4}$$

From (B.4) and Bellman equation (3.7), the first-order condition (FOC) for $y_{i,t}$ is

$$\hat{R}^I_{i,t+1}(1+\xi^*_t) - \phi - A_{t+1}(1+\xi^*_t)^2 y_{i,t}\frac{1}{\chi_t} = 0 \iff y_{i,t} = \frac{\chi_t\big(\hat{R}^I_{i,t+1}(1+\xi^*_t) - \phi\big)}{A_{t+1}(1+\xi^*_t)^2}. \tag{B.5}$$

From (B.4) and (B.5),

$$\mathrm{E}\left[V_{t+1}(W_{i,t+1})|\mathcal{F}^I_{i,t}\right] = -\exp\left(-\frac{1}{2}A^2_{t+1}(1+\xi^*_t)^2 y^2_{i,t}\frac{1}{\chi_t} - A_{t+1}(1+r)(W_{i,t} - c_{i,t}) - B_{t+1}\right). \tag{B.6}$$

Market clearing implies $y_{i,t} = S/(1+\xi^*_t)$ in equilibrium. Plugging this into (B.6),

$$\mathrm{E}\left[V_{t+1}(W_{i,t+1})|\mathcal{F}^I_{i,t}\right] = -\exp\left(-\frac{1}{2}A^2_{t+1}S^2\frac{1}{\chi_t} - A_{t+1}(1+r)(W_{i,t} - c_{i,t}) - B_{t+1}\right). \tag{B.7}$$

Thus, Bellman equation (3.7) is rewritten as

$$V_t(W_{i,t}) = \max_{c_{i,t}}\left\{-\exp(-\nu c_{i,t}) - \exp\left(-\psi_t - A_{t+1}(1+r)(W_{i,t} - c_{i,t})\right)\right\}, \tag{B.8}$$

$$\text{where} \quad \psi_t \equiv -\ln\beta + \frac{1}{2}A^2_{t+1}S^2\frac{1}{\chi_t} + B_{t+1}. \tag{B.9}$$

The FOC for $c_{i,t}$ is

$$\begin{aligned}
&\nu\exp(-\nu c_{i,t}) - \exp(-\psi_t)A_{t+1}(1+r)\exp\left(-A_{t+1}(1+r)(W_{i,t} - c_{i,t})\right) = 0\\
\iff\ & \ln\nu - \nu c_{i,t} = -\psi_t + \ln\left(A_{t+1}(1+r)\right) - A_{t+1}(1+r)(W_{i,t} - c_{i,t})\\
\iff\ & \psi_t + \ln\left(\frac{\nu}{A_{t+1}(1+r)}\right) + A_{t+1}(1+r)W_{i,t} = \left(\alpha + A_{t+1}(1+r)\right)c_{i,t}\\
\iff\ & c_{i,t} = \frac{A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}W_{i,t} + \frac{1}{\nu + A_{t+1}(1+r)}\left(\psi_t + \ln\left(\frac{\nu}{A_{t+1}(1+r)}\right)\right).
\end{aligned} \tag{B.10}$$

Plugging (B.10) back into (B.8), we have

$$\begin{aligned}
V_t(W_{i,t}) &= -\exp\left(-\frac{\nu A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}W_{i,t} - \frac{\nu}{\nu + A_{t+1}(1+r)}\left(\psi_t + \ln\left(\frac{\nu}{A_{t+1}(1+r)}\right)\right)\right)\\
&= -\exp\left(-\frac{\nu A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}W_{i,t} - \frac{\nu}{\nu + A_{t+1}(1+r)}\psi_t\right)\\
&\quad\times\left(\left(\frac{A_{t+1}(1+r)}{\nu}\right)^{-\frac{A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}}\left(1 + \frac{A_{t+1}(1+r)}{\nu}\right)\right).
\end{aligned} \tag{B.11}$$

Taking log to (B.11),

$$-A_tW_{i,t} - B_t = -\frac{\nu A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}W_{i,t} - \frac{\nu}{\nu + A_{t+1}(1+r)}\psi_t$$
$$- \frac{A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}\ln\left(\frac{A_{t+1}(1+r)}{\nu}\right) + \ln\left(1 + \frac{A_{t+1}(1+r)}{\nu}\right). \tag{B.12}$$

From (B.12) we have

$$A_t = \frac{\nu A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)} \tag{B.13}$$

and

$$B_t = \frac{\nu}{\nu + A_{t+1}(1+r)}\psi_t + \frac{A_{t+1}(1+r)}{\nu + A_{t+1}(1+r)}\ln\left(\frac{A_{t+1}(1+r)}{\nu}\right) - \ln\left(1 + \frac{A_{t+1}(1+r)}{\nu}\right). \tag{B.14}$$

Using (B.9), (B.14) is rearranged as

$$B_t = \frac{\nu}{\nu + A_{t+1}(1+r)}\left(\begin{array}{c} B_{t+1} - \ln\beta + \frac{1}{2}A_{t+1}^2S^2\frac{1}{\chi_t} \\ +\frac{A_{t+1}(1+r)}{\nu}\ln\left(\frac{A_{t+1}(1+r)}{\nu}\right) - \frac{\nu + A_{t+1}(1+r)}{\nu}\ln\left(\frac{\nu+A_{t+1}(1+r)}{\nu}\right)\end{array}\right). \tag{B.15}$$

Solving (B.13) backward from the terminal value $A_{T+1} = \nu$, we have

$$A_t = \nu\left(1 + \left(\frac{1}{1+r}\right) + \left(\frac{1}{1+r}\right)^2 + \cdots + \left(\frac{1}{1+r}\right)^{T+1-t}\right)^{-1}$$
$$= \frac{\nu}{1+a_t}. \tag{B.16}$$

Using (B.16), (B.15) is rewritten as

$$B_t = m_t(B_{t+1} + n_t), \tag{B.17}$$

$$\text{where}\quad m_t \equiv \frac{a_t}{1+a_t}, \tag{B.18}$$

$$n_t \equiv -\ln\beta + \frac{1}{2}\left(\frac{\nu S}{(1+r)a_t}\right)^2\frac{1}{\chi_t} + \frac{1}{a_t}\ln\left(\frac{1}{a_t}\right) - \frac{1+a_t}{a_t}\ln\left(\frac{1+a_t}{a_t}\right). \tag{B.19}$$

Solving (B.17) backward from the terminal value $B_{T+1} = 0$, we have

$$\begin{aligned} B_t =& m_t n_t + m_t m_{t+1} n_{t+1} + m_t m_{t+1} m_{t+2} n_{t+2} + \cdots + m_t m_{t+1} \cdots m_T n_T \\ =& \sum_{s=t}^{T} \left(\prod_{k=t}^{s} m_k \right) n_s, \end{aligned} \tag{B.20}$$

which is equivalent to (3.9) in the main text. □

C Proof of Lemma 3

First, we determine investor i's purchase order on an arbitrary off-the-equilibrium path where expert i is deviating from his equilibrium strategy.

Lemma 6. *If expert i plays $(\xi_{i,0}, ..., \xi_{i,T})$ when investor i believes that he plays $(\xi_0^*, ..., \xi_T^*)$, then investor i's purchase order in period $t = 1, ..., T$ is*

$$y_{i,t} = y_t + y_{i,t}^{+}, \tag{C.1}$$

$$\textit{where} \quad y_t \equiv \frac{\chi_t \big(\hat{R}_{t+1}(1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2} \tag{C.2}$$

$$\textit{and} \quad y_{i,t}^{+} \equiv \frac{\chi_t \left(1 + a_{t+1}(1-\lambda_{t+1})\right)}{A_{t+1}(1+\xi_t^*)} \sum_{s=1}^{t} \left(\prod_{k=s+1}^{t} \lambda_k \right) (1-\lambda_s) \left(\frac{\xi_{i,s-1} - \xi_{s-1}^*}{1+\xi_{s-1}^*} \right) R_s.^{14} \tag{C.3}$$

Proof of Lemma 6: In order to prove Lemma 6, we prove the following two claims.

Claim C.1. *If investor i believes that the payoff history up to period t is $\mathcal{H}_{i,t}^{I} = (\delta_{i,1}^{I}, ..., \delta_{i,t}^{I})$, her estimate of $\bar{\delta}$ in an arbitrary period t is*

$$\hat{\delta}_{i,t}^{I} = \hat{\delta}_t + \sum_{s=1}^{t} \left(\prod_{k=s+1}^{t} \lambda_k \right) (1-\lambda_s)(\delta_{i,s}^{I} - \delta_s). \tag{C.4}$$

Proof of Claim C.1: See Appendix C of Sato (2014). (*End of proof of Claim C.1.*)

Claim C.2. *Investor i's expected excess asset return is*

$$\hat{R}_{i,t+1}^{I} = \hat{R}_{t+1} + \left(1 + a_{t+1}(1-\lambda_{t+1})\right) (\hat{\delta}_{i,t}^{I} - \hat{\delta}_t). \tag{C.5}$$

[14] In (C.3), we abuse notation and set $\prod_{k=t+1}^{t} \lambda_k \equiv 1$.

Proof of Claim C.2: From (B.2), the expected excess return conditional on the true history $\mathcal{H}_t$ is

$$\hat{R}_{t+1} \equiv \mathrm{E}[R_{t+1}|\mathcal{H}_t] = (1 + a_{t+1}(1 - \lambda_{t+1}))\,\hat{\delta}_t + a_{t+1}\lambda_{t+1}\hat{\delta}_t^I - b_{t+1} - (1+r)P_t. \tag{C.6}$$

The expected excess return from investor i's perspective (conditional on her inferred history $\mathcal{H}_{i,t}^I$) is

$$\hat{R}_{i,t+1}^I \equiv \mathrm{E}[R_{t+1}|\mathcal{F}_{i,t}^I] = (1 + a_{t+1}(1 - \lambda_{t+1}))\,\hat{\delta}_{i,t}^I + a_{t+1}\lambda_{t+1}\hat{\delta}_t^I - b_{t+1} - (1+r)P_t. \tag{C.7}$$

From (C.6) and (C.7) we obtain (C.5). (*End of proof of Claim C.2.*)

Now Claims C.1 and C.2 can be used to rearrange the investor's order (3.11) as follows:

$$\begin{aligned}
y_{i,t} =& \frac{\chi_t\big(\hat{R}_{i,t+1}^I(1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2} \\
=& \frac{\chi_t\big(\hat{R}_{t+1}(1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2} + \frac{\chi_t}{A_{t+1}(1+\xi_t^*)}(\hat{R}_{i,t+1}^I - \hat{R}_{t+1}) \\
=& \frac{\chi_t\big(\hat{R}_{t+1}(1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2} + \frac{\chi_t\,(1 + a_{t+1}(1-\lambda_{t+1}))}{A_{t+1}(1+\xi_t^*)} \sum_{s=1}^{t} \left(\prod_{k=s+1}^{t} \lambda_k\right)(1-\lambda_s)(\delta_{i,s}^I - \delta_s).
\end{aligned} \tag{C.8}$$

Substituting (3.13) into (C.8) then yields

$$\begin{aligned}
y_{i,t} =& \frac{\chi_t\big(\hat{R}_{t+1}(1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2} + \frac{\chi_t\,(1 + a_{t+1}(1-\lambda_{t+1}))}{A_{t+1}(1+\xi_t^*)} \sum_{s=1}^{t} \left(\prod_{k=s+1}^{t} \lambda_k\right)(1-\lambda_s)\left(\frac{\xi_{i,s-1} - \xi_{s-1}^*}{1+\xi_{s-1}^*}\right) R_s \\
=& y_t + y_{i,t}^+,
\end{aligned}$$

as required. (*End of proof of Lemma 6.*)

Now we prove Lemma 3. To simplify the expert's period-t objective, note the following points.

- The fee on the current order, $\phi y_{i,t}$, can be omitted from the original objective function (2.1) because, from (3.11), $y_{i,t}$ is independent of the expert's actual choice of $\xi_{i,t}$.
- By Lemma 6, the future order $y_{i,t+\tau}$ ($\tau = 1, 2, ..., T-t$) is linear in $y_{t+\tau}$ and $y_{i,t+\tau}^+$. Since (2.1) is linear in $y_{i,t+\tau}$, it follows that (2.1) is linear in $y_{t+\tau}$ and $y_{i,t+\tau}^+$. This implies that $y_{t+\tau}$ can be omitted from (2.1) because the expert cannot influence $y_{t+\tau}$ by his choice of $\xi_{i,t}$. That is, only $y_{i,t+\tau}^+$ is relevant for his choice of $\xi_{i,t}$.
- Conjecture 2—which is verified later—implies that the expert's current action ($\xi_{i,t}$) does not affect his own future actions ($\xi_{i,t+1}, ..., \xi_{i,T}$) both on and off the equilibrium path. Thus, his costs of

reneging in future periods can be omitted from (2.1).

Taking these points into account, the expert's period-t maximization problem reduces to

$$\max_{\xi_{i,t}\in[0,\infty)} -\kappa\xi_{i,t} + \mathrm{E}\left[\sum_{\tau=1}^{T-t}\beta^{\tau}\phi y^{+}_{i,t+\tau}\middle|\mathcal{F}^{E}_{i,t}\right],$$

$$\text{where}\quad y^{+}_{i,t+\tau} \equiv \frac{\chi_{t+\tau}\,(1+a_{t+\tau+1}(1-\lambda_{t+\tau+1}))}{A_{t+\tau+1}(1+\xi^{*}_{t+\tau})}\sum_{s=1}^{t+\tau}\left(\prod_{k=s+1}^{t+\tau}\lambda_k\right)(1-\lambda_s)\left(\frac{\xi_{i,s-1}-\xi^{*}_{s-1}}{1+\xi^{*}_{s-1}}\right)R_s. \tag{C.9}$$

Since $y^{+}_{i,t+\tau}$ is a linear function of $(\xi_{i,0},...,\xi_{i,t+\tau-1})$, the marginal effect of the expert's current action $\xi_{i,t}$ on $y^{+}_{i,t+\tau}$ is independent of his actions in other periods, $(\xi_{i,0},...,\xi_{i,t-1},\xi_{i,t+1},...,\xi_{i,t+\tau-1})$. Hence, in $y^{+}_{i,t+\tau}$ given by (C.9), only the term corresponding to $s=t+1$ is relevant for the choice of $\xi_{i,t}$. Thus, an equivalent problem is

$$\max_{\xi_{i,t}\in[0,\infty)} -\kappa\xi_{i,t}+\mathrm{E}\left[\sum_{\tau=1}^{T-t}\beta^{\tau}\phi\frac{\chi_{t+\tau}\,(1+a_{t+\tau+1}(1-\lambda_{t+\tau+1}))}{A_{t+\tau+1}(1+\xi^{*}_{t+\tau})}\left(\prod_{k=t+2}^{t+\tau}\lambda_k\right)(1-\lambda_{t+1})\left(\frac{\xi_{i,t}-\xi^{*}_{t}}{1+\xi^{*}_{t}}\right)R_{t+1}\middle|\mathcal{F}^{E}_{i,t}\right].$$

This is rewritten as

$$\max_{\xi_{i,t}\in[0,\infty)} -\kappa\xi_{i,t}+\phi\left(\frac{\xi_{i,t}-\xi^{*}_{t}}{1+\xi^{*}_{t}}\right)\Omega_t\hat{R}_{t+1}, \tag{C.10}$$

$$\text{where}\quad \Omega_t \equiv (1-\lambda_{t+1})\sum_{\tau=1}^{T-t}\beta^{\tau}\frac{\chi_{t+\tau}\,(1+a_{t+\tau+1}(1-\lambda_{t+\tau+1}))}{A_{t+\tau+1}(1+\xi^{*}_{t+\tau})}\left(\prod_{k=t+2}^{t+\tau}\lambda_k\right)\quad \text{for } t=0,...,T-1$$

and $\Omega_T\equiv 0$. Choosing $\xi_{i,t}\geq 0$ to maximize (C.10), the FOC is given by (3.14) in the main text. □

D Proof of Proposition 1

Statements 1–5 are proved in the main text. Statement 6 follows by rearranging (B.10) with (B.16). □

E Dynamics of Ω_t

For $t=T$, we have $\Omega_T=0$. For $t=0,...,T-1$, from (3.15) we have

$$\Omega_t=(1-\lambda_{t+1})\sum_{\tau=1}^{T-t}\beta^{\tau}\left(\prod_{k=t+2}^{t+\tau}\lambda_k\right)M_{t+\tau},\quad \text{where}\ M_{t+\tau}\equiv\frac{\chi_{t+\tau}\,(1+a_{t+\tau+1}(1-\lambda_{t+\tau+1}))}{A_{t+\tau+1}(1+\xi^{*}_{t+\tau})}.$$

Let us denote $\prod_{k=t+2}^{t+1} \lambda_k \equiv 1$ (by abuse of notation). Then we have

$$\begin{aligned}
\frac{\Omega_t}{1-\lambda_{t+1}} =& \beta M_{t+1} + \beta^2 \lambda_{t+2} M_{t+2} + \beta^3 \lambda_{t+2}\lambda_{t+3} M_{t+3} + \cdots + \beta^{T-t}\lambda_{t+2}\lambda_{t+3}\cdots\lambda_T M_T, \\
\frac{\Omega_{t+1}}{1-\lambda_{t+2}} =& \beta M_{t+2} + \beta^2 \lambda_{t+3} M_{t+3} + \beta^3 \lambda_{t+3}\lambda_{t+4} M_{t+4} + \cdots + \beta^{T-t-1}\lambda_{t+3}\lambda_{t+4}\cdots\lambda_T M_T.
\end{aligned}$$

These two equations yields the difference equation of Ω_t for $t = 0, ..., T-1$:

$$\frac{\Omega_t}{1-\lambda_{t+1}} = \beta M_{t+1} + \beta \frac{\lambda_{t+2}}{1-\lambda_{t+2}} \Omega_{t+1}. \tag{E.1}$$

Note that (A.5) implies $1-\lambda_{t+1} = (1-\lambda_{t+2})/\lambda_{t+2}$. Using this, (E.1) is rewritten as

$$\Omega_t = \beta \left(\Omega_{t+1} + \left(\frac{1-\lambda_{t+2}}{\lambda_{t+2}} \right) \frac{\chi_{t+1}\left(1 + a_{t+2}(1-\lambda_{t+2})\right)}{A_{t+2}(1+\xi_{t+1}^*)} \right). \tag{E.2}$$

From (E.2) and the terminal value $\Omega_T = 0$, we obtain $\{\Omega_t\}_{t=0}^T$ by backward induction.

F Proof of Lemma 5

First, suppose $\phi = 0$. From (3.18), $\xi_t^* = 0$ for all t so (5.1) implies $U^* = 0$. Second, (3.18) implies that we can pick a very small $\phi > 0$ such that $\xi_t^* = 0$ for all t. For such a ϕ, we have $U^* > 0$.

Last, suppose $\phi \to \infty$. We want to show $U^* < 0$. To do so, let us prove the following claim.

Claim F.1. $\Omega_0, ..., \Omega_{T-1}$ *are all positive and finite even if* $\phi \to \infty$.

Proof of Claim F.1: We know that $\Omega_T = 0$, and thus $\xi_T^* = 0$. So, from (4.1),

$$\Omega_{T-1} = \beta \left(\frac{1-\lambda_{T+1}}{\lambda_{T+1}} \right) \frac{\chi_T(1 + a_{T+1}(1-\lambda_{T+1}))}{A_{T+1}}$$

is positive and finite. Thus, if $\phi \to \infty$, (3.18) implies $\xi_{T-1}^* \to \infty$. This means, from (4.1), $\Omega_{T-2} = \beta\Omega_{T-1}$, which is positive and finite. So (3.18) implies $\xi_{T-2}^* \to \infty$, meaning that $\Omega_{T-3} = \beta\Omega_{T-2}$, which is positive and finite. Continuing this way until $t = 0$, the result follows. (*End of proof of Claim F.1.*)

Now, (5.1) is rewritten as

$$U^* = \sum_{\tau=0}^{T-1} \beta^\tau \frac{\phi S}{1+\xi_\tau^*} + \beta^T \phi S - \kappa \sum_{\tau=0}^{T-1} \beta^\tau \xi_\tau^* \tag{F.1}$$

because $\xi_T^* = 0$. For ϕ large enough, $\xi_t^* > 0$ for $t = 0, ..., T-1$. So, using (3.18), (F.1) is

$$
\begin{aligned}
U^* = & \sum_{\tau=0}^{T-1} \beta^\tau \frac{2S\kappa}{\Omega_\tau} \left(\frac{A_{\tau+1}S}{\chi_\tau} + \sqrt{\frac{A_{\tau+1}^2 S^2}{\chi_\tau^2} + \frac{4\kappa}{\Omega_\tau}} \right)^{-1} \\
& + \beta^T \phi S - \kappa \sum_{\tau=0}^{T-1} \beta^\tau \left(\frac{\phi \Omega_\tau}{2\kappa} \left(\frac{A_{\tau+1}S}{\chi_\tau} + \sqrt{\frac{A_{\tau+1}^2 S^2}{\chi_\tau^2} + \frac{4\kappa}{\Omega_\tau}} \right) - 1 \right) && \text{(F.2)} \\
= & \sum_{\tau=0}^{T-1} \beta^\tau \frac{2S\kappa}{\Omega_\tau} \left(\frac{A_{\tau+1}S}{\chi_\tau} + \sqrt{\frac{A_{\tau+1}^2 S^2}{\chi_\tau^2} + \frac{4\kappa}{\Omega_\tau}} \right)^{-1} \\
& + \beta^T \phi \left[S - \sum_{\tau=0}^{T-1} \beta^{\tau - T} \left(\frac{\Omega_\tau}{2} \left(\frac{A_{\tau+1}S}{\chi_\tau} + \sqrt{\frac{A_{\tau+1}^2 S^2}{\chi_\tau^2} + \frac{4\kappa}{\Omega_\tau}} \right) - \frac{\kappa}{\phi} \right) \right]. && \text{(F.3)}
\end{aligned}
$$

Claim F.1 implies that the first line of (F.3) is positive and finite even if $\phi \to \infty$. Claim F.1 also implies that the expression inside the square brackets of the second line of (F.3) is negative if T is large enough. Thus, $U^* \to -\infty$ as $\phi \to \infty$. $\square$

G Proof of Proposition 3

The conjectures about the equilibrium price and the HF expert's strategy remain the same as Conjecture 1 and Conjecture 2 of Section 3, respectively. The investors' out-of-equilibrium belief is still (3.5).

HF investor i chooses $y_{i,t}$ and $c_{i,t}$ to maximize $-\mathrm{E}\left[\sum_{\tau=0}^{T-t} \beta^\tau \exp\left(-\nu c_{i,t+\tau}\right) | \mathcal{F}_{i,t}^I\right]$, subject to the dynamic budget constraint $W_{i,t+1} = Q_{i,t+1} - \phi y_{i,t} + (1+r)(W_{i,t} - c_{i,t} - P_t y_{i,t})$. Guess and later verify that her value function is $V_t(W_{i,t}) = -\exp(-A_t W_{i,t} - B_t)$, where A_t is given by (3.8) and B_t is a deterministic variable. Using the dynamic budget constraint and conjectured value function, we have

$$
\mathrm{E}\left[V_{t+1}(W_{i,t+1}) | \mathcal{F}_{i,t}^I\right] = -\exp \left(\begin{array}{c} -A_{t+1}\Big(\hat{R}_{i,t+1}^I (1+\xi_t^*) y_{i,t} - \phi y_{i,t} + (1+r)(W_{i,t} - c_{i,t}) \\ -\frac{1}{2} A_{t+1} (1+\xi_t^*)^2 y_{i,t}^2 \frac{1}{\chi_t}\Big) - B_{t+1} \end{array} \right). \tag{G.1}
$$

From (G.1) and the Bellman equation, the FOC for $y_{i,t}$ is

$$
\hat{R}_{i,t+1}^I (1+\xi_t^*) - \phi - A_{t+1}(1+\xi_t^*)^2 y_{i,t} \frac{1}{\chi_t} = 0 \quad \Longleftrightarrow \quad y_{i,t} = \frac{\chi_t \big(\hat{R}_{i,t+1}^I (1+\xi_t^*) - \phi\big)}{A_{t+1}(1+\xi_t^*)^2}. \tag{G.2}
$$

From (G.1) and (G.2),

$$\mathrm{E}\left[V_{t+1}(W_{i,t+1})|\mathcal{F}^I_{i,t}\right] = -\exp\left(-\frac{1}{2}A^2_{t+1}(1+\xi^*_t)^2 y^2_{i,t}\frac{1}{\chi_t} - A_{t+1}(1+r)(W_{i,t}-c_{i,t}) - B_{t+1}\right). \tag{G.3}$$

OF investor i chooses $\tilde{y}_{i,t}$ and $\tilde{c}_{i,t}$ to maximize $-\mathrm{E}\big[\sum_{\tau=0}^{T-t}\beta^\tau \exp\left(-\nu\tilde{c}_{i,t+\tau}\right)|\tilde{\mathcal{F}}^I_{i,t}\big]$, subject to the dynamic budget constraint $\tilde{W}_{i,t+1} = R_{t+1}\tilde{y}_{i,t} - \phi\tilde{y}_{i,t} + (1+r)(\tilde{W}_{i,t} - \tilde{c}_{i,t})$. Guess and later verify that her value function is $\tilde{V}_t(\tilde{W}_{i,t}) = -\exp(-A_t\tilde{W}_{i,t} - \tilde{B}_t)$, where $\tilde{B}_t$ is a deterministic variable. Since the OF expert never reneges on the investor's order, the investor correctly infers δ_t always, both on and off the equilibrium path. Thus, $\hat{R}^I_{i,t+1} = \hat{R}_{t+1}$ holds always. Hence,

$$\mathrm{E}\big[\tilde{V}_{t+1}(\tilde{W}_{i,t+1})\big|\tilde{\mathcal{F}}^I_{i,t}\big] = -\exp\left(-A_{t+1}\left(\hat{R}_{t+1}\tilde{y}_{i,t} - \phi\tilde{y}_{i,t} + (1+r)(\tilde{W}_{i,t}-\tilde{c}_{i,t}) - \frac{1}{2}A_{t+1}\tilde{y}^2_{i,t}\frac{1}{\chi_t}\right) - \tilde{B}_{t+1}\right). \tag{G.4}$$

From (G.4) and the Bellman equation, the FOC for $\tilde{y}_{i,t}$ is

$$\hat{R}_{t+1} - \phi - A_{t+1}\tilde{y}_{i,t}\frac{1}{\chi_t} = 0 \qquad \Longleftrightarrow \qquad \tilde{y}_{i,t} = \frac{\chi_t(\hat{R}_{t+1}-\phi)}{A_{t+1}}. \tag{G.5}$$

From (G.4) and (G.5),

$$\mathrm{E}\big[\tilde{V}_{t+1}(\tilde{W}_{i,t+1})\big|\tilde{\mathcal{F}}^I_{i,t}\big] = -\exp\left(-\frac{1}{2}A^2_{t+1}\tilde{y}^2_{i,t}\frac{1}{\chi_t} - A_{t+1}(1+r)(\tilde{W}_{i,t}-\tilde{c}_{i,t}) - \tilde{B}_{t+1}\right). \tag{G.6}$$

In equilibrium, $\hat{R}^I_{i,t+1} = \hat{R}_{t+1}$ for all i. So plugging (G.2) and (G.5) into the market clearing condition,

$$\begin{aligned}
&\int_0^\gamma (1+\xi_{i,t})y_{i,t}\mathrm{d}i + \int_\gamma^1 \tilde{y}_{i,t}\mathrm{d}i = S \\
\Longleftrightarrow\ & \frac{\gamma\chi_t(\hat{R}_{t+1}(1+\xi^*_t)-\phi)}{A_{t+1}(1+\xi^*_t)} + \frac{(1-\gamma)\chi_t(\hat{R}_{t+1}-\phi)}{A_{t+1}} = S \\
\Longleftrightarrow\ & \hat{R}_{t+1} = \frac{A_{t+1}S}{\chi_t} + \frac{\gamma\phi}{1+\xi^*_t} + (1-\gamma)\phi. \qquad \text{(G.7)}
\end{aligned}$$

Plugging (G.7) into (G.2) and (G.5), we have

$$y_{i,t} = \frac{S}{1+\xi^*_t} + \frac{\phi(1-\gamma)\chi_t\xi^*_t}{A_{t+1}(1+\xi^*_t)^2} \tag{G.8}$$

$$\text{and} \quad \tilde{y}_{i,t} = S - \frac{\phi\gamma\chi_t\xi^*_t}{A_{t+1}(1+\xi^*_t)}. \tag{G.9}$$

Plugging (G.8) into (G.3), and (G.9) into (G.6), we have

$$\mathrm{E}\left[V_{t+1}(W_{i,t+1})|\mathcal{F}^I_{i,t}\right] = -\exp\left(-\frac{1}{2\chi_t}\left(A_{t+1}S + \frac{\phi(1-\gamma)\chi_t\xi^*_t}{1+\xi^*_t}\right)^2 - A_{t+1}(1+r)(W_{i,t}-c_{i,t}) - B_{t+1}\right) \tag{G.10}$$

$$\text{and} \quad \mathrm{E}\big[\tilde{V}_{t+1}(\tilde{W}_{i,t+1})\big|\tilde{\mathcal{F}}^I_{i,t}\big] = -\exp\left(-\frac{1}{2\chi_t}\left(A_{t+1}S - \frac{\phi\gamma\chi_t\xi^*_t}{1+\xi^*_t}\right)^2 - A_{t+1}(1+r)(\tilde{W}_{i,t}-\tilde{c}_{i,t}) - \tilde{B}_{t+1}\right). \tag{G.11}$$

Thus, (G.10) implies that the Bellman equation for each HF investor is

$$V_t(W_{i,t}) = \max_{c_{i,t}}\{-\exp(-\nu c_{i,t}) - \exp\left(-\psi_t - A_{t+1}(1+r)(W_{i,t}-c_{i,t})\right)\} \tag{G.12}$$

$$\text{with} \quad \psi_t \equiv -\ln\beta + \frac{1}{2\chi_t}\left(A_{t+1}S + \frac{\phi(1-\gamma)\chi_t\xi^*_t}{1+\xi^*_t}\right)^2 + B_{t+1}, \tag{G.13}$$

and (G.11) implies that the one for each OF investor is

$$\tilde{V}_t(\tilde{W}_{i,t}) = \max_{\tilde{c}_{i,t}}\left\{-\exp(-\nu\tilde{c}_{i,t}) - \exp\left(-\tilde{\psi}_t - A_{t+1}(1+r)(\tilde{W}_{i,t}-\tilde{c}_{i,t})\right)\right\} \tag{G.14}$$

$$\text{with} \quad \tilde{\psi}_t \equiv -\ln\beta + \frac{1}{2\chi_t}\left(A_{t+1}S - \frac{\phi\gamma\chi_t\xi^*_t}{1+\xi^*_t}\right)^2 + \tilde{B}_{t+1}. \tag{G.15}$$

Now, following the same steps in Appendix B (from (B.8) to (B.20)), we obtain the values of B_t and $\tilde{B}_t$ as well as consumptions $c_{i,t}$ and $\tilde{c}_{i,t}$ as presented in Proposition 3.

Each HF expert's optimization problem is identical to Section 3.5. The equilibrium ξ^*_t given $\hat{R}_{t+1}$ is given by (3.16). Solving the system of equations (G.7) and (3.16) for two unknown $\hat{R}_{t+1}$ and ξ^*_t yields their values presented in Proposition 3. Having obtained $\hat{R}_{t+1}$, the price P_t is readily obtained following the same steps in Section 3.6. □