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with Extraction and Exploration Technologies**

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This paper provides a new non-renewable resource extraction model with both extraction and exploration technologies. We show how these technological changes affect efficient non-renewable resource extraction differently. Policy-makers therefore need to carefully choose the type of technology by considering their properties.

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# Non-renewable Resource Extraction

## with Extraction and Exploration Technologies

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### Abstract

This paper provides a new non-renewable resource extraction model with both extraction and exploration technologies. We show how these technological changes affect efficient non-renewable resource extraction differently. Policy-makers therefore need to carefully choose the type of technology by considering their properties.

*Keywords:* Non-renewable resource extraction; Resource exploration; Technological Change; Extraction technology; Exploration technology

*JEL classification:* Q3; Q5

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## 1. Introduction

Production in the electronic industry, electric industry and many other industries depends on scarce resources called “minor metals” found in specific locations<sup>1</sup>. To secure these resources, technologies play a crucial role in terms both of resource extraction and resource exploration<sup>2</sup>. In fact, many countries consider improvement of these two technologies to be important policy measures<sup>3</sup>.

In the resource economics literature, earlier studies explain the effects of resource exploration and technological change on resource extraction (for resource exploration, see Stewart (1979), Pindyck (1982), Arrow and Cheng (1982) and Cairns (1990) and for technological change, see

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<sup>1</sup> A typical example is rare earth. Rare earth is essential for battery and magnet production, but its reserves are low and 97% is located in China (U.S. Geological Survey, 2013).

<sup>2</sup> If substitution is possible, it also becomes important to secure resources. Im et al. (2006), Chakravorty (2008) and Chakravorty et al. (2011) focus on substitution among multiple resources.

<sup>3</sup> For example, the strategies of major countries are summarized in Critical Materials Strategy 2011 issued by U.S. department of energy ([http://energy.gov/sites/prod/files/DOE\\_CMS2011\\_FINAL\\_Full.pdf](http://energy.gov/sites/prod/files/DOE_CMS2011_FINAL_Full.pdf)).

Slade (1982), Farzin (1995) and Lin and Wagner (2007)). However, as far as we know, no existing study considers resource exploration and technological change in same model while paying attention to exploration technology.

This paper provides a theory for examining efficient extraction of non-renewable resources incorporating both resource exploration and technological change. We show how two types of technological changes affect efficient non-renewable resource extraction differently.

## 2. Resource extraction and exploration with technological change

We generalize the Pindyck (1982) monopolistic non-renewable resource extraction and exploration model by incorporating two types of technological change. A monopolistic producer chooses a level of production  $q_t$  from a resource stock  $R_t$  given an inverse demand function  $p_t(q_t)$  at time  $t$ . Time is discrete and runs through the interval  $t \in [0, \infty]$ . The average extraction cost is given by  $C^1(R_t, z_t^1)$ , where  $z_t^1$  is an index of the state of extraction technology at time  $t$ . Following Farzin (1995), we do not assume investment for technological change, but we do assume  $z_t^1 - z_{t-1}^1 > 0$  which implies an incremental improvement over time. The average extraction cost increases as the resource becomes depleted and decreases as the extraction technology is improved, and thus the average cost function satisfies  $C_{R_t}^1 < 0$  and  $C_{z_t^1}^1 < 0$ . Throughout this paper, subscripts other than  $t$  denote partial derivatives.

Moreover, we assume that increases in the existing resource occur in response to the producer's explorative efforts denoted by  $w_t$ . We assume that the cost of explorative effort is linear as in Stewart (1979), but also assume depletion of exploration. That is, the impact of one unit of explorative effort decreases based on the cumulative discoveries up to that time. The cost of exploration is expressed by  $kw_t$  for a constant  $k > 0$  and the total increase in resources is expressed by the discovery function  $f(w_t, X_t, z_t^2)$ , where  $X_t$  is the cumulative discoveries up to time  $t$  and  $z_t^2$  indicates the state of exploration technology at time  $t$ . From our assumptions, the discovery function satisfies  $f_{w_t} > 0$ ,  $f_{X_t} < 0$ . We further assume that exploration technology increases discoveries and improves incrementally over time, i.e.,  $f_{z_t^2} > 0$  and  $z_t^2 - z_{t-1}^2 > 0$ .

A monopolistic producer maximizes the sum of the present discounted value of net profit:

$$\max_{q_t, w_t} \sum_{t=0}^{\infty} \rho^t [p_t(q_t)q_t - C^1(R_t, z_t^1)q_t - kw_t] \quad (1)$$

subject to

$$R_{t+1} - R_t = f(w_t, X_t, z_t^2) - q_t; \quad X_{t+1} - X_t = f(w_t, X_t, z_t^2), \quad (2)$$

where  $\rho > 0$  is the market interest rate. The Lagrangian of this problem is:

$$L = \sum_{t=0}^{\infty} \rho^t \left\{ p_t(q_t)q_t - C^1(R_t, z_t^1)q_t - kw_t + \rho \lambda_{t+1}^1 (R_t + f(w_t, X_t, z_t^2) - q_t - R_{t+1}) + \lambda_{t+1}^2 (X_t + f(w_t, X_t, z_t^2) - X_{t+1}) \right\} \quad (3)$$

The first-order conditions are:

$$\frac{\partial L}{\partial q_t} = \rho^t (MR_t - C^1(R_t, z_t^1) - \rho \lambda_{t+1}^1) = 0, \quad (4)$$

$$\frac{\partial L}{\partial w_t} = \rho^t (-k + \rho(\lambda_{t+1}^1 + \lambda_{t+1}^2) f_{w_t}) = 0, \quad (5)$$

$$\frac{\partial L}{\partial R_t} = \rho^t (-C_{R_t}^1 q_t + \rho \lambda_{t+1}^1 - \lambda_t^1) = 0, \quad (6)$$

$$\frac{\partial L}{\partial X_t} = \rho^t (\rho \lambda_{t+1}^2 (1 + f_{X_t}) - \lambda_t^2) = 0, \quad (7)$$

$$\frac{\partial L}{\partial \lambda_{t+1}^1} = \rho^t (R_t + f(w_t, X_t, z_t^2) - q_t - R_{t+1}) = 0, \quad (8)$$

$$\frac{\partial L}{\partial \lambda_{t+1}^2} = \rho^t (X_t + f(w_t, X_t, z_t^2) - X_{t+1}) = 0, \quad (9)$$

where  $MR_t := p_t^{1q_t} + p_t(q_t)$ , i.e.,  $MR_t$  is the marginal revenue by resource extraction at time  $t$ . Finally, transversality conditions for the dynamics of extraction and explorative efforts are:

$$\lim_{t \rightarrow \infty} \lambda_t^1 R_t = 0, \quad (10)$$

$$\lim_{t \rightarrow \infty} \lambda_t^2 = 0. \quad (11)$$

Equation (10) holds with complementary slackness (Farzin, 1995). Equation (11) means that there are no additional costs associated with the cumulative discoveries  $X_t$  as  $t \rightarrow \infty$ . Efficient extraction and exploration for the monopolistic producer are characterized by Eqs. (4)–(11).

### 3. The difference between extraction and exploration technologies

First, we examine the effect of the extraction technology. Define  $\alpha_t := MR_t - C^1(R_t, z_t^1)$ , i.e.,  $\alpha_t$  is the extraction rent for a monopolistic producer at time  $t$ . Then, by Eqs. (4), (6) and (8), we have the following modified Hotelling rule for resource extraction:

$$\frac{\alpha_t - \alpha_{t-1}}{\alpha_{t-1}} = \delta + \frac{C_{R_t}^1 (R_t - R_{t+1})}{\alpha_{t-1}} + \frac{C_{R_t}^1 f(w_t, X_t, z_t^2)}{\alpha_{t-1}}. \quad (12)$$

The LHS of Eq. (12) is the rate of extraction rent change and the RHS is the sum of the interest rate, reserve dependent cost effects and exploration effects, respectively. By the exploration effect, the monopolistic producer extracts their reserves in such a way that the extraction rent rises at less than the interest rate minus the reserve dependent cost effect (Pindyck, 1982).

To identify the effect of technological change of extraction, rearranging Eq. (12) by linear approximation of the difference in average extraction cost:

$$MR_t - MR_{t-1} = \delta \alpha_{t-1} + C_{R_t}^1 (R_t + f(w_t, X_t, z_t^2) - R_{t+1}) + C_{R_t}^1 (R_t - R_{t-1}) + C_{z_t^1}^1 (z_t^1 - z_{t-1}^1). \quad (13)$$

Equation (13) characterizes the dynamics of marginal revenue of extraction. The last term on the RHS of Eq. (13) is multiplied by the technological change of extraction. Thus the extraction

technology changes the structure of the dynamics of the marginal revenue of extraction. Because we assume that  $C_{z_t^1}^1(z_t^1 - z_{t-1}^1) < 0$ , the marginal revenue of extraction rises more slowly as extraction technology advances. The level of marginal revenue also decreases because planned reserves would increase with technological progress. The same thing can be said about resource price (we show numerical examples later).

Next, we examine the effect of exploration technology. Define  $\beta_t := (MR_t - C^1(R_t, z_t^1)) - \frac{k}{f_{w_t}}$ , i.e.,  $\beta_t$  is the exploration rent for a monopolistic producer at time  $t$ . By rearranging Eqs. (5), (7) and (9), we have the following Hotelling rule for resource exploration:

$$\frac{\beta_t - \beta_{t-1}}{\beta_{t-1}} = \delta - \frac{\beta_t}{\beta_{t-1}} \frac{f_{X_t}(X_{t+1} - X_t)}{f_{w_t}}. \quad (14)$$

This expression does not characterize efficient resource extraction but characterizes efficient resource exploration. If the accumulated discoveries do not affect the increase in resources, the second term on the RHS of Eq. (14) vanishes. Then, under efficient exploration by a monopolistic producer, exploration rent increases according to the interest rate.

To see the effect of the technological change of exploration, rearranging Eq. (14) by linear approximation of the difference of marginal discoveries by explorative efforts gives:

$$\alpha_t - \alpha_{t-1} = \delta \beta_{t-1} - \beta_t \frac{f_{X_t}(X_{t+1} - X_t)}{f_{w_t}} - \frac{k}{f_{w_t} f_{w_{t-1}}} \left( f_{w_t X_t}(X_t - X_{t-1}) + f_{w_t z_t^2}(z_t^2 - z_{t-1}^2) \right). \quad (15)$$

By Eqs. (15) and (12), we have:

$$\beta_t = \frac{f(w_t, X_t, z_t^2)}{f_{X_t}(X_{t+1} - X_t)} \left( \frac{-k}{f_{w_t} f_{w_{t-1}}} \left( \delta f_{w_t} + f_{w_t X_t}(X_t - X_{t-1}) + f_{w_t z_t^2}(z_t^2 - z_{t-1}^2) \right) - C_{R_t}^1(R_t + f(w_t, X_t, z_t^2) - R_{t+1}) \right). \quad (16)$$

Notice that the LHS of Eq. (16) is the value of the exploration rent rather than the difference. We can substitute Eq. (16) into Eq. (13) to find an expression for  $MR_t - MR_{t-1}$  depending on  $z_t^2 - z_{t-1}^2$ . However, the structure of the dynamics remains as in Eq. (13). Thus, technological change of exploration does not change the structure of the dynamics of the marginal revenue of extraction. This point is crucially different from the case for extraction technology. We summarize the above discussion in the following proposition.

**Proposition.** Extraction and exploration technologies effect on efficient extraction for a monopolistic producer differently. Progress in extraction technology drops marginal revenue of extraction and resource price by changing the structure of those dynamics. Progress in exploration technology drops marginal revenue of extraction and resource price remaining the structure of those dynamics.

## 4. Numerical examples

We illustrate some numerical examples in three scenarios (*no progress*, *extraction progress*,

*exploration progress*) using the specified model. For simplicity, we only consider the time interval  $t \in [0, 29]$  and assume that technology changes once every 15 years or once every 10 years. Under the *no progress* scenario, two technologies sustain the constant level. Under the *extraction progress* and *exploration progress* scenarios, only one of the technologies will improve at  $t = 15$  or at  $t = 10$  and  $t = 20$ . Following Pindyck (1982), we specify the demand function, the average extraction cost function and the discovery function as:

$$q_t = a - bp_t, \quad a, b > 0, \quad (17)$$

$$C^1(R_t, z_t^1) = \frac{A}{R_t z_t^1}, \quad A > 0, \quad (18)$$

$$f(w_t, X_t, z_t^2) = \alpha w_t^\beta \exp\left(-\frac{\gamma}{z_t^2} X_t\right), \quad \alpha, \beta, \gamma > 0. \quad (19)$$

Fig. 1

Figure 1 (a) illustrates the time paths for the resource price, Fig. 1 (b) illustrates the time paths for the marginal revenue of extraction and Fig. 1 (c) illustrates the time paths for the explorative efforts when technology changes only at  $t = 15$ . The *no progress* scenario shows the typical path for the Hotelling rule, where resource price and marginal revenue rise over time. Under the *extraction progress* scenario, the paths for resource price and marginal revenue change, starting from a lower level and rising more slowly after the technological progress. Conversely, under the *exploration progress* scenario, explorative efforts increase drastically with technological progress. However, the paths for resource price and the marginal revenue of extraction shift slightly downward.

Figure 1 (d), (e) and (f) illustrates the time paths when technology changes twice. The time paths for resource price and the marginal revenue of extraction change with every improvement in extraction technology. By contrast, those paths again just shift downward under the *exploration technology* scenario. As our economic model shows in the previous section, technological change of exploration does not affect the structure of the dynamics of resource price or marginal revenue of extraction.

## 5. Conclusion

We have examined the dynamics of non-renewable resource extraction for a monopolistic producer with resource exploration and two types of technological change. Our analysis finds that extraction and exploration technologies have different effects on efficient resource extraction. Extraction technology changes the structure of the dynamics of resource price. Exploration technology only changes the value of resource price.

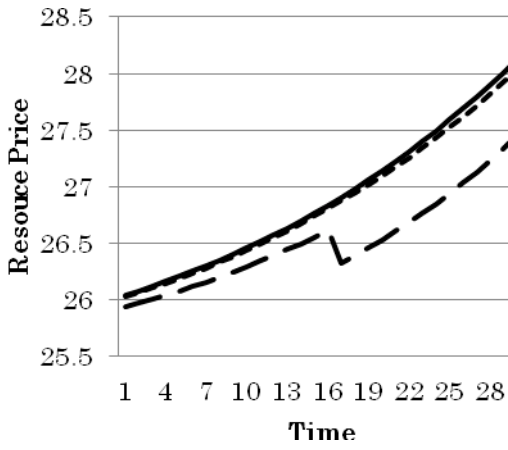
We remark that this difference does not always determine the superiority of a technology. While extraction technology can lead to a large change in resource price, it may increase risk for the

demand on resources. So far, discussion of the effect of technology has proceeded without discriminating the type of technology in both theory and policymaking. However, the difference between two technologies is expected to affect a number of issues on efficient resource use. Policy-makers need to carefully choose the type of technology by observing their characteristics.

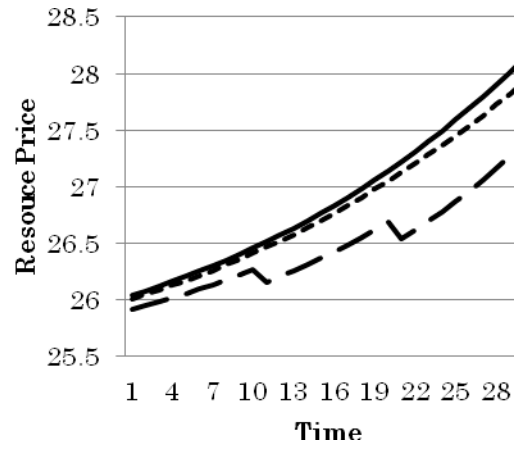
## References

- Arrow, K.J., Cheng, S., 1982. Optimal Pricing, Use and Exploration of Uncertain Natural Resource Stocks. *J. Environ. Econ. Manag.* 9, 1–10.
- Cairns, R.D., 1990. The Economics of Exploration for Non-renewable Resources. *J. Econ. Surv.* 4, 361–395.
- Chakravorty, U., 2008. Ordering the Extraction of Polluting Nonrenewable Resources. *Am. Econ. Rev.* 98, 1128–1144.
- Chakravorty, U., Moreaux, M., Leach, A., 2011. Would Hotelling kill the electric car? *J. Environ. Econ. Manag.* 61, 281–296.
- Farzin, Y.H., 1995. Technological Change and the Dynamics of Resource Scarcity Measures. *J. Environ. Econ. Manag.* 29, 105–120.
- Hotelling, H., 1931. The economics of exhaustible resources. *J. Polit. Econ.* 39, 137–175.
- Im, E.I., Chakravorty, U., Roumasset, J., 2006. Discontinuous extraction of a nonrenewable resource. *Econ. Lett.* 90, 6–11.
- Lin, C.-Y.C., Wagner, G., 2007. Steady-state growth in a Hotelling model of resource extraction. *J. Environ. Econ. Manag.* 54, 68–83.
- Pindyck, R.S., 1978. The Optimal Exploration and Production of Nonrenewable Resources. *J. Polit. Econ.* 86, 841–861.
- Slade, M.E., 1982. Trends in natural-resource commodity prices: an analysis of the time domain. *J. Environ. Econ. Manag.* 9, 122–137.
- Stewart, M.B., 1979. Market structure and the search for exhaustible resources. *Econ. Lett.* 2, 85–90.
- U.S. Geological Survey, Mineral Commodity Summaries 2013, Washington DC.
- U.S. Department of Energy, Critical Materials Strategy 2011, Washington DC.

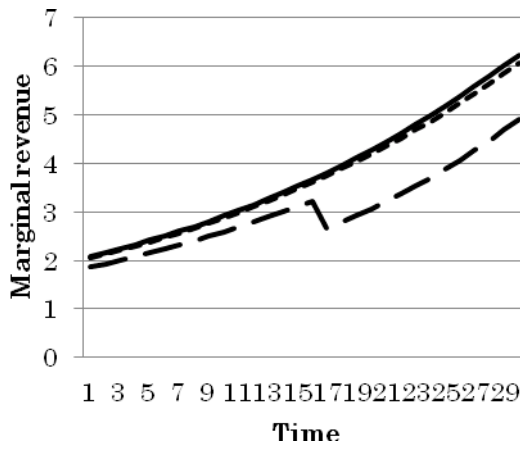
(a)



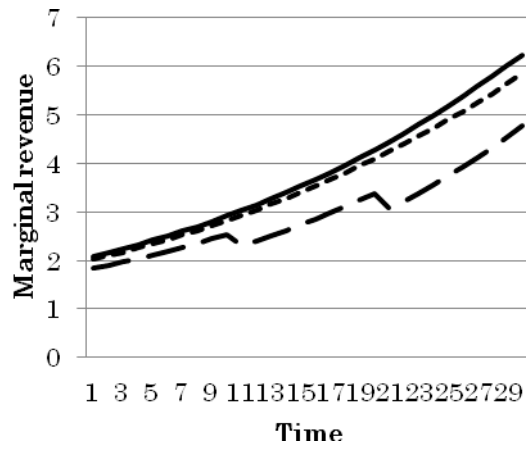
(d)



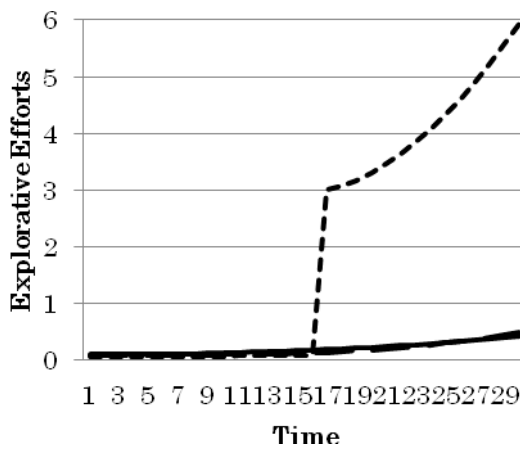
(b)



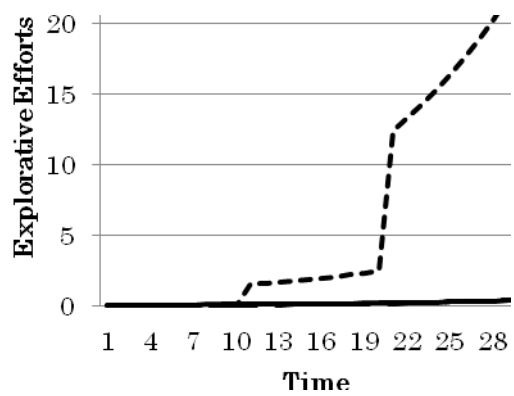
(e)



(c)



(f)



———— No progress    - - - - Extraction progress    - · - · - · Exploration progress



Figure 1. Note that  $\delta = 0.05$ ,  $\lambda_{30}^1 = 5$ ,  $k = 0.5$ ,  $a = 25$ ,  $b = 0.5$ ,  $A = 250$ ,  $\alpha = 2$ ,  $\beta = 0.5$ ,  $\gamma = 0.5$  for all scenarios.

(a) Time paths for the resource price when technology changes once every 15 years. In the *no progress* scenario,  $z_t^1 = 1$  and  $z_t^2 = 1$  for  $t \in [0, 29]$ . In the *extraction progress* scenario,  $z_t^1 = 1$  for  $t \in [0, 14]$ ,  $z_t^1 = 10$  for  $t \in [15, 29]$  and  $z_t^2 = 1$  for  $t \in [0, 29]$ . In the *exploration progress* scenario,  $z_t^1 = 1$  for  $t \in [0, 29]$ ,  $z_t^2 = 1$  for  $t \in [0, 14]$  and  $z_t^2 = 10$  for  $t \in [15, 29]$ . (b) Time paths for the marginal revenue of resource extraction for the same parameters as used in (a). (c) Time paths for explorative efforts for the same parameters as used in (a). (d) Time paths for the resource price when technology changes once every 10 years. In the *no progress* scenario,  $z_t^1 = 1$  and  $z_t^2 = 1$  for  $t \in [0, 29]$ . In the *extraction progress* scenario,  $z_t^1 = 1$  for  $t \in [0, 15]$ ,  $z_t^1 = 2$  for  $t \in [16, 29]$  and  $z_t^2 = 1$  for  $t \in [0, 29]$ . In the *exploration progress* scenario,  $z_t^1 = 1$  for  $t \in [0, 29]$ ,  $z_t^2 = 1$  for  $t \in [0, 15]$ ,  $z_t^2 = 10$  for  $t \in [16, 29]$  and  $z_t^2 = 50$  for  $t \in [16, 29]$ . (e) Time paths for the marginal revenue of resource extraction for the same parameters as used in (d). (f) Time paths for explorative efforts for the same parameters as used in (d).