KEIO UNIVERSITY
KEIO/KYOTO MARKET QUALITY RESEARCH PROJECT
(Global Center of Excellence Project)

KEIO/KYOTO GLOBAL COE DISCUSSION PAPER SERIES

DP2008-043

Entry, Exit, and Endogenous Growth

Katsuhiko Hori*
Shiro Kuwahara**

Abstract

This paper presents an endogenous growth model, in which entry, exit, and growth are endogenously determined through the rational behavior of agents, to investigate the effects of growth-enhancing policies on the exit rate of firms, and on the unemployment rate as well. Unlike standard Schumpeterian growth models, the exit of firms in our model is not simply the result of side effects of entry of newcomers with state-of-the-art technologies, but according to the literature of dynamic industry model, it occurs due to the fact that firms facing heterogeneous productivity shocks. Therefore, the exit rate is influenced by various kinds of economic factors and does not have a simple positive relationship with growth-enhancing policies in our paper. The main results are as follows: a subsidy to entry raises the exit rate. On the other hand, a subsidy to R&D raises growth rates, while the effects on the exit rate depend on the degree of the intertemporal elasticity of substitution.

*Katsuhiko Hori
Institute of Economic Research, Kyoto University
**Shiro Kuwahara
Graduate School of Systems & Information Engineering, University of Tsukuba

KEIO/KYOTO MARKET QUALITY RESEARCH PROJECT
(Global Center of Excellence Program)
Graduate School of Economics and Graduate School of Business and Commerce,
Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345 Japan
Kyoto Institute of Economics,
Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501 Japan

Entry, Exit, and Endogenous Growth*

Katsuhiko Hori† Shiro Kuwahara‡

First Draft: December, 2005
This version: March 20, 2009

Abstract

This paper presents an endogenous growth model, in which entry, exit, and growth are endogenously determined through the rational behavior of agents, to investigate the effects of growth-enhancing policies on the exit rate of firms, and on the unemployment rate as well. Unlike standard Schumpeterian growth models, the exit of firms in our model is not simply the result of side effects of entry of newcomers with state-of-the-art technologies, but according to the literature of dynamic industry model, it occurs due to the fact that firms facing heterogeneous productivity shocks. Therefore, the exit rate is influenced by various kinds of economic factors and does not have a simple positive relationship with growth-enhancing policies in our paper. The main results are as follows: a subsidy to entry raises the exit rate. On the other hand, a subsidy to R&D raises growth rates, while the effects on the exit rate depend on the degree of the intertemporal elasticity of substitution.

JEL classification: L16, O32, O41
Keywords: exit, R&D-based growth, subsidies

---

*The authors would like to thank Akihisa Shibata, Katsuya Takii, Katsunori Yamada, and Yuuho Yamashita for their useful comments. Furthermore, the authors acknowledge the financial support of the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Grant-in-Aid for the 21st Century Centre of Excellence (COE) Program "Interfaces for Advanced Economic Analysis," the Global COE program “Raising Market Quality · Integrated Design of Market Infrastructure,” and the Japan Society for the Promotion of Science (wakate B). Any remaining errors are, of course, the sole responsibility of the authors.

†E-mail: hori@kier.kyoto-u.ac.jp, Institute of Economic Research, Kyoto University.

‡E-mail: kuwahara@sk.tsukuba.ac.jp, Graduate School of Systems & Information Engineering, University of Tsukuba.

1 Introduction

This paper presents a stochastic dynamic model of the entry and exit of firms, incorporating an endogenous growth mechanism and analyzes the effects of various kinds of subsidy policies. The Schumpeterian growth models, initiated by Aghion and Howitt [1992] and Grossman and Helpman [1991], and constitute the most substantial literature on endogenous growth, incorporate private firms' profit maximization activities and long-run growth, entry, and exit, which are among the most basic behavior of firms. However, in the literature, the entry of firms is formulated as the creation of higher quality goods, and exit is assumed to occur through the drastic innovation, which generate the alternations of the monopolist firms. In this situation, the exit is simply generated by the result of entry of new innovator. Thus, entry and exit are like two sides of a coin, the result of which is that those previous studies undermine the values of the firm-specific factors of the exit: while, in the standard Schumpeterian growth models, the exit of firms should always correlate to the growth rate positively, and thus, growth-enhancing policies necessarily raise the exit rate of firms since technological progress is simply generated by the result of exit of firms, our model treats the entry and exit separately and shows that growth-enhancing policies relate to the exit more sensitively. In this sense, the present study is a trial analyzing relationships between the exit rate and subsidy policies which have effects on the long-run growth rate.

Regarding the literature on dynamic industry models of entry and exit, Brock [1972] and Smith [1974] were the first papers to present dynamic models of entry. However, these models show that the entry of firms ceases in the limit and they do not consider the exit of firms. Jovanovic [1982] and Hopenhayn [1992] constructed stochastic dynamic models of perfectly competitive economies with heterogeneous firms to consider exit as well as entry. More recently, Melitz [2003] and Asplund and Nocke [2006] investigated entry and exit in monopolistically competitive setups. Although all of these studies analyzed entry and exit, they did not consider entry and exit in connection with economic growth, and thus, the effects of growth on entry and exit. However, from the point of the long-run perspective, growth implications are generally regarded as important in dynamical settings, and studies of entry, exit, and endogenous growth are still rare; thus, we incorporate the endogenous growth mechanism into our model with entry and exit and analyze the relationship between entry, exit, and growth through subsidy policies.

For the purpose of the investigations described the above, the model in this paper consists of three types of company:assemblers, entrepreneurial firms, and research institutes; assemblers produce final goods by using intermediate goods manifactured by entrepreneurial firms, which produce intermediate goods by using labor and firm-specific technology, and research institutes innovate higher quality intermediate goods and provides them for entrepreneurial firms. In this process, the most important agents are entrepreneurial firms, because this type of firms is founded by the entry cost to develop goods with new type good, acing firm-specific uncertainty and monopolistic competition, and facing cost shock. When a firm is hit by a large negative shock, it may yield a negative firm value, and in such a case, the firm will be forced to exit the sector. Thus, the arrangement of the entrepreneurial firms illustrates the firm life cycle, and and, our model does not treat the exit as merely the tails side of the coin; rather, it contains the main mechanism of the firm life cycle, including entry, product improvement, and exit. This allows us to conduct a simultaneous investigation of the long-run dynamic effects of economic growth and the entry and exit rates, as well as the effects of subsidy policies.[1]

The obtained main results are as follows. The increment of entry raises the exit rate while reducing the economic growth rate. Therefore, subsidy to entry raises the exit rate while reducing the economic growth rate. The increment of R&D activities raises the economic growth rate while effects on entry and exit depends on the degree of intertemporal substitution. If the degree of intertemporal substitution is smaller than unity, then the increment of R&D activities raises increases the exist of firms. Accordingly, we also obtain the result that subsidy to R&D raises the growth rates while the effects on entry and exit rates depend on the degree of intertemporal substitution.

In many cases, exit of firms causes unemployment, which is one of interesting matters among economists. As shown in Section 5, we can easily extend our model to introduce unemployment and discuss the relationship between economic growth and unemployment. The relationship between economic growth and unemployment has mainly considered in two opposite directions. One is from search theory á la Pissarides [2000], where an increase of the growth rate reduces the unemployment rate by raising the possibility of gaining from future technological advances, and the other is from Schumpeterian growth theory á la Aghion and Howitt [1994], where an increase of the growth rate raises the unemployment rate by raising firm turnover. In this connection, the extension of our model, as in Aghion and Howitt [1994] but more simply, bridges the gap between the two opposite view to the relationship between growth and unemployment. Since the unemployment rate corresponds to the exit rate in our extension, the effect of R&D

[1]One of the main issues pertaining to existing endogenous growth literature is of the elimination of scale effects. See, for example, Jones [1995], Young [1998], Segerstrom [1998], Dinopoulos and Thompson [1998], Peretto [1998], Howitt [1999], and Segerstrom [2000]. Our model contains a similar structure to that of Young [1998], Dinopoulos and Thompson [1998], Peretto [1998], Howitt [1999], and Segerstrom [2000] in that it incorporates both variety expansion and quality improvements. However, our main objective is to analyze the exit behavior of firms, which is not included in these works.

subsidy on unemployment rate also depends on the degree of the intertemporal elasticity of substitution. Accordingly, we can obtain the result that the R&D subsidy to enhance the growth rate has negative or positive effects on employment.

The paper is organized as follows. Section 2 setups the model. Section 3 determines the balanced growth of the economy. Section 4 discuss the policy implications. Section 5 introduces unemployment. Finally, Section 6 concludes the paper.

2 The Model

There are three types of firms: assemblers, entrepreneurial firms, and research institutes. An assembler operates under a perfectly competitive market and produces the final goods, which are used as consumption, input for developing new types of the intermediate goods and improving the qualities of the current intermediate goods, and a fixed factors for production of intermediate goods.

An entrepreneurial firm plays the most critical role in the present study. It is founded to produce a new type of intermediate good with its own brand by employing labor to produce this original intermediate good in a monopolistically competitive market. Each entrepreneurial firm in the manufacturing sector faces an idiosyncratic fixed cost shock to produce its own brand of the intermediate good, and some of them exit the sector if they confront higher fixed costs so that their values are negative.

A research institute operates for the purpose of improving the quality of the current intermediate goods in exchange for paying the R&D expenditure. We assume the free entry of this research business; therefore, the new or advanced technology is competitively developed by firms in the R&D sector. The time is discrete and the final good is taken as numéraire.

2.1 Assembler

Each firm produces the final good in a perfectly competitive market using a continuum of intermediate goods supplied by entrepreneurial firms. Specifically, the production function in period t is defined as follows:

$$Y_t = \int_{S_t} A_{it} x_{it}^{\alpha} \mathrm{d}i X^{1-\alpha}. \tag{1}$$

Here, Y_t denotes the GDP; A_{it} is the quality index of the intermediate good of firm i in period t; x_{it} is the input of intermediate good i; and X is a fixed factor such as land. Hereafter, the total amount of the fixed factor is normalized to one; $X = 1$. S_t denotes an index set of surviving firms. This is because some fraction of the firms producing the intermediate goods exits the manufacturing sector and only the surviving firms produce the intermediate goods. How the exit of the firms occurs will be shown later. Therefore, the first order condition (FOC) for the profit maximization of firm i in the final goods sector is derived as follows:

$$p_{it} = \alpha A_{it} x_{it}^{\alpha-1}. \quad (2)$$

2.2 Entrepreneurial Firm

2.2.1 Description of the structure

Entrepreneurial firms play a central role in the present study on entry and exit. We designate these types of firms as entrepreneurial firms because they are established through entrepreneurs who are developing new enterprises. Each firm has an inherent technology to produce its own intermediate goods, which are produced by labor provided by households and used by the assembler. Since the intermediate goods are differentiated, the firm that newly creates an intermediate good has monopoly power. We assume that the patent affects only one period and the firm confronts an expiration of the patent at the next period [2] and entry to obtain the monopolistic profit if it exists.

Firms must pay a fixed cost for production.[3] The volatility in the fixed cost depends on the firm-specific shock in each period, e.g., a recall for defective products, a fire in a factory, an injunction to stop producing goods by a patent infringement assertion, etc. Inevitably, it implies a large fixed cost forces the firm to exit and only firms incurring lower fixed costs operate.

After a period following the development of a new technology, the patent of the technology expires and it becomes the task of research institutions to improve. Because the patent perishes one period after the technology is developed, the entrepreneurial firm is threatened by potential newcomers, but entry of these potential competitors is excluded in the following manner. Because it is also necessary for a research institution to pay a marginal cost to enter the production sector, institutions prefer selling patents to entrepreneurial firms. The price of a patent is determined at the limit price in order to prevent the research institution from paying a cost to enter the manufacturing sector; thus, the cost is close to the R&D cost for developing the higher quality intermediate good. In this case, the existence of entry cost gives the advantage to the entrepreneurial firm over potential

[2]Equivalently, we can assume that the monopoly power of each firm disappears in the next periods of the firm's founded period because other firms can imitate the idea through expansion of the knowledge.

[3]The term "fixed" is used in reference to the production quantity, and it is stochastically fluctuated at every period.

newcomers. Thus, an entrepreneurial firm continues to monopolize the intermediate good as long as the expected firm's value is positive. Thus, each surviving entrepreneurial firm purchases the elaborated technology from the research institution that develops the advanced technology for the intermediate good, and maintains its position of the monopolist.

2.2.2 Profit maximization

In this paper, we assume that each firm has a one-to-one production technology and the average fixed cost for production proportionately rises with the quality of the intermediate good in the current period. Thus, entrepreneurial firm i maximizes

$$\Pi_{it} = p_{it}x_{it} - w_t x_{it} - A_{it} f_{it},$$

under the consideration of (2). Here, Π_{it} is the profit of firm i in period t; w_t is the wage rate in period t; and f_{it} is an idiosyncratic shock to the fixed cost of firm i in period t, that is assumed to be drawn from a distribution $G(\cdot)$. Throughout the paper, we assume the following.

Assumption. *The distribution function $G(\cdot)$ is independent, identical, differentiable, and satisfying that*

1. $G(0) = 0$
2. $G(f) < 1$ *for all* $f \in (0, \infty)$.

We put this assumption to guarantee the existence of the exit level of the fixed cost. Thus, the FOC of this profit-maximization problem is the optimization of x_{it}, which yields

$$w_t = \alpha^2 A_{it} x_{it}^{\alpha-1}. \tag{3}$$

By using (3), the profit of firm i in period t is given as

$$\Pi_{it} = [\alpha(1-\alpha)x_{it}^{\alpha} - f_{it}]A_{it}. \tag{4}$$

Therefore, the expected profit of the firm, denoted by $\bar{\Pi}_{it}$, is written as

$$\bar{\Pi}_{it} = \mathrm{E}_{t-1}[\Pi_{it} | f_{it} \leq \bar{f}_{it}] G(\bar{f}_{it}) = \left[\alpha(1-\alpha)x_{it}^{\alpha} - \psi(\bar{f}_{it})\right] A_{it} G(\bar{f}_{it}), \tag{5}$$

where $\bar{f}_{it}$ is the exit level of fixed cost in period t and $\psi\left(\bar{f}_{it}\right)$ is the conditional expectation of fixed cost on the case that the firm survives:

$$\psi\left(\bar{f}_{it}\right) = \mathrm{E}_{t-1}[f_{it} | f_{it} \leq \bar{f}_{it}] = \frac{\int_{f \leq \bar{f}_{it}} f \mathrm{d}G(f)}{G(\bar{f}_{it})}.$$

Moreover, we denote the quality-adjusted profit, π_{it}, and its expectation, $\bar{\pi}_{it}$, as $\pi_{it} = \Pi_{it}/A_{it}$ and $\bar{\pi}_{it} = \bar{\Pi}_{it}/A_{it}$, respectively.

Finally, a firm will cease operations when its expected value is negative since we assume that entrepreneurial firms face the fixed cost shocks.[4]

2.3 Research Institution

Research institutions improve the brand powers or qualities of the current intermediate goods in exchange for paying costs, and competitively sell them to entrepreneurial firms. In this paper, new designs are assumed to be effective only during one period, and after that, they are eroding or widely available. Hence, adoption of new designs make entrepreneurial firms earn monopoly rents only within a single period. In this paper, the quality improvement of each intermediate good is assumed to be increasing in the aggregate research expenditures, measured by the final good, devoted by research institutions. Specifically, the quality improvement of intermediate good produced by firm i, $q_{it} = A_{it+1}/A_{it}$, is given as

$$q_{it} = \gamma \frac{Z_{qit}}{A_{it}}, \tag{6}$$

where Z_{qit} is the R&D expenditure for improving the design or brand of intermediate good i in period t. Equation (6) implies that a more advanced design requires more expenditure to develop.

Since the entrepreneurial firm obtains the monopoly rent within a period by adopting a new design, the firms bid up an aggregate patent price, P_{Ait}, equal to the expected value of entrepreneurial firm i in perioed $t+1$, discounted to the current period. Since this implies that the firm's value is equal to its monopoly rent in period $t+1$, P_{Ait} is given as[5]

$$P_{Ait} = \frac{1}{1+r_t} \bar{\Pi}_{it+1},$$

where r_t denotes the interest rate in period t. Therefore, the free-entry condition can be written as

$$\frac{1}{1+r_t} \bar{\Pi}_{it+1} = \left(1 - s_q\right) Z_{qit}, \tag{7}$$

where s_q is a subsidy rate for R&D.

[4]This is because rational investors do not have any incentive to invest in a firm with a negative value; an investor who invests in such a firm to keep its business loses his or her money.

[5]See Appendix A.

2.4 Entry of Entrepreneur

To develop new designs and brands of intermediate goods, each entrepreneur must pay a cost, which is measured by the final good. The entrepreneur who develops a new design of an intermediate good enters the manufacturing sector in the following period and earns a profit, unless it faces a high fixed cost shock. See Figure 1, in which timings of entry, realization of fixed costs, and exit are depicted.

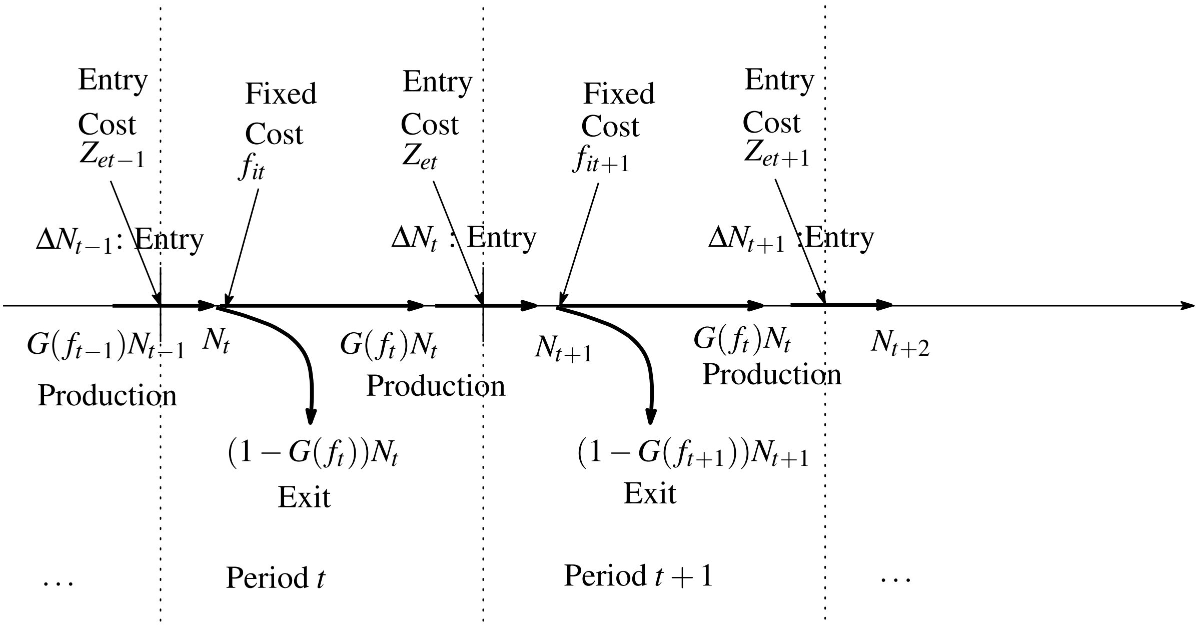


Figure 1: Timing of entry, exit, and the number of firms

Specifically, we denote the *gross* numbers of firms, i.e., the total number of surviving and exiting firms, by N_t. Thus, the *net* number of firms can be written as $N_t G(\bar{f}_t)$ since $G(\bar{f}_t)$ represents the share of surviving firms. Moreover, the entry cost per firm is assumed to be proportionately increasing in the productivity of the intermediate goods. Therefore, an amount of entry costs of each entrepreneur at time t is given as δA_t, where A_t denotes the average productivity of the intermediate goods. The entry cost is increasing in A_t since a higher quality of the intermediate goods in the economy implies that it is more difficult to enter the sector.

Since the entrepreneurs paying entry costs in period t expect to earn a series of profits after period $t+1$, the free-entry condition of the entry can be written as

$$\frac{1}{1+r_t}\bar{\Pi}_{t+1} = (1-s_e)\,\delta A_t, \tag{8}$$

where s_e is the subsidy rate for entry and $\bar{\Pi}_{t+1}$ is the expected value of a entrepreneurs entering the manufacturing sector. Since rational entrepreneurs equate their expected value for entry with the average value of the existing firms, it is also an average profit among entrepreneurial firms by Appendix A.

2.5 Households

Each household lives forever and has additive separable preferences on consumption with a positive and constant intertemporal elasticity of substitution, $\eta > 0$, and subjective discount factor, $\beta \in (0,1)$. Thus, a household maximizes

$$U = \sum_{t=0}^{\infty} \left[\beta^t (1+n)^t \frac{\eta}{\eta - 1} c_t^{1-\frac{1}{\eta}} \right],$$

subject to intertemporal budget constraints. Here, c_t is an amount of consumption per capita in period t and n is the population growth rate which is assumed to be constant over time. Thus, the FOC of this utility maximization problem yields the following Euler equation:

$$\frac{c_{t+1}}{c_t} = \beta^{\eta} \left(1 + r_{t+1}\right)^{\eta}. \tag{9}$$

2.6 Market-Clearing Conditions

The market-clearing conditions for the labor and final goods markets are respectively given as

$$\int_{S_t} x_{it} \mathrm{d}i = L_t = L_0 (1+n)^t \tag{10}$$

and

$$Y_t = C_t + \int_{S_t} \left(Z_{qit} + A_{it} f_{it}\right) \mathrm{d}i + \delta A_t \Delta N_t. \tag{11}$$

Here, L_t denotes the amount of labor at time t; C_t is an aggregate amount of consumption at time t: $C_t = c_t L_t$; ΔN_t is the number of entrepreneurs entering the sector. Since the number of entrepreneurs entering the sector is the difference between the gross number of firms in period $t+1$ and the net number of firms in period t, the number of entrepreneurs can be written as

$$\Delta N_t = N_{t+1} - N_t G\left(\bar{f}_t\right). \tag{12}$$

3 Balanced Growth

In this section, we determine the balanced growth path in a symmetric equilibrium. A symmetric equilibrium in our economy is characterized by the conditions that the initial qualities and R&D costs are equal among intermediate goods; $A_{i0} = A_0$ for all $i \in S_0$ and $Z_{qit} = Z_{qt}$ for all $i \in S_t$ and t. This also implies that $A_{it} = A_t$ for all $i \in S_t$ and t, and thus, it follows from (3) that $x_{it} = x_t$ for all $i \in S_t$ and t. Moreover, from (4), the exit levels of the fixed costs are the same across firms; $\bar{f}_{it} = \bar{f}_t$ for all $i \in S_t$, and thus, the expected profits are equal to the average profit; $\bar{\Pi}_{it} = \bar{\Pi}_t$ for all $i \in S_t$ and t. Therefore, the combination of (8) and (7) yields the quality-adjusted expenditure to the R&D:

$$\hat{z}_q = \frac{\delta (1 - s_e)}{1 - s_q} \quad \text{for all } t, \tag{13}$$

where we denote the effective expenditure for R&D in a symmetric equilibrium as $\hat{z}_q = Z_{qit}/A_{it}$.

Moreover, the balanced growth path in a symmetric equilibrium is characterized by the conditions that per-firm population, the exit level of fixed cost, and the quality-adjusted R&D cost are constant over time; $l_t = l$ and $\bar{f}_t = \bar{f}$ for all t, where $l_t = L_t/N_t$. Henceforce, subscriptions i and t are eliminated whenever variables have the same values across firms and over periods. Under these assumptions, combining (10) with (1), (3), (4), and (5) gives

$$\hat{y} = l^{\alpha} G\left(\bar{f}\right)^{1-\alpha}, \tag{14}$$

$$w_t = \alpha^2 A_t l^{\alpha-1} G(\bar{f})^{1-\alpha}, \tag{15}$$

$$\pi_{it} = \alpha(1-\alpha)\frac{\hat{y}}{G\left(\bar{f}\right)} - f_{it}, \tag{16}$$

and

$$\bar{\pi} = \alpha(1-\alpha)\hat{y} - \psi\left(\bar{f}\right) G\left(\bar{f}\right), \tag{17}$$

where $\hat{y} = Y_t / \left(A_t N_t\right)$, $\pi_{it} = \Pi_{it}/A_t$, and $\bar{\pi} = \bar{\Pi}_t/A_t$. Here, it should be noted that, from (15), w_t is increasing in the exit level of fixed cost, taken l and A_t as given. Moreover, noting that $l_t = l$ in the balanced growth path, (12) can be rewritten as

$$\frac{\Delta N_t}{N_t} = 1 - G\left(\bar{f}\right) + n.$$

Therefore, from the above equation, we find that the growth rate of the number of entry firms is decreasing in the exit level of fixed cost. The exit rate can be written as $1 - G\left(\bar{f}\right)$, implying that the growth rate of the number of entry firms is increasing in the exit rate. Equation (9), together with (6), (11), and the fact that $N_{t+1}/N_t = L_{t+1}/L_t = 1+n$, implies that

$$1+r = \beta^{-1}\left(\gamma\hat{z}_q\right)^{\frac{1}{\eta}}. \tag{18}$$

Therefore, from (6), (13), and the fact that $N_{t+1}/N_t = 1+n$, (9), and (14) imply that the gross growth rates of Y_t, as well as that of C_t, on the balanced growth path can be written as

$$g = \frac{\delta\gamma(1-s_e)}{1-s_q}(1+n). \tag{G}$$

We now focus on determining the condition for the exit level of fixed cost under balanced growth. As derived in Appendix B, the condition for the exit level of fixed cost is given as

$$\Gamma\left(\bar{f}\right) = \beta^{-1}\delta^{\frac{1}{\eta}}\gamma^{\frac{1}{\eta}-1}(1-s_e)^{\frac{1}{\eta}}(1-s_q)^{1-\frac{1}{\eta}}, \tag{E}$$

where $\Gamma\left(\bar{f}\right)$ is the expected difference between the exit level and the actual level of fixed cost:

$$\Gamma\left(\bar{f}\right) = \int_{f\leq\bar{f}}\left(\bar{f}-f\right)\mathrm{d}G(f) = \left(\bar{f}-\psi\left(\bar{f}\right)\right)G\left(\bar{f}\right).$$

Here, it is easily checked that $\Gamma'(f) = G(f) > 0$, $\Gamma(0) = 0$, and $\Gamma(f) \to \infty$ as $f \to \infty$ by Assumption. Therefore, it should be noted that (G) uniquely determines the exit level of the fixed cost on the balanced growth, $\bar{f}$. Equation (G) and (E) characterize the balanced growth path of the economy and Figure 2 illustrates the determination of the exit level on the $\bar{f}$-$\bar{\pi}$ plain.

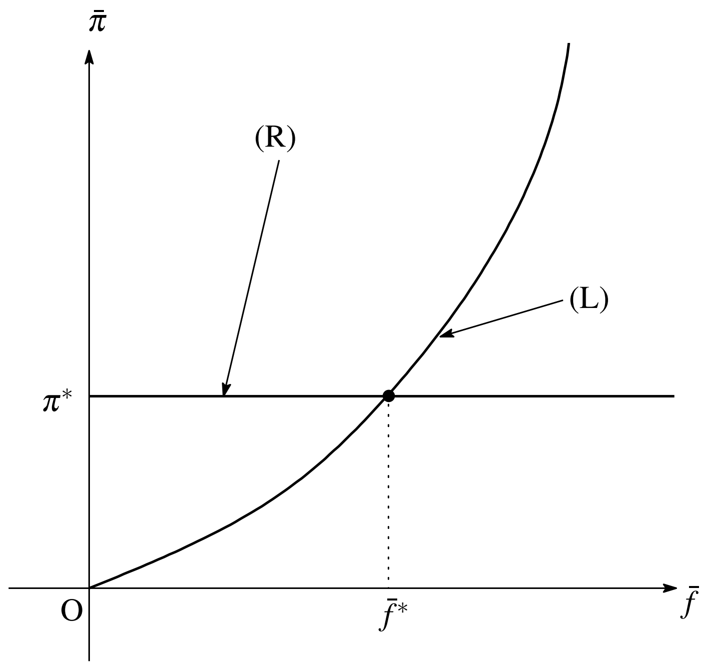
4 Effects of Subsidies on Balanced Growth

In this section, we give the properties of the balanced growth of the economy.

Proposition 1. *The growth rate on the balanced growth, given by (G), is increasing in the rate of subsidy to R&D, s_q. On the other hand, an increase of s_q raises the exit level of the fixed cost on the balanced growth, given by (E), if the degree of the intertemporal elasticity of substitution, η, is less than 1, and vice versa.*

$$\frac{\mathrm{d}g}{\mathrm{d}s_q} > 0 \quad \textit{and} \quad \frac{\mathrm{d}\bar{f}}{\mathrm{d}s_q} \lesseqgtr 0 \qquad \textit{if} \quad \eta \gtreqless 1.$$

Proof. It is obvious from (G) and (E). □



(L): $\Gamma\left(\bar{f}\right)$
(R): $\beta^{-1}\delta^{\frac{1}{\eta}}\gamma^{\frac{1}{\eta}-1}(1-s_e)^{\frac{1}{\eta}}(1-s_q)^{1-\frac{1}{\eta}}$

Figure 2: Determination of the balanced growth path

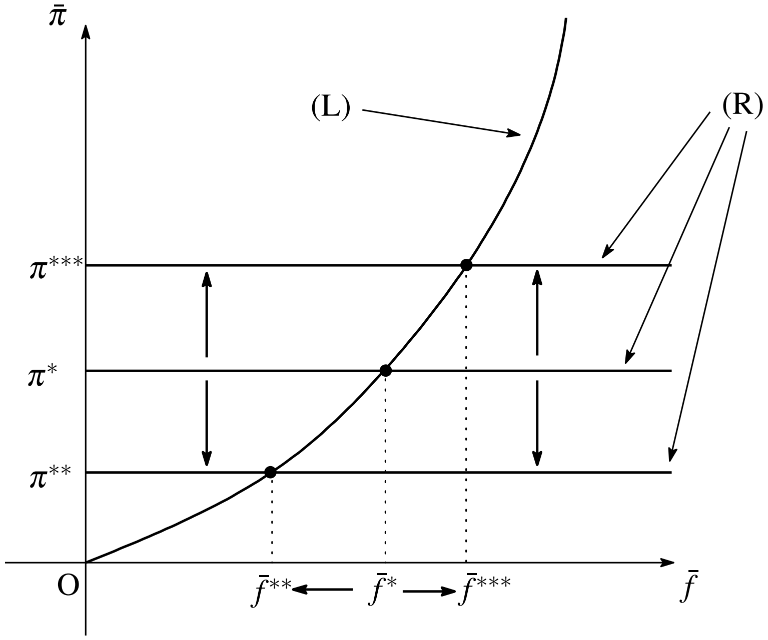


Figure 3: The effects of an increase of s_q on $\bar{f}$

Figure 3 shows how the effect of subsidy to R&D depends on the degree of the intertemporal elasticity of substitution. If $\eta > 1$, an increase of the subsidy to R&D raises the value of the right-hand side of (E), which makes line (R) in the figure move down. This results in the decrease of the exit level from $\bar{f}^*$ to $\bar{f}^{**}$, yielding the increase of the exit rate. However, if $\eta < 1$, an increase of the subsidy to R&D makes line (R) move up and raises the exit level from $\bar{f}^*$ to $\bar{f}^{***}$. Hence, in this case, the exit rate is decreasing in the subsidy rate to R&D. Finally, if $\eta = 1$, which implies the log utility function, there is no effect of a subsidy to R&D on the exit rate. In either case, we find that the effect of a subsidy to R&D on the growth rate is positive from (G).

Next, we give the proposition for the effect of subsidy to entry.

Proposition 2. *On the balanced growth, the growth rate, given by (G), and the exit level of the fixed cost, given by (E), are decreasing in the rate of subsidy to entry, s_e.*

$$\frac{\mathrm{d}g}{\mathrm{d}s_e} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\bar{f}}{\mathrm{d}s_e} < 0.$$

Proof. It is also obvious from (G) and (E). □

The effect of an increase in subsidy to entry is illustrated in Figure 4. Since

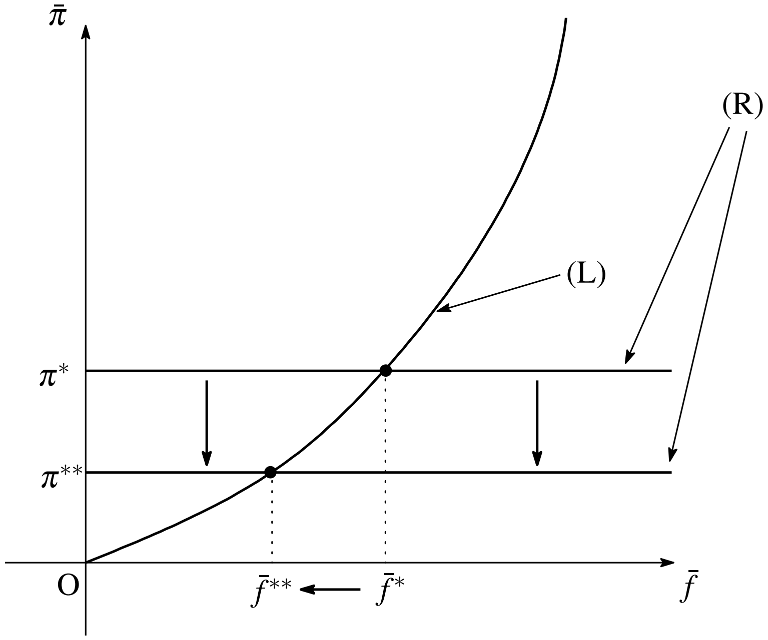


Figure 4: The effect of an increase of s_e on $\bar{f}$

the value of the right-hand side of (E) declines as s_e rises, line (R) moves down and the exit level falls from $\bar{f}^*$ to $\bar{f}^{**}$. Therefore, subsidy to entry raises the exit rate.

5 An Extension - Introducing Unemployment

The relationship between growth and unemployment has mainly been considered in two opposite directions. One is a direction along the line of search theory á la Pissarides [2000], where the growth and unemployment rates are negatively correlated, and the other is a direction along the line of Schumpeterian growth theory á la Aghion and Howitt [1994], where they are positively correlated. In this section, we introduce unemployment into our model and as in Aghion and Howitt [1994], but more simply, bridge the gap between these two opposite directions.

So far, we implicitly assume that workers engaged in exiting firms can instantaneously find their new employers through labor market. In contrast, to introduce unemployment, this section assumes that workers engaged in exiting firms can find their new employer only a period after they lose their jobs. In this case, market-clearing condition for labor market (10) is replaced to the following equa-

tion:
$$\int_{S_t} x_{it} \mathrm{d}i = \int_{S_t} L_{it} \mathrm{d}i,$$
where L_{it} denotes the labor engaged in surviving firm i. Therefore, this modification implies that the unemployment rate in this case is given by $1 - G(\bar{f})$. On the other hand, it does not change the steady-state values of $\bar{f}$ and g, which are given by (G) and (E). Therefore, our results given in Proposition 1 in the previous section implies that R&D subsidy raises the growth rate, but its effect on the unemployment rate, as well as the exit rate, depends on the degree of the intertemporal elasticity of substitution.

Corollary. *An increase of the rate of subsidy to R&D reduces the unemployment rate on the balanced growth if the degree of the intertemporal elasticity of substitution is less than one, and vice versa.*

6 Conclusion

In this paper, we presented a model where entry, exit, and growth, are endogenously determined through the rational behavior of agents. Contrary to the standard Schumpeterian endogenous growth theory where growth and exit of firms are inevitably correlated, we emphasized that the relationship between growth and exit should be more sensitive. Especially, we analyzed the effects of subsidy policies on growth and exit and had the following results: a subsidy to entry raises the exit rate but reduces the growth rate. However, a subsidy to R&D promotes economic growth but the effect on the exit rate depends on the magnitude of the intertemporal elasticity of substitution: if it is greater than one, subsidy to R&D raises the exit rate, and vice versa. Since our extension introducing unemployment gives the unemployment rate as the same rate of exit, the results bridge the gap between the two opposite approaches to investigation for the relationship between growth and unemployment.

Appendix

A The Value of an Entrepreneurial Firm

Let us denote the value of firm i in period t by V_{it}, which is given by

$$V_{it} = \Pi_{it} - P_{Ait} + \frac{1}{1+r_t} \mathrm{E}_t \left[V_{it+1} \right]. \tag{19}$$

Since the free-entry condition can be written as

$$P_{Ait} = \frac{1}{1+r_t}\mathrm{E}_t\left[V_{it+1}\right],$$

equation (19) implies that $V_{it} = \Pi_{it}$.

B Exit level of the Fixed Cost

Here we derive the condition for the exit level of fixed cost under balanced growth. Since a firm cease operations when its value, equal to profit in this model, becomes negative, it follows from Appendix A that the exit condition is given as $\pi_{it} \leq 0$. Hence, by using (16), the condition for the exit level of fixed cost under balanced growth can be written as

$$\alpha(1-\alpha)\hat{y} - \bar{f}G\left(\bar{f}\right) = 0.$$

Using (17) and the definition of $\Gamma(\bar{f})$, we can rewrite the above equation as

$$\Gamma\left(\bar{f}\right) = \bar{\pi}.$$

Therefore, eliminating $\bar{\pi}$ from the above equation by using (7), we have

$$\Gamma\left(\bar{f}\right) = \frac{\left(1-s_q\right)(1+r)}{\gamma}.$$

Therefore, combining the above equation and (18), together with (13),

$$\Gamma\left(\bar{f}\right) = \frac{\left(1-s_q\right)}{\gamma}\underbrace{\frac{1}{\beta}\left[\gamma\frac{\delta(1-s_e)}{1-s_q}\right]^{\frac{1}{\eta}}}_{\text{Euler eq.}}.$$

Rearranging the above equation, we have (E). It should be noted that γ and s_q have two opposite effects on the exit level of fixed cost on the balanced growth path: one is direct effect from shutdown condition and the other is indirect effect through interest rate determied by the Euler equation.

References

P. Aghion and P. Howitt. A Model of Growth through Creative Destruction. *Econometrica*, 60(2):323–351, 1992.

P. Aghion and P. Howitt. Growth and Unemployment. *The Review of Economic Studies*, 61(3):477–494, 1994.

M. Asplund and V. Nocke. Firm Turnover in Imperfectly Competitive Markets. *Review of Economic Studies*, 73(2):295–327, 2006.

W. Brock. On Models of Expectations that Arise from Maximizing Behavior of Economic Agents over Time. *Journal of Economic Theory*, 5(3):348–376, 1972.

E. Dinopoulos and P. Thompson. Schumpeterian Growth without Scale Effects. *Journal of Economic Growth*, 3(4):313–335, 1998.

G. M. Grossman and E. Helpman. Quality Ladders in the Theory of Growth. *Review of Economic Studies*, 58(1):43–61, 1991.

H. A. Hopenhayn. Entry, Exit, and Firm Dynamics in Long Run Equilibrium. *Econometrica*, 60(5):1127–1150, 1992.

P. Howitt. Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy*, 107(4):715, 1999.

C. I. Jones. R&D-Based Models of Economic Growth. *Journal of Political Economy*, 103(4):759, 1995.

B. Jovanovic. Selection and the Evolution of Industry. *Econometrica*, 50(3): 649–670, 1982.

M. J. Melitz. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71(6):1695–1725, 2003.

P. F. Peretto. Technological Change and Population Growth. *Journal of Economic Growth*, 3(4):283–311, 1998.

C. A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, 2000.

P. S. Segerstrom. Endogenous Growth Without Scale Effects. *American Economic Review*, 88:1290–1310, 1998.

P. S. Segerstrom. The Long-Run Growth Effects of R&D Subsidies. *Journal of Economic Growth*, 5(3):277–305, 2000.

V. L. Smith. Optimal Costly Firm Entry in General Equilibrium. *Journal of Economic Theory*, 9(4):397–417, 1974.

A. Young. Growth without Scale Effects. *Journal of Political Economy*, 106(1): 41–63, 1998.