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1 Introduction

The trade and geography literature, with seminal papers by Krugman (1991), Krugman-Venables (1995) and Venables (1996), highlights how trade integration may lead to concentration or agglomeration of firms to larger countries or regions (for surveys see Fujita et al. 1999, Baldwin et al. 2003, and Combes et al. 2008). An important development of the trade and geography literature is the introduction of heterogeneous firms. The tendency to agglomeration typically differs among firms, and Baldwin and Okubo (2006) show that the most productive firms are the first to agglomerate to the core in the 'footloose capital' (FC) model by Martin and Rogers (1995). This is the simplest version of the trade and location models, which assumes that physical capital is the geographically mobile factor, and that it moves in response to differences in nominal return. The price index does therefore not enter the migration decision, which simplifies the calculations. The model does not generate agglomeration when starting from a symmetric equilibrium.

Our paper introduces spatial sorting of heterogeneous firms in the 'footloose entrepreneur' (FE) model by Forslid (1999) and Forslid and Ottaviano (2002). We here also apply quasilinear...
utility à la Pfluger (2004) and Borck and Pfluger (2006). Our model generates agglomeration from a uniform space contrary to the FC model. We also show that our model, contrary to the FC model with heterogeneous firms, generates spatial sorting in reverse productivity order with the least productive entrepreneur being the first to relocate. The reason for this is that the advantage of the lower price index in the larger region is the same for all entrepreneurs (because of the quasilinear utility), while the local competition effect is more important for large firms (more productive entrepreneurs). The agglomeration of low skilled entrepreneurs to large cities may be consistent with the location pattern in several locations in the third world (see e.g. Gollin et al. 2016).

Finally, the introduction of heterogeneous firms (entrepreneurs) implies a less drastic agglomeration pattern compared to the standard model with homogenous firms. This is the case because the gains of agglomerating to the core region are different for firms of different productivity, which implies that firms will find it optimal to relocate for different levels of trade costs. The somewhat muted agglomeration tendency agrees well with the data, since instances of abrupt or catastrophic agglomeration are rare in practice (Brakman et al. 2004, Davis and Weinstein 2002, 2008, and Redding et al. 2011).\footnote{Using WWII data from Germany, Bosker et al. (2007) do find instances of multiple equilibria.}

\section{The Model}

This paper uses the heterogenous firm version of the FE-model by Forslid (1999) and Forslid and Ottaviano (2002), where we use quasilinear utility as in Pfluger (2004) and Borck and Pfluger (2006).

\subsection{Basics}

There are two regions, Home and Foreign (denoted by *), and two factors, human capital $H$ and labour $L$. Human capital or entrepreneurs move between regions and bring with them their business. Workers can move freely between sectors but are immobile between regions. There are two sectors $M$ (manufacturing) and $A$ (services). The A-sector produces a freely traded homogeneous good with a constant-returns technology using only labour. The M-sector produces differentiated manufactures with increasing-returns technologies using both human capital and labour. Firm productivities in the M-sector are distributed according to a cumulative density function $G(a)$.

All individuals in a region have the utility function

\[ U = \mu \ln C_M + C_A, \quad (1) \]

where $\mu \in (0, 1)$ is a constant and $C_A$ is consumption of the homogenous good, and where the region subscript is suppressed. Differentiated goods enter the utility function through the index...
$C_M$, defined by

$$C_M = \left[ \int_0^N c_i^{(\sigma-1)/\sigma} \, di \right]^{\sigma/(\sigma-1)},$$

(2)

$N$ being the mass of varieties consumed, $c_i$ the amount of variety $i$ consumed, and $\sigma > 1$ the elasticity of substitution.

Each consumer spends $\mu$ on manufactures, and the total demand for a domestically produced variety $i$ is therefore

$$x_i = \frac{p_i^{-\sigma}}{P^{1-\sigma}} \cdot \mu,$$

(3)

where $p_i$ is the price of variety $i$, and $P$ is the price index.

The unit factor requirement of the homogeneous A-sector good is one unit of labour. This good is freely traded, and since it is also chosen as the numeraire, we have

$$p_A = w = 1,$$

(4)

$w$ being the wage of workers in all regions.

Each firm has a fixed cost in human capital. We normalise the fixed cost so that one entrepreneur is associated with one firm. Firms (entrepreneurs) are differentiated in terms of their marginal cost, and the firm-specific marginal production costs $a_i$ are distributed according to the cumulative distribution function $G(a)$. The total cost of producing $x_i$ units of manufactured commodity $i$ in a region is

$$TC_i = \pi + a_ix_i,$$

(5)

where $\pi$ is the return to human capital (i.e. to an entrepreneur).

Distance is represented by trading costs. Shipping the manufactured good involves a frictional trade cost of the “iceberg” type: for one unit of good from region $j$ to arrive in region $k$, $\tau_{jk} > 1$ units must be shipped. Trade costs are also assumed to be equal in both directions so that $\tau_{jk} = \tau_{kj}$.

Profit maximisation by manufacturing firms leads to the price

$$p_i = \frac{\sigma}{\sigma - 1} a_i.$$

(6)

Firm heterogeneity in labour requirements, $a_i$, is probabilistically allocated among firms (entrepreneurs). In order to analytically solve the model, we assume the following cumulative density function of $a$:

$$G(a) = \frac{a^\rho - a_0^\rho}{a_0^\rho - a^\rho},$$

(7)

where $\rho$ is a shape parameter and $a_0^\rho$ is a scaling factor. We assume the distribution to be truncated at $a$, where $0 < a < a_0$, so that the productivity of firms is bounded, and we normalise so that $a_0 = 1$.

\[2\text{This is essentially a Pareto distribution that has been truncated (see Forslid and Okubo 2015).}\]
In the short run, the allocation of $H$ is taken to be fixed. The model is closed by the M-sector market-clearing condition, where the left-hand side is the nominal return to human capital, which equals a firm’s operating profit, and the right-hand side follows from the demand functions in (3). The nominal return to entrepreneur $i$ in Home is:

$$\pi_i = \frac{\mu}{\sigma} \left( \frac{(L + H)}{\Delta} + \frac{\phi(L^* + H^*)}{\Delta^*} \right) a_i^{1-\sigma},$$

where

$$\Delta \equiv H \int_{a}^{1} a_1^{1-\sigma} dG(a) + \phi H^* \int_{a}^{1} a_1^{1-\sigma} dG(a).$$

The object $\phi_{jl} = \tau_{jl}^{1-\sigma}$, ranging between 0 and 1, stands for "freeness" of trade between $j$ and $l$ (0 is autarchy and 1 is zero trade costs). These equilibrium conditions hold under the condition that the A-sector, which pins down the wage, is active in all regions.

### 2.2 Stability analysis

In the long run, entrepreneurs respond to the incentives provided by the difference in real return that can be attained in the two regions.
\[ V - V^* = (\pi_j - \mu \ln P) - (\pi_j^* - \mu \ln P^*) \]
\[ = \frac{\mu}{\sigma} (1 - \phi) (B - B^*) a^{1-\sigma} - \frac{\mu}{1 - \sigma} (\ln \Delta - \ln \Delta^*), \]  
where \( B \equiv \frac{L + H}{\Delta} \), and \( B^* \equiv \frac{L^* + H^*}{\Delta^*} \).

### 2.2.1 The Break Point

As is customary, a symmetric allocation of resources is always an equilibrium since everything is symmetric in the model, and we will use this equilibrium as the starting point. We will investigate whether this equilibrium is stable when perturbed a little. The experiment is to move one entrepreneur over to the other region, and then to see if the utility of the entrepreneur rises or falls as a consequence of this move. If it rises, the symmetric is unstable, otherwise it is stable. However, since entrepreneurs are heterogeneous, it matters which entrepreneur that moves. More precisely, the effect on demand (the demand-link) is identical because of the quasilinear preferences, whereas the effect on the price index depends on the productivity of the moving entrepreneur. We will assume that the entrepreneur with the highest gains from moving will move first.\(^3\)

The welfare effect for a marginal entrepreneur \( i \) that moves is given by
\[
\frac{d(V - V^*)}{dH_i} = \frac{\mu}{\sigma} (1 - \phi) \left( \frac{dB}{dH_i} - \frac{dB^*}{dH_i} \right) a_i^{1-\sigma} + \frac{\mu}{\sigma - 1} \left( \frac{d\Delta}{\Delta dH_i} - \frac{d\Delta^*}{\Delta^* dH_i} \right). \tag{11}
\]
This expression illustrates the agglomeration forces in action when one (infinitesimal) entrepreneur moves: The first term shows the expenditure shifting as well as the competition effect. The second term shows the price index effect or the supply link, which is positive since a larger region has a lower price index. Using that \( \frac{dB}{dH_i} = \frac{1}{\Delta} - \frac{L + H}{\Delta^2} \frac{d\Delta}{dH_i}, \frac{dB^*}{dH_i} = \frac{1}{\Delta^*} \frac{dH^*}{dH_i} - \frac{L + H^*}{\Delta^2} \frac{d\Delta^*}{\Delta^* dH_i} \) and evaluating the expression at the symmetric equilibrium, where \( \frac{d\Delta^*}{dH_i} = - \frac{d\Delta}{dH_i} \), gives
\[
\left. \frac{d(V - V^*)}{dH_i} \right|_{H=H^*} = 2\frac{\mu}{\sigma} (1 - \phi) \frac{1}{\Delta} a_i^{1-\sigma} - 2\frac{\mu}{\sigma} (1 - \phi) \frac{L + H}{\Delta^2} \frac{d\Delta}{dH_i} a_i^{1-\sigma} + 2 \frac{\mu}{\sigma - 1} \frac{d\Delta}{\Delta dH_i}. \tag{12}
\]

The effect on the price index of the movement of one infinitesimal entrepreneur with productivity \( a_i \) is given by
\[
\frac{d\Delta}{dH_i} = \frac{(1 - \phi) \lim_{\delta \to 0} \frac{1}{\delta} H \int_{a_i}^{a_i+\delta} a_k^{1-\sigma} dG(a)}{g(a)H}. \tag{13}
\]
Using l’Hopital’s rule gives
\[
\frac{d\Delta}{dH_i} = \frac{\rho}{1 - \rho a - \rho a^{-1}} (1 - \phi) a_i^{1-\sigma} = (1 - \phi) a_i^{1-\sigma}, \tag{14}
\]
\(^3\)We could formalize this by having firms (or entrepreneurs) bidding for transportation from a transport sector with limited capacity. The highest bidder will be the firm with the highest gain from moving.
and using this in (12) and noting that 

\[ \Delta = \frac{\rho}{1-\sigma+\rho} \frac{1-a^{1-\sigma+\rho}}{1-a^\sigma} (H + \phi H^*) = \xi (1 + \phi) H, \]

where

\[ \xi = \frac{\rho}{1-\sigma+\rho} \frac{1-a^{1-\sigma+\rho}}{1-a^\sigma} \]

at symmetry gives

\[ \frac{d(V - V^*)}{dH_i} \bigg|_{H=H^*} = 2^{\frac{a^\sigma}{\sigma}} \frac{L + H}{1 + \phi} \xi (1 - \frac{L + H}{1 + \phi} H \xi (1 - \phi) a_i^{1-\sigma} + \frac{\sigma}{\sigma - 1} a_i^{1-\sigma}). \]

As usual we assume that \( 1 - \sigma + \rho > 0 \), to ensure that the integrals in \( \Delta \) converge.

The breakpoint, the level of trade freeness at which the symmetric equilibrium becomes unstable, is found by solving \( \frac{d(V - V^*)}{dH_i} \bigg|_{H=H^*} = 0 \) for \( \phi^B \):

\[ \phi^B = \frac{L + H}{\xi H} a_i^{1-\sigma} \left( 1 - \frac{L + H}{1 + \phi} H \xi (1 - \phi) a_i^{1-\sigma} + \frac{\sigma}{\sigma - 1} a_i^{1-\sigma} \right). \]

It is seen from (16) that \( \phi^B < 1 \), and the existence of \( \phi^B > 0 \) for \( a \in [a, 1] \), guaranteed by the condition that \( \frac{H}{L} < \frac{\sigma - 1}{\sigma} \), which is the "no-black-hole" condition in this model, prevent full agglomeration from always being the equilibrium outcome.

Furthermore, from (16) \( \phi^B \) decreases in \( a \), which means that the first firm to deviate from the symmetric equilibrium, given that this equilibrium exists, will be the least productive (the firm that has the highest \( a \)). Interestingly, this sorting pattern is the opposite to that of the footloose capital (FC) model where the most productive firms are the first to move to the core (see Baldwin and Okubo 2006). Here instead the least productive entrepreneur moves first because the advantage of the lower price index in the larger region is the same for all entrepreneurs, while the local competition effect is more important for large firms (more productive entrepreneurs).

Another important difference between the models is that our model generates agglomeration starting from symmetry. In the FC model, there is no agglomeration force when markets are symmetric. Agglomeration and spatial sorting only occur when markets are different enough in size, and a larger market favours the relocation of high productive firms with high sales volumes.

The trade cost at which the symmetric equilibrium ceases to be stable, the break-point, is found by setting \( a_i = 1 \) in (16):

\[ \phi^B = \frac{L + H}{\xi H} a_i^{1-\sigma} - \frac{\sigma - 1}{\sigma} a_i^{1-\sigma} = 1. \]

The importance of firm heterogeneity can be seen by varying \( a \). Since \( \frac{d\phi^B}{da} < 0 \) we have from (17) that \( \frac{d\phi^B}{da} > 0 \). That is, more heterogeneity (a lower \( a \)) decreases the breakpoint. Thus, heterogeneity leads to an earlier agglomeration process when trade costs fall. However, as we shall see below, it also delays full agglomeration. The agglomerations process is thus more drawn out. For \( a = 1 \) the productivity distribution collapses to one point, and we have returned to the case of homogeneous firms with the standard breakpoint.

The sorting pattern is illustrated in Figure 2, where firms with a marginal cost above \( a_R \) sort to the larger home market.

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4 Note that our breakpoint corresponds to that of Pfuger (2004) for \( a_i = 1 \), since \( \lim_{a \to 1} \xi = 1 \).
2.2.2 The Sustain Point

Next we derive the sustain point, $\phi^S$, where full agglomeration in Home just becomes unstable. The condition for this is that $V - V^* = 0$ at full agglomeration in Home:

$$V - V^* = \frac{\mu}{\sigma}(1 - \phi^S)^2 \left( \frac{L + H + H^*}{\Delta} - \frac{L^*}{\phi^S \Delta} \right) \frac{1}{H + H^*} a^{1-\sigma} - \frac{\mu}{1-\sigma} (\ln \Delta - \ln \phi^S \Delta)$$

$$= \frac{\mu}{\sigma}(1 - \phi^S)^2 \left( L + H + H^* - \frac{L^*}{\phi^S} \right) \frac{1}{H + H^*} a^{1-\sigma} - \frac{\mu}{1-\sigma} \ln \phi^S = 0.$$  \hspace{1cm} (18)

Note that the last term $-\frac{\mu}{\sigma-1} \ln \phi^S$ is always positive for any $\phi^S \in (0, 1)$. Thus, the first term must be negative at the sustain point, which means that $(L + H + H^* - \frac{L^*}{\phi^S}) < 0$. The most productive firm ($a$), with the highest $a_i^{1-\sigma}$, will have the most negative first term. Therefore, it will be the first to move away from the agglomeration if trade freeness falls. Thus, the last to move into the core agglomeration are also the first to leave if the movement in trade costs is reversed. The sustain point is therefore determined by the relation:

$$\frac{\mu}{\sigma}(1 - \phi^S)^2 \left( L + H + H^* - \frac{L^*}{\phi^S} \right) \frac{1}{H + H^*} \frac{a^{1-\sigma}}{\xi} - \frac{\mu}{1-\sigma} \ln \phi^S = 0.$$  \hspace{1cm} (19)

The effect of firm heterogeneity is determined by the term \( \frac{a^{1-\sigma}}{\xi} = \frac{(1-\sigma+\rho)(1-\sigma^\rho)}{\rho(a^{1-\sigma^\rho})}. \) As seen from this expression, \( \frac{d}{da} \frac{a^{1-\sigma}}{\xi} < 0 \) for $0 < a < 1$. This means that $\phi^S$ increases in firm heterogeneity.
for $0 < a < 1$. The first term in (19) vanishes when $a = 0$, in which case $\phi^S = 1$. That is, full agglomeration cannot occur before free trade when we allow for infinitely productive firms. The relocation process is thus more drawn out when firms are heterogeneous. Firm heterogeneity, in this sense, delays the agglomeration process.\footnote{The sustain point is another difference as compared to the FC-model in Baldwin and Okubo (2006), which has the same sustain point as in the standard core-periphery model by Krugman (1991).}

### 2.3 Long-run equilibrium

Having investigated the properties of the model at the break- and sustain points, we now turn to the migration pattern as the model reaches its long equilibrium. Generally, the value of migrating depends on the productivity of the migrating entrepreneur and the entrepreneurs that have already migrated. The problem is manageable because the entrepreneurs here move in order of increasing productivity. The value of migrating for an entrepreneur with the marginal cost $a_R$ is

$$v(a_R) = (\pi(a_R) - \mu \ln P(a_R)) - (\pi^*(a_R) - \mu \ln P^*(a_R)) = \frac{\mu}{\sigma} (1 - \phi) (B(a_R) - B^*(a_R)) a_R^{1-\sigma} - \frac{\mu}{1-\sigma} (\ln \Delta(a_R) - \ln \Delta^*(a_R)),$$

where $a_R$ is the marginal cost of the entrepreneur that is next in line for migrating. $B(a_R)$ and $B^*(a_R)$ are given by:

$$B = \frac{L + H + \int_a^{a_R} H^* dF(a)}{\Delta(a_R)}, \quad B^* = \frac{L^* + H^* - \int_a^{a_R} H^* dF(a)}{\Delta^*(a_R)},$$

and

$$\Delta(a_R) = H^* \int_a^{a_R} a^{1-\sigma} dF(a) + \phi H^* \int_a^{a_R} a^{1-\sigma} dF(a) + \phi H^* \int_a^{a_R} a^{1-\sigma} dF(a),$$

$$\Delta^*(a_R) = H^* \int_a^{a_R} a^{1-\sigma} dF(a) + \phi H^* \int_a^{a_R} a^{1-\sigma} dF(a) + \phi H^* \int_a^{a_R} a^{1-\sigma} dF(a).$$

The long-run equilibrium is defined by $v(a_R) = 0$. This equation cannot be solved analytically and we therefore proceed by simulation. Figure 3 shows a simulation of $a_R$ for $\sigma = 5, \rho = 7, a = 0.1, \mu = 0.2, L = 200$, and $H = H^* = 25$, the breakpoint for these parameter values $\phi^B = 0.26$ and the sustain point $\phi^S = 0.8$. The corresponding bifurcation diagram is shown in Figure 4. It is seen how the introduction of heterogeneous firms leads to a supercritical pitchfork bifurcation contrary to the subcritical (tomahawk) bifurcation in the standard
FE-model. Here, there is no jump or catastrophic relocation at the breakpoint (the bifurcation point). The reason for this is simply that the heterogeneous firms here have heterogeneous gains from migrating, and this implies that they will find it optimal to relocate for different levels of trade costs. This relocation pattern may correspond well to real world data, since instances of abrupt or catastrophic agglomeration are rare in practice (see e.g. Brakman et al. 2004 and Davis and Weinstein 2002, 2008, and Redding et al. 2011).

We now turn to the welfare consequences of this migration pattern.

3 Welfare

Indirect utility of the immobile factor labour is given by

$$V = w - \mu + \mu \ln \mu + \frac{\mu}{\sigma - 1} \log \Delta. \quad (23)$$

Figure 5 (\(\sigma = 5, \rho = 7, \mu = 0.2, L = 200, \) and \(H = H^* = 25\)) shows how an index of labour welfare, as defined by (23), is developing in the two regions when trade is liberalized. The figure shows graphs for three different values of \(\alpha\). When \(\alpha\) approaches unity, there is less and less firm heterogeneity as the productivity distribution is compressed. It is seen in the figure how this leads to a later but more drastic agglomeration. Thus, the introduction of heterogeneous
Figure 4: Bifurcation diagram
firms gives a more gradual and less abrupt localization pattern. It also means that the range of trade costs for which welfare is different in the two regions is larger. A higher $a$ also means a lower average productivity. This explains why the welfare index curves in Figure 5 are lower when $a$ is higher.

Welfare for the immobile factor in the periphery falls at the breakpoint but thereafter climbs as trade liberalization proceeds. A sufficiently deep integration always benefits all factors of production.

4 Conclusion

This paper introduces agglomeration in the footloose entrepreneur version of the core-periphery model, when there are heterogenous firms. This model generates agglomeration when trade is liberalized, starting from a fully symmetric equilibrium contrary to the footloose capital model. The model also has an opposite spatial sorting pattern to the footloose capital model. Here, small and low productive firms are the first to agglomerate to the core. This location pattern may be consistent with large cities in the third world.

The introduction of heterogeneous firms also implies that agglomeration starts earlier, but that it is less drastic. This may be an attractive feature of the model, since the catastrophic agglomeration predicted by the standard model is rarely observed in real world data.
References


