Household Portfolios in a Secular Stagnation World: Evidence from Japan*

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Abstract

We document low stock market participation rates and high proportions of money in Japanese household portfolios. To replicate these facts, we introduce a money demand motive in a life-cycle portfolio choice model and calibrate the model's structural parameters to match Japanese household financial data. Using counterfactual analysis we find that low expected stock returns, low expected inflation and high fixed costs of stock market participation are the main determinants of Japanese household portfolios.

JEL Classification: E41, G11.

1 Introduction

Japan experienced a stock market boom (bust) in the 1980s (1990s) followed by a series of financial crises. The Bank of Japan’s response to the collapse of the bubble and subsequent deflation is an early version of the kinds of quantitative easing programmes now pursued by other major central banks. Despite this policy stimulus, Japan has gone through two decades of low growth, low interest rates and low inflation.

In this paper we set out to understand the way these macroeconomic events affected Japanese household financial decisions.\(^1\) This is interesting because the Japanese experience can provide some indication of how the developed world may be affected by the onset of such a 'secular stagnation' episode characterised by persistently weak rates of inflation and economic growth.

Using data from the Japanese Survey of Household Finance (SHF) from 1981 to 2014, we start by documenting several key household portfolio facts that are unique to Japan. First, stock market participation is considerably lower than in the US: in 2014, 15.5% of all households participate in the stock market. Second, conditional on participation, stockholders hold a relatively small share of wealth in stocks as a percentage of total financial wealth, and a relatively large share of financial wealth in bonds and money. Third, whether one focusses on stockholders or non-stockholders, the share of wealth allocated to cash-like financial instruments is very high. For instance, even for stockholders the share of liquid bank accounts in total financial assets is between 20% and 40% (depending on the age group). Fourth, the gap between the average wealth of stockholders relative to that of non-stockholders is much smaller in Japan than in the US.

What can account for these facts? To answer this question we rely on

\(^1\)For a review of Japanese household finance, see Iwaisako (2009) and references therein.
counterfactual analysis based on a structural, quantitative, life-cycle portfolio choice model with an explicit role for inflation and money demand. Understanding money demand is essential to match Japanese household portfolios given the strong prevalence of money-like financial instruments in Japanese portfolios. Portfolio choice models that incorporate monetary assets are not readily available, however. In the recent large literature on portfolio choice, households are assumed to choose between different real assets (typically bonds, stocks and/or housing), ignoring the fact that all transactions in the data are actually done in nominal terms. These models therefore cannot study the effects of deflation or inflation on household money demand, portfolio choice and stock market participation decisions. On the other hand, canonical models in the monetary economics literature that allow for nominal assets follow the Baumol-Tobin analysis and typically focus on the distinction between money and bonds as a proxy for all other assets in the household portfolio (see, for example, Mulligan and Sala-i-Martin (2000), Alvarez and Lippi (2009) and Lippi and Secchi (2009)). Instead, we make explicit the choice between money (that earns a zero nominal return) and other assets like bonds and stocks that earn the historically observed rates of return.

Given that our purpose is to develop a tractable, quantitative, model that can be confronted with the data, we introduce money demand through the shopping time approach. Specifically, we assume that money provides liquidity services: a higher amount of money lowers the cost from having to undertake a given transaction for consumption purposes. Everything else we assume is similar to recent life-cycle models that feature intermediate con-

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2See Campbell (2006) for a recent excellent survey.

3Introducing money in a model can vary in complexity from the decentralized search Kiyotaki and Wright (1989) setup, to other money demand models, such as cash-in-advance (Lucas and Stokey (1987)), money-in-the-utility function (Sidrauski (1967)) and shopping-time approaches (McCallum and Goodfriend (1987)). For recent applications of shopping time models, see, for example, Mulligan and Sala-i-Martin (2000).
sumption and stochastic uninsurable labor income in the tradition of Deaton (1991) and Carroll (1997), and as extended in the life cycle portfolio choice literature by Cocco, Gomes, and Maenhout (2005), for instance. Our setting nicely nests life-cycle portfolio models where bonds and stocks are real assets and money does not circulate in the economy. We also introduce a fixed cost to participate in the stock market to generate low stock market participation for one group of households.

We calibrate the structural parameters of the model by matching key data moments from the Japanese Survey of Household Finances. Using fixed costs of stock market entry and preference heterogeneity, the calibrated model matches quantitatively limited stock market participation, and the share of wealth in money, bonds and stocks over the later parts of the life cycle. Understanding the portfolio choices associated with that part of the life cycle becomes extremely important in counterfactual analysis as most wealth accumulation takes place at that stage of the life cycle. Armed with this model we can now run counterfactual experiments to better understand the key drivers of Japanese household portfolios. Our counterfactual analysis can informatively address our key questions: why do Japanese households hold so few stocks and such high money balances?

Perhaps the single most widely discussed aspect of Japan’s economic per-

\[4\] Attanasio, Banks, Meghir, and Weber (1999), Gourinchas and Parker (2002) and Cagetti (2003) extend this tradition and estimate the structural parameters of life cycle models with a single real asset (a riskless bond).

\[5\] Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Polkovnichenko (2007) and Wachter and Yogo (2010) study real assets (real bonds and real stocks) and therefore the effects of inflation on consumption, wealth accumulation, money demand and portfolio choices cannot be analyzed. A recent exception is Campanale, Fugazza, and Gomes (2015) that focusses on asset illiquidity rather than money.

\[6\] This follows a large recent literature on household portfolio choices to generate limited stock market participation, see for example, Vissing-Jorgensen (2002), Haliassos and Michaelides (2003), Gomes and Michaelides (2005), Alan (2006), Paiella (2007), Attanasio and Paiella (2010), Bonaparte, Cooper, and Zhu (2012), Alvarez, Guiso, and Lippi (forthcoming) and Cooper and Zhu (2013).

\[7\] Haliassos and Bertaut (1995) originally asked this question for American households.
formance since 1990 has been its persistently low level of inflation which has averaged close to zero for almost 15 years. It has been argued that a low level of inflation encourages investors to hold nominal assets (such as money) rather than real assets (such as equities). Our counterfactual experiments confirm this intuition. Had inflation in Japan averaged 2% (as in the US) stock market participation would have risen from 15.3% in the baseline simulation to around 20%. Moreover, the share of stocks in young and middle aged stockholders’ portfolios would have been significantly higher mainly at the expense of lower money holdings, while the share of stocks in elderly households’ portfolios would remain unaffected. Therefore, low inflation plays an important role in keeping the share of money in Japanese household financial assets very high and contributes to crowding out stocks and bonds from household portfolios.

The second legacy of Japan’s long stagnation since 1990 has been its poor history of realized (and plausibly expected) stock returns as compared to other countries. In the baseline calibration, the mean equity premium for Japan is set to 1.8%. Increasing this to 4% (a typical choice in many life-cycle models calibrated to US data) raises the mean financial wealth to income ratio substantially and increases the stock market participation rate to 50%, a rate that is very close to the recent US experience.

Another important feature of the calibrated structural model is the relatively high cost of stock market entry. Reducing this fixed cost from our estimate (9%) to 5% (as estimated for the US in the literature) leads to an increase in the stock market participation rate from 15.3% to 43% indicating that frictions in equity market participation can be a very important factor in limiting Japanese households’ investment in equities. Our interpretation of the fixed cost centres on non-monetary factors. Such factors include low trust in the stock market (Guiso, Sapienza, and Zingales (2008)), Kinari and Tsutsui
The final interesting aspect of the Japanese household portfolio data is the puzzling low mean wealth of stockholders relative to non-stockholders at least in comparison with the US Survey of Consumer Finances (SCF). As before, low realized (and expected) stock returns play an important role in explaining this fact. Just as in the work of Piketty (2014), returns to capital (the stock market) have important long term implications for the wealth distribution. In the US, realized equity returns have been high, benefitting stock owners. In Japan, by comparison, realized stock returns have been low, and the wealth of those who own stocks relative to those who do not, has not risen to the same extent, generating lower wealth differences. This is especially interesting given the finite nature of the life cycle: one need not rely on an infinite horizon model to generate substantial differences in the wealth distribution. Moreover, these substantial wealth differences can arise even from a relatively low mean differential in expected stock returns (2%).

Overall, our results carry important lessons for other post financial crisis economies today. Persistently poor macroeconomic performance associated with low expected stock returns and low expected inflation can have a very far reaching impact on household behaviour. Persistently low expected inflation can discourage stock market participation and increase the share of money in the household portfolio, crowding out investments in the stock market. Low expected returns also discourage stock market participation and reduce wealth accumulation by those willing to hold stocks, while at the same time compressing the wealth distribution.

The rest of the paper is organized as follows. Section 2 discusses some stylized facts regarding money holdings over the life cycle. Section 3 presents
the model, and Section 4 reports the calibrated parameters. Section 5 presents
the benchmark numerical results and Section 6 conducts several comparative
statics. Section 7 concludes.

2 The Japanese Economy and Household Fi-

nance: 1981-2014

The Japanese economy experienced an asset price bubble in the late 1980s. Its
collapse was followed by a banking crisis of 1997 and a long period of stagnation
– the so-called lost two decades. Figure 1 shows the time path of stock prices
which have been very weak for the last 25 years. During this period, inflation
also remained extremely low, as shown in Figure 2.

The lost two decades were also characterised by very low short and long
term interest rates. For example, Figure 3 shows the interest rates of time
deposits, a typical financial assets many Japanese households own. To some
extent the low short term interest rates reflected the Bank of Japan’s (hereafter
BoJ) attempts to stimulate the economy and fight against deflation. But to
some extent, they also reflected the phenomenon of ‘secular stagnation’ - a
period of zero short term interest rates, low growth and low inflation brought
about by a combination of debt overhang and aging (and shrinking) working
age population. In recent years, Summers (2014) has raised the possibility
that the entire developed world may fall into a period of ‘secular stagnation’
which makes the Japanese experience instructive for other nations as well.

Our focus in this paper is on the way Japanese household financial port-
folios were affected by these economic developments. We use the Survey of
Household Finance (SHF) - an annual survey of household financial assets by
the Central Council for Financial Services Information. The survey asks 8000
households of two or more family members. The response rate is roughly 50% every year. Similar to the Survey of Consumer Finances of the United States, the SHF is a series of repeated cross sections. The SHF asks its respondents the amount of financial assets and liabilities. The survey also asks households’ financial portfolios — the outstanding amounts of currency, current deposits and time deposits, life insurance, non-life insurance, personal annuity insurance, bonds, stocks and investment trusts, workers’ asset formation savings, and other financial products. It also asks household characteristics such as age of household head and annual income. The Council allows us to use the data from 1981 to 2014.

We decompose financial assets into three categories — money, stocks and bonds. We define money as the sum of currency and current deposits. Households report the amount of stock they own. All other assets (mostly consisting of time deposits) are classified as “bonds”.

Figure 4 shows the time path of stock market participation rate during our sample. It has fluctuated around an average of around 15%, which is consistent with Fujiki, Hirakata, and Shioji (2012) who uses the same data set. The participation rate in 2014 is 15.5%. This is remarkably lower than the participation rate of the United States (around 50%). One of our objectives is to explain this low rate of stock market participation. Recent work explains this fact using a fixed cost to prevent households from participating in the stock

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8Investment trusts include both bond and equity, but the Survey does not decompose bond trusts and equity trusts. Since most of the investment trusts have been bond trusts, we just include investment trusts in the bond category.

9This number counts the percentage of the households who reply that they own stocks. Those number do not include the households who reply that they own investment trusts. The fraction of households who own investment trusts is around 10% in recent years. Investment trusts include both bond and equity, so the value 15% may underestimate stock market participation. The survey does not decompose bond trusts and equity trusts, but since much of the investment trusts are bond trusts, we believe that the participation rate is not overly underestimated. Another commonly used Japanese survey data, the National Survey of Family Income and Expenditure, shows a somewhat higher value (around 19%).
Life Cycle Financial Wealth Accumulation Relative to Mean Labor Income in Japan

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Wealth/Income)</th>
<th>Mean (Wealth/Income)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Stockholders</td>
<td>Stockholders</td>
</tr>
<tr>
<td>20-34</td>
<td>0.61</td>
<td>2.11</td>
</tr>
<tr>
<td>35-45</td>
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<td>1.12</td>
<td>2.47</td>
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<td>56-65</td>
<td>2.09</td>
<td>5.16</td>
</tr>
<tr>
<td>66-75</td>
<td>4.32</td>
<td>10.44</td>
</tr>
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</table>

Table 1: Mean financial wealth relative to labor income for the non-stockholders/stockholders in the 2014 SHF data.

One of the well-known stylized facts in the life cycle portfolio choice literature is that financial wealth is correlated with stock market participation (see Campbell (2006) for a recent survey). Japan is no exception to this finding. Table 1 reports mean financial wealth of households who own stocks (stockholders) and who do not (non-stockholders) in 2014. In the table mean financial wealth is reported for each age group and it is normalised by labour income. The table shows the stark dichotomy between households that hold stocks and households that do not. Consistent with the finding of Fujiki, Hirakata, and Shioji (2012), Table 1 shows that stockholders are richer.

For comparison, in Table 2 below we give the same set of numbers from the 2007 US Survey of Consumer Finances (SCF). Two differences immediately stand out from the table. Japanese stockholders aged below 55 are richer than stockholders in the US.

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10 This follows a large recent literature on household portfolio choices to generate limited stock market participation, see for example, Vissing-Jorgensen (2002), Haliassos and Michaelides (2003), Gomes and Michaelides (2005), Alan (2006), Paiella (2007), Attanasio and Paiella (2010) and Bonaparte, Cooper, and Zhu (2012).

11 We do not include home equity in our measures of household financial wealth: we want to think of the available financial assets and how they are allocated across money, stocks and bonds. In the structural model we will use a data-driven housing expenditures life-cycle function to exogenously subtract the expenditures that go in housing from labour income, as is done in Gomes and Michaelides (2005) and Love (2010).

12 The data shows that there is a jump after retirement as retirement income drops below mean working life labor income.
<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Wealth/Income)</th>
<th>Mean (Wealth/Income)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Stockholders</td>
<td>Stockholders</td>
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<tr>
<td>20-34</td>
<td>0.31</td>
<td>0.85</td>
</tr>
<tr>
<td>35-45</td>
<td>0.26</td>
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<tr>
<td>46-55</td>
<td>0.56</td>
<td>2.65</td>
</tr>
<tr>
<td>56-65</td>
<td>0.77</td>
<td>7.02</td>
</tr>
<tr>
<td>66-75</td>
<td>2.48</td>
<td>15.7</td>
</tr>
</tbody>
</table>

Table 2: Mean financial wealth relative to labor income for the non-stockholders/stockholders in the 2007 SCF data.

their US counterparts but the opposite holds for those aged above 55. More significantly, Japanese non-stockholders are considerably wealthier than US non-stockholders. Hence, the wealth difference between stockholders and non-stockholders are much smaller in Japan than in the US.

We next go deeper into the way household portfolio change over the life cycle. We compute the mean asset allocations across money, bonds and stocks for the households that hold all three assets (who we define as stockholders) and also for the households that hold only money and bonds (non-stockholders).

Figure 5 shows the time path of the mean portfolio shares of money ($\alpha_m$), bonds ($\alpha_b$) and stocks ($\alpha_s$) between 1981 and 2014. Bonds is the largest component of household wealth, and it consists mostly of time deposits. The share of bonds has been declining during the 2000s when inflation fell, while the share of money has increased.

Figure 6 shows the time path of the mean portfolio shares of the stockholders. The share of stocks increased during the asset price bubble period of the late 1980s, reaching 32.3% in 1989. After the collapse of the bubble, the share of stocks declined and has been around 20% in recent periods. As inflation declined, the share of money has risen from around 10% (the 1990s) to around 20% (the 2010s). Figure 7 shows that non-stockholders substituted bonds for money as inflation declined in the 2000s.
Life Cycle Portfolio Choice in Japan

Table 3 reports the mean portfolio shares for money ($\alpha_m$), bonds ($\alpha_b$) and stocks ($\alpha_s$) for the five age groups and across stockholders and non-stockholders in 2014. By definition, non-stockholders hold no equities and we can observe that their portfolios are heavily dominated by bonds but also with a heavy reliance on money accounts. The share of money declines over the life cycle, falling from 46.7% for the 20-34 age group to 31.7% for the over 55s.

The life-cycle profiles for stockholders display a tendency for the share of wealth in money balances to decrease over the working life while the shares of stocks and bonds increase. What is immediately apparent, however, is that money is a key component of household portfolios, despite the rate of return dominance of other assets, with all age groups devoting a substantial percentage of their financial wealth to money holdings.

As one might expect, the share of money in stockholders’ portfolios is lower than in non-stockholders’ portfolios for all age group. This is qualitatively similar to the data from the 2007 US Survey of Consumer Finances (Table 4) although there the difference in the share of money in stockholders’ and non-stockholders’ portfolios is much larger.

Existing literature on Japanese household finance investigates possible factors behind the low stock market participation and the low share of stock in household financial portfolio. For example, Kitamura and Uchino (2010) ar-

<table>
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<th>Age Group</th>
<th>$\alpha_m$</th>
<th>$\alpha_b$</th>
<th>$\alpha_s$</th>
<th>$\alpha_m$</th>
<th>$\alpha_b$</th>
<th>$\alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>46.7</td>
<td>53.2</td>
<td>0.0</td>
<td>38.4</td>
<td>45.0</td>
<td>16.5</td>
</tr>
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<td>35-45</td>
<td>38.5</td>
<td>61.5</td>
<td>0.0</td>
<td>29.8</td>
<td>49.2</td>
<td>21.0</td>
</tr>
<tr>
<td>46-55</td>
<td>28.5</td>
<td>71.5</td>
<td>0.0</td>
<td>19.2</td>
<td>58.0</td>
<td>22.8</td>
</tr>
<tr>
<td>56-65</td>
<td>31.7</td>
<td>68.2</td>
<td>0.0</td>
<td>18.5</td>
<td>60.0</td>
<td>21.6</td>
</tr>
<tr>
<td>66-75</td>
<td>28.0</td>
<td>72.0</td>
<td>0.0</td>
<td>17.7</td>
<td>58.9</td>
<td>23.3</td>
</tr>
</tbody>
</table>
Life Cycle Portfolio Choice in US

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Non-Stockholders</th>
<th>Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>75.8 24.2 0.0</td>
<td>28.6 29.8 41.5</td>
</tr>
<tr>
<td>35-45</td>
<td>67.9 32.1 0.0</td>
<td>16.7 35.3 48.0</td>
</tr>
<tr>
<td>46-55</td>
<td>62.5 37.5 0.0</td>
<td>14.4 37.6 48.0</td>
</tr>
<tr>
<td>56-65</td>
<td>59.2 40.8 0.0</td>
<td>13.5 38.7 47.8</td>
</tr>
<tr>
<td>66-75</td>
<td>63.3 36.7 0.0</td>
<td>16.2 42.0 41.8</td>
</tr>
</tbody>
</table>

Table 4: Mean financial portfolios for the non-stockholders/stockholders in the 2007 SCF data.

Kasuga and Matsuura (2005) empirically show that liquidity constraints may make the Japanese households invest less in stocks. Kinari and Tsutsui (2009) argue that lack of trust with security companies and financial literacy are important factors behind low holdings of risky assets. Iwaisako, Ono, Shu, and Tokuda (2015) focus on the negative effects of housing on stockholdings.

Regarding money demand, Fujiki and Shioji (2006) use the SHF data to investigate how concerns about soundness of the financial institutions in the early 2000s and the low nominal interest rates affect household money demand. Consistently with the standard theory of money demand, households increase money holdings facing low nominal interest rates. Household with low financial wealth are more likely to hold more cash, which is consistent with a prediction of our theoretical model. Finally, they show that households who are concerned with soundness of the financial institutions increase cash holdings.

All the existing literature mentioned above uses reduced form regression approach. In contrast, we employ a structural approach — we construct a life-cycle portfolio model to investigate factors behind low stock market participation, low share of stocks and high share of money in household financial portfolio.
3 The Model

The model we use is a nominal version of the life-cycle models that are extensively used in the household portfolio literature. Agents work when they are young, and receive a pension after retirement. They are subject to uninsurable labor income risk and borrowing constraints. Households hold three assets: money, bonds and stocks; all three are traded in nominal terms. In order to introduce money, we extend the standard life-cycle portfolio choice models by introducing transaction frictions.

3.1 Preferences

Time is discrete and $t$ denotes adult age which, following the typical convention in the literature, corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 ($T$) periods (age 100). The probability that a consumer/investor is alive at time $(t + 1)$ conditional on being alive at time $t$ is denoted by $\xi_t$ ($\xi_0 = 1$). Finally, the consumer/investor has a bequest motive.

Households have Epstein-Zin-Weil utility functions (Epstein and Zin (1989), Weil (1990)) defined over one single non-durable consumption good. Let $C_{it}$ and $X_{it}$ denote respectively real consumption and nominal wealth (cash on hand) of agent $i$ at time $t$. Then the real cash on hand is defined as $X_{it}/P_t$ where $P_t$ denotes the price level at time $t$. The preferences of household $i$ are defined over real consumption by

$$V_{it}^j \left( \frac{X_{it}}{P_t} \right) = \left\{ (1 - \beta)C_{it}^{1-1/\psi_j} + \beta \left( E_t \left[ \xi_t V_{it+1} \left( \frac{X_{it+1}}{P_{t+1}} \right)^{1-\rho_j} \right] + (1 - \xi_t)\varphi_j \left( \frac{X_{it+1}}{P_{t+1}} \right)^{1-\rho_j} \right) \right\}^{\frac{1}{1-1/\psi_j}} \tag{1}$$
where $\rho$ is the coefficient of relative risk aversion, $\beta$ is the discount factor, and $\varphi_j$ determines the strength of the bequest motive. Following Vissing-Jorgensen (2002) we assume that different households are heterogeneous in their inter-temporal elasticity of substitution, $\psi$. Our economy is populated with two equally-sized groups $j$, respectively, with high ($\psi_H$) and low ($\psi_L$) intertemporal elasticity of substitution. We also utilize ex ante heterogeneity in risk aversion (high ($\rho_H$) and low ($\rho_L$)) following Gomes and Michaelides (2005) to generate a stronger variation in precautionary saving motives over the life cycle (and therefore more variation in wealth accumulation profiles). Finally, households are heterogeneous in their bequest motive ($\varphi_H > \varphi_L$).

### 3.2 Labor Income Process

Following the standard specification in the literature, the labor income process before retirement is given by

$$ Y_{it} = Y^p_{it}U_{it} $$

$$ Y^p_{it} = \exp(f(t, Z_{it}))Y^p_{it-1}N_{it} $$

where $f(t, Z_{it})$ is a deterministic function of age and household characteristics $Z_{it}$, $Y^p_{it}$ is a permanent component with innovation $N_{it}$, and $U_{it}$ a transitory component. We assume that $\ln U_{it}$ and $\ln N_{it}$ are independent and identically distributed with mean $\{-0.5 \cdot \sigma^2_u, -0.5 \cdot \sigma^2_n\}$, and variances $\sigma^2_u$ and $\sigma^2_n$, respectively. The log of $Y^p_{it}$ evolves as a random walk with a deterministic drift, $f(t, Z_{it})$. For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period $K$, corresponding to age 65 ($K = 46$). Earnings in retirement ($t > K$) are given by $Y_{it} = \lambda Y^p_{iK}$, where $\lambda$ is the replacement ratio ($\lambda = 0.6$).
Durable goods, and in particular housing, can provide an incentive for higher spending early in life.\textsuperscript{13} We exogenously subtract a fraction of labor income every year allocated to durables (housing). This empirical process is estimated from the National Survey of Family Income and Expenditure.

### 3.3 Specification of shopping cost technology

In order to motivate money holdings, we assume transaction frictions. Our approach is related to shopping time models, first proposed by McCallum and Goodfriend (1987). We modify slightly that specification to incorporate it more easily in the portfolio choice literature. In shopping time models, transaction costs are modeled in terms of foregone time: money can help reduce transaction time. As shown in Lucas (2000), there is a connection between the shopping time models and the inventory-theoretic studies of money (Baumol (1952), Tobin (1956)). More broadly speaking, the transaction cost can include not only a shopping cost but also a cost of selling illiquid assets to finance consumption. Different versions assume different trade-offs in the presence of transactions frictions. For example, Lucas (2000) assumes that agents face a trade-off between hours spent on production and transactions. Ljungqvist and Sargent (2004) (Ch. 24) assume a trade-off between transaction time and leisure.

To generate money holdings, we assume a shopping cost transaction friction (proportional to $Y_{it}^p$), a direct physical cost in consumption goods:

$$\Omega_i Y_{it}^p = \Omega(C_{it}, M_{it}/P_t; \varepsilon)Y_{it}^p, \quad \Omega_C > 0, \quad \Omega_M < 0$$

The cost is increasing in real consumption and decreasing in real money bal-

\textsuperscript{13}Iwaisako (2009) and Iwaisako, Ono, Shu, and Tokuda (2015) emphasises the role of owner-occupied housing on Japanese household financial portfolio.
ances. In the benchmark case we assume

\[ \Omega_{it}Y_{it} = \varepsilon \left( \frac{C_{it}}{M_{it}/P_t} \right) Y_{it}, \quad \varepsilon \geq 0 \]  \hspace{1cm} (4)

It will be convenient later on to divide variables by the permanent component of labor income. In this case, the shopping cost per unit of \( Y_{it} \) is given by

\[ \Omega_{it} = \varepsilon \left( \frac{c_{it}}{m_{it}} \right), \quad \varepsilon \geq 0 \]  \hspace{1cm} (5)

where \( c_{it} = \frac{C_{it}}{Y_{it}} \) and \( m_{it} = \frac{M_{it}}{Y_{it}/P_t} \). Our preferred interpretation is that the transaction cost represents an opportunity cost of time which is why we assume that it is proportional to the permanent component of labor income. The functional form (4) is consistent with Lucas (2000) who shows that the implied money demand function is consistent with the demand function of Baumol (1952) and Tobin (1956). The parameter \( \varepsilon \) measures the severity of transaction frictions. A large \( \varepsilon \) means it takes more resources to do transactions and it can be different over the life cycle or across agents.

We model transaction costs as a direct physical cost in terms of consumption goods. An advantage of our approach is that we can treat money in exactly the same way as we treat bonds and stocks because there is no additional margin between money holding decisions and leisure (or labor supply) decisions. Therefore our model maintains the basic structure of the models used in the portfolio choice literature, making the model computationally tractable and making the results easily comparable to those obtained in the literature. The presence of the permanent component of income in our shopping cost formulation (4) is consistent with the spirit of the shopping time technology specification which relates the cost of illiquidity to the wage rate faced by the household. Also, our modeling approach maintains the basic properties of the
shopping time models — money demand will be increasing in consumption and decreasing in nominal interest rates.

### 3.4 Financial Assets and Constraints

The agent has options to hold three kinds of assets: fiat money ($M_{it}$), nominal bonds ($B_{it}$) and nominal stocks ($S_{it}$). We let $X_{it}$ be nominal “cash on hand” that the agent can use for consumption and portfolio decisions. The budget constraint is given by

$$X_{it} = P_tC_{it} + S_{it} + B_{it} + M_{it} + 1_{t(.)}P_tFY_{it}^p.$$  

(6)

where the indicator function $1_{t(.)}$ becomes one when the fixed cost to participate in the stock market is incurred (equal to $P_tFY_{it}^p$ in nominal terms).

We assume that the shopping cost is deducted at the beginning of the next period. Then, the evolution of $X_{it}$ is given by

$$X_{it+1} = R_{t+1}^a S_{it} + R_{t+1}^b B_{it} + M_{it} + P_{t+1}Y_{it+1} - P_{t+1}\Omega_{it}Y_{it}^p$$

(7)

where $R_{t+1}^a$ and $R_{t+1}^b$ respectively denote the nominal returns of stocks and bonds. Note that the nominal return of fiat money is unity. Finally, $Y_{it+1}$ is real income at time $t + 1$.

Following the portfolio choice literature, we prevent households from borrowing against their future labor income (this prevents households from counterfactually leveraging up to invest in the stock market to take advantage of the equity premium). More specifically we impose the following restrictions:

$$B_{it} \geq 0$$
\[ S_{it} \geq 0 \]
\[ M_{it} \geq 0 \]

We have one continuous state variable: \( X_{it} \) and the control variables are \( C_{it} \), \( M_{it} \), \( S_{it} \) and \( B_{it} \).

Moreover, we use the initial wealth distribution from the data to initialize the simulation when households first enter the labor force at age 20. We refer to this initial financial wealth as bequests later on in the paper.

### 3.5 Normalizing by Prices and Growth

Let lower case letters denote real variables normalized by the permanent component of labor income (\( Y_{it}^p \)). For example the normalized real cash on hand is defined as \( x_{it} = X_{it}/(Y_{it}^p P_t) \)^{14}. The evolution of the state variable is then given by

\[
x_{it+1} = \frac{r_{it+1}^s}{g_{it+1}} s_{it} + \frac{r_{it+1}^b}{g_{it+1}} b_{it} + \frac{r_{it+1}^m}{g_{it+1}} m_{it} + U_{it+1} - \frac{\Omega_{it}}{g_{it+1}}
\]

where

\[
r_{it+1}^s \equiv R_{it+1}^s \pi_{t+1}^{-1}, \quad r_{it+1}^b \equiv R_{it+1}^b \pi_{t+1}^{-1}, \quad r_{it+1}^m \equiv \pi_{t+1}^{-1}
\]

are respectively the real returns of stocks, bonds and money, where \( \pi_{t+1} \equiv P_{t+1}/P_t \) denotes gross inflation, and \( g_{it+1} \equiv Y_{it+1}^p/Y_{it}^p \) is the gross growth rate of the permanent component of labor income.

The representation of consumer preferences in terms of stationary (normalized) units is given in Appendix B.

---

^{14}The only lower-case variable not normalized by \( P_t \) is consumption \( (C_{it}) \) which was in real terms from the beginning.
4 Parameter Calibration

We calibrate the model to the Japanese economy. The life cycle income profile and labour income risks are in line with Abe and Inakura (2007), Abe, Inakura, and Yamada (2007) and Abe and Yamada (2009): $\sigma_u = 0.1$, $\sigma_n = 0.1^{15}$. The replacement ratio of pension, is set roughly equal to its current value $\lambda = 0.6^{16}$. The correlation between permanent idiosyncratic labor income shocks and the stock return ($\rho_{sn}$) is set to 0.15 based on the relatively scant evidence that exists with regards to this choice as discussed in Gomes and Michaelides (2005).

As explained in Section 3.2, we exogenously subtract a fraction of income every year allocated to housing. The empirical process is estimated from the National Survey of Family Income and Expenditure. For each age group, we compute the fraction of housing-related expenditure (mortgage payments and rents) as a fraction of income, and take a polynomial interpolation with respect to age.

We will use exogenous processes for stock and bond returns, inflation and the aggregate component of labor income. Given that we focus our analysis on the effects of financial crisis and economic stagnation of the “lost two decades” on household portfolio decisions, we use the period 1995 to 2014 in order to compute descriptive statistics and correlations between these variables. Table 5 reports the first two moments that describe the distributions for inflation, real bond returns, real stock returns and real aggregate wage growth.

In the model we assume an i.i.d. process for real asset returns, real wage growth and inflation. As shown in Table 5, real stock returns average 2.1%.

\footnote{Abe and Inakura (2007) use micro data from the Japanese Panel Surveys of Consumers (JPSC) and estimated permanent and transitory income risks from balanced and unbalanced panels. They report that the the standard deviations of transitory and permanent income shocks are respectively in the rage of 0.099-0.135 and 0.091-0.156.}

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>S. D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>0.06</td>
<td>0.80</td>
</tr>
<tr>
<td>Real Bond Returns</td>
<td>0.31</td>
<td>0.69</td>
</tr>
<tr>
<td>Real Stock Returns</td>
<td>2.11</td>
<td>25.9</td>
</tr>
<tr>
<td>Real Wage growth</td>
<td>0.00</td>
<td>1.70</td>
</tr>
</tbody>
</table>

Table 5: We report the means and standard deviations (S.D.) of key inputs in the annual frequency decision model.

per annum with a standard deviation equal to almost 26%. It is relatively standard in the household finance literature to assume that due to annual management fees investors earn a lower equity premium than the historically observed Japanese equity returns of 4.1%. Also it is worth noting that even if we ignore management fees, Japanese equity returns are less than half of US equity returns over the same period (8.4%). In addition, the volatility of Japanese equity returns has been considerably higher than in the US. Together these two factors are likely to be important in explaining the reason why so few Japanese households have found it attractive to invest in stocks.

The bond return process is similarly calibrated with a mean return equal to 0.31% and a standard deviation equal to 0.69%. Since a large part of “bond” in household portfolio is actually time deposits, we calibrate the bond return by the average real rate of return on time deposits of maturity 1-2 years. Again average real bond returns have been much lower in Japan compared to the US (2.4% mean bond return). This reflects the persistent weakness of the Japanese economy and the attempts by the Bank of Japan (BoJ) to stimulate growth by keeping short term interest rates low and through a number of quantitative easing measures.

We also need to take a stance on the correlations across inflation and asset returns. These are shown in Table 6. The contemporaneous correlation between inflation (inverse of real money returns) and real bond returns is esti-
<table>
<thead>
<tr>
<th>Variable</th>
<th>Money Returns</th>
<th>Bond Returns</th>
<th>Stock Returns</th>
<th>Wage growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Money Returns</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Bond Returns</td>
<td>0.81</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Stock Returns</td>
<td>-0.02</td>
<td>-0.12</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Real Wage growth</td>
<td>0.24</td>
<td>0.52</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 6: We report the correlation matrix of key inputs in the decision model. All variables are real, and the sample period is 1995-2014.

estimated in this sample to be -0.81, and there is almost zero correlation inflation and stock returns over this period. There is also a positive correlation between wage growth and both bond and stock returns.

To calibrate the model we need to decide which moments to match. The key variables of interest for our purposes are the mean holdings of financial wealth over the life cycle for stock holders and non-stockholders and the mean participation rate. Conditional on the participation status we then have asset allocations between money, bonds and stocks sorted by age, and for non-stockholders the allocations between bonds and money. This gives a total of twenty six moment conditions.

Given the large number of preference parameters we follow the empirical evidence in Vissing-Jorgensen (2002) and the calibration in Gomes and Michaelides (2005) and use two different values for the elasticity of intertemporal substitution, higher for the households more likely to participate in the stockmarket and lower for the rest. We also utilize heterogeneity in risk aversion rather than the discount rate to generate low propensities to save, thereby generating poorer households who are therefore less likely to incur the fixed stock market participation cost. In our baseline calibration we choose \( \psi_H = 0.5, \psi_L = 0.3, \rho_H = 5.0 \) and \( \rho_L = 1.3 \).

Our preference parameters for non-stockholders might appear strange given that they have lower risk aversion (on average) than stockholders. The incen-
Table 7: Calibrated structural parameters. $\psi_H$ ($\psi_L$) is the elasticity of intertemporal substitution for the more (less) wealthy households, $\rho_H$ ($\rho_L$) is the relative risk aversion coefficient for the more (less) wealthy households, $\sigma_U$ is the standard deviation of the transitory labor income shocks, $\sigma_N$ is the standard deviation of the permanent labor income shocks and $\rho_{sn}$ is the correlation between the permanent labor income shocks and stock returns.

tive to save, however, is what matters in incurring the fixed cost to participate in the stock market and this is monotonically related to the precautionary savings motive (Haliassos and Michaelides (2003)). Our calibration is consistent with the low risk aversion parameters for non-participants found in Alan (2006), Paiella (2007) and Attanasio and Paiella (2010). We also set the bequest parameter for the households that are likely not to accumulate substantial financial wealth equal to zero, consistent with De Nardi, French, and Jones (2010) that find evidence for bequest motives in the richer part of the population but not for poorer households. For compactness purposes Table 7 lists those calibrated parameters and the standard deviations of the permanent and transitory labor income shocks as used by Carroll (1997) and Cocco, Gomes, and Maenhout (2005).

For the remaining structural parameters there is much less guidance from the empirical literature. We calibrate those parameters in order for the model to match the variables of interest — wealth to income ratio and financial portfolio that are reported in Tables 1 and 3. At this stage we also introduce
another layer of heterogeneity to be consistent with the empirical regularity that households that remain poorer during the working part of the life cycle also remain poor during retirement. We therefore calibrate different parameters for the shopping cost $\{\varepsilon\}$ and the bequest parameter for the wealthier households $\varphi_H$, while keeping the fixed cost parameter $F$ constant across the two groups. We also calibrate a constant discount factor ($\beta$) across the two groups.

Shopping technology costs should reflect the opportunity cost of time as emphasized by Aguiar and Hurst (2005). On the size of the fixed cost in the context of this model, the closest paper is Alan (2006) and our non-stockholders would be similar to the non-participants modelled through a fixed cost in that paper.

Those calibrated parameters for our model are given in Table 8. The parameter values are consistent with previous estimates in the literature. There is some evidence for a bequest motive needed because financial wealth is not fully decumulated during retirement, this is consistent with, among others, De Nardi (2004). There are no estimates for the equivalents of the shopping cost parameters (from microeconomic data) against which we can compare our results. The implied shopping cost varies between 0.0025 and 0.5 percent of mean annual labor income that we view as a reasonable transaction cost and is consistent with Lucas (2000).

The fixed cost to generate non-participation is considerably higher than the low costs that have either been calibrated (Gomes and Michaelides (2005)) or estimated (Alan (2006), Bonaparte, Cooper, and Zhu (2012)) to generate stock market non-participation for poorer households in the US. However, the large value of $F$ makes sense given that we are trying to match a 15.5% participation rate. For example Favilukis (2013) estimates a very similar fixed cost when
Table 8: \( \beta \) is the annual discount factor, \( \varphi_H \) is the bequest parameter for the wealthier households, \( \varepsilon_H \ (\varepsilon_L) \) is the shopping cost parameter for the more (less) wealthy households, and \( F \) is the fixed cost incurred to participate in the stock market.

This cost should be interpreted as a short cut for anything ranging from inertia, behavioral biases, low trust in the stock market (Guiso, Sapienza, and Zingales (2008)), observation and transaction costs stemming from rational inattention (Alvarez, Guiso, and Lippi (forthcoming)), or repeated costs from having a stock trading account (small annual trading costs can add up over a few years as in Bonaparte, Cooper, and Zhu (2012)).

The bequest parameter heterogeneity is consistent with recent estimates (De Nardi (2004)) that find a stronger bequest motive for richer households. In the context of this model the stronger bequest motive for richer households reflects the slow decumulation of financial wealth during retirement for this segment of the population. The calibrated discount factor is within the range of recent estimates (Attanasio, Banks, Meghir, and Weber (1999), Gourinchas and Parker (2002), Cagetti (2003)) from structural estimation of life cycle consumption models. Finally, the shopping technology is higher for the richer households than for the poorer ones. Without this heterogeneity poorer households spend a substantial amount of their small financial wealth in shopping costs, a potentially counterfactual implication. Using this heterogeneity allows us to replicate the selected moments across the two groups without large
shopping costs arising from holding money.

5 Benchmark Results

What are the policy functions and life cycle profiles implied by these parameter estimates? Figure 8 shows the portfolio choice policy functions of the low risk aversion- low EIS group against the continuous state variable (cash on hand, $x$). We show the policy functions for the young (age 25), middle-aged (age 55) and retirees (age 85). The solid lines show the portfolio choices of households that have already paid the fixed cost to participate in the stock market. The dashed lines show the choices of non-participants. The vertical axis plots portfolio shares as a percentage of financial wealth invested in each asset (between zero and one due to the no borrowing/no short sale constraints). The three age groups mainly hold money when their cash on hand is small. In particular, the young and the middle-aged agents invest almost their entire assets in money when they are poor. This is consistent with the data that shows that the young and poor agents tend to hold more money.

In line with the portfolio choice literature, the bond share is increasing in cash on hand while the stock share is decreasing in cash on hand. This is because human wealth is similar to a bond and therefore only households with a high level of financial wealth desire to increase their exposure to bonds. This feature of the standard life cycle model generates unrealistically high stock shares at lower levels of financial wealth. The presence of transactions frictions in our model helps to correct this. In our framework, poorer stockholders hold a lot of money in their portfolios.

The dashed lines indicate the policy functions for the non-participants. These households need to accumulate assets in money and bonds before incur-
ring the fixed participation cost to invest in the stock market. They therefore accumulate money first (to minimize the shopping cost), and then bonds and then if they accumulate a sufficient amount of wealth to justify incurring the fixed participation cost, they do so. At that point they reduce their bond holdings and invest in the stock market.

A similar picture arises from Figure 9 that plots the portfolio policy functions for the higher risk aversion - higher EIS group. The first difference from the previous figure is that the participation takes place faster even though these households are more risk averse. Because of a stronger precautionary saving motive they accumulate more saving and have a stronger incentive to participate in the stock market. The second difference is that the share of wealth in stocks is more quickly declining as a function of cash on hand again because of the higher risk aversion.

Figure 10 conditions on participation status (some of the low risk aversion households will have had lucky labor income draws and participate in the stockmarket) and shows the simulated paths of consumption, financial wealth and income over the life cycle. We simulate the model economy with 10,000 individuals in each age cohort starting with the initial financial wealth taken from the 2014 SHF and take the mean of each variable. The non-stockholders do not accumulate much financial wealth, and their consumption tracks labor income over the life-cycle. The stockholders accumulate a higher amount of financial wealth and therefore their consumption is decoupled from labor income.

Figure 11 shows simulated portfolio choices over the life cycle. The young non-stockholders hold mainly money, and the money share decreases as the agents become older, reaching a minimum at retirement. At that point money holdings increase again as the limited financial wealth is decumulated during
retirement. To minimize on shopping costs during retirement, the financial portfolio is re-allocated towards money and bonds are rapidly crowded out from the portfolio.

Stock market participants are richer households that invest heavily in stocks. In the beginning of working life most households keep their wealth in money in order to minimize shopping costs. Only very rich households that have received large bequests\(^ {17} \) find it profitable to pay the fixed cost and invest in stocks. This is why the share of stocks is so high (and the share of money so low) early on in stockholders’ lives.

As we can see from Figure 12, by around age 38, other high-saving households accumulate enough wealth to begin investing in stocks (this can be seen from the sharp pick up in the fraction of stock market participants). This leads to a decline in the average wealth to income ratio of stockholders of age 30-40 as can be seen from the middle panel of Figure 10. As a result of the entry of poorer households into the stockholder population, the average share of money in stockholders’ portfolios rises and the average share of stocks and bonds falls.

As Figure 12 shows, from age 45 onwards, participation does not change as sharply and most of the evolution in stockholder portfolios is driven by changes to existing stockholders’ portfolios. The share of wealth allocated to stocks increases until age 40 as households get richer. This occurs because wealth grows much faster than consumption over this period, meaning that households need to devote a much smaller fraction of their portfolios to money holdings while still keeping shopping costs down. Consequently richer households invest more heavily in stocks - a prediction that the model without money typically cannot generate unless extended in other directions (Wachter and Yogo (2010),

\(^{17}\)We start from an initial wealth distribution taken from the data in the beginning of life. We think of this initial wealth distribution as arising from a bequest motive.
Mean Financial Wealth/Income: Data versus Model

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Non-Stockholders</th>
<th>Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>20-34</td>
<td>0.61</td>
<td>0.20</td>
</tr>
<tr>
<td>35-45</td>
<td>0.89</td>
<td>0.34</td>
</tr>
<tr>
<td>46-55</td>
<td>1.12</td>
<td>0.73</td>
</tr>
<tr>
<td>56-65</td>
<td>2.09</td>
<td>2.14</td>
</tr>
<tr>
<td>66-75</td>
<td>4.32</td>
<td>3.05</td>
</tr>
</tbody>
</table>

Table 9: Actual versus predicted moments for mean financial wealth relative to mean labor income for the nonstockholders/stockholders. The model is compared to the 2014 SHF data.

for example). The stock holdings predicted by the model is in line with the data, which is shown in the lower panel of Figure 12. The average stock participation rate implied by the model is 15.3% versus 15.5% in the 2014 SHF data.

How do other predicted moments compare with the actual ones? We first go through the mean wealth to mean labor income ratios which are given in Table 9. The model matches the household wealth to income profile relatively well. In particular, it captures the fact that stockholders are considerably richer than non-stockholders.

We next present the moments for the portfolio shares. We start with the profiles for non-stockholders, given in Table 10. The model overpredicts the share of wealth held in money relative to the data especially in the early parts of the life cycle (ages 20-54). Correspondingly, the model underpredicts the share of wealth held in the form of bonds. However the model does capture the fact that the share of wealth in bonds is rising over the life cycle.

Next we move on to the comparison between data and model for stockholders (Table 11). Here, the main failure of the model is that it cannot match bond-holdings for the first two age groups. Equivalently, as in the real model without time-varying investment opportunities, the model overpredicts
Table 10: Actual versus predicted moments for mean financial portfolios for the nonstockholders. The model is compared to the 2014 SHF data.

Table 11: Actual versus predicted moments for mean financial portfolios for the stockholders. The model is compared to the 2014 SHF data.

6 Explaining the investment decisions of Japanese households

Having obtained a reasonably good match to the data, we now proceed to use the model to analyse the investment decisions of Japanese households. We do this by identifying several key drivers of household portfolios which are very
different in Japan compared to the US. We then do counterfactual experiments by moving these key drivers, one by one, to their US values in order to see how the portfolios of Japanese households change as a result. We are trying to explain three key facts: (a) Why do so few Japanese hold stocks and why is the share of stocks so low in the portfolios of those that do? (b) Why do Japanese savers hold such a high fraction of their wealth in monetary form? (c) Why is the difference in the wealth of Japanese stockholders and non-stockholders so much smaller than in the US data?

We look at three main sets of drivers - macroeconomic factors (inflation, interest rates and stock returns), demographic factors and the fixed cost of stock market participation. The latter consists of both the monetary cost but also of factors such as ‘trust’ and financial literacy that have been shown to be important in the US.

6.1 Macroeconomic factors

6.1.1 Low inflation

In the first counterfactual experiment, we move the rate of inflation from its very low level of 0.06% in Japan to the US level of 2%. Deflation in Japan has been a much talked about phenomenon in part because of the worry that it may have encouraged Japanese savers to hold nominal instead of real assets. Indeed the counterfactual simulation with our model confirms that, to some extent such fears were not completely unfounded.

The higher rate of inflation indeed brings about an increase in stock participation from 15.3% in the baseline simulation to 20.2% - an increase of almost 5 percentage points. In addition, the share of stocks held by younger households (below 45 years of age) also increases substantially, rising by more than
Table 12: Effects of higher inflation on portfolio choice: counterfactual simulation in which inflation rate is set to the US level of 2.0%

10 percentage points. On the other hand, the share of stock held by older households (age 46 and above) is little affected.

The higher level of inflation discourages households from holding low yielding monetary assets and the share of portfolios devoted to money declines very significantly relative to the baseline. In contrast to stockholdings, the shrink in money holdings is observed in all age groups. It however remains higher than in the US, indicating that low inflation is not the only reason for the popularity of monetary accounts amongst Japanese households.

6.1.2 Low interest rates

Due to the series of financial problems suffered by Japan since the 1990s, its short term interest rates have been close to zero for almost 20 years now. This fact, combined with Japan’s low level of inflation has ensured that the real bond return has averaged 0.31% compared to 2.4% in the US over the same period. In this counterfactual simulation, we examine the model’s implications for Japanese household portfolios in a world in which they had faced the US’s real bond return of 2.4% while the equity risk premium is kept at the Japanese rate of 1.8%.

Not surprisingly, facing a higher bond rate the households’ share of money
Table 13: Effects of higher base rate on portfolio choice: counterfactual simulation in which the bond rate is set to the US value of 2.4%, keeping the risk premium constant at the Japanese value of 1.8%.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>$\alpha_m$</th>
<th>$\alpha_b$</th>
<th>$\alpha_s$</th>
<th>$\alpha_m$</th>
<th>$\alpha_b$</th>
<th>$\alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>82.2</td>
<td>17.8</td>
<td>-</td>
<td>38.1</td>
<td>12.5</td>
<td>49.4</td>
</tr>
<tr>
<td>35-45</td>
<td>56.2</td>
<td>43.8</td>
<td>-</td>
<td>32.7</td>
<td>17.3</td>
<td>50.0</td>
</tr>
<tr>
<td>46-55</td>
<td>26.2</td>
<td>73.8</td>
<td>-</td>
<td>14.0</td>
<td>60.6</td>
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<tr>
<td>56-65</td>
<td>6.5</td>
<td>93.5</td>
<td>-</td>
<td>6.7</td>
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<td>66-75</td>
<td>27.1</td>
<td>72.9</td>
<td>-</td>
<td>6.4</td>
<td>78.3</td>
<td>15.3</td>
</tr>
</tbody>
</table>

in their financial portfolio decreases, as is shown in Table 13. In this counterfactual simulation we keep inflation rate at 0.06%, so the implied level of the nominal interest rate is 2.46%. A higher nominal interest rate represents a higher opportunity cost of money holdings, which induce households to invest less in monetary assets. Higher interest rates make both non-stockholders and stockholders richer, as is shown in Table 14. However, the stock market participation rate declines significantly to 4.3%. Note that, in this counterfactual simulation, the equity premium is kept low at the Japanese value of 1.8%, and also the correlation matrix and asset return volatility are kept constant at the Japanese values. Facing a higher bond rate while keeping the premium constant, equity becomes less attractive. As a result agents accumulate wealth by investing in safe and high-yielding bond.

The higher interest rate has a rather significant effect on wealth accumulation by both stockholders and non-stockholders. It benefits all savers helping financial wealth to rise relative to income.  

\(^{18}\)If we increase the base rate with a higher equity premium (US value of 4.0%), then the stock market participation rate increases. This is because a higher interest rate helps households accumulate wealth, giving them more incentive to pay for the fixed cost to participate in the stock market.
Life Cycle Financial Wealth Accumulation: Effects of higher base rate

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Wealth/Income) Non-Stockholders</th>
<th>Mean (Wealth/Income) Stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>0.22</td>
<td>1.28</td>
</tr>
<tr>
<td>35-45</td>
<td>0.51</td>
<td>1.31</td>
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<tr>
<td>46-55</td>
<td>1.45</td>
<td>3.02</td>
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<tr>
<td>56-65</td>
<td>3.60</td>
<td>6.73</td>
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<tr>
<td>66-75</td>
<td>6.26</td>
<td>12.94</td>
</tr>
</tbody>
</table>

Table 14: Effects of higher base rate on wealth accumulation: counterfactual simulation in which the bond rate is set to the US value of 2.4%, keeping the risk premium constant at the Japanese value of 1.8%.

Life Cycle Portfolio Choice: Effects of Equity Premium

<table>
<thead>
<tr>
<th>Age Group</th>
<th>(\alpha_m)</th>
<th>(\alpha_b)</th>
<th>(\alpha_s)</th>
<th>(\alpha_m)</th>
<th>(\alpha_b)</th>
<th>(\alpha_s)</th>
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<td>20-34</td>
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<td>-</td>
<td>32.0</td>
<td>0.0</td>
<td>68.0</td>
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<td>-</td>
<td>19.4</td>
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<td>54.0</td>
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Table 15: Effects of higher equity premium on portfolio choice. counterfactual simulation in which the equity premium is set to the US value of 4%

6.1.3 Low equity premia

Another important legacy of the crisis in Japan has been a poor record of stock market returns which have averaged less than 2% per year in excess of the real bond return as compared to 4% for the US. Indeed, our counterfactual simulations show that the expected profitability of investing in the stock market is a key quantitative determinant of portfolio choice. When we use a 4% risk premium in the model which is otherwise calibrated to the Japanese economy, we get a remarkable increase in participation. It increases from 15.3% in the baseline simulation to 51.8% which is actually slightly higher than the US figure. Stockholders’ portfolio share allocated to money actually becomes extremely similar to that of US stockholders while their share of equities become even higher.
Table 16: Effects of higher equity premium on wealth accumulation. Counterfactual simulation in which the equity premium is set to the US value of 4%

Another very interesting consequence of the higher equity premium is to increase dramatically the wealth of stockholders relative to non-stockholders. For example, Table 16 shows that the relative mean wealth/income ratio of the age 66-75 group is 6.68 versus 0.24, while that of the baseline simulation (Table 9) is 8.84 versus 3.05. To some extent, of course, this is the mechanical effect of higher participation. The richest non-stockholders switch to participating but they are still poorer than those who were already stockholders in the benchmark. As a result, the relative wealth of stockholders compared to non-stockholders rises substantially.

But there is another important impact of the higher equity premium. Stockholders’ rate of return on wealth rises substantially relative to non-stockholders widening the wealth difference between the two groups in a way that is reminiscent of the discussion in Piketty (2014). This demonstrates another important consequence of the poor stock returns following the financial crisis. They have compressed the wealth distribution by helping to hold back the net worth of stockholders.
6.2 Demographic factors and the pension system

Crisis related factors are not the only drivers of Japanese household portfolios. Demographic factors in particular may be important due to the way they affect wealth accumulation. And Japan is sufficiently different from other countries, especially due to its high longevity and its relatively less generous welfare state (including state pension provision).

Fujiki, Hirakata, and Shioji (2012) uses the SHF data to empirically investigate how age affects stockholding decisions. They find that, after controlling for household characteristics such as income, wealth, and education, the effects of age per se are significant but not very large. In this subsection we examine the effect of household life spans and the generosity of the pension system. We set those two to the US levels keeping the other factors constant at the Japanese values.

Figure 12 shows that the survival probability of the Japanese household is higher in particular for between age 60-80. Higher life expectancy gives households more incentive to save when they are young. This incentive to save is especially strong because the pension replacement ratio of Japan (0.6) is lower than that of the US (0.68).

Tables 17 and 18 show that, consistent with the findings of Fujiki, Hirakata, and Shioji (2012), the overall effects of longevity and the pension system are relatively modest. Both life cycle portfolio and wealth accumulation are similar to the benchmark case reported in Tables 10 and 11. The stock market participation rate is somewhat higher at 16.4%.
Life Cycle Portfolio Choice: Effects of longevity and pension

<table>
<thead>
<tr>
<th>Age Group</th>
<th>$\alpha_m$</th>
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<td>-</td>
<td>26.5</td>
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Table 17: Effects of longevity and pension on portfolio choice. Counterfactual simulation in which the longevity and pension replacement ratio are set to the US values.

Life Cycle Financial Wealth Accumulation: Effects of longevity and pension

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean (Wealth/Income)</th>
<th>Mean (Wealth/Income)</th>
</tr>
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<td></td>
<td>Non-Stockholders</td>
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<td>7.16</td>
</tr>
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</table>

Table 18: Effects of longevity and pension on wealth accumulation. Counterfactual simulation in which the longevity and pension replacement ratio are set to the US values.
Table 19: Effects of Steeper Income Profile on Portfolio: Life cycle income profile set to that of the US

<table>
<thead>
<tr>
<th>Age Group</th>
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<th>$\alpha_b$</th>
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<td>-</td>
<td>39.2</td>
<td>37.9</td>
<td>22.9</td>
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6.3 Income evolution over the life cycle

We also examine the impact of income growth over the life cycle in Japan. We do this by solving the model with the deterministic age related income component taken from the US rather than the Japanese data. The life cycle income profile is affected by several factors: time, cohort and age related effects. Even though the Japanese life-time employment system involves deferred wages, the slower economic growth since the 1990s makes the Japanese life cycle income profile flatter than that of the US. Tables 19 and 20 show the impact of this on household portfolios and on wealth accumulation.

A steeper income profile reduces the desired saving of the young who would like to bring consumption forwards in time. However, the large increase in income over time implies that future savings are expected to be high and, as a result, investing in the stock market becomes more attractive already today. As a result, stock participation rises to 19.5% in this counterfactual simulation.

Furthermore, since the net present value of future labour income behaves more like a bond (aggregate risk is low and its correlation with stock returns is low), having higher human (bond-like) wealth incentivises households to hold more stocks and less money.
Life Cycle Financial Wealth Accumulation: Effects of Income Profile

<table>
<thead>
<tr>
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<th>Mean (Wealth/Income)</th>
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</thead>
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Table 20: Effects of Steeper Income Profile on Wealth Accumulation: Life cycle income profile set to that of the US

Life Cycle Portfolio Choice: Effects of Financial Development

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<th>( \alpha_s )</th>
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<td>-</td>
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<td>35.1</td>
<td>48.8</td>
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<td>-</td>
<td>25.2</td>
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</table>

Table 21: Effects of Financial Development on Portfolio Choice: Fixed cost of stock market participation is set to the US value

6.4 The fixed cost of stock market entry

Finally we check how the high one-off cost of entering the stock market affects Japanese household portfolios. We change \( F \) from our Japanese calibration value of 0.09 to the 0.05 value used by Gomes and Michaelides (2005) for the US. The result is shown in Table 21. Stock market participation increases strongly to 42.9%. This shows that the fixed cost of stock market entry has been a very significant factor holding back Japanese participation in the equity market.

Our model cannot tell us what lies behind the high fixed cost of stock market entry. Some of it may reflect lower financial development though this appears an unlikely explanation for such a large difference in the fixed cost. There is more evidence that it may reflect a lower degree of trust in the stock
market or in corporations more generally. For example, Giannetti and Wang (2015) have shown that the experience of corporate scandals in the US has had a significant negative impact on the propensity of households to hold stocks.

Following the collapse of the Japanese bubble economy of the 1980s, a number of well-known corporations have been implicated in accounting or corruption scandals. Little direct evidence exists on how such scandals affected Japanese households’ attitudes to the stock market but there is evidence that their trust in business more generally is very low in international perspective. Kinari and Tsutsui (2009) argue that lack of trust with security companies and financial literacy are important factors behind low holdings of risky assets. Inoguchi (2002) reports comparative survey evidence on how much people in different countries trust various institutions (including 'Business Firms'). Interestingly, it turns out that the Japanese are one of the most mistrustful nations in the survey with only 20-30% of respondents saying they trust business firms. Only South Korean respondents trust business less (10-20%). In contrast, nations with a higher stock participation ratio (the US, the UK and Sweden for example) have 40-50% of people saying they trust corporations.

Kitamura and Uchino (2010) argue that financial literacy is important as a cost of stock market participation. Observable household characteristics such as income, occupation and the level of financial wealth can explain only 30-50% of the difference in the stock market participation rate of college graduates and the others. Their results imply the existence of the cost of stock market participation that stems from financial literacy. Using the SHF data, Fujiki, Hirakata, and Shioji (2012) also show that financial literacy has a positive effect on the stock market participation of younger households.

In general, what the size of our fixed cost estimate reveals is that (all else

\footnote{Some examples of such scandals include Nomura Securities, Yamaichi Securities, Sumitomo Corporation}
equal) Japanese households have a higher hurdle for investing in the stock market compared to their US counterparts. Exploring this issue further would be a very interesting avenue for future research.

7 Conclusion

This paper sets out to understand the nature of Japanese household portfolios as measured in the Japanese Survey of Household Finances. We benchmark the investment and saving choices of Japanese households with those of their more widely studied US counterparts (obtained from the 2007 US Survey of Consumer Finances).

We find that Japanese households behave very different from US ones. Very few participate in the stock market and, even those that do participate, invest a relatively low share of their savings in equities. Instead, Japanese households hold a high share of wealth in monetary form and in bonds. Finally, the wealth difference between Japanese stockholders and non-stockholders is not as great as in the US. This last fact is especially puzzling given the low participation rate.

In order to model the investment decisions of Japanese households we extend the standard life cycle buffer stock model to include a motive to hold money. We choose a tractable approach to modelling money demand, making use of the shopping cost model. The main idea of the model is that shopping requires resources but money helps economise on such 'shopping costs'. This is a parsimonious and tractable approach that is suitable for empirical applications.

We calibrate the model to match a large set of moments from the 2014 Japanese Survey of Household Finances. The model matches the data rela-
tively well and we use it to understand why Japanese households invest their wealth the way they do. Our approach is to perform counterfactual simulations that, one by one, move key drivers of investment and saving decisions to US values and observe the way portfolios change as a result.

We find that Japan actually has structural demographic characteristics that are conducive to high stock ownership and wealth accumulation. Households have a higher life expectancy and the pension system offers a less generous replacement ratio than the US system. Both of these factors increase desired saving and make stock market participation more attractive.

Nevertheless, two main factors offset the effect of demographics and keep stock participation among Japanese households low. First of all, real stock returns have been much lower in Japan than in the US and, second, our calibrated fixed cost of stock market participation is double the cost estimated by other studies for the US. Moving either of those factors to US levels is capable of getting participation close to the 50% observed in the US.

Even when we condition on participation, the share of stocks in Japanese household portfolios is low. Again, our counterfactual simulations indicate that the low equity returns in Japan have played an important role in this as they reduce the attractiveness of investing in stocks. But second, low inflation has made money more attractive, crowding out stocks and bonds. The low level of Japanese inflation can also explain why the share of money in household portfolios is so high. Moving inflation to US or European historical levels (around 2%) moves the share of money and stocks much closer to those observed in the US SCF.

Low real interest rates have been another important factor behind the money dominated portfolios of Japanese households. They have kept the wealth of savers low and since poorer households hold more money and less
stocks, this has kept the share of monetary assets high in Japanese portfolios while reducing the share of stocks.

The final interesting aspect of the Japanese household portfolio data is the puzzling low wealth of stockholders relative to non-stockholders at least in comparison with the US. Again, poor stock returns play an important role in explaining this. Just as in the work of Piketty (2014), returns to capital have important long term implications for the wealth distribution. In the US, equity returns have been high, benefitting stock owners. In Japan, stock returns have been low in comparison, and the wealth of those who own stocks relative to those who do not, has not risen to the same extent.

Overall, our results carry important lessons for other post financial crisis economies today. Persistently poor macroeconomic performance with low interest rates, low stock returns and low inflation can have a very far reaching impact on household behavior. Persistently low interest rates damage the wealth of savers and compresses the wealth distribution. Under the presence of any kinds of fixed costs of stock market participation or entrepreneurship, this could lead to a large decrease in household participation in public equity or in own business creation. Instead, low wealth and low inflation may encourages households to keep their wealth in the form of money.

**Appendix A  Data**

The household data are taken from the Survey of Household Finance. The participation rate is computed as the ratio of households who report positive amounts of stockholdings out of the total number of respondents. The survey asks households’ financial portfolios — the outstanding amounts of currency, current deposits and time deposits, life insurance, non-life insurance, personal
annuity insurance, bonds, stocks and investment trusts, workers’ asset formation savings, and other financial products. Money is defined as currency plus current deposits. Stocks in our model is the amount of stocks respondents report. We define the all the other assets as “bonds.” Investment trusts include both bond and equity, but the Survey does not decompose bond trusts and equity trusts. Since most of the investment trusts have been bond trusts, we just include investment trusts in the bond category.

Inflation is a year-on-year change in the Consumer Price Index. The stock returns are annual returns of TOPIX (Total Return Index). Bond returns are the average of time deposits of maturity 1-2 years (from Nikkei NEEDS FinancialQuest). The real wage growth is computed from wage index reported in the Monthly Labour Survey produced by the Ministry of Health, Labour and Welfare.

Appendix B  The Normalized Value Function

Let \( v^j_{it} \equiv V_{it}/Y^p_{it} \) be the normalized value of individual \( i \) at age \( t \). Households also differ according to their preferences, denoted by \( j = H, L \). Households of type \( H \) have a high risk aversion (\( \rho_H \)), high EIS (\( \psi_H \)) and a high bequest motive (\( \varphi_H \)). Households of type \( L \) have low risk aversion (\( \rho_L \)), low EIS (\( \psi_L \)) and no bequest motive (\( \varphi_L = 0 \)). \( g_{it+1} \equiv Y^p_{it+1}/Y^p_{it} \) is the growth rate of the
permanent component of income for the household.

\[ v^j_{it}(x_{it}, I_t = a) = \begin{cases} (1 - \beta)c^{1-1/\psi_j}_{it} \\ + \beta E_t \left[ \xi_t(v^{j}_{it+1}(x_{it+1}, I_{t+1} = a))^{1-\rho_j}(Y_{it+1}^p/Y_{it}^p)^{1-\rho_j} + (1 - \xi_t)\varphi_j(x_{it+1})^{1-\rho_j}(Y_{it+1}^p/Y_{it}^p)^{1-\rho_j} \right] \right]^{1-1/\psi_j} \]

\[ = \begin{cases} (1 - \beta)c^{1-1/\psi_j}_{it} + \\ \beta E_t \left[ \xi_t(v^{j}_{it+1}(x_{it+1}, I_{t+1} = a))g_{it+1})^{1-\rho_j} + (1 - \xi_t)\varphi_j(x_{it+1}g_{it+1})^{1-\rho_j} \right] \right]^{1-1/\psi_j} \]

for \( a = 0,1 \). The continuous state is \( x_{it} \) (normalized cash on hand) and its evolution is given by (8). \( x_{it} \) is the cash on hand which is at the disposal of the household before the payment of the stock market participation cost \( F \) (this is important later). The state also includes participation status (denoted by \( I_t \)) where 1 denotes participation and 0 denotes non-participation.

**Appendix C  Numerical Solution**

We exploit the scale-independence of the maximization problem and rewrite all variables as ratios to the permanent component of labor income \( (Y_{it}^p) \). The laws of motion and the value function can then be rewritten in terms of these normalized variables, and we use lower case letters to denote them. This normalization allows us to reduce the number of state variables to three: liquid wealth, participation status, and age. The problem is solved as follows.

For households who are already stock market participants, there is no par-
ticipation decision. Their value function is given by:

\[
v^j_t(x_{it}, I_t = 1) = \max_{c_t, \alpha_t^j, b_t} \left\{ (1 - \beta)c_t^{1-1/\psi_j} + \beta \left( E_t \left\{ \left( \frac{Y_{it+1}^p}{Y_{it}^p} \right)^{1-\rho_j} \left( \xi_t[v_{it+1}^j(x_{it+1}, I_{t+1} = 1)]^{1-\rho_j} \right) \right) \right) \right\}^{1-1/\psi_j}
\]

Non-participants decide whether or not to incur the fixed cost \( F \) at time (age) \( t \) and this immediately comes out of their cash on hand \( x_{it} \). They compare the two value functions associated with direct stock market participation or continued non-participation:

\[
v^j_t(x_{it}, I_t = 0) = \max_{0,1} \{ v^j_t(x_{it}, I_t = 0), v^j_t(x_{it} - F, I_t = 1) \}
\]

where \( I_t = 1 \) denotes stock market participation. The value of remaining a non-participant is given by:

\[
v^j_t(x_{it}, I_t = 0) = \max_{c_t, \alpha_t^j} \left\{ (1 - \beta)c_t^{1-1/\psi_j} + \beta \left( E_t \left\{ \left( \frac{Y_{it+1}^p}{Y_{it}^p} \right)^{1-\rho_j} \left( \xi_t[v_{it+1}^j(x_{it+1}, I_{t+1} = 0)]^{1-\rho_j} \right) \right) \right) \right\}^{1-1/\psi_j}
\]

where

\[
x^0_{it+1} = \frac{r_{it+1}}{g_{it+1}} b_{it} + \frac{r_{it+1}^m}{g_{it+1}} m_{it} + U_{it+1} - \frac{\omega_{it}}{g_{it+1}}
\]

is normalized cash-on-hand in period \( t + 1 \) conditional on the decision not to begin stock market participation at time \( t \).
For those who decide to participate

\[ v_t^j(x_{it} - F, I_t = 1) = \max_{\alpha_s^t, \alpha_b^t} \left\{ (1 - \beta) v_{it}^{1-1/\psi} \right\} \]

\[ + \beta \left( E_t \left\{ \left( \frac{Y_{it+1}^p}{Y_{it}^p} \right)^{1-\rho} \right\} \right) \]

\[ \left( \xi_t [v_{it+1}^j(x_{it+1}^1, I_{t+1} = 1)]^{1-\rho} \right) \]

\[ + (1 - \xi_t) \varphi_j (x_{it+1}^1)^{1-\rho} \]

where

\[ x_{it+1}^1 = \frac{r_{t+1}^s}{g_{it+1}} s_{it} + \frac{r_{t+1}^b}{g_{it+1}} b_{it} + \frac{r_{t+1}^m}{g_{it+1}} m_{it} + U_{it+1} - \frac{\omega_{it}}{g_{it+1}} \]

is normalized cash-on-hand in period \( t+1 \) conditional on the decision to begin stock market participation at time \( t \). The main difference between \( x_{it+1}^0 \) and \( x_{it+1}^1 \) lies in the fact that \( x_{it+1}^1 \) includes returns from holding stocks \( \left( \frac{r_{t+1}^s}{g_{it+1}} s_{it} \right) \).

We solve the model recursively backwards starting from the last period for households of each type \( j = H, L \). In the last period \( t = T \) the policy functions are trivial and the value function corresponds to the bequest function.

We need to solve for four control variables in every year for stock-holders: current consumption \( (c_t) \), the fraction of the portfolio allocated to stocks \( (\alpha_s^t) \) and bonds \( (\alpha_b^t) \) (the fraction of saving allocated to money \( \alpha_m^t \) can be determined as the residual) and the participation decision.

For every age \( t \) prior to \( T \), and for each point in the state space, we optimize using grid search. From the Bellman equation the optimal decisions are given as current utility plus the discounted expected continuation value \( (E_t v_{it+1}^j(\cdot)) \), which we can compute since we have just obtained \( v_{it+1}^j \). We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the risky asset returns. Cubic splines are used to perform the interpolation of the value func-
tion for points which do not lie on the state space grid, with more points used at lower levels of wealth where the value function has high curvature. Once we have computed the value of each alternative we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation, we obtain this period’s value function ($v_t^j(.)$), which is then used to solve the previous period’s maximization problem. This process is iterated until $t = 1$. 


References


Figure 1. Stock price index in Japan (1985-2014)

Nikkei Stock Price Index
Figure 2: Inflation in Japan

Inflation 1980-2015 (%)

% change from the previous year
Figure 3. Time Deposit Rate (1-2 years): 1994-2014
Figure 4. Stock Market Participation Rate (1981-2014)
Figure 5. Household financial portfolio: 1981-2014

All households
Figure 6. Stockholders’ Financial Portfolio: 1981-2014
Figure 7. Non-stockholders’ Financial Portfolio: 1981-2014
Figure 8: Policy functions of low IES-low RRA household
Figure 9. Policy functions of high IES-high RRA household
Figure 10. Consumption, financial wealth and income over the life cycle.
Figure 11. Portfolio choice over the life cycle
Figure 12. Stock market participation rate

SHF data (2007-2014)
Figure 13: Japanese and US longevity