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This study develops a method for decomposing the demand for illiquid securities into the commitment motive and the interest motive, and characterizes the commitment motive by the interest rate and the degree of time inconsistency. By this analysis, the present study demonstrates the following two results. First, if the interest rate of illiquid securities is higher than that of liquid securities, then the more time-inconsistent a consumer is, the larger the commitment motive is, in comparison with the interest motive. Second, if the interest rate of illiquid securities is lower than that of liquid securities, then a time-inconsistent consumer's demand for illiquid securities is only from the commitment motive.

Keywords: Illiquid securities; Time-inconsistent preferences; Commitment device
JEL classification: D11; D91

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1 Introduction

It is well known that a consumer demands illiquid securities, such as long-term bonds, because illiquid securities provide higher interest than do liquid securities, such as short-term bonds. In addition to this interest motive, as Laibson (1997) points out, a consumer who has time-inconsistent preferences demands illiquid securities from the commitment motive, i.e., in order to commit himself to his future consumption plan that is made in the current period by using them. However, we do not know how large the commitment motive is, in comparison with the interest motive.

This study develops a method for decomposing the demand for illiquid securities into the commitment motive and the interest motive, and characterizes the commitment motive by the interest rate and the degree of time inconsistency. By this analysis, the present study demonstrates the following two results. First, if the interest rate of illiquid securities is higher than that of liquid securities, then the more time-inconsistent a consumer is, the larger the commitment motive is, in comparison with the interest motive. Second, if the interest rate of illiquid securities is lower than that of liquid securities, then a time-inconsistent consumer's demand for illiquid securities is only from the commitment motive.

Kocherlakota (2001) also constructs a model, in which a time-inconsistent consumer demands securities. However, unlike the present study, his study is not concerned with the distinction between the commtiment motive and the interest motive.

The structure of the present study is as follows: Section 2 explains the method and presents the models. Section 3 demonstrates the results.

2 The method and models

2.1 The method

The method for decomposing the demand for illiquid securities into the commitment motive and the interest motive is as follows. First, we measure the demand for illiquid securities. Second, we replace illiquid securities with virtual securities, called v-securities, and measure the demand for v-securities. v-securities are the same as illiquid securities, except that only illiquid securities work as a commitment device. Third, we calculate the difference between the demand for illiquid securities and the demand for v-securities. Then, the difference represents the commitment motive. This is because illiquid securities and v-securities differ in that only illiquid securities work as a commitment device. Finally, the demand for v-securities represents the interest motive of the demand for illiquid securities. This is because v-securities and liquid securities differ only in that v-securities provide higher interest rate than do liquid securities, and the interest rate of v-securities is assumed to be the same as that of illiquid securities.

We call the case in which illiquid securities exist Model (I) and call the case in which v-securities exist Model (II). Model (I) is the same three-period consumption model as Model (II), except that there are illiquid securities only in Model (I) and that there are v-securities only in Model (II).

2.2 Preferences

In each of the models, a consumer supplies one unit of labor in the first period and receives labor income $y > 0$ in the second period. He consumes one good in the second and third periods. The preferences over consumption are common to both models.

A consumer's preferences are as follows: In the first period, the preferences over consumption are represented by

$$J_1 = \sqrt{c_2} + \delta\sqrt{c_3}, \tag{1}$$

where c_2 is the consumption in the second period, c_3 is the consumption in the third period, and $\delta \in (0, 1]$ is a discount parameter. In the second period, the preferences over consumption become

$$J_2 = \sqrt{c_2} + \beta\delta\sqrt{c_3}, \tag{2}$$

where $\beta \in (0, 1)$ represents time inconsistency in preferences between the first period and the second period. The smaller β is, the more time inconsistent a consumer is. In the third period, the preferences over consumption are represented by

$$J_3 = \sqrt{c_3}, \tag{3}$$

where c_3 is the consumption in the third period. This structure of preferences is a simplified version of the quasi-hyperbolic preferences, which were introduced by Phelps and Pollak (1968) and used by Laibson (1997).

2.3 Securities

Illiquid securities are sold in the first period, and the return (i.e., the principal and interest) accrues in the third period. Since the time at which a consumer receives labor income is the second period, his purchase of illiquid securities in the first period means that he makes a contract to purchase them in the first period. Liquid securities and v-securities are sold in the second period, and the return accrues in the third period. In both models, it is assumed that a consumer cannot sell any types of securities in all of the periods.

2.4 The budget constraints in Model (I)

A consumer's budget constraints in Model (I) are as follows: In the first period, a consumer purchases z units of illiquid securities within his income:

$$z \leq y, \ z \geq 0. \tag{4}$$

In the second period, he receives $y - z$. He consumes c_2 units of the good and purchases x units of liquid securities:

$$c_2 + x \leq y - z, \ c_2 \geq 0, \ x \geq 0. \tag{5}$$

In the third period, he receives the principals and interests on liquid and illiquid securities. He consumes c_3 units of the good:

$$c_3 \leq Qx + Rz, \ c_3 \geq 0, \tag{6}$$

where Q is the gross interest rate of liquid securities and R is the gross interest rate of illiquid securities.

2.5 The budget constraints in Model (II)

A consumer's budget constraints in Model (II) are as follows. Since there are no illiquid securities in this model, a consumer cannot purchase securities in the first period. In the second period, he receives labor income y. He consumes c_2 units of the good and purchases x units of liquid securities and v units of v-securities within his income:

$$c_2 + x + v \leq y, \ c_2 \geq 0, \ x \geq 0, \ v \geq 0. \tag{7}$$

In the third period, he receives the principals and interests on liquid securities and v-securities. He consumes c_3 units of the good:

$$c_3 \leq Qx + Rv, \ c_3 \geq 0, \tag{8}$$

where Q is the gross interest rate of liquid securities and R is the gross interest rate of v-securities, which is the same as that of illiquid securities.

2.6 Consumer behavior

This study assumes that a consumer is "sophisticated." That is, a consumer recognizes time inconsistency in preferences and acts in consideration of the future changes in preferences. Under this assumption, a consumer's actual consumption path is solved backward.[1]

The actual consumption path in Model (I) is the outcome of the following procedure. In the third period, a consumer chooses consumption c_3 so as to maximize J_3 subject to (6), taking a securities allocation (x, z) as given. The optimal consumption choice in the third period becomes a function $c_3(x, z)$. In the second period, he chooses consumption c_2 and securities x so as to maximize J_2 subject to (5) and $c_3(x, z)$, taking z as given. The optimal choices in the second period become functions $c_2(z)$ and $x(z)$. In the first period, he chooses illiquid securities z so as to maximize J_1 subject to (4), $c_2(z)$ and $c_3(x(z), z)$. The actual consumption path in Model (I) is given by $(c_2(z^\star), c_3(x(z^\star), z^\star))$, where $z^\star$ is the optimal value of z.

The actual consumption path in Model (II) is the outcome of the following procedure. In the third period, a consumer chooses consumption c_3 so as to maximize J_3 subject to (8), taking a securities allocation (x, v) as given. The optimal consumption choice becomes a function $c_3(x, v)$. In the second period, he chooses consumption c_2, securities x, and securities v so as to maximize J_2 subject to (7) and $c_3(x, v)$. The actual consumption path in Model (II) is given by $(c_2^\star, c_3(x^\star, v^\star))$, where $c_2^\star$, $x^\star$, and $v^\star$ are the optimal values of c_2, x, and v, respectively.

2.7 The demand for illiquid securities

This subsection explains why a consumer demands illiquid securities in Model (I). First, a consumer demands illiquid securities from the commitment motive. By assumption, a consumer realizes in the first period that his preferences change to J_2 in the second period, and that his actual consumption path does not necessarily coincide with his initial consumption plan, which is made in the first period so as to maximize J_1 subject to his lifetime budget constraint. Hence, to solve this problem, he demands illiquid securities in the first period, thereby committing himself to his future consumption. This is the commitment motive. Second, if the interest rate of illiquid securities is higher than that of liquid securities, a consumer demands illiquid securities from the interest motive. Hence, a consumer demands illiquid securities from the two motives.

[1] See, for example, O'Donoghue and Rabin (1999).

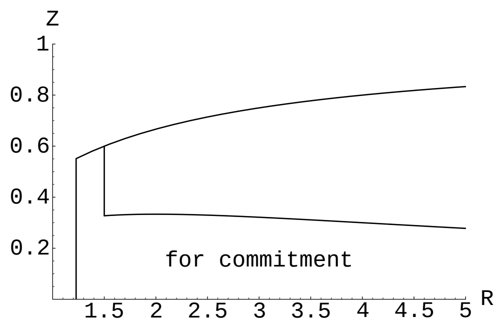

Figure 1: The demand for illiquid securities in the case of $\beta = 0.5$

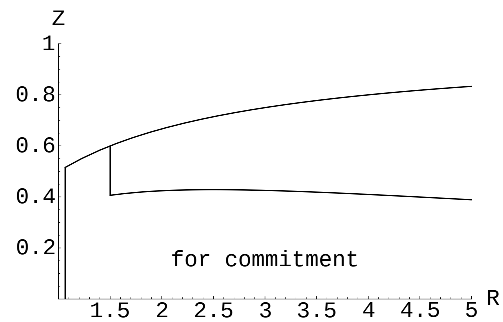

Figure 2: The demand for illiquid securities in the case of $\beta = 0.4$

3 Results

Section 2.1 explains that the demand for illiquid securities, $z^\star$, is decomposed into the commitment motive, $z^\star - v^\star$, and the interest motive, $v^\star$. This section presents actual decomposition and demonstrates the results.

The demand for illiquid securities when $Q = 1.5$, $y = 1.0$, and $\delta = 1.0$ is illustrated in Figures 1 and 2. Figure 1 plots the demand in the case of $\beta = 0.5$; figure 2 plots the demand in the case of $\beta = 0.4$. The lower part of each graph indicates the commitment motive, which is equal to $z^\star - v^\star$. The upper part of each graph indicates the interest motive, which is equal to $v^\star$.

The first result of this study is that if the interest rate of illiquid securities is higher than that of liquid securities, as a consumer becomes more time inconsistent, the ratio of the commitment demand to the total demand rises.[2] For example, if $R = 2$, the ratio in the case of $\beta = 0.5$ is 0.5; the ratio in the case of $\beta = 0.4$ is 0.64. This result implies that in this case, the more time-inconsistent a consumer is, the larger the commitment motive is, in comparison with the interest motive. Proposition 1 in the Appendix demonstrates this result formally.

The second result of this study is that if the interest rate of illiquid securities is lower than that of liquid securities, the ratio of the commitment demand to the total demand is 1. This implies that in this case, a time-inconsistent consumer's demand for illiquid securities is only from the commitment motive, i.e., $v^\star = 0$.

Finally, if a consumer is time consistent (i.e., $\beta = 1$), there is no commitment motive, i.e., $z^\star - v^\star = 0$. He demands illiquid securities from the interest motive, only if the interest rate of illiquid securities is higher than that of liquid securities.

[2] Even if a consumer consumes the good in the first period, this result still holds.

Appendix

Let $\hat{z}$ be the ratio of $z^\star - v^\star$ to $z^\star$, where $z^\star$ and $v^\star$ are the demand for illiquid securities and v-securities, respectively.

Proposition 1. *For any Q, y, and δ, if $R > Q$, as β decreases, $\hat{z}$ rises.*

Proof. Let $\bar{R} = \frac{2\beta Q + (\beta\delta Q)^2 - \beta^2 Q}{1+(\beta\delta)^2 Q}$. It is shown that $\bar{R} < Q$,

$$z^\star = \begin{cases} \frac{\delta^2 R}{1+\delta^2 R} y & \text{if} \quad \bar{R} < R \\ 0 & \text{if} \quad R < \bar{R}, \end{cases} \tag{9}$$

and

$$v^\star = \begin{cases} \frac{(\beta\delta)^2 R}{1+(\beta\delta)^2 R} y & \text{if} \quad Q < R \\ 0 & \text{if} \quad R < Q. \end{cases} \tag{10}$$

Hence, it holds that

$$\hat{z} = \frac{z^\star - v^\star}{z^\star} = \begin{cases} \frac{1-\beta^2}{1+\beta^2\delta^2 R} & \text{if} \quad Q < R \\ 1 & \text{if} \quad \bar{R} < R < Q. \end{cases} \tag{11}$$

If $R > Q$, $\frac{\partial \hat{z}}{\partial \beta} = -\frac{2\beta(1+\beta^2\delta^2 R) + 2\beta\delta^2 R(1-\beta^2)}{(1+\beta^2\delta^2 R)^2} < 0$. □

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