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Illiquid Securities and Time-Inconsistent Preferences

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Abstract

This paper characterizes the effectiveness of illiquid securities as a commitment device. That is, it derives a condition under which illiquid securities enable a consumer who has time-inconsistent preferences to commit himself to his future consumption choices. By this analysis, it is shown that the value of his initial endowment is important for illiquid securities to work as a commitment device.

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Illiquid Securities and Time-Inconsistent Preferences

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This paper characterizes the effectiveness of illiquid securities as a commitment device. That is, it derives a condition under which illiquid securities enable a consumer who has time-inconsistent preferences to commit himself to his future consumption choices. By this analysis, it is shown that the value of his initial endowment is important for illiquid securities to work as a commitment device.
Keywords: Illiquid securities; Time-inconsistent preferences; Commitment device
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1 Introduction

Laibson (1997) demonstrates that illiquid securities, such as long-term bonds, work as a partial commitment device. That is, illiquid securities enable a consumer with time-inconsistent preferences to commit himself to his future consumption choices provided his income satisfies certain time patterns. However, the function of illiquid securities as a partial commitment device has not been fully characterized with respect to the time patterns of a consumer's income.

The purpose of this paper is to provide a complete characterization of the effectiveness of illiquid securities as a commitment device. For this purpose, the present paper constructs a simple consumption model, in which the relationship between a time pattern of income and a consumption path of a time-inconsistent consumer when he uses illiquid securities as a commitment device is tractable. By using this model, the present paper derives a condition in a time pattern of income under which illiquid securities enable a time-inconsistent consumer to commit himself to his future consumption choices.

This analysis demonstrates that only if a consumer has large initial income, he can commit himself to his future consumption choices. This implies that only in this case, illiquid securities can serve as a perfect commitment device. Hence, the value of the initial income is important for illiquid securities to work as a commitment device.

This study is related to that of Bernheim, Ray, and Yeltekin (1999), which, like the present study, examined the effect of an exogenous wealth change on consumption-savings decisions of a time-inconsistent consumer.[1] However, their study does not consider the role of illiquid securities as a commitment device, which is highlighted in this study.

2 The model

The model of this paper is a three-period consumption model, which is a short-period version of Laibson's model. It assumes that there is one kind of good, that there is no uncertainty, and that a consumer lives for three periods and consumes the good in the second and third periods.

The preferences of a consumer are as follows: In the first period, the

[1]The other related study is by Diamond and Kőszegi (2002). Unlike the present study, theirs incorporates a retirement choice into a model with a time-inconsistent consumer. By using the model, their study examines the effect of an exogenous wealth change on retirement and consumption-savings decisions of a time-inconsistent consumer.

preferences over consumption are represented by

$$J_1 = \ln c_2 + \delta \ln c_3, \tag{1}$$

where c_2 is the consumption in the second period, c_3 is the consumption in the third period, and $\delta \in (0, 1]$ is a discount parameter. In the second period, the preferences over consumption become

$$J_2 = \ln c_2 + \beta\delta \ln c_3, \tag{2}$$

where $\beta \in (0, 1)$ represents the time inconsistency in preferences between the first period and the second period. In the third period, the preferences over consumption are represented by

$$J_3 = \ln c_3, \tag{3}$$

where c_3 is the consumption in the third period.

This structure of preferences is a simplified version of the quasi-hyperbolic preferences, which were introduced by Phelps and Pollak (1968) and used by Laibson (1997). In general, the quasi-hyperbolic preferences in a three-period model are given by

$$\begin{cases} \hat{J}_1 = u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) \\ \hat{J}_2 = u(c_2) + \beta\delta u(c_3) \\ \hat{J}_3 = u(c_3), \end{cases} \tag{4}$$

where $\hat{J}_i$ represents the preferences in the ith period and c_i is the consumption in the ith period. If $u(c_1)$ is set to zero, the period utility function is assumed to be logarithmic, and $\hat{J}_1$ is replaced by $J_1 = \frac{\hat{J}_1}{\beta\delta}$, thence the preferences of the model of this paper are derived.

Following Laibson's model, this paper assumes that there are two types of securities: illiquid securities and liquid securities. There are many identical firms, and each firm sells both types of securities in order to finance its project. The project of a firm begins in the second period, and the return on that project accrues in the third period. The rate of return is constant, relative to the size of the project, and the rates of returns of all firms are identical. In order to finance its project, a firm can sell illiquid securities in the first period and liquid securities in the second period. Moreover, following Laibson's model, this paper assumes that a consumer cannot sell securities in every period. Hence, a consumer cannot borrow.

Under these assumptions, if a consumer purchases illiquid securities in the first period, he receives the principal and interest two periods later. If a consumer purchases liquid securities in the second period, he receives the principal and interest one period later. Moreover, as in Laibson's model, the interest accruing to an illiquid security must be the same as that accruing to a liquid security. This implies that the sum of the lifetime income of a consumer becomes independent of the types of securities in which he invests.

A consumer's budget constraints are as follows: In the first period, a consumer has an initial endowment $y_1 > 0$. He purchases z_1 units of illiquid securities and keeps x_1 units of the good on hand:

$$z_1 + x_1 \leq y_1,\ z_1 \geq 0,\ x_1 \geq 0. \tag{5}$$

In the second period, he supplies one unit of labor to a firm and receives labor income $y_2 > 0$. He consumes c_2 units of the good, purchases z_2 units of liquid securities, and keeps x_2 units of the good on hand:

$$c_2 + z_2 + x_2 \leq y_2 + x_1,\ c_2 \geq 0,\ z_2 \geq 0,\ x_2 \geq 0. \tag{6}$$

In the third period, he receives the returns on the securities (i.e., the principal and interest) in which he invested in the first and second periods, $(1+r)(z_1+z_2)$, where $r > 0$ is a common interest rate. He consumes c_3 units of the good:

$$c_3 \leq (1+r)(z_1 + z_2) + x_2,\ c_3 \geq 0. \tag{7}$$

3 The condition for commitment

This section derives a condition under which illiquid securities enable a time-inconsistent consumer to commit himself to his future consumption choices. We assume that a time-inconsistent consumer recognizes the time inconsistency in preferences and acts in consideration of the future changes in preferences. Under this assumption, the actual consumption path of a time-inconsistent consumer is solved backward.[2] In the third period, a consumer chooses consumption c_3 to maximize J_3 subject to (7), taking z_1, z_2, and x_2 as given. In the second period, he chooses securities z_2, consumption c_2, and x_2 to maximize J_2 subject to (6) and the optimal consumption choice in the third period, taking z_1 and x_1 as given. In the first period, he chooses securities z_1 and x_1 to maximize J_1 subject to (5) and the optimal choices in the second and third periods. The actual consumption path is the outcome of this procedure.

In general, if a consumer has time-inconsistent preferences, the actual consumption path of the consumer differs from his initial consumption plan,

[2]This is a prevalent approach when a consumer recognizes time inconsistency in preferences. This type of consumer is called sophisticated. See O'Donoghue and Rabin (1999).

which is the one made in the first period. However, by purchasing illiquid securities in the first period, a time-inconsistent consumer may be able to commit himself to his future consumption choices. As Laibson points out, whether he can actually commit or not depends on time patterns of his income. The following proposition gives the condition.

Proposition 1. *1. If $\frac{y_2}{y_1} \leq \frac{1}{\delta}$, then the actual consumption path of a time-inconsistent consumer coincides with the initial consumption plan of the consumer. Otherwise, the actual consumption path is different from the initial consumption plan.*

2. Moreover, if $\frac{1}{\beta\delta} \leq \frac{y_2}{y_1}$, the actual consumption path coincides with the consumption path in the case in which there is no commitment device.

This proposition states the following results: If the value of the income ratio is less than or equal to $\frac{1}{\delta}$, illiquid securities work as a commitment device, and a time-inconsistent consumer can commit himself to his future consumption choices perfectly. However, if the value of the ratio is greater than $\frac{1}{\delta}$, the effectiveness of illiquid securities as a commitment device falls, and a time-inconsistent consumer cannot commit himself perfectly. In particular, if the value of the income ratio is greater than or equal to $\frac{1}{\beta\delta}$, illiquid securities do not play a role as a commitment device, and the actual consumption path coincides with the path in the case in which there is no commitment device.

This analysis demonstrates that only if a consumer has large initial income, he can commit himself to his future consumption choices. This implies that only in this case, illiquid securities can serve as a perfect commitment device. Hence, the value of the initial income is important for illiquid securities to work as a commitment device. Moreover, because of this, their effectiveness depends on the time pattern of income.

Appendix: Proof of Proposition 1

Consider the actual consumption path of a time-inconsistent consumer. In the second period, c_2, z_2, and x_2 are chosen so as to maximize the following problem, taking z_1 and x_1 as given:

$$(c_2^\star(z_1, x_1), z_2^\star(z_1, x_1), x_2^\star(z_1, x_1)) = \arg \max_{(c_2, z_2, x_2) \in \mathbb{R}_+^3} \ln c_2 + \beta\delta \ln c_3^\star(z_1, z_2, x_2)$$

$$\text{s.t. } c_2 + z_2 + x_2 \leq y_2 + x_1,$$

where the objective function is J_2 and the budget constraint is (6). Since c_3 is chosen in the third period, it is not a control variable but a function. In the first period, z_1 and x_1 are chosen so as to maximize the following problem:

$$(z_1^\star, x_1^\star) = \arg \max_{(z_1,x_1)\in\mathbb{R}_+^2} \ln c_2^\star(z_1,x_1) + \delta \ln c_3^\star(z_1,x_1)$$

$$\text{s.t. } z_1 + x_1 \le y_1,$$

where c_2 and c_3 are functions, and $c_3^\star(z_1,x_1) = c_3^\star(z_1, z_2^\star(z_1,x_1), x_2^\star(z_1,x_1))$. The objective function is J_1 and the budget constraint is (5). The actual consumption path $(c_2,c_3) = (c_2^\star(z_1^\star,x_1^\star), c_3^\star(z_1^\star,x_1^\star))$ becomes

$$(c_2,c_3) = \begin{cases} (\frac{1}{1+\delta}(y_1+y_2), \frac{(1+r)\delta}{1+\delta}(y_1+y_2)) & \text{if} \quad \frac{y_2}{y_1} \le \frac{1}{\delta} \\ (y_2, (1+r)y_1) & \text{if} \quad \frac{1}{\delta} \le \frac{y_2}{y_1} \le \frac{1}{\beta\delta} \\ (\frac{1}{1+\beta\delta}(y_1+y_2), \frac{(1+r)\beta\delta}{1+\beta\delta}(y_1+y_2)) & \text{if} \quad \frac{1}{\beta\delta} \le \frac{y_2}{y_1}. \end{cases} \tag{8}$$

If $\frac{y_2}{y_1} \le \frac{1}{\delta}$, the actual consumption path coincides with the initial consumption plan, which is the solution to the following problem: $\max_{(c_2,c_3)} \ln c_2 + \delta \ln c_3$ subject to $c_2 + \frac{1}{1+r}c_3 \le y_1 + y_2$. In this problem, the objective function is J_1 and the constraint is the lifetime budget constraint, which is derived from (5), (6), and (7). If $\frac{1}{\beta\delta} \le \frac{y_2}{y_1}$, the actual consumption path coincides with the consumption path in the case in which there is no commitment device.[3]

[3]Except that z_1 must be zero, the consumption path in this case is solved in the same manner as in the case in which there are illiquid securities.

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