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Abstract

This paper presents an overlapping generations model of growth and the environment in which government's environmental investment is financed by issuing public debt. Under this framework, the paper considers the effects of public debt-financed environmental investment on the allocation of capital and environmental quality as well as on the welfare of generations. The following three results are shown. First, there are two non-trivial, dynamically inefficient steady state equilibria: one is a saddle and the other is a sink. Second, the environmental investment financed by public debt enhances (degrades) capital accumulation at the saddle (sink) equilibrium. In other words, non-environmental welfare (i.e., utility from consumption) is deteriorated (improved) at the saddle (sink) equilibrium. Third, under a certain condition, the investment improves (degrades) environmental welfare (i.e., environmental quality) at the saddle (sink) equilibrium. These results imply the difficulty of public debt-financed environmental policy for attaining a double dividend, that is, improvements of both non-environmental and environmental welfare in each equilibrium.

Key words: Public debt, Double dividend, Overlapping generations, Environmental quality, Capital accumulation, Environmental investment

JEL Classification Number: H50, H63, Q28.

1 Introduction

This paper considers the effects of public debt-financed environmental policy on the allocation of capital and environmental quality as well as on the welfare of generations in an overlapping-generations model based on Diamond (1965). Although a large number of studies have analyzed the intergenerational effects of environmental policy in a framework of overlapping generations, most of them have been related to environmental tax policy.[1] In fact, the tax policy would succeed in protecting the environment enjoyed by future generations, but the policy imposes a burden on a current generation that does not necessarily benefit from environmental preservation. This intergenerational trade-off is one of the reasons why environmental tax policy is often opposed by currently living voters.

For the purpose of resolving such an intergenerational trade-off, an environmental investment financed by public debt has been proposed and has already been carried out in several countries (UNEP(1997)). This debt policy implies that, by financing environmental investment with the issue of public debt, people in a current generation can contribute to future environmental preservation by transferring the burden of the current environmental investment to future generations that will benefit from environmental preservation. The current paper therefore aims to examine the possibility of debt-financed environmental policy for cleaning up the problem of intergenerational trade-off with respect to environmental preservation and welfare.

To the best of our knowledge, only the following three papers have considered public debt-financed environmental policy: John et al. (1995), Bovenberg and Heijdra (1998), and Bovenberg and Heijdra (2002). John et al. (1995), who utilize the two-period overlapping-generations model of Diamond (1965), indicate the possibility of public debt-financed environmental policy as an instrument of achieving an intergenerationally efficient capital allocation and environmental protection. However, they leave the analysis of environmental debt policy for future research. Bovenberg and Heijdra (1998) use the continuous time overlapping-generations model of Yaari (1965), Blanchard (1985), and Buiter (1988) by including the environment into the model, and show that the incorporation of public debt into an environmental tax scheme can produce Pareto improvements for every generation, and that otherwise a current generation is made worse off by the environmental tax burden. Their analysis focuses mainly on pollution

[1] See, for example, John and Pecchenino (1994), John et al. (1995), Ono (1996), Fisher and Marrewijk (1997), Zhang (1999), Jouvet, Michel, and Vidal (2000), Jones and Manuelli (2001), Wendner (2001), Ono and Maeda (2002), and Ono (2003).

tax rather than public debt. The study of Bovenberg and Heijdra (2002) is different from their former study in that they investigate the effect of public abatement (i.e., government's environmental investment) financed by issuing public debt. They consider the intergenerational effects of such policy.

Based on these previous studies, this paper contributes to the literature on public debt-financed environmental policy in the following sense. We conduct an analysis of public debt-financed environmental investment within the framework of John et al. (1995), which has not been fully done in a two-period overlapping-generations framework. In other words, our analysis is the counterpart to that of Bovenberg and Heijdra (2002) who analyze public debt-financed environmental investment in a continuous time overlapping-generations framework. Although the policy we analyze is similar to that in Bovenberg and Heijdra (2002), our result is quite different from theirs, as is shown below.

We show the following three results which might be important in designing an environmental policy financed by public debt. First, there are two dynamically inefficient steady-state equilibria: one is a sink with a higher level of capital and the other is a saddle with a lower level of capital. The multiplicity of dynamically inefficient equilibria in our analysis is quite different from the unique and dynamically efficient equilibrium in Bovenberg and Heijdra (2002). Second, public debt-financed environmental policy enhances (degrades) capital accumulation at the saddle (sink) equilibrium. In other words, non-environmental welfare (i.e., utility from consumption) is deteriorated (improved) by the policy at the saddle (sink) equilibrium. Third, under a certain condition, the policy improves (degrades) environmental welfare (i.e., environmental quality) at the saddle (sink) equilibrium. These results imply the difficulty of the public-debt financed environmental policy for attaining a double dividend, that is, improvements of both non-environmental and environmental welfare. Our analysis therefore shows the difficulty of public debt-financed environmental policy for solving intergenerational conflict concerning environmental preservation and welfare.

Our result showing opposite welfare implications of the policy between two equilibria is quite different from that of Bovenberg and Heijdra (2002) who show decisive welfare consequences due to a unique equilibrium. The difference of our result from that of Bovenberg and Heijdra (2002) comes mainly from the respective models' specifications. Bovenberg and Heijdra (2002) adopted a continuous-time overlapping generations model while we use a two-period overlapping generations model. Moreover, they included lump-sum taxes for financing government spending while we do not; we consider government's environmental investment financed by only public debt issue in order to focus our attention on the effect of public debt-financed environmental policy. Due to such different modeling strategies, we obtain a different result from that of Bovenberg and Heijdra (2002).

The analysis of this paper proceeds as follows. Section 2 develops a two-period overlapping generations model of growth and the environment based on the work of John et al. (1995). Although each generation undertakes environmental maintenance, the activity is not sufficient to preserve environmental quality into the future since each generation is non-altruistic. The long-lived government therefore implements in each period an environment investment financed by public debt. Section 3 characterizes the equilibrium allocation in the steady-state. Section 4 analyzes, for each steady state equilibrium, how the levels of capital, environmental quality, and welfare change when the long-lived government changes its environmental policy. Section 5 provides concluding remarks. All the proofs are included in the appendix.

2 The Model

The model presented here is based on John and Pecchenino (1994) and John et al. (1995). Consider an infinite-horizon economy composed of perfectly competitive firms and finitely-lived agents. A new generation (called generation t) is born in each period $t = 0, 1, 2, ...$, and lives for two periods, youth and old age. We assume no population growth and normalize the size of each generation as unity. Agents are identical in each generation.

Technology

There is only one kind of good in each period. This is produced by the production process using capital and labor. The production occurs within a period according to a constant returns to scale production function which is invariant though time. The output produced at time t, Y_t, is governed by a neoclassical production function,

$$Y_t = F(K_t, L_t) = L_t f(k_t);\ k_t = K_t/L_t,$$

where K_t is an aggregate amount of capital invested, L_t is an aggregate amount of labor employed, and k_t is per capita capital. We assume the following with respect to the intensive form of the production function:

Assumption 1: The intensive form of the production function $f(\cdot) : \Re_+ \to \Re_+$ is continuous, strictly increasing and strictly concave on $\Re_+$ and twice continuously differentiable on $\Re_{++}$ with the following constraints:

(i) $f(0) = 0, \lim_{k \to 0} f'(k) = \infty$, and $\lim_{k \to \infty} f'(k) = 0$;

(ii) $f'(k)k$ is increasing in k.

The first assumption is the Inada condition, which is often utilized in the literature of economic growth. The second assumption is necessary for showing the stability of the steady state equilibrium (in the next section). This assumption is satisfied when the production function is specified by the Cobb-Douglas technology.

Preferences and Endowment

Agents born in period t have preferences over consumption in old age, c_{t+1}, and an index of the quality of the environment when they consume, E_{t+1}.[2] Their preferences are represented by the utility function $u(c_{t+1}, E_{t+1})$ where $u : \Re_t^2 \to \Re$ satisfies the following assumption.

Assumption 2: The utility function $u : \Re_t^2 \to \Re$ is continuous and strictly increasing in each argument on $\Re_+^2$, and quasi-concave and twice continuously differentiable on $\Re_{++}^2$ with the following constraints:

(i) u is homothetic;

(ii) $\lim_{c \to 0} u_c = \infty$ for all $E > 0$ and $\lim_{E \to 0} u_E = \infty$ for all $c > 0$;[3]

(iii) both c and E are normal goods; and

(iv) c and E are gross substitutes.

Assumption (i) makes enables us to write the saving function as a separable form (see (15) below). Assumption (ii) ensures interior solutions. Assumption (iii) implies that the individual's demands for consumption and environmental quality are increasing in wage. Assumption (iv) means that saving is increasing in the interest rate.

Young agents in generation t are each endowed with one unit of labor which they supply to firms inelastically. They divide their wage, w_t, between

[2] We follow John and Pecchenino (1994) and John et al. (1995) by assuming that individuals have defined preferences over consumption in old age and environmental quality in old age. Including environmental quality in youth does not affect the qualitative result of this paper since, by the assumption of environmental quality, environmental investment undertaken by current young agents do not affect the current environmental quality. On the other hand, including consumption in youth may affect the qualitative result since it has an effect on saving decisions. We abstract from saving decisions, that is, consumption in youth, in order to simplify the analysis and to focus our attention on the interaction between debt-financed environmental policy and welfare.

[3] Note that $u_c \equiv \partial u / \partial c$ and $u_E = \partial u / \partial E$.

savings for consumption in old age, s_t, and investment in the environment, m_t. The budget constraint in youth is therefore $s_t + m_t \leq w_t$. In old age, agents supply their savings to firms and earn the gross return $1 + r_{t+1}$. The budget constraint in old age is therefore $c_{t+1} \leq (1 + r_{t+1})s_t$.

Environmental Quality

The environment is modeled as a renewable resource deteriorating at the fixed rate $a \in [0, 1]$ in every period. It is improved by environmental preservation activities undertaken by both the long-lived government and the short-lived agents.[4] Specifically, we assume that

$$E_{t+1} = (1 - a)E_t + m_t + g,$$

where m_t is the amount of environmental investment by agents in generation t and g is that by the long-lived government: g corresponds to public abatement in Bovenberg and Heijdra (2002). This assumption implies that there are two types of maintenance activities in this economy: m_t imposed by short-lived agents and g by the long-lived government. The former (m_t) has an effect on a young generation but not on an old generation. On the other hand, the latter (g) has an effect on both generations since it affects an interest rate (R_t) for an old generation and wage (w_t) for a young generation. This paper therefore treats g parametrically in the sense that the amount of environmental investment engaged in by the government is the same in every period, and then considers the long-run consequence of that investment on the allocation of capital and environmental quality as well as welfare.

Utility Maximization

Following John and Pecchenino (1994) and John et al. (1995), we assume that, in each period t, the young choose environmental investment m_t in order to maximize the utility on the condition that the old are not made worse off by this decision. Given the wage, w_t, the return on saving, $1+r_{t+1}$, environmental quality in period t, E_t, and the maintenance investment by the long-lived government, g, the lifetime choice problem of the agents in generation t is:

$$\max u(c_{t+1}, E_{t+1})$$
$$\text{subject to}$$

[4]An example is the total area of land under forest. It declines by deforestation at the rate a in every period, while it is reforested by individuals' contributions, who enjoy forest (obtain utility from forest) by taking therapeutic walks in the forest, and by the government's tree planting.

$$
\begin{aligned}
& s_t + m_t \leq w_t, && (1)\\
& c_{t+1} \leq (1 + r_{t+1}) s_t, && (2)\\
& E_{t+1} = (1 - a)E_t + m_t + g, && (3)\\
& s_t, m_t \geq 0,
\end{aligned}
$$

where (1) and (2) are the budget constraints and (3) is the environmental equation. This problem is rewritten as

$$
\begin{aligned}
& \max u(c_{t+1}, E_{t+1})\\
& \text{subject to}\\
& \frac{c_{t+1}}{1 + r_{t+1}} + E_{t+1} \leq \widetilde{w}_t, && (4)
\end{aligned}
$$

where $\widetilde{w}_t$ is the social wealth defined by

$$
\widetilde{w}_t = (1 - a)E_t + w_t + g. \tag{5}
$$

Throughout this paper, we focus on the positive maintenance case, $m > 0$.[5] Then, the first-order condition of the above problem for the interior solution is

$$
(1 + r_{t+1}) u_c(c_{t+1}, E_{t+1}) = u_E(c_{t+1}, E_{t+1}). \tag{6}
$$

Under Assumption 2(ii) ensuring interior solutions, (4) and (6) lead to optimal choices of consumption and environmental quality:

$$
\begin{aligned}
c_{t+1} &= c(1 + r_{t+1}, \widetilde{w}_t),\\
E_{t+1} &= E(1 + r_{t+1}, \widetilde{w}_t),
\end{aligned}
$$

and the corresponding saving function is

$$
s_t = \frac{c(1 + r_{t+1}, \widetilde{w}_t)}{1 + r_{t+1}} \equiv s(1 + r_{t+1}, \widetilde{w}_t). \tag{7}
$$

Profit Maximization

[5] If the environmental quality is sufficiently high in period 0, successive generations beginning with generation 0 will choose zero investment in the environment. Environmental quality continues to decrease over time because of the lack of maintenance investment. Then, in some period t, the short-lived government representing generation t will find it worthwhile to invest in the environment. The economy thus finally switches to a situation with positive maintenance. See John and Pecchenino (1994) for a detailed analysis of the equilibrium path including zero maintenance.

Production takes place in a competitive fashion. The first-order conditions for profit maximization are:

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t), \tag{8}$$
$$\rho_t = f'(k_t) \equiv \rho(k_t), \tag{9}$$

where ρ is the rental price of capital. Due to the assumed condition of perfect competition, these conditions imply factor markets clearing.

The government

The long-lived government in this particular economy undertakes a constant level of maintenance investment for the environment, g. Moreover, it can issue public debt with a one-period maturity in order to finance maintenance investment. Thus, the government budget constraint in period t is

$$b_{t+1} = R_t b_t + g, \tag{10}$$

where b_t is the value of public debt maturing during period t, and R_t is the gross rate of return. As regards the left-hand side of the above constraint, b_{t+1}, is the revenue from debt issue.

Capital Market Clearing

A market clearing condition for capital is $k_{t+1}+b_{t+1} = s_t$ which expresses the equality of the total savings by young agents in period t, s_t, to the sum of the stocks of capital and public debt. Since the market for capital is competitive, the following arbitrage condition holds under the assumption of a full depreciation of capital:

$$1 + r_{t+1} = R_{t+1} = \rho_{t+1}. \tag{11}$$

The Initial Old

In period 0, there are both young agents in generation 0 and the initial old agents in generation -1. Each agent in generation -1 is endowed with k_0 units of capital, earns the return $(1 + r_0)k_0$, and consumes it.

3 Equilibrium

Definition 1. An *equilibrium* is a sequence of prices $\{w_t, \rho_t, R_t\}_{t=0}^{\infty}$, a sequence of allocations $\{c_t, E_t\}_{t=0}^{\infty}$, and a sequence of capital stock $\{k_t\}_{t=0}^{\infty}$ and public debt $\{b_t\}_{t=0}^{\infty}$ with the initial condition (E_0, k_0, b_0) and exogenous parameters $\{a, g\}$, such that given these prices and allocations, an agent's utility is maximized, firms' profits are maximized, the government budget constraint is satisfied, and markets clear.

In what follows, we first characterize the equilibrium allocation of capital and debt $\{k_t, b_t\}$. We then examine the existence and multiplicity of equilibria. Finally, the stability of the equilibria is considered.

3.1 Characterization of the Equilibrium

Summarizing (1) - (11), the equilibrium is fully characterized by a sequence $\{E_t, k_t, b_t\}_{t=0}^{\infty}$ with the initial condition (E_0, k_0, b_0), such that the sequence satisfies the following three equations:

$$s(R(k_{t+1}), \widetilde{w}(k_t, E_t; g)) = k_{t+1} + b_{t+1}, \tag{12}$$

$$b_{t+1} = R(k_t)b_t + g, \tag{13}$$

$$E_{t+1} = \widetilde{w}(k_t, E_t; g) - s(R(k_{t+1}), \widetilde{w}(k_t, E_t; g)). \tag{14}$$

Note that $R(k_{t+1}) = 1 + r(k_{t+1}) = \rho(k_{t+1})$ is the gross rate of return on savings, which depends on capital, and that $\widetilde{w}(k_t, E_t; g)$ is the social wealth depending on k_t, E_t, and g (see the definition of social wealth, (5)). Equation (12) is the capital market clearing condition, and equation (13) is the government budget constraint. Equation (14) is the environmental equation derived from (1), (3), and (5).

Under the assumption of the homothetic utility function (Assumption 2(i)), the saving function $s(R(k_{t+1}), \widetilde{w}(k_t, E_t; g))$ can be written in a separable form as

$$s(R(k_{t+1}), \widetilde{w}(k_t, E_t; g)) = h(R(k_{t+1}))\widetilde{w}(k_t, E_t; g), \tag{15}$$

where $0 < h < 1$ and $h'(\cdot) > 0$ hold. Thus, from (12) and (14), the environmental equation can be written as

$$E_t = \left(h(R(k_t))^{-1} - 1\right)(k_t + b_t) \equiv E(k_t, b_t). \tag{16}$$

The direct calculation leads to $\partial E/\partial k > \partial E/\partial b > 0$. The equilibrium is therefore characterized by a sequence $\{k_t, b_t\}_{t=0}^{\infty}$ with the initial condition (k_0, b_0), such that the sequence satisfies the following two equations:

$$h(R(k_{t+1}))[w(k_t) + (1 - a)E(k_t, b_t) + g] = k_{t+1} + R(k_t)b_t + g, \tag{17}$$

$$b_{t+1} = R(k_t)b_t + g. \tag{18}$$

3.2 Existence and Multiplicity of Equilibria

In what follows, we analyze the properties of the model represented by the above two equations, (17) and (18). Equation (17) is rewritten as

$$h(R(k_{t+1}))[\Omega(k_t) + (1-a)\left(h(R(k_t))^{-1} - 1\right)b_t + g] = k_{t+1} + R(k_t)b_t + g, \tag{19}$$

where

$$\Omega(k_t) \equiv w(k_t) + (1-a)\left(h(R(k_t))^{-1} - 1\right) k_t. \tag{20}$$

Define a function F as

$$F(k_{t+1}, k_t, b_t) \equiv h(R(k_{t+1}))\left[w(k_t) + (1-a)\left(h(R(k_t))^{-1} - 1\right)(k_t + b_t) + g\right] - k_{t+1} - R(k_t)b_t - g.$$

Then, we obtain[6]

$$\frac{\partial F}{\partial k_{t+1}} = h'R'\left[w(k_t) + (1-a)\left(h(R(k_t))^{-1} - 1\right)(k_t + b_t) + g\right] - 1 < 0.$$

Thus, (17) can be solved for k_{t+1}. This solution is expressed as

$$k_{t+1} = \phi(k_t, b_t). \tag{21}$$

We also have

$$\begin{aligned}
\frac{\partial F}{\partial k_t} &= h(R(k_{t+1}))\left[w'(k_t) + (1-a)\left(h(R(k_t))^{-1} - 1\right) - h'R'h(R(k_t))^{-2}(k_t + b_t)(1-a)\right] \\
-R'(k_t)b_t &> 0, \\
\frac{\partial F}{\partial b_t} &= h(R(k_{t+1}))(1-a)\left(h(R(k_t))^{-1} - 1\right) - R(k_t).
\end{aligned}$$

Phase Diagram

We proceed with our analysis of the equilibrium using a phase diagram. We initially draw the phase diagram by which (21) yields the following equation:

$$\begin{aligned}
k_{t+1} &\geq k_t \Leftrightarrow F(k_t, k_t, b_t) \geq F(k_{t+1}, k_t, b_t) = 0 \\
&\Leftrightarrow [R(k_t) - (1-a)\{1 - h(R(k_t))\}]\, b_t \leq h(R(k_t))\Omega(k_t) - \{1 - h(R(k_t))\}g - (k_t)^2.
\end{aligned}$$

Note that $F(k_t, k_t, b_t) \geq F(k_{t+1}, k_t, b_t)$ holds from $\partial F/\partial k_{t+1} < 0$, and that $F(k_{t+1}, k_t, b_t) = 0$ holds from the definition of F. To derive the equation in the second line, we use

$$w(k_t) + (1-a)\{h(R(k_t))^{-1} - 1\}(k_t + b_t) = \Omega(k_t) + (1-a)\{h(R(k_t))^{-1} - 1\}b_t.$$

A function $Z(k; g)$ is defined as

$$Z(k; g) \equiv h(R(k))\Omega(k) - \{1 - h(R(k_t))\}g - k.$$

Let $k^+(g)$ be the non-trivial solution for k of $Z(k; g) = 0$, and $k^+ \equiv k^+(0)$.

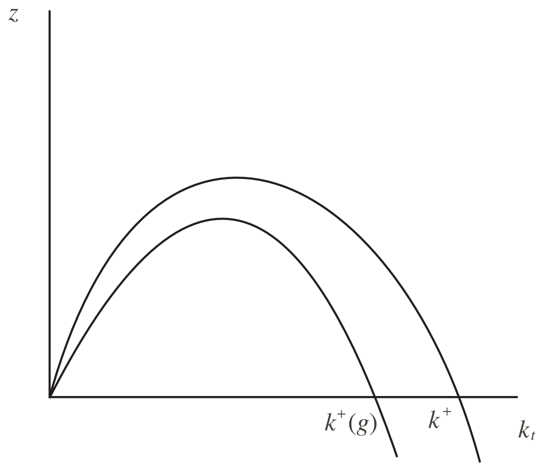

Figure 1:

Assumption 3. There exists a unique $k^+ \in (0, +\infty)$.

Under this assumption, the graph of $Z(k; g)$ is depicted in Figure 1. Since $\lim_{k\to 0} h(R(k)) = 1$,[7] $\lim_{k\to 0} Z(k; g) = 0$.

Let $\hat{k}$ denote the solution of

$$R(k) - (1-a)(1 - h(R(k))) = 0.$$

It is easy to show that $\hat{k}$ exists uniquely. We also have $R(k) - (1-a)(1 - h(R(k))) \gtrless 0$ for $k \lessgtr \hat{k}$. A function $B(k)$ is defined for all $k > 0$ with $k \neq \hat{k}$ as

$$B(k) \equiv \frac{Z(k; g)}{R(k) - (1-a)(1 - h(R(k)))}.$$

Then, it holds that

$$k_{t+1} > k_t \Leftrightarrow b_t \lessgtr B(k_t) \text{ for } k_t \lessgtr \hat{k}. \tag{22}$$

By using (22), we can draw the locus of $\{k_t\}$.

We can now draw the phase diagram by which (18) yields:

$$b_{t+1} > b_t \Leftrightarrow b_t < C(k_t) \equiv \frac{g}{1 - R(k_t)},$$

[6] $h'R'$ is a simplified expression of $h'(R(k))R'(k) = dh/dk$.

[7] Since E_{t+1} cannot be less than $(1-a)E_t + g > 0$, we have $\lim_{R_{t+1}\to\infty} s_t = w_t$ from Assumption 2(ii).

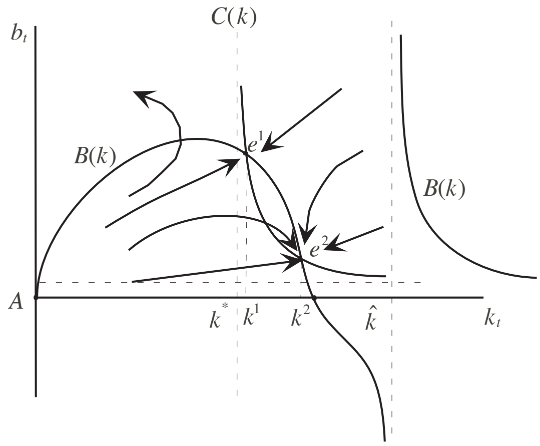

Figure 2:

since we focus on the dynamically inefficient equilibria where $R < 1$ holds. If $R \geq 1$ holds at equilibria, then the amount of public debt becomes negative. We abstract from such a case since we are considering the environmental policy financed by public debt.

Steady State Equilibria

A *steady state equilibrium* is a sequence $\{k, b\}$ that is stationary along the equilibrium path. The steady state equilibrium is calculated as $\{k, b\}$ which satisfies

$$h(R(k))\{w(k) + (1-a)E(k,b) + g\} = k + R(k)b + g, \tag{23}$$

$$b = R(k)b + g. \tag{24}$$

Let k^* be the solution for $R(k) = 1$: the steady state equilibrium is dynamically efficient (inefficient) if and only if $k \leq (>) k^*$. Actually, two cases should be considered: $k^+(g) < \hat{k}$ and $k^+(g) > \hat{k}$. Although the shape of the phase diagram seems to be different for these two cases, the dynamic properties are the same. Therefore, we consider only the case in which $k^* < k^+(g) < \hat{k}$. The phase diagram of this case becomes the one depicted in Figure 2.

As shown in Figure 2, in general there are two non-trivial, dynamically inefficient steady state equilibria with positive environmental investment by the long-lived government. These steady states are shown as e^1 and e^2 in Figure 2. Let k^1 and k^2 be the level of capital at e^1 and e^2, respectively. We denote e^1 as k^1- equilibrium and e^2 as k^2- equilibrium.

3.3 Stability of the Steady State Equilibria

This subsection examines the stability of the dynamically inefficient equilibria. That is, it considers whether the competitive equilibrium converges to a steady state, and to which of the steady states the economy converges.

Proposition 1. *k^1- equilibrium is a saddle and k^2- equilibrium is a sink.*

Proof. See the appendix.

Proposition 1 shows the stability of each steady state equilibrium. The steady state equilibrium with a low k (k^1- equilibrium) represents a saddle equilibrium and the equilibrium with a high k (k^2- equilibrium) is considered as a sink. There is a possibility that two economies with very similar initial capital stock converges to very different steady state equilibria. The initial values of capital and public debt play crucial roles in determining the long-run consequences for the economy.

4 Effects of Public Debt-financed Environmental Policy

This section analyzes how capital, public debt, environmental quality, and welfare are affected when the government increases the amount of the environmental investment, g.

4.1 Effects on Capital and Public Debt

This subsection considers the effects of the environmental investment, g, on capital and public debt.

From (23) and (24), we have

$$\Sigma \begin{pmatrix} dk/dg \\ db/dg \end{pmatrix} = \begin{pmatrix} h \\ 1 \end{pmatrix},$$

where

$$\Sigma = \begin{pmatrix} 1-q-hw'-h'R'(w-(1-a)(k+b)+g) & 1-q \\ -R'b & 1-R \end{pmatrix}. \tag{25}$$

Direct calculation leads to:

$$\frac{dk}{dg} = \frac{1}{\mathrm{Det}(\Sigma)}(h(1-R)+q-1), \tag{26}$$

$$\frac{db}{dg} = \frac{1}{\mathrm{Det}(\Sigma)}\left(1-q-hw'+hR'b-\frac{h'R'}{h}a(k+b)\right), \tag{27}$$

where $q \equiv (1-a)(1-h)$. Then, we have:

Proposition 2.

(i) *An increase in g leads to a higher (lower) level of capital at k^1- equilibrium (k^2 - equilibrium).*

(ii) *An increase in g leads to a higher level of public debt at k^2 - equilibrium, while it leads to either a higher or lower level of public capital at k^1- equilibrium.*

Proof. See the appendix.

As the government's environmental investment g increases, the constant-capital phase line shifts downward while the constant-public-debt phase line shifts upward. This movement will force capital down and public debt up at the stable steady state (k^2 - equilibrium). An increase in g at k^2 - equilibrium improves the non-environmental welfare (i.e., utility from consumption) of some generations without harming other ones. On the other hand, at the saddle steady state, a greater amount of environmental investment g produces the counterintuitive outcome of more capital. An increase in g at k^1-equilibrium degrades the non-environmental welfare of some generations.

4.2 Effects on Environmental Quality

This subsection considers the effects of environmental investment, g, on environmental quality.

Proposition 3.

(i) *At k^1- equilibrium, an increase in g leads to higher environmental quality.*

(ii) *At k^2- equilibrium, an increase in g leads to either higher or lower environmental quality. In particular, it lowers the environmental quality if $w'(\cdot)dk/dg + 1 < 0$.*

Proof. See the appendix.

The effect of environmental investment, g, on the environment is calculated as follows (see the proof of Proposition 3 in Appendix):

$$\frac{dE}{dg} = \frac{1}{1-q}\left[-\frac{\partial s}{\partial R}R'\frac{dk}{dg} + \left(1 - \frac{\partial s}{\partial \tilde{w}}\right)\left(w'\frac{dk}{dg} + 1\right)\right]. \qquad (28)$$

From (28), we can see the *price effect* and the *income effect* of the environmental policy, g. The first term in the bracket in (28), $(-1)\left(\partial s/\partial R\right) R' \left(dk/dg\right)$, is the effect on the environment through the interest rate. The change in the level of capital affects the interest rate, which, in turn, affects savings. We call this effect the "price effect". Since $(-1)\,(\partial s/\partial R)\,R' > 0$ in each equilibrium and $dk/dg > (<) 0$ at $k^1(k^2)$- equilibrium, the first-term is positive (negative) at $k^1(k^2)$- equilibrium. The second term in the bracket in (28), $(1 - \partial s/\partial \tilde{w})\,(w'dk/dg + 1)$, is the effect on the environment through the change in income. The second term in the second bracket, 1, is the direct effect of an increase in g, and the first term in the second bracket, $w'dk/dg$, is the indirect effect caused by the change in the wage rate. We call this effect the "income effect". The sign of the first bracket, $(1 - \partial s/\partial \tilde{w})$, is positive in each equilibrium and the sign of the second bracket, $(w'dk/dg + 1)$, is positive (ambiguous) at $k^1(k^2)$- equilibrium.

When the level of environmental quality increases by one unit at a certain period, the social wealth of that generation increases by the same unit. As a result, the agent's voluntary environmental investment increases by $(1-a)(1-h) = q$ units in the next period. This process continues forever, and the total effect amounts to $1/(1-q)$. This explains why $1/(1-q)$ is placed in front of the bracket term in (28).

Implication for the Double Dividend

The above result also has implications for the double dividend hypothesis. It is said that a policy reform attains the *double dividend* if it improves both non-environmental and environmental welfare (Goulder 1995). In the current model economy, an increase of the government's environmental investment generally fails to attain the double dividend. At k^1- equilibrium, an increase in the environmental investment g degrades non-environmental welfare while it improves environmental welfare. The double dividend is impossible at k^1- equilibrium. On the other hand, at k^2- equilibrium, an increase in the environmental investment g improves non-environmental welfare while it may or may not degrade environmental welfare. In particular, under a certain condition, it degrades environmental welfare (see Proposition 3). Thus, the double dividend is generally impossible at k^2- equilibrium.

4.3 Effects on Total Welfare

This subsection considers the total welfare effects of environmental investment, g. In particular, we consider how agents along the steady state equilibrium are made better off or worse off by an increase in environmental investment.

From the environmental equation (3), the budget constraint in youth (1), and the government budget constraint (10), the environmental quality at the steady state becomes $(1/a)(w - k - Rb)$. From the budget constraint in old age (2) and the capital market clearing condition (12), consumption is $R(k+b)$. Hence, the utility can be written as

$$u = u(R(k+b), a^{-1}(w-k-Rb)).$$

Then, we can calculate the effect of environmental investment on welfare as:

$$\begin{aligned}\frac{du}{dg} &= u_c \cdot \left(R'(k+b)+R)\frac{dk}{dg}+R\frac{db}{dg}\right)+u_E \cdot a^{-1}\left((w'-1-R'b)\frac{dk}{dg}-R\frac{db}{dg}\right) \\ &= u_c \cdot (1-R)\left((k+b)R'\frac{dk}{dg}+R\frac{db}{dg}\right)-u_E \cdot (a^{-1}-1)\left((1+R'(k+b))\frac{dk}{dg}+R\frac{db}{dg}\right).\end{aligned}$$

This equation finally rewritten as

$$\frac{du}{dg} = u_c \cdot (1-R)\left(-w'\frac{dk}{dg}+\left(bR'\frac{dk}{dg}+R\frac{db}{dg}\right)\right)+u_E \cdot (1-a)\frac{dE}{dg}. \quad (29)$$

We should note that $-w'dk/dg$ in (29) measures the net transfer from the young to the old through the change in the wage rate due to the change in g, and $bR'\frac{dk}{dg}+R\frac{db}{dg}$ measures the net transfer from the young to the old through the interest rate payments of the public debt. Hence, the sum of these two factors amounts to the total net transfer from the young to the old. As in Diamond (1965), the term $1-R$ is the factor which measures dynamic efficiency. Therefore, the first term can be interpreted as the term which measures the change in utility due to the change in the dynamic efficiency of the economy. We call this term the *dynamic efficiency effect*. In the second term of (29), dE/dg is the change in environmental quality due to the change in g. An improvement of environmental quality has a positive intergenerational externality, which amounts to $(1-a)dE/dg$. Hence, the change in utility from this effect becomes $u_E(1-a)dE/dg$. We call this term the *intergenerational externality effect*.

In sum, at k^1- equilibrium, an increase in the environmental investment g has a negative dynamic efficiency effect and a positive intergenerational externality effect. At k^2- equilibrium, an increase in the environmental investment g has a positive dynamic efficiency effect, and the intergenerational externality effect is ambiguous. Therefore, the total effect on utility is indecisive at either steady state equilibrium.

Difficulty of Pareto-ranking

In a standard overlapping generations model where a constant amount of government spending is financed by issuing public debt, there are two-nontrivial, dynamically inefficient steady state equilibria which are Pareto-ranked (see, for example, Azariadis (1993, Chapter 20). The saddle equilibrium with a lower level of capital is Pareto-superior to the sink equilibrium with a higher level of capital. Contrary to the result in the standard model, the equilibria in the current model are not generally Pareto-ranked due to the presence of the environment. We should take into account environmental quality as well as the level of capital when we evaluate the efficiency of the equilibrium.

5 Conclusion

The government might consider investing in environmental preservation financed by issuing public debt, since the costs and benefits of such an activity seem to be balanced. However, we should consider the effect of such government activities in a dynamic setting, since issuing public debt affects interest rates which, in turn, have effects on several aspects of the economy.

By means of rigorous analysis, we have shown that there are two nontrivial steady state equilibria: one is a saddle and the other is a sink. At the saddle equilibrium, the effect on the environment is positive, i.e., the government's environmental policy to preserve the environment by issuing public debt improves environmental quality in the long run. However, this policy worsens the dynamic efficiency of the economy, which makes the total effect on welfare ambiguous. At the sink equilibrium, the effect of the government's policy on the environment, as analyzed in this paper, is ambiguous. Under a certain condition, however, this effect is more likely to be negative, that is, the government's policy for the purpose of improving environmental quality may worsen it. Although at this equilibrium, the policy improves the dynamic efficiency of the economy, the total effect is also ambiguous.

Our theoretical analysis suggests that we should be careful when deciding upon the design and magnitude of a policy which has dynamic implications such as the environmental policies analyzed in this paper. Otherwise, the policy may end up producing results that are opposite of the initial aim.

Appendix
Proof of Proposition 1

Linearizing (17) and (18) around the steady state leads to

$$\begin{pmatrix} k_{t+1}-k \\ b_{t+1}-k \end{pmatrix} = \Delta|_{k=k,b=b} \begin{pmatrix} k_t-k \\ b_t-b \end{pmatrix},$$

where

$$\Delta \equiv \begin{pmatrix} \phi_k(k,b) & \phi_b(k,b) \\ R'(k)b & R(k) \end{pmatrix}.$$

By direct calculation, we obtain

$$\begin{aligned} \phi_k(k,b) &= \frac{hw' + (q - h'R'h^{-1}(k+b)) - R'b}{1 - h'R'h^{-1}(k+b)}, \\ \phi_b(k,b) &= \frac{q-R}{1-h'R'h^{-1}(k+b)}, \end{aligned}$$

where

$$q \equiv (1-a)(1-h).$$

The determinant of Δ becomes

$$\mathrm{Det}(\Delta) = \frac{hRw' + q(R-R'b) - (1-a)(k+b)h'R'h^{-1}R}{1-h'R'h^{-1}(k+b)}. \tag{30}$$

Since $1 - h'R'h^{-1}(b+k) > 0$, we have $\mathrm{Det}(\Delta) > 0$. We also have

$$\begin{aligned} 1 - \mathrm{Tr}(\Delta) + \mathrm{Det}(\Delta) &= (1-R)\left(1 - \frac{hw' + q - h'R'h^{-1}(1-a)(k+b)}{1-h'R'h^{-1}(k+b)}\right) \\ &\quad + \frac{R'b(1-q)}{1-h'R'h^{-1}(k+b)}, \end{aligned} \tag{31}$$

where $\mathrm{Tr}(\Delta)$ is the trace of Δ.

In what follows, we prove three lemmas which will be utilized for the proof of Proposition 1.

Lemma A1. $1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta) < 0$ *at* k^1*- equilibrium and* $1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta) > 0$ *at* k^2*- equilibrium.*

Proof. Consider $B(k)$ and $C(k)$ in Figure 2. The slope of $B(k)$ evaluated at the steady state equilibrium becomes

$$\frac{db_t}{dk_t} = \frac{hw' + q - 1 - R'b + h'R'(w - (1-a)(k+b) + g)}{R-q}. \tag{32}$$

The slope of $C(k)$ at the steady state equilibrium becomes

$$\frac{db_t}{dk_t} = \frac{R'b}{1-R}. \tag{33}$$

At k^1- equilibrium, the slope of $B(k)$ is larger than that of $C(k)$, and at k^2- equilibrium, the slope of $C(k)$ is larger than that of $B(k)$. Therefore, in order to prove the lemma, it is sufficient to show that

$$1 - \mathrm{Tr}(\Delta) + \mathrm{Det}(\Delta) < 0 \Leftrightarrow [\text{ the slope of } B(k) > \text{the slope of } C(k)].$$

Using (32) and (33), [the slope of $B(k)$ > the slope of $C(k)$] is equivalent to

$$\frac{z}{R-q} > \frac{R'b}{1-R},$$

where

$$z \equiv hw' + q - 1 - R'b + h'R'(w - (1-a)(k+b) + g).$$

The above inequality becomes

$$R'b(R-q) - z(1-R) < 0. \tag{34}$$

Using (25) and z defined above, we obtain

$$\begin{aligned} \mathrm{Det}(\Sigma) &= (1-R)(-z-R'b) + R'b(1-q) \\ &= (R-q)R'b - z(1-R). \end{aligned}$$

Thus, we have

$$\mathrm{Det}(\Delta)¡0 \Leftrightarrow [\text{the slope of } B(k) > \text{the slope of } C(k)].$$

Hence, the assertion of the lemma is proved since

$$\mathrm{Det}(\Sigma) = \{1 - h'R'h^{-1}(k+b)\}\{1 - \mathrm{Tr}(\Delta) + \mathrm{Det}(\Delta)\}.$$

||

Lemma A2. $q + h'h^{-1}((1-a)R-1) > 0 \Rightarrow q - 1 + (1-R)ah'h^{-1} < 0.$

Proof. From $q + h'h^{-1}((1-a)R-1) > 0$, we have

$$h'h^{-1} < \frac{q}{1-(1-a)R}.$$

Then, we obtain

$$q - 1 + (1-R)ah'h^{-1} < q - 1 + \frac{q}{1-(1-a)R}(1-R)a.$$

Hence, what we must show in order to prove the lemma is that the right-hand side of the above inequality is negative. Since this term decreases with respect to R, we can see the maximum value of this term by putting $R = q$ (i.e., the infimum of R) in the range we are considering.[8] The maximum value of the right-hand side is:

$$(1-q)\left(-1+\frac{aq}{1-(1-a)q}\right).$$

The first term in the above is positive, and it is easy to see that the second term in the above is negative. This completes the proof of this lemma. ||

Lemma A3. $0 < \text{Det}\ (\Delta) < 1$ *at* k^2*- equilibrium.*

Proof. From (31), $1-\text{Tr}(\Delta)+\text{Det}(\Delta) > 0$ is equivalent to

$$bR'(q-1+(1-R)ah'h^{-1}) < (1-R)(1-q-hw'-akh'R'h^{-1}). \tag{35}$$

From (30), Det $(\Delta) < 1$ is equivalent to

$$\begin{aligned} & bR'(q-1+h'h^{-1}((1-a)R-1)) \\ > \;& R(hw'+q+akh'R'h^{-1}-1)+(1-R)(kh'R'h^{-1}-1). \end{aligned} \tag{36}$$

In the case of $q+h'h^{-1}((1-a)R-1) < 0$, the left-hand side of (36) becomes positive. Since the right-hand side of (36) is negative[9], this inequality holds. What we need to show is that (35) implies (36) in the case of $q+h'h^{-1}((1-a)R-1) > 0$.

From Lemma A2, the condition (35) is rewritten as

$$b < \frac{1}{R'} \times \frac{(1-R)(1-q-hw'-akh'R'h^{-1})}{q-1+(1-R)ah'h^{-1}},$$

and the condition (36) is rewritten as

$$b < \frac{1}{R'} \times \frac{R(hw'+q)-1+kh'R'h^{-1}(1-R(1-a))}{q+h'h^{-1}((1-a)R-1)}.$$

Hence, using $R' < 0$, what we need to show is that

$$\frac{(1-R)(z-y)}{-z+(1-R)ah'h^{-1}} > \frac{R(y-z)+(1-R)(h'R'h^{-1}k-1)}{(1-z)+((1-a)R-1))h'h^{-1}}, \tag{37}$$

[8] Since the case $k \in (k^+, \hat{k})$ is considered here, we have $q < R < 1$.

[9] The inequality $w' < R$ holds from Assumption 1(ii). The inequality $R < 1$ holds since the steady state equilibrium considered here is dynamically inefficient. We thus have

$$hw'+q < hR+q < h+q < 1.$$

Hence, the right-hand side of (36) becomes negative.

where $z \equiv 1-(1-a)(1-h)$ and $y \equiv hw' + akh'R'h^{-1}$. Note that $z > y$. The inequality (37) is equivalent to[10]

$$(1-R)(zh'R'h^{-1}k-y)-h'h^{-1}(1-R)^2(z-y-a(h'R'h^{-1}k-1)) < z(z-y). \tag{38}$$

It is easy to verify that $zh'R'h^{-1}k-y<0$ by direct substitution. Then, the left-hand side of (38) is strictly negative. Since the right-hand side of (38) is positive, (38) holds. ∥

We are ready to prove Proposition 1. From Lemma A1, at k^1- equilibrium, we have $1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta) < 0$. Since $\mathrm{Det}(\Delta) > 0$, k^1- equilibrium becomes a saddle. From Lemma A1, at k^2- equilibrium, we have $1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta) > 0$. Since $\mathrm{Tr}(\Delta) > 0$, we also have $1+\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta) > 0$. From Lemma A3, $0 < \mathrm{Det}(\Delta) < 1$ at k^2- equilibrium. Hence, k^2- equilibrium becomes a sink. ∥

Proof of Proposition 2

We first show that $\mathrm{Det}(\Sigma) = (1-h'R'h^{-1}(k+b))(1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta))$. From the definition of Σ, we have

$$\mathrm{Det}(\Sigma) = (1-R)[1-q-hw'-h'R'(w-(1-a)(k+b)+g)] + R'b(1-q)].$$

From the definition of Δ, we have

$$\begin{aligned} & (1-h'R'h^{-1}(k+b)(1-\mathrm{Tr}(\Delta)+\mathrm{Det}(\Delta)) \\ = \; & (1-R)[1-(h'R'/h)(k+b)-hw'-q+(h'R'/h)(1-a)(k+b)] + R'b(1-q) \\ = \; & (1-R)[1-q-hw'-h'R'(a/h)(k+b)] + R'b(1-q). \end{aligned}$$

Then, it is sufficient to show

$$(a/h)(k+b) = w-(1-a)(k+b)+g. \tag{39}$$

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[10] (37) is rewritten as

$$\begin{aligned} & (1-R)(z-y)(1-z+h'h^{-1}((1-a)R-1) \\ < \; & ((1-R)ah'h^{-1}-z)(R(y-z)+(1-R)(h'R'h^{-1}k-1), \end{aligned}$$

which is equivalent to

$$\begin{aligned} & (z-y)((1-R)-z-h'h^{-1}(1-R)^2) \\ < \; & ((1-R)^2ah'h^{-1}-(1-R)z)(h'R'h^{-1}k-1). \end{aligned}$$

We can see that this inequality is equivalent to (38).

At the steady state, we have $E = (1/a)(m + g)$ and $hE = (1 - h)(k + b)$. Then, the right-hand side of (39) becomes

$$\begin{aligned} & w - (1-a)(k+b) + aE - m \\ = \ & a(k+b) + aE; \text{ since } w - m = s = b + k) \\ = \ & a(k+b) + a((1-h)/h)(k+b) \\ = \ & a(k+b)/h, \end{aligned}$$

which is equivalent to the left-hand side of (39). Thus, we can show that Det(Σ)= $(1 - h'R'h^{-1}(k + b))$(1−Tr(Δ)+Det(Δ)).

From Lemma A1, Det(Σ) < 0 at k^1- equilibrium and Det(Σ) > 0 at k^2- equilibrium. Since we are focusing on the case where $k^* < k^+(g) < \hat{k}$, we have $q < R < 1$. Then, the sign of the equation in the parentheses of (26) is negative, which implies that the sign of dk/dg is the opposite of the sign of Det(Σ). Hence, we have $dk/dg > 0$ at k^1- equilibrium and $dk/dg < 0$ at k^2- equilibrium.

The sign of the equation in the parentheses of (27) becomes positive when the curve $B(k)$ in Figure 2 is downward-sloping.[11] At k^2- equilibrium, we have

$$\text{the slope of } B(k) < \text{ the slope of } C(k) < 0,$$

which implies that $db/dg > 0$. At k^1- equilibrium, however, whether (i) is downward-sloping or upward-sloping is ambiguous. Hence, the sign of db/dg is indecisive. $\|$

Proof of Proposition 3

The amount of saving at the steady state is

$$s = h(R(k))[(1-a)E + w(k) + g]. \tag{40}$$

Then, from the environmental equation (3) and the budget constraint in youth (1), we have

$$\begin{aligned} E &= (1-a)E + w(k)Lg - s \\ &= (1 - h(R(k)))[(1-a)E + w(k) + g]; \text{ from (40)}. \end{aligned}$$

[11] From (32), $B(k)$ being downward-sloping is equivalent to

$$hw' + q - 1 - R'b + h'R'(w - (1-a)(k+b) + g) < 0.$$

Using (39) and $R' < 0$, the above inequality is equivalent to the sign of the equation in the parentheses of (27) being positive.

By totally differentiating the above equation, we obtain

$$(1-q)dE = \{(1-h)w' - h'R'w\}dk + (1-h)dg.$$

Since we have $\partial s/\partial \tilde{w} = h$ and $\partial s/\partial R = h'\tilde{w}$, we finally obtain

$$\frac{dE}{dg} = \frac{1}{1-q}\left[-\frac{\partial s}{\partial R}R'\frac{dk}{dg} + \left(1 - \frac{\partial s}{\partial \tilde{w}}\right)\left(w'\frac{dk}{dg} + 1\right)\right].$$

At k^1- equilibrium, $dk/dg > 0$ holds (Proposition 2). The first term in the square bracket, $-\frac{\partial s}{\partial R}R'\frac{dk}{dg}$, is positive since $R' = f'' < 0$ and $\partial s/\partial R > 0$ (from Assumption 2(iv) (gross substitutes)). The second term is also positive since $\partial s/\partial w \in (0,1)$ (from Assumption 2(iii) (normal goods)) and $w'dk/dg + 1$. Hence, $dE/dg > 0$ at k^1- equilibrium. On the other hand, at k^2- equilibrium, the sign of dE/dg is ambiguous since the sign of dk/dg is ambiguous. However, it is easy to see that, in case of $w'dk/dg + 1 < 0$, $dk/dg < 0$ holds at k^2- equilibrium. Hence, $dE/dg < 0$ at k^2- equilibrium if $w'dk/dg + 1 < 0$. $\quad \|$

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