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Environmental Externality on Production in an OLG Economy

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Abstract

In this paper, we introduce the environmental externality into the Diamond (1965) model. The environmental externality affects on the production negatively. We define a socially optimal allocation and a competitive equilibrium, and obtain the first-order necessary conditions. In competitive equilibrium, both consumers and firms have no incentives to maintain the environment, hence competitive equilibrium allocation can not be socially optimal. Therefore we propose a tax scheme. Our model requires two types of taxes in order to achieve a social optimum.

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In this paper, we introduce the environmental externality into the Diamond (1965) model. The environmental externality affects on the production negatively. We define a socially optimal allocation and a competitive equilibrium, and obtain the first-order necessary conditions. In competitive equilibrium, both consumers and firms have no incentives to maintain the environment, hence competitive equilibrium allocation can not be socially optimal. Therefore we propose a tax scheme. Our model requires two types of taxes in order to achieve a social optimum.

Keywords: overlapping generations economy; environmental externality; socially optimal allocation; optimal tax scheme

JEL Classification: Q52, O44, E22, D21

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1 Introduction

In this paper, we introduce the environmentally externality into the Diamond (1965) model. The environmental externality affects on the production negatively. We define a social optimal allocation and a competitive equilibrium, and obtain the first order necessary conditions. In competitive equilibrium, both consumers and firms have no incentives to maintain the environment, hence competitive equilibrium allocation can not be socially optimal. Therefore we propose a tax scheme. Our model requires two types of taxes in order to achieve a social optimum.

2 The model

We consider a discrete-time overlapping generations (OLG) model with production. In each period, one agent enters the economy and lives for two periods. So, there is no population growth. The productivity of a representative firm is affected by the level of pollution, and the production activity itself is a source of increasing the pollution level. The pollution level does not appear in the utility function of agents. The pollution affects the productivity of the firm only.

Consider an agent who is young in period t and old in period $t+1$. Let c_t^1 and c_{t+1}^2 denote the levels of her consumption in periods t and $t+1$, respectively. We assume that her utility function is time-separable,

$$u(c_t^1, c_{t+1}^2) := u(c_t^1) + \rho u(c_{t+1}^2)$$

where $\rho \in (0,1)$ is the common discount factor. We assume that u is strictly increasing and concave with $\lim_{c \to +0} u'(c) = +\infty$.

The total output of the representative firm in period t is

$$y_t = f(k_t)g(p_t)$$

where k_t and p_t are the capital-labor ratio and the pollution level in period t, respectively. We assume that f is twice differentiable, strictly increasing, and concave with $f(0) = 0$ and $\lim_{k \to +0} f'(k) = +\infty$. Similarly, g is twice differentiable, strictly decreasing, and convex with $g(0) = 1$ and $\lim_{p \to +\infty} g(p) = 0$. Note that $g(p_t)$ represents the externality of the pollution.

When the pollution gets worse, the amount of output decreases.

Let m_t denote the amount of "maintenance". Then the pollution is accumulated according to the following equation: [1]

$$p_{t+1} = \alpha p_t + \beta h(k_t) - \gamma m_t, \tag{1}$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and h is twice differentiable, strictly increasing, and concave with $h(0) = 0$ and $\lim_{k \to +0} h'(k) = +\infty$. The production makes the pollution worse in the next period. Maintenance in period t reduces the pollution in the next period. In case $\alpha < 1$, the pollution has its natural purification. In case $\alpha > 1$, the pollution gets worse by itself. Note that β and γ represent the maintenance technology. If β is small or γ is large, the economy has a good maintenance technology. We assume $p_t \geq 0$ for all $t \geq 1$, for simplicity. [2] In case that $p_t = 0$, there is no pollution in the economy. We also assume that p_1 and k_1 are given.

Let us say that an allocation $(c_t^1, c_t^2, k_t, m_t, p_t)_{t=1}^{\infty}$ is *feasible* if and only if it satisfies (1) and the following resource constraint:

$$c_t^1 + c_t^2 + k_{t+1} - (1-\delta)k_t + m_t \leq f(k_t)g(p_t) \tag{2}$$

for all $t \geq 1$. We also say that an allocation $(c_t^1, c_t^2, k_t, m_t, p_t)_{t=1}^{\infty}$ is *stationary* if and only if there exist $(c^1, c^2, k, m, p) \in \mathbb{R}_+^5$ such that $(c_t^1, c_t^2, k_t, m_t, p_t) = (c^1, c^2, k, m, p)$ for all $t \geq 1$. We refer to such (c^1, c^2, k, m, p) as a *stationary allocation.*

We say that a feasible stationary allocation $(\bar{c}^1, \bar{c}^2, \bar{k}, \bar{m}, \bar{p})$ is socially optimal if it solves the following social planner's problem:

$$\begin{aligned} \max_{(c^1, c^2, k, m, p) \in \mathbb{R}_+^5} \quad & u(c^1) + \rho u(c^2), \\ \text{subject to} \quad & c^1 + c^2 + k - (1-\delta)k + m \leq f(k)g(p), \\ & p = \alpha p + \beta h(k) - \gamma m. \end{aligned}$$

To obtain the first-order necessary conditions, we define the Lagrangian of this maximiza-

[1] John and Pecchenino (1994) consider a similar accumulation process.

[2] To be precise, (1) should be rewritten as $p_{t+1} = \max\{0, \alpha p_t + \beta h(k_t) - \gamma m_t\}$.

tion problem L as

$$L(c^1, c^2, k, m, \lambda, \mu) := u(c^1) + \rho u(c^2) + \lambda\{f(k)g(p) - c^1 - c^2 - \delta k - m\}$$
$$+ \mu\{(1-\alpha)p - \beta h(k) + \gamma m\}. \tag{3}$$

If $(\bar{c}^1, \bar{c}^2, \bar{k}, \bar{m}, \bar{p})$ is an interior solution to the problem above, then there exist $\lambda \in \mathbb{R}_+$ and $\mu \in \mathbb{R}$ such that

$$\begin{aligned}
\frac{\partial L}{\partial c^1} &= u'(\bar{c}^1) - \lambda = 0, \\
\frac{\partial L}{\partial c^2} &= \rho u'(\bar{c}^2) - \lambda = 0, \\
\frac{\partial L}{\partial k} &= \lambda\{f'(k)g(p) - \delta\} - \mu\beta h'(k) = 0, \\
\frac{\partial L}{\partial m} &= -\lambda + \mu\gamma = 0, \\
\frac{\partial L}{\partial p} &= -\lambda f(k)g'(p) - \mu(1-\alpha) = 0.
\end{aligned}$$

For a stationary social optimal allocation, the following conditions are necessary.

$$u'(\bar{c}^1) = \rho u'(\bar{c}^2), \tag{4}$$

$$f'(\bar{k})g(\bar{p}) - \frac{\beta}{\gamma}h'(\bar{k}) = \delta, \tag{5}$$

$$f(\bar{k})g'(\bar{p}) = -\frac{1-\alpha}{\gamma}, \tag{6}$$

$$\bar{c}^1 + \bar{c}^2 + \bar{k} - (1-\delta)\bar{k} + \bar{m} = f(\bar{k})g(\bar{p}), \tag{7}$$

$$\bar{p} = \frac{1}{1-\alpha}\left\{\beta h(\bar{k}) - \gamma\bar{m}\right\}. \tag{8}$$

3 The results

It is clear that without any tax schemes, agents do not pay the cost of maintenance, neither does the firm if we assume that it maximizes its profit in each period. So, without any tax schemes, competitive equilibrium cannot be socially optimal.

Here, we introduce the three kinds of taxes (as in Ono, 1996), a consumption tax $\tau_c \in [0, 1]$, a lump-sum tax $\tau_w \geq 0$, and a tax on production $\tau_f \in [0, 1]$. We suppose that the levied tax is utilized as maintenance. We define a competitive equilibrium allocations under taxation as follows.

We say that a feasible allocation $(c_t^{1*}, c_{t+1}^{2*}, k_t^*, m_t^*, p_t^*)_{t=1}^{\infty}$ is a *competitive equilibrium allocation under taxation* if for all $t \geq 0$, there exist $w_t > 0$, $r_t > 0$, $\tau_{c,t} \in [0,1]$, $\tau_{w,t} \geq 0$, and $\tau_{f,t} \in [0,1]$ that satisfy the following conditions:

i) Agents maximize their utility under the budget constraints given $w_t, r_t, \tau_{c,t}$, and $\tau_{w,t}$:

$$\begin{aligned}
&\max_{(c_t^1, c_{t+1}^2, s_t) \in \mathbb{R}_+^3} && u(c_t^1) + \rho u(c_{t+1}^2), \\
&\text{subject to} && (1+\tau_c)c_t^1 + s_t \leq w_t - \tau_w, \\
& && (1+\tau_c)c_{t+1}^2 \leq (1 + r_{t+1} - \delta)s_t;
\end{aligned}$$

ii) The representative firm maximizes its profit in each period:

$$\begin{aligned}
&\max_{(K_t, L_t) \in \mathbb{R}_+^2} && (1-\tau_f) f\left(\frac{K_t}{L_t}\right) g(p_t) L_t - r_t K_t - w_t L_t, \\
&\text{where} && k_t = \frac{K_t}{L_t};
\end{aligned}$$

iii) The markets for the capital goods and labor clear:

$$\begin{aligned}
s_t &= k_t, \\
L_t &= 1;
\end{aligned}$$

iv) The amount of maintenance is decided on the levied taxes:

$$m_t = (c_t^1 + c_t^2)\tau_c + \tau_w + f(k_t) g(p_t) \tau_f;$$

For a stationary competitive equilibrium allocation, the following conditions are necessary.

$$\frac{1}{1+r^*-\delta}(1+\tau_c)u'(c^1) = \rho u'(c^2), \tag{9}$$

$$r^* = (1-\tau_f)f'(k^*)g(p^*), \tag{10}$$

$$w^* = (1-\tau_f)\left\{f(k^*)g(p^*) - f'(k^*)g(p^*)k^*\right\}, \tag{11}$$

$$(c^{1*}+c^{2*})(1+\tau_c) + k^* - (1-\delta)k^* + \tau_w = f(k^*)g(p^*)(1-\tau_f), \tag{12}$$

$$p^* = \frac{1}{1-\alpha}\left\{\beta f(k^*) - \gamma T\right\}. \tag{13}$$

A stationary competitive equilibrium can be a social optimum by setting $\bar{k} = k^*$ and $\bar{m} = m^*$.

Proposition 1. *An interior socially optimal allocation* $(\bar{c}^1, \bar{c}^2, \bar{k}, \bar{m}, \bar{p})$ *can be realized as a stationary competitive equilibrium if* τ_c, τ_w, *and* τ_f *satisfy*

$$\bar{m} = (\bar{c}^1 + \bar{c}^2)\tau_c + \tau_w + f(\bar{k})g(\bar{p})\tau_f, \tag{14}$$

$$\frac{1+\tau_c}{1+(1-\tau_f)f'(\bar{k})g(\bar{p})}u'(\bar{c}^1) = \rho u'(\bar{c}^2). \tag{15}$$

τ_c adjusts the level of the consumption between a young and an old generation. τ_f restricts the firm's production, but does not affect the ratio between r_t and w_t. τ_w levies the difference from the required amount of maintenance.

Next, we examine the optimal τ_f. From (10), we can rewrite r^* as

$$r^* = (1-\tau_f)f'(\bar{k})g(\bar{p}) =: r(\tau_f). \tag{16}$$

And from (5), (10) and (16),

$$\phi(\tau_f) := 1 - \tau_f - \frac{r(\tau_f)}{\delta}\left\{1 - \frac{\beta}{\gamma g(\bar{p})}\right\}. \tag{17}$$

Then, an optimal τ_f is the fixed point of (17).

First, let $\tau_f = 0$, by using (16), (17) and (5),

$$\begin{aligned}
\phi(0) &= 1 - \frac{r(0)}{\delta}\left\{1 - \frac{\beta}{\gamma g(\bar{p})}\right\} \\
&= 1 - \frac{f'(\bar{k})g(\bar{p})}{\delta}\left\{1 - \frac{\beta}{\gamma g(\bar{p})}\right\} \\
&= 1 - \frac{f'(k)}{\delta}\left\{g(\bar{p}) - \frac{\beta}{\gamma}\right\} \\
&= 1 - \frac{\delta}{\delta} \\
&= 0.
\end{aligned}$$

Next, let $\tau_f = 1$, by using (16) and (17),

$$\phi(1) = 1 - 1 - \frac{r(1)}{\delta}\left\{1 - \frac{\beta}{\gamma g(\bar{p})}\right\} = 0.$$

Therefore, $\tau_f = 0$ is the fixed point of (17), and then $\tau_f = 0$ is an optimal. We can achieve social optimal with only two taxes.

Proposition 2. *A social optimum can be achieved with τ_c and τ_w.*

4 Concluding Remarks

In competitive equilibrium, agents have no incentives to maintain the environment, for the pollution does not affect their utility. The firm also does not reduce the pollution, because it maximizes its profit in each period. Then, we introduce a tax scheme to achieve a social optimum. In the literature such as Ono (1996), three taxes are required for improving the equilibrium. However, in this paper, it is shown that the social optimum can be achieved with two taxes.

References

[1] Batina, R. G., Ihori, T., (2000), *Consumption Tax Policy and the Taxation of Capital Income*, Oxford University Press.

[2] Diamond, P. A., (1965), "National debt in a neoclassical growth model, " *The American Economic Review*, 55(5): 1126–1150.

[3] John, A., Pecchenino, R., (1994), "An overlapping generations model of growth and the environment," *Economic Journal*, 104: 1393–1410.

[4] John, A., Pecchenino, R., Shimmelpfennig, D., Schreft, S., (1995), "Short-lived agents and the long-lived environment," *Journal of Public Economics*, 58: 127–141.

[5] Jouvet, P.-A., Michel, P., Vidal, J.-P., (2000), "Intergenerational altruism and the environment," *Scandinavian Journal of Economics*, 102(1): 135–150.

[6] Jouvet, P.-A., Michel, P., Rotillon, G., (2005), "Optimal growth with pollution: how to use pollution permits?" *Journal of Economic Dynamics & Control*, 29: 1597–1609.

[7] Michel, P., Thibault, E., Vidal, J.-P., (2004), "Intergenerational altruism and neoclassical growth models," Working Paper Series No. 386, European Central Bank.

[8] Ono, T., (1996), "Optimal tax schemes and the environmental externality," *Economics Letters*, 53: 283–289.

[9] Ono, T., (2002), "Effects of emission permits on growth and the environment," *Environmental and Resource Economics*, 21: 75–87.

[10] Prieur, F., Jean-Marie, A., Tidball, M., (2013), "Growth and irreversible pollution: are emission permits a means of avoiding environmental and poverty traps?" *Macroeconomic Dynamics*, 17(2): 261–293.