Improving the Government Debt Market Quality by Determining the Optimal Structure of Government Debt Portfolio

Ahmad Danu Prasetyo* and Naoyuki Yoshino**

Abstract
Recently, there is an upward tendency for switching external debts to domestic borrowings in many developing countries. While the domestic government bonds market development could reduce the sovereign exposure to currency risk, there are also potential risks faced by the government; namely: higher domestic interest rates, maturity mismatch, and crowding out effect to the private issuers. In this paper we develop a simple general equilibrium model to determine the optimal share for domestic and external government bonds in a sovereign country. We emphasize the important role of the demand side in forming the optimal structure of government bonds. We found that, at ceteris paribus, domestic government bond correlates negatively to external government bond at a constant rate. In addition, the back testing simulation results that the government has to reduce the level of its external debt. Through a dynamic recursive simulation, it is suggested that, in the long run, the Indonesian government must not hold any external debt while the Debt-to-GDP ratio shall be maintained at 16%-17% level.

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JEL Classifications: E60, H63

Keywords: Optimal structure, optimal proportion, domestic government debts, external government debts.

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1. **Background**

Deficit in primary balance has become a generic policy for triggering the economy and generating growth. Governments, especially in poor and developing countries, need higher amount of investments to build public facilities and to satisfy their consumptions and expenses compared to the tax revenues that could be attained. Barro (1999) mentioned that the government should run budget deficits at times of temporarily high public outlays. Moreover, the budget deficits should be high at times of temporary economic distress and low (typically negative) in good times. In order to cover the deficit, the government seeks for funding sources other than tax revenues; including financial aids and public debts from domestic and foreign countries.

In the past, many poor and developing countries received grants and soft-loans from developed countries and international fund institutions to finance their primary gaps. However, many economists criticize this approach. They argued that these aids are not effective for triggering the economic development in the poor and developing countries; this is indicated by the poor growth record of the grantee countries (Loser, 2004). On the other hand, there is no such thing as free lunch; there are economic and political motives behind external aids and debts (Makmun, 2005).

Recently, there is a tendency for switching the external debts into domestic debts in many developing countries. On one side, this policy can reduce the government exposure to interest rates and currency risks. In addition, it could also reduce the dependence on external assistance and on external shocks (Arnone and Presbitero, 2008). Nevertheless, there also risks embedded in the domestic debts issuance. Compare to external debts, domestic debts tend to have higher rates and short-maturity. Higher interest rates imply higher debt services, which aggravate fiscal imbalances and decrease the government's ability to sustain debts. In addition, they eventually lead to a fall in the real demand for government bonds, due to the increase in default risk (Akemann and Kanczuk, 2002). Alternatively, sovereigns that are switching from external to domestic debts could be trading a currency mismatch for a maturity mismatch since few of them are able to issue long-term domestic debts at a reasonable interest rate (Panizza, 2008).

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1. However, investors might assumed that domestic and external debt rates are indifferent since, as mentioned by Barro (1999), the extra premium required on domestically denominated issues (even if indexed) may justify the extra riskiness of the foreign currency debt.
Muhdi (2007) found that emphasizing on domestic debts as a financing resource had an implication to crowd out private investments. Panizza (2008) explained that this is happened because institutional investors and banks were absorbing "too much" government debts. In addition, he also mentioned that there are political reasons that may make domestic debts more difficult to restructure. For some sovereigns, the problem is more fundamental. They are not able to raise the amount needed from domestic sources due to lack of liquidity and saving. Hence, they enter the international capital market since it can provide a large amount of funds without crowding out lending to private sectors or recurring to inflationary finance. However, these countries could not issue the debts in their own currency in this market, they have to borrow in foreign currency; this is also known as “original sin” (Panizza, 2008; Muhdi, 2007).

Debt’s sustainability has been a central issue in government’s fiscal policy. Unsustainable debt was one of main causes why the economic recovery were so slow in Indonesia compare to other countries impacted by Asian financial crisis in 1998 (Suhud, 2004), and has triggered the sovereign debt crisis in euro area just recently. Some economists believe that the structure of the government debt (which is associated with its holder) is one of the keys for a sustainable debt. For example, Tokuoka (2010) explained that the large pool of household savings and the stable domestic institutional investor base have contributed to keeping yields steady despite the rapid rise in government debt in Japan.

Because of these pros and contras of the domestic and external debts issuance and the important role of the structure of government debt in order to maintain its sustainability, it is tempting to investigate the optimal share of domestic and external debts in the government debt’s portfolio. Our terminology of external debts refers to the debt that is denominated in foreign currencies; in this paper we convert all external debts into US dollar denomination.

We develop a simple general equilibrium model to determine the optimal share for domestic and external government bonds in a sovereign country. We emphasize the important role of the demand side in forming the optimal structure of government debts. We found that, at ceteris paribus, domestic government bond correlates negatively to external government bond at a constant rate. In addition, the back testing simulation results that the government has to reduce the level of its external debt. Through a dynamic recursive simulation, it is suggested that, in the
long run, the Indonesian government must not hold any external debt while the Debt-to-GDP ratio shall be maintained at 16%-17%.

The rest of this paper is structured as follows: in the next section, we review the existing literature about domestic and external government debts’ sustainability, and also the government debts management in Indonesia. The development of the model will be discussed in the third section. Section 4 will discuss about the empirical evidence in Indonesia. Lastly, the concluding remarks will appear in section 5

2. Literature review

2.1. Domestic Government Debts Sustainability

The study of government’s fiscal sustainability, especially concerning the sustainability of its debts, has flourished in past decade. IMF (2006) defined debts sustainability as “a situation in which a borrower is expected to be able to continue servicing its debts without an unrealistically large correction to the balance of income and expenditure”. However, there is no exact parameter to measure a sustainable debt. Some economists just set a threshold that serves as warning indicator, while other economists set a dynamic parameter that depends on the economic situation at the moment.

Maybe the most famous debt’s threshold is stated in the Maastricht Treaty by the European Council. This treaty is arranged to establish the criteria for European Union member states to enter the third stage of European Economic and Monetary Union (EMU) and adopt the euro as their currency. One of the criteria in the treaty is to maintain the ratio of gross government-debt-to-GDP not exceeding 60% at the end of the preceding fiscal year. Even if the target cannot be achieved due to the specific conditions, the ratio must have sufficiently diminished and must be approaching the reference value at a satisfactory pace. Other threshold is stated by Aiyagar and McGrattan (1997). Using the U.S. economics data, they reach a conclusion that the optimal debt ratio is 2/3 of the country’s GDP.

On the dynamic parameter study, Makmun (2005) argued that when a sovereign is obliged to pay annual payment, the total output produced in the country plus the capital inflows (including net use of foreign exchange reserve) must exceed the domestic consumptions plus the interest of current investments. Ferrucci and Penalver (2003) develop a probabilistic method in forecasting
the debt’s sustainability. They found that with strictly positive probability, debts will be unsustainable when the variance of expected debt-to-GDP ratio is infinite. Thus, a finite variance of expected debt-to-GDP ratio is a prerequisite in order to sustain the government debt.

Other economists set the parameterization of debt’s sustainability depend on the Debt-to-GDP ratio. The current debt would be considered as sustainable when its present value of the debt-to-GDP ratio converges into zero; this parameterization is also known as the inter-temporal budget constraint (Akyuz, 2007). Inter-temporal budget constraint is often formulated with respect to conditions for solvency which requires that the present discounted value of future primary budget balances should at least equal to the value of outstanding debt. This constrain implies that a sovereign that is able to run a larger primary surplus can have a higher initial stock of debt while maintaining long-term sustainability. Alternatively, a country that is growing fast can run a lower primary surplus for a given stock of debt and interest rate. Other implication is the public sector cannot be a debtor, and the private sector cannot be a creditor, in present-value terms; any debt incurred should eventually be fully payable. This would require the government to constantly increase taxes and reduce spending on goods and services. In addition, the theoretical concept of sustainability based on solvency is problematic because it does not impose specific constraint on debts and deficits at any point in time. Since current deficits are collateralized by surpluses in some distant future, any level of debts and deficits could be compatible with the present-value budget constraint. On the other hand, both the underlying economic conditions as reflected by the growth adjusted interest rate and the fiscal policy stance are vary over time and highly uncertain. (Akyuz, 2007)

Another approach is static budget constraint; that is when the public sector is able to finance its current expenditures with its revenues and new borrowing, and meet or rollover its maturing liabilities; that is, if it is not liquidity constrained (Akyuz, 2007). As mentioned, this parameter would depend on the demand side of the market. This approach, in our knowledge, is less explored in the debt sustainability study. In practice, a holistic assessment is needed so the government could flexibly apply the best strategy in issuing its debts. Waluyanto (2009; in Abimanyu et al, 2009) mentioned that the traditional paradigm to see public debts as a residual had caused an upward revision without accommodating the demand of the market. As the result, the government acts as an opportunistic issuer – applies the front loading strategy to get more
available funds. This make the investor seems to be less confidence on government, even though a clear calendar-of-issuance of public debts has been announced.

2.2. External Government Debt Sustainability

The formal definition of the external government debts sustainability is “the ability of a country to meet the current and future external obligations without running into arrears, recourse to debt-rescheduling and eventually a drastic balance-of-payments adjustment” (Akyuz, 2007). Like in the domestic debts sustainability, there is no exact parameterization on how to measure the external debts sustainability. For example, IMF (2002) consider an external debt ratio of 40% as a useful benchmark, however many economists relate the external debts sustainability with the level of trade surplus.

The amount of trade surplus desired is not directly linked to policy, but influenced by a host of variables operating on imports and exports, particularly the exchange rate and the rate of growth (Akyuz, 2007). Fisher in 1933 propounded a paradox regarding the external debt sustainability: the heavily indebted countries are trapped in a net transfer problem – that is when debt services are higher than new debts - while in the same time are experiencing depreciation in unit value and term of trade of their export commodities. The unit value and term of trade of export commodities can be depreciated as the foreign reserves are exhausted due to high debt services (Suhud, 2004). In fact, increasing external debts itself will potentially depreciate the local currency, thus the total redemption will increase (Muhdi, 2007).

Another characteristic of external debts is they are subject to the shock of external events. An external event will result in an increase in borrowing costs and a capital outflow (Loser, 2004). Creditors are less willing to roll over loans when there is an expectation that the debtor will be unable to repay in the future (Ferucci and Penalver, 2003). At the extreme cases, debt overhang is followed by massive capital flight as investors’ confidence is eroded (Muhdi, 2007). However, sudden stops in lending or rolling over debt do not always signal solvency problems. Investor behavior and risk appetites tend to vary over time without any significant change in the economic fundamental. Furthermore, country’s history of default and the nature of its government and institution also play a significant role in influencing the investor’s confidence (Akyuz, 2007)
Despite these drawbacks, the preference for the domestic debts issuance is not always superior to the external debts issuance. As shown by Arnone and Presbitero (2008), the ex-post evaluation of the sustainability condition of 14 heavily indebted poor countries shows that the inclusion of domestic debts makes the evolution of debts not always sustainable, as, instead, is the results obtained looking exclusively at the external public debts.

2.3. Government Debt Management in Indonesia

Indonesia is one of the most severe countries that are impacted by financial crisis in mid-1997 in South East Asia. One of the factors that made it hard to struggle out from the crisis was the increase of accumulative government debts (Suhud, 2004). These government debts especially took form of external debts, which is denominated in foreign currency. As the IDR was depreciated drastically against USD, the debt services had multiplied over a short period. Indonesian government took several actions in response, including seeking soft loans from bilateral and international institutions, rescheduling the existing debts, privatizing its assets, and issuing domestic government bonds in order to bail out the collapsed banking system. During 1998-2000, Indonesian government had issued totally IDR 643.8 Trillion of domestic government bonds. By the end of year 2000, the bonds holders (which are the national banks whom bailed out by the government) started to trade these bonds to other investors, which initiated the development of domestic government bonds market.

In 2002, Indonesian government has issued Government’s Bond Act No. 24/2002 which contains the government’s objective of bonds management and other general concerns related to government bond issuance. It is then elaborated in more specific in Financial Minister Decree about Government’s Debt Management Strategy No.447/KMK.06/2005, which stated: “In more specific, the objectives of debt management are (1) To finance the gap in primary balance and to maintain fiscal sustainability which is in line with the macroeconomic condition, and within the lowest cost, (2) To increase the prudence in debt management in order to minimize the risks embedded, (3) To be persistent with all of the scheduled plans and estimated costs”. Those objectives are then translated into several actions, i.e. (1) Maintaining the debt issuance below 1% of GDP, (2) Prioritizing the issuance of domestic debts and maintaining the external debts in a balanced proportion in order to reduce the crowding out effect in the local market, (3) Issuing debts with longer maturities and continuously rearranging the maturity profile through buyback
and debt-switch in order to mitigate the refinancing risk. Furthermore, it also established a debt management office under the ministry of finance to monitor the debts development and to implement strategies for sustaining debts. Thus, the government has started to manage its debts more professionally.

Table 1 Statistics of Indonesian government debts

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Balance Surplus/Deficit</td>
<td>-4.12</td>
<td>-88.62</td>
<td>-133.75</td>
<td>-124.66</td>
<td></td>
</tr>
<tr>
<td>Domestic Debt Sources Net flows</td>
<td>46.73</td>
<td>52.45</td>
<td>66.29</td>
<td>90.24</td>
<td></td>
</tr>
<tr>
<td>Stock of Domestic Non-Tradable Debt (Domestic Loans)</td>
<td>0.17</td>
<td>0.81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock of Domestic Tradable Debt (Domestic Bonds)</td>
<td>737.13</td>
<td>783.86</td>
<td>836.31</td>
<td>902.43</td>
<td>992.03</td>
</tr>
<tr>
<td>External Debt Sources Net flows</td>
<td>200.6</td>
<td>-98.54</td>
<td>20.07</td>
<td>37.04</td>
<td></td>
</tr>
<tr>
<td>Stock of External Non-Tradable Debt (External Loans)</td>
<td>586.36</td>
<td>730.25</td>
<td>611.2</td>
<td>612.45</td>
<td>615.83</td>
</tr>
<tr>
<td>Stock of External Tradable Debt (External Bonds)</td>
<td>65.93</td>
<td>122.64</td>
<td>143.15</td>
<td>161.97</td>
<td>195.63</td>
</tr>
</tbody>
</table>

Source: Debt Management Office, Indonesian Ministry of Finance

Domestic debts recently have become the main source to cover the primary deficit. From table 1, it can be seen that, except in 2008, the additional amount of domestic debts are higher compared to the additional amount of external debts. In 2008, the government borrowed a significant amount of external debts as a preventive action for facing the impact of sub-prime mortgage crisis.

3. Development of the Model

3.1. General overview of the markets

Demand in the domestic debts market is forming an upward slope since the quantity of debt demanded will increase as the interest rate increase. The domestic debts’ interest rate (r) is endogenously determined as when the market is cleared. We assume that the demand in external debts market is perfectly elastic. Thus, the external debts’ interest rate is considered as an exogenous variable.
In addition, we assume that domestic investors are only investing in the domestic government debts; they are restricted to access the external government debts. Likewise, foreign investors are only investing in the external government debts. From the demands point of view, the two markets are independent to each other since domestic investors will only consider domestic factors while investing into domestic government debts, whereas foreign investors could neglect the role of domestic interest rate and exchange rate risks since the external debt is made in foreign currency. Both markets, however, are still linked from the supply side as the government determines the optimal proportion between the domestic and external debt issuance.

The supply for both domestic and external government debts are solely determined by the government. One of the main considerations for the government in determining the amount of government debts is the budget constraint of its primary balance. As it needs the debts to cover the deficit in the primary balance, it would accept debts at any given interest rate. This made the supply for both domestic and external government debts are perfectly inelastic.

3.2. Supply Side of Domestic and External Government Debts

The government revenues come from tax, which is equal to domestic income (Y) multiplied by tax rate (τ), and raising new debts, which come from domestic debts ($B^D$) and external debts ($B^E$).
On the other side, the government has to spend its expenditure \((G)\) and to pay the debt service of previous debts. The debt service of previous debt comes from the principal payments and the interest payment, where \(r\) denotes the interest rate for domestic borrowings and \(r'\) denotes the interest rate for external borrowings respectively. In this paper, we assume that the government only issue fix-rated-short-term-bonds for both domestic and external bonds. It means that the interest rate to be paid in the end of observed period is determined in the beginning of the period. Since the external debts are made in foreign currency, therefore it has to be multiplied by current exchange rate \((e)\). Mathematically, the government budget constraint is formulated as following:

\[
\tau Y_t + B_t^D + B_t^E e_t = G_t + (1 + r_{t-1})B_{t-1}^D + (1 + r_{t-1}^f)B_{t-1}^E e_t
\]

Many economists assume that the government bonds would be perfectly rolled over into the next period (Ferruci and Penalver, 2003; Penalver and Thwaites, 2006). However, this assumption is not applicable in reality. Ceteris paribus, the stock of debt would naturally increase as the debt services are integrated in the new debts issuance. These accumulative debts would raise a general concern towards the debt sustainability in the long run. For developing countries, such as Indonesia, debt services are the ultimate source of the foreign reserves’ outflows. Indonesian government spending is less likely to act as a stimulus for the economic growth due to the debt-service-to-revenue ratio is very high (Makmun, 2005; Suhud, 2004). Thus, the government objective function is not only to minimize the interest payment in the future, but also to minimize the new borrowings.

\[
\min_{B_t^D, B_t^E} B_t^D - B_{t-1}^D + (B_t^E - B_{t-1}^E)e_t + r_t B_t^D + r_t^f B_t^E E(e_{t+1})
\]

Subject to equation (1)

Solving the first order condition would give us an equation that must be satisfied by the government. It should maintain the level of domestic debts and external debt, thus the domestic debts’ interest rate should be equal to the external debts’ interest rate multiplied by the ratio of expected exchange rate in the future to the current level of exchange rate. This equation means that suppose that exchange rate is expected to be fixed, then the domestic debts’ interest rate should be equal to the external debts’ interest rate.
\[ r_t^* = r_t^f \frac{E(e_{t+1})}{e_t} \]  

(3)

The current expectations of future exchange rate reflect past expectations and an "error-adjustment" term, in which current expectations are raised (or lowered) according to the gap between actual exchange rate and previous expectations.

\[ E(e_{t+1}) = E(e_t) + \zeta (e_t - E(e_t)) \]  

(4)

Where \( \zeta \) denotes the subjective preference of the importance of the error adjustment. Rearranging equation (4) yields:

\[ E(e_{t+1}) = \zeta e_t + (1 - \zeta)E(e_t) \]  

(5)

By rearranging equation (3) and take it backward for one period, we could obtain the previous expected exchange rate function.

\[ E(e_t) = \frac{r_{t-1}}{r_t^f} e_{t-1} \]  

(6)

Substituting equation (6) into equation (5), the current expectations of future exchange rate is then formulated as:

\[ E(e_{t+1}) = \zeta e_t + (1 - \zeta)\frac{r_{t-1}}{r_t^f} e_{t-1} \]  

(7)

Up to this point, we have not determined the optimal value of domestic and external debts should be maintained by the government. Except the external government debts interest rate, all variables are endogenously determined in the market. Therefore, in order to determine what are the optimal level of debts should be maintained by the government, we should look up the demand side of the markets.
3.3. Demand Side of Domestic Government Debts

From the infamous IS-LM model, we understand that the household’s wealth \((W)\) is the summation of the amount of distributed money \((M)\) and the stock of government debts \((B)\). By using this relationship, we could obtain the demand for government bond.

\[
B_t^D = W_t - M_t
\]  
(8)

There are many instruments in the money market where households could put their money in. In fact, risk averse investors are always comparing the returns of the assets given the risks. In this paper we include deposit interest rate as the next best assets for the returns of government debts. Thus, we expand the formula for determining the demand for domestic government debts as follows:

\[
B_t^D = \alpha_0 + \alpha_1 (r_t^{dep} - r_t) + \alpha_2 W_t + \epsilon_r
\]  
(9)

We could determine the level of domestic debts’ interest rate by rearranging the formula for clearing condition in equilibrium, as shown in equation (11).

\[
B_t^{D*} = B_t^D
\]  
(10)

\[
r_t = r_t^{dep} + \delta_0 + \delta_1 W_t - \delta_2 B_t^{D*} + \epsilon_r
\]  
(11)

\[
\delta_0 = \frac{-\alpha_0}{\alpha_1}; \quad \delta_1 = \frac{-\alpha_2}{\alpha_1}; \quad \delta_2 = \frac{-1}{\alpha_1}
\]

3.4. The role of exchange rate

The exchange rate plays an important role as the external debts are made in foreign currency denomination. Once it is attained, the external debt will be converted into domestic currency to fund some parts of government’s spending. Then it will be converted back into foreign currency denomination when it is due.

On the other hand, government’s decision in determining the level external debts affects the value of exchange rate. In the foreign currency market, the stock of foreign currency is supplied from exports and capital inflow, whereas the demand for the foreign currency is determined by imports and capital outflow. The value of exports and imports is a function of exchange rate. Capital inflow comes from the new external debt, whereas the capital outflow is the amount of
existing external debt to be paid plus its interest. Mathematically, it is explained by equation (8) and (9).

\[ Q_{e,t}^x = E_{x,t} + B_{e}^F e_t = x_0 + x_1 e_t + B_{e}^F e_t + \epsilon_x \] (12)

\[ Q_{e,t}^d = l m_t + (1 + r_{t-1})B_{e-1}^F e_t = m_0 + m_1 e_t + (1 + r_{t-1})B_{e-1}^F e_t + \epsilon_{lm} \] (13)

At the equilibrium, the value of exchange rate is formulated as:

\[ e_t = \frac{x_0 - m_0 + \epsilon_x - \epsilon_{lm}}{(1 + r_{t-1})B_{e-1}^F - B_e^F - x_1 + m_1} \] (14)

### 3.5. The optimal amount of Domestic and External Government Debts

At equilibrium, the optimal domestic government debts is formulated as

\[ \delta_2 B_{e,t}^d = r_{t}^{dep} + \delta_0 + \delta_1 W_t + \epsilon_r - r_t \] (15)

Substituting equation (3) into equation (15) yields:

\[ \delta_2 B_{e,t}^d = r_{t}^{dep} + \delta_0 + \delta_1 W_t + \epsilon_r - r_{t}^f E(e_{t+1})/e_t \] (16)

By further substituting equation (7) and (14) into equation (16) yields the optimal amount of domestic government debts.

\[ B_{e,t}^{d*} = -\alpha_1 \left( r_{t}^{dep} + \delta_0 + \delta_1 W_t + \epsilon_r - r_{t}^f \zeta \right) \left( -r_{t}^f (1 - \zeta) \frac{r_{t-1} + (1 + r_{t-1})B_{e-1}^F - B_e^F - x_1 + m_1}{x_0 - m_0 + \epsilon_x - \epsilon_{lm}} \right) \] (17)

As can be seen in equation (17) the optimal amount of domestic government debt depends on the value of external government debt. Therefore, it has to be solved in advance. As dictated by the government budget function in equation (1), the external government debt is the residual funds needed by the government to cover its budget deficit after it raising the debts internally.

Mathematically it is explained by equation (18).

\[ B_e^F e_t = G_t + (1 + r_{t-1})B_{e-1}^F + (1 + r_{t-1})B_{e-1}^F e_t - \tau Y_t - B_e^D \] (18)

The domestic income is a function of domestic government debt interest rate and exchange rate as follow:

\[ Y_t = \varphi_0 + \varphi_1 r_t + \varphi_2 e_t + \epsilon_y \] (19)

Substituting equation (3),(14),(17) and (19) into equation (18) we found that the optimal external debts is the roots of a quadratic function of interacting variables.
\[ B_t^F = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  

Where:

\( a = -j \)
\( b = (ih + h^2 + 2jg) \)
\( c = -gh - (1 + r_{t-1}^F)B_{t-1}^F - t \varphi_2) \) 
\( g = (1 + r_{t-1}^F)B_{t-1}^F - x_1 + m_1 \)
\( h = (x_0 - m_0 + \varepsilon_x - \varepsilon_{lm}) \)
\( i = G_t + (1 + r_{t-1})B_{t-1}^P - \tau(\varphi_0 + \varepsilon_Y - \varphi_1 r_t^F \zeta) + \alpha_1 (r_t^{dep} + \delta_0 + \delta_1 W_t + \varepsilon_r - r_t^F \zeta) \)
\( j = -(\alpha_1 + \tau \varphi_0) r_t^F (1 - \zeta) r_{t-1}^F e_{t-1} \)

Since the government is constrained by its primary budget, then the relationship between the domestic and external debts is expected to be negative. Mathematically, the relationship between the two debts is given as follows:

\[ \frac{dB_t^P}{dB_t^F} = -\frac{\alpha_1 r_t^F (1 - \zeta) r_{t-1}^F e_{t-1}}{x_0 - m_0 + \varepsilon_x - \varepsilon_{lm}} \]

4. **Empirical Analysis**

4.1. The data

We do an empirical analysis to support the theoretical framework described in the previous section. We use Indonesian macroeconomic data to get a more perceptive analysis. We gather the annual and monthly data of domestic government debts, external government debts, averaged domestic debts interest rate, averaged T-Bill interest rate in US, GDP, IDR/USD exchange rate, exports and imports. We assume that the subjective preference of the importance of the error adjustment is equal to 0.5, while the tax rate is assumed to be fixed at 10%. The statistical summary of the data is shown in table 2.

<table>
<thead>
<tr>
<th>Units</th>
<th>Year Range</th>
<th>Interval</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
</table>

Table 2 Statistical summary of the data
### Table 3: Regression result of the difference of domestic government debts’ interest rates and deposits rates and household wealth to domestic government debts

<table>
<thead>
<tr>
<th></th>
<th>coefficient</th>
<th>Std. Error</th>
<th>t</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>5.325E+14</td>
<td>1.558E+13</td>
<td>34.174</td>
<td>***</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-5.960E+13</td>
<td>5.406E+13</td>
<td>1.103</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>.035</td>
<td>.002</td>
<td>19.642</td>
<td>***</td>
</tr>
</tbody>
</table>

R$^2 = 0.991$

### Table 4: Regression result of exchange rates to exports

<table>
<thead>
<tr>
<th></th>
<th>coefficient</th>
<th>Std. Error</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>-7.771E+13</td>
<td>4.173E+13</td>
<td>-1.862</td>
</tr>
</tbody>
</table>

### Table 5: Regression result of exchange rates to imports

<table>
<thead>
<tr>
<th></th>
<th>coefficient</th>
<th>Std. Error</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$</td>
<td>-6.497E+13</td>
<td>4.200E+13</td>
<td>-1.547</td>
</tr>
<tr>
<td>$m_1$</td>
<td>9.003E+10</td>
<td>8.107E+09</td>
<td>11.105</td>
</tr>
</tbody>
</table>

R$^2 = 0.746$
Table 6 Regression result of domestic government debts’ interest rates and exchange rates to GDP

<table>
<thead>
<tr>
<th></th>
<th>coefficient</th>
<th>Std. Error</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_0$</td>
<td>1.477E+15</td>
<td>7.223E+14</td>
<td>2.045 *</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>-9.394E+15</td>
<td>2.934E+14</td>
<td>-3.202 ***</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>3.538E+11</td>
<td>5.737E+10</td>
<td>6.167 ***</td>
</tr>
<tr>
<td>$R^2$</td>
<td>= 0.704</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Denotes significant at 10% confidence level
*** Denotes significant at 1% confidence level

4.2. The simulations

We conduct two kinds of simulations, back-testing and recursive dynamic simulation, in order to comprehend the optimal level of domestic and external government debts in Indonesia. Back-testing simulation is conducted for 2004-2011 sampling period. The actual values of exogenous and lagged variables are used to calculate the optimal level of domestic and external government debts for each year. From this simulation, it can be seen that the actual level of external government debt is higher than the optimal level for every year in the sampling period. While the actual level of domestic government debt is almost as high as the optimal level.

Figure 2: Comparison between optimal level of external government debt and the actual level
At deeper analysis, we comprehend that household’s wealth plays a significant role in shaping the demand of the government debts. As the household’s wealth increase at a rapid pace, the government is allowed to increase the amount of domestic debt; thus reducing the proportion of external debt. In the ratio level, Indonesian government should maintain the domestic debts 1.15 to 1.54 times of the external debts.
We also run a recursive dynamic simulation to estimate the optimal value of domestic and external government debt given an initial condition as shown in table 7. Government spending and household’s wealth is assumed to be growing at 7% rate. Stochastic estimation is applied to measure exogenous variables and error terms. The parameters of the stochastic estimation are shown in table 8.

**Table 7 Initial condition for recursive dynamic simulation**

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>G₀</td>
<td>6.67E+14</td>
</tr>
<tr>
<td>B₀</td>
<td>9.92E+14</td>
</tr>
<tr>
<td>B₁₀</td>
<td>8.94E+10</td>
</tr>
<tr>
<td>r₀</td>
<td>6.61%</td>
</tr>
<tr>
<td>r₁₀</td>
<td>1.74%</td>
</tr>
<tr>
<td>e₀</td>
<td>8770.43</td>
</tr>
<tr>
<td>W₀</td>
<td>1.31E+16</td>
</tr>
</tbody>
</table>

**Table 8 Parameters for stochastic estimation**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Average</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>1.74%</td>
<td>0.90%</td>
</tr>
<tr>
<td>rdep</td>
<td>5.80%</td>
<td>0.10%</td>
</tr>
<tr>
<td>εₕ</td>
<td>-1.94E+12</td>
<td>1.03E+13</td>
</tr>
<tr>
<td>εₓ</td>
<td>3.17E+14</td>
<td>4.22E+14</td>
</tr>
<tr>
<td>εₘ</td>
<td>3.14E+14</td>
<td>4.39E+14</td>
</tr>
<tr>
<td>εₚ</td>
<td>8.73E+14</td>
<td>1.79E+15</td>
</tr>
</tbody>
</table>

From the simulation, it can be seen that the optimal stock of external government debt keeps decreasing, even goes to zero at the 8th period. This means that the government is not allowed to hold external debt for long run. On contrary, the domestic government debt is allowed to be increased gradually. The debt-to-GDP ratio will be decreased as the stock of external debt decrease; however, it should be maintained at 16%-17% level.

**Figure 5 Recursive dynamic simulation for external government debt**
5. Conclusion

This paper is purposed to determine the optimal proportion for domestic and external government debts. We develop a simple general equilibrium model with emphasizing the important role of the demand side in forming the optimal structure of government debts. We derive IS-LM model in determining the demand of domestic government debts, while assuming
the interest rate for external government debt is exogenous. In the supply side, we set the
government objective is to minimize the interest payment and also the new issued bonds

We found that the optimal supply of domestic bond is a function of deposits interest rate, wealth,
external bond’s interest rate, and External Bond’s supply. In addition, the external bond’s supply
is a factor of quadratic function of inter-related variables. Given other variables are being equals;
domestic government bond correlates negatively to external government bond at a constant rate.

By using the data in Indonesian government bond’s market, the back testing simulation results
that the government has to reduce the level of its external debt, while it is allowed to increase the
amount of domestic government bonds at the current level. We found that household’s wealth
plays a significant role in shaping the demand of the government debts. As the household’s
wealth increase at a rapid pace, the government is allowed to increase the amount of domestic
debt; thus reducing the proportion of external debt. The dynamic simulation results that the
Indonesian government should decrease the level of the external debt significantly that it has to
be zero at the 8th period. Along with the reduction of the external government debt, the Debt-to-
GDP ratio will be decreased. The debt-to-GDP ratio shall be maintained at 16%-17% level,
which is when the government runs domestic sourced debt only.

References


Global Economy Series, Publisher: Third World Network, Malaysia, 2010.


Evans, George W. and Seppo Honkapohja (2001), Learning and Expectations in Macroeconomics. Princeton University Press


