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Measuring the Tax Revenue Elasticity to Output in Dynamic Stochastic General Equilibrium Model[1]

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Abstract

We use structural method, that is, Dynamic Stochastic General Equilibrium (DSGE) model with fiscal stabilization rules, for calculating the tax revenue elasticity rate and estimate more plausible value of it. In the short-run, the tax revenue elasticity to output takes negative value and, in medium-run, it takes quite large positive values (from 2.3 to 4) in both permanent and temporary positive productivity shocks. On the other hand, in the long-run, under permanent positive productivity shock remains nearly the value of the tax revenue elasticity converses to about 2.3. But, under temporary one, it decreases and reverses it.

Keywords: Tax revenue elasticity to output, Dynamic stochastic general equilibrium (DSGE) model, Fiscal stabilization rules, Temporary productivity shock, Permanent productivity shock.

JEL classification: E17, H20, E20.

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1. Introduction

Fiscal reconstruction has been one of the most major political issues in Japan for several decades. Especially, the most excited discussion is the magnitude of Japanese tax revenue elasticity rate to income. If this magnitude is sufficiently large, economic growth enables to reconstruct fiscal deficit without tax rate increase, because economic growth lets tax revenue increase larger than itself. This paper investigates this magnitude using Dynamic Stochastic General Equilibrium (DSGE) model which is structural model.

Some institutions and researchers estimate Japanese tax revenue elasticity rate to income, such as OECD (2000), CAO (2007a, b) and Kitaura and Nagashima (2007). However, there are some politicians and economists who insist that the tax revenue elasticity is larger than these estimations and fiscal reconstruction can be archived by economic growth without tax rate increase. At least, both discussions depend on their ideology, and do not have a structural economic model. Especially, they deal with output growth as reduced idea, which means that economic growth consists of various components, such as total factor productivity (TFP) growth and fiscal and monetary policy. Therefore, we propose new method for calculating the tax revenue elasticity using structural model, that is, Dynamic Stochastic General Equilibrium (DSGE) model. DSGE model is used by central banks and policy institutions to analyze economic policy, such as monetary and fiscal policy. DSGE model is structural and more comprehensive than previous researches.

This paper investigates the magnitude of tax revenue elasticity to output when positive productivity shock occurs; i.e. I consider the situation of growth situation and investigate how much economic growth contribute tax revenue increase. Productivity shock is equal to total factor productivity (TFP) shock which is the major source of business cycle and long-run economic growth. Therefore, I focus on this shock in this paper but do not focus on other shocks, such as fiscal and monetary policies[2]. As an example of temporary positive productivity shock, I illustrate the temporal deregulation. And a am example of permanent one, I illustrate innovations, such as introduction of new more productive technologies. I obtain three results.

[2] Needless to say, both fiscal and monetary policy shocks affect output and tax revenue elasticity. But, both shocks are temporary and smaller effect to business cycle than productivity. Moreover, these shocks may cause side effect of the government debt, such as lack of fiscal discipline and inflation-financing.

First, short-run impulse responses in both temporary and permanent are negative. Second, medium-run impulse responses are quite large; these values are larger than 2.5 in both shocks. Third, the long-run impulse responses are different; the impulse responses to temporary shock diminish and reverse, on the other hand, the impulse responses to permanent shock are much smoother. Moreover, large value of elasticity can explain not only economic growth or expanding tax base, but also increasing tax rates following the rules of fiscal authority.

The rest of this paper is organized as follows. In section 2, I set out the model in detail. Section 3 calibrates the model, i.e. sets the parameters and simulates the model. Section 4 concludes.

2. The Model

We bring the model of Iwata (2009), which is an extended variant of the medium-scale DSGE model developed Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003), which features various real and nominal rigidities: habit formation, investment adjustment cost, variable capital utilization, stick price and wage á la Calvo (1983), and indexation in prices and wages. Concretely, Iwata (2009) includes non-Ricardian households and distortionary taxation, i.e. labor and capital income and consumption tax.

2.1. Households

There is a continuum of households indexed by $n \in [0,1]$. A fraction 1-ω of this continuum of households indexed by $i \in [0,1-\omega)$ has access to financial markets and acts Ricardian; which means that the households maximize their lifetime utilities by choosing consumption and saving which is equal to investment to financial assets in the form of government bonds, capital stock and utilization rate of capital stock. The remaining households, indexed by $j \in [1-\omega,1]$ do not have access to financial markets and acts non-Ricardian; which means that the households simply consume all of their current disposal income.

2.1.1. Ricardian households

Each number of Ricardian households i maximizes its lifetime utility by choosing consumption $C_t^R(i)$, investment $I_t(i)$, government bonds $B_t(i)$, next

period's capital stock $K_t(i)$ and intensity of the capital stock utilization $z_t(i)$, given the following lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \left(\frac{1}{1-\sigma_c} \left(C_t^R(i) - hC_{t-1}^R \right)^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l} L_t^R(i)^{1+\sigma_l} \right),$$

where β is the discount factor, σ_c denotes the inverse of the intertemporal elasticity of substitution and σ_l is the inverse of the elasticity of labor supply (that is to say, labor effort) with respect to real wages. $L_t^R(i)$ represents the labor supply of the Ricardian household i. h measures the degree of habit formation in consumption. In this utility function, there are two serially correlated shocks, a preference shock ε_t^b and a labor supply shock ε_t^l. These shocks are considered and are assumed to follow a first-order autoregressive process with an i.i.d. normal error terms: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, $\varepsilon_t^l = \rho_l \varepsilon_{t-1}^l + \eta_t^l$.

The Ricardian household faces a real terms flow budget constraint:

$$\begin{aligned} &\left(1+\tau_t^c\right) C_t^R(i) + I_t(i) + \Psi(z_t(i)) K_{t-1}(i) + \frac{B_t(i)}{R_t P_t} \\ &= \left(1-\tau_t^n\right) w_t(i) L_t^R(i) + \left(1-\tau_t^k\right) r_t^k z_t(i) K_{t-1}(i) + \left(1-\tau_t^k\right) \frac{D_t(i)}{P_t} + \frac{B_{t-1}(i)}{P_t}, \end{aligned} \tag{1}$$

where $\Psi(z_t(i))$ represents the cost associated with variations in the degree of capital utilization $z_t(i)$. $\tau_t^c, \tau_t^n, \tau_t^k$ denote consumption, labor and capital income tax rates, respectively. $D_t(i)$ denotes dividends distributed by firms to the Ricardian household i. P_t is aggregate price level, R_t is riskless return on government bonds, $w_t(i)$ is real wage, and r_t^k is real rental rate of capital. $K_{t-1}(i)$ and $B_{t-1}(i)$ denote capital stock and government bonds of the current period that their decisions are made at time t-1. For simplicity, we assume that a consumption tax is livied on private consumption expenditure alone. A lump-sum tax (or transfer) is omitted similar to Iwata (2009).

The physical capital accumulation law of motion for the Ricardian

household is expressed as follows:

$$K_t(i) = (1-\delta)K_{t-1}(i) + \left[1 - S\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\right] I_t(i), \tag{2}$$

where $\Psi(z_t(i))$ is the depreciation rate, S(・) represents the adjustment cost function in investment, ε_t^i is a shock to investment cost function and this shock follows a first-order autoregressive (AR(1)) process as follow:

$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$,

where η_t^i is an i.i.d-normal error term. Following Iwata (2009), I assume the capital utilization rate in steady state as $\bar{z} = 1$, and the corresponding cost as $\Psi(\bar{z}) = 0$. In adding, I assume that the investment adjustment cost function satisfies: $S(1) = S'(1) = 0.$

I let Λ_t and $\Lambda_t Q_t$ denote the Lagrange multipliers, the first-order conditions with respect to $C_t^R(i), B_t(i), I_t(i), K_t(i)$ and $z_t(i)$ are expressed as follows:

$$(1+\tau_t^c)\Lambda_t = \varepsilon_t^b \left(C_t^R(i) - hC_{t-1}^R(i)\right)^{-\sigma_c}, \tag{3}$$

$$\beta R_t E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{P_t}{P_{t+1}}\right] = 1, \tag{4}$$

$$\begin{aligned} &Q_t\left[1 - S\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\right] - Q_t S'\left(\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)}\right)\frac{\varepsilon_t^i I_t(i)}{I_{t-1}(i)} \\ &= -\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} S'\left(\frac{\varepsilon_{t+1}^i I_{t+1}(i)}{I_t(i)}\right)\frac{\varepsilon_{t+1}^i I_{t+1}(i)^2}{I_t(i)^2}\right] + 1, \end{aligned} \tag{5}$$

$$Q_t = -\beta E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left((1-\delta)Q_{t+1} + \left(1-\tau_{t+1}^k\right)r_{t+1}^k z_{t+1}(i) - \Psi\left(z_{t+1}(i)\right)\right)\right] + \eta_t^q, \quad (6)$$

$$\left(1-\tau_t^k\right)r_t^k = \Psi'\left(z_t(i)\right). \quad (7)$$

Q_t represents the shadow value of additional unit of capital, which is the same meaning of Tobin's Q. Letting an over-bar denote a steady-state value, it can be shown that $1/\beta = \overline{R} = 1-\delta+\left(1-\overline{\tau}^k\right)\overline{r}^k + \delta\overline{\tau}^k$ and $\overline{Q} = 1$.

2.1.2. Non-Ricardian households

Non-Ricardian households are modeled as non-optimizing agents following the original assumption in Campbell and Mankiw (1989) and Galí et al (2007). Since the members of non-Ricardian household's *j* do not have access to financial markets, they simply consume all of their after-tax disposal income. I denote consumption and labor income of non-Ricardian households as $C_t^{NR}(j)$ and $L_t^{NR}(j)$, the period-by-period budget constraint they face is given by:

$$\left(1+\tau_t^c\right)C_t^{NR}(j) = \left(1-\tau_t^n\right)w_t(j)L_t^{NR}(j). \quad (8)$$

2.1.3. Wage setting

Similar to Iwata (2009), I assume that the members of Ricardian households act as wage setter for differentiated labor services $L_t^R(i)$ in monopolistically competitive markets[3]. And, the member of non-Ricardian households are assumed to set their wages $W_t^{NR}(j)$ for their differentiated labor services $L_t^{NR}(j)$ to be equal to the average wage of Ricardian households. Since all households face the same labor demand schedule, the following equations are satisfied: $W_t^R(i) = W_t^{NR}(j) = W_t(n)$ and $L_t^R(i) = L_t^{NR}(j) = L_t(n)$.

[3] I use the setting of Erceg et al (2006), Forni et al (2009) and Coenen and Straub (2005).

An independent and perfectly competitive employment agency bundles differentiated labor $L_t(n)$ into a single type of effective labor input L_t using the following technology:

$$L_t = \left[\int_0^1 L_t(n)^{\frac{1}{1+\lambda_{w,t}}} dn \right]^{1+\lambda_{w,t}},$$

where $\lambda_{w,t}$ is the wage markup which follows a i.i.d-normal process with drift λ_w an i.i.d.-normal error term: $\lambda_{w,t} = \lambda_w + \eta_t^w$. The employment agency solves:

$$\max_{L_t(n)} W_t \left[\int_0^1 L_t(n)^{\frac{1}{1+\lambda_{w,t}}} dn \right]^{1+\lambda_{w,t}} - \int_0^1 W_t(n) L_t(n) dn,$$

where $W_t = w_t P_t$ is aggregate nominal wage index. The labor demand schedule for each differentiated labor service is then expressed as:

$$L_t(n) = \left(\frac{W_t(n)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t.$$

Putting the labor demand to the bundler technology of the employment agency gives:

$$W_t = \left[\int_0^1 W_t(n)^{-\frac{1}{\lambda_{w,t}}} dn \right]^{-\lambda_{w,t}},.$$

With probability $1-\xi_w$, each Ricardian household i is assumed to be allowed to reset its wage optimally. On the other hand, with probability ξ_w, each Ricardian household i is assumed to remain its wage. Therefore, the optimal wage $W_t^{R*}(i)$ is given by:

$$W_t^{R*}(i) \equiv \arg\max_{W_t^R(i)} E_0 \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[\frac{1}{1-\sigma_c} \left(C_t^R(i) - h C_{t-1}^R(i) \right)^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l} \left(\left(\frac{W_t^{R*}(i)}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s} \right)^{1+\sigma_l} \right],$$

subject to

$$\left(1+\tau_{t+s}^c\right) C_{t+s}^R(i) + I_{t+s}(i) + \Psi\left(z_{t+s}(i)\right) K_{t+s-1}(i) + \frac{B_{t+s}(i)}{R_{t+s} P_{t+s}}$$
$$= \left(1-\tau_{t+s}^n\right) w_{t+s}(i) L_{t+s}^R(i) + \left(1-\tau_{t+s}^k\right) r_{t+s}^k z_{t+s}(i) K_{t+s-1}(i) + \left(1-\tau_{t+s}^k\right) \frac{D_{t+s}(i)}{P_{t+s}} + \frac{B_{t+s-1}(i)}{P_{t+s}}.$$

Since we know that $W_t^R(i) = W_t^{NR}(j) = W_t(n)$, aggregate nominal wage law of motion is then expressed as:

$$W_t = \left[(1-\xi_w)(W_t^*(n))^{-\frac{1}{\lambda_{w,t}}} + \xi_w \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}(n) \right) \right], \tag{9}$$

where $W_t^*(n) = W_t^{R*}(i)$.

2.2. Firms

There are two types firms: perfectly competitive final-good firms and monopolistic competitive intermediate-good firms indexed by $j \in [0,1]$. The final-good firm produces the good Y_t combining the differentiated intermediate goods $y_t(f)$ produced by the firm *f*.

2.2.1. Final-good firms

The final-good producing firm combines intermediate goods using the following CES-bundler technology:

$$Y_t = \left[\int_0^1 y_t(f)^{\frac{1}{1+\lambda_{w,t}}} df \right]^{1+\lambda_{w,t}},$$

Where $\lambda_{p,t}$ is the wage markup which follows a i.i.d-normal process with

drift λ_p an i.i.d.-normal error term: $\lambda_{p,t} = \lambda_p + \eta_t^p$. The employment agency

solves:

$$\max_{y(f)} W_t \left[\int_0^1 y_t(f)^{\frac{1}{1+\lambda_{p,t}}} df \right]^{1+\lambda_{p,t}} - \int_0^1 p_t(f) y_t(f) df,$$

where $p_t(f)$ is the price of the intermediate good $y_t(f)$. Then, the demand function for the intermediate good is expresses as:

$$y_t(f) = \left(\frac{p_t(f)}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t.$$

Putting the demand to the CES bundler technology of the final-good firm gives a pricing rule:

$$P_t = \left[\int_0^1 p_t(f)^{-\frac{1}{\lambda_{p,t}}} df \right]^{-\lambda_{p,t}}.$$

2.2.2. Intermediate-good firms

Each intermediate-good firm f produces its differential output using an increasing-return-to-scale Cobb-Douglas technology:

$$y_t(f) = \varepsilon_t^a \bar{k}_{t-1}(f)^{\alpha} l_t(f)^{1-\alpha} - \Phi.$$

where $\bar{k}_{t-1}(f)$ is the effective capital stock at time t given $\bar{k}_{t-1}(f) = z_t k_{t-1}(f)$, $l_t(f)$ is the effective labor input bundled by the employment agency and Φ represents a fixed cost. ε_t^a represents a technology shock which follows a AR(1) process with i.i.d.-normal error term: $\varepsilon_t^a = \rho_a \varepsilon_t^a + \eta_t^a$.

Taking the real rental cost of capital r_t^k and aggregate real wage w_t as given, cost minimization subject to the production technology tields marginal cost and labor demand:

$$mc_t = \frac{w_t^{1-\alpha}\left(r_t^k\right)^{\alpha}}{\varepsilon_t^a \alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \tag{10}$$

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{z_t K_{t-1}}{L_t}. \tag{11}$$

Aggregate nominal profits are given by:

$$D_t = P_t Y_t - P_t mc_t (Y_t + \Phi). \tag{12}$$

2.2.3. Price setting

Similar to the case of wag setting, I assume sluggish price adjustment which consists of the staggered price contracts á la Calvo (1983). A fraction $1-\xi_p$ of intermediate-good firms can reset (that is to say, re-optimize) their prices. On the other hand, a fraction ξ_p cannot obtain the opportunity of resetting their prices. Therefore, I can obtain the price indexation scheme as follows:

$$p_t(f) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} p_t(f),$$

where γ_p measures the degree of indexation.

An intermediate-good firm *f*, which is allowed to reset, know the probability ξ_p^s that the price it chooses in this period will still be in effect *s* periods in the future. Taking aggregate nominal price index P_t and Y_t as given, the optimal price $p_t^*(f)$ is chosen as:

$$p_t^*(f) = \underset{p_t(f)}{\arg\max}\, E_0 \sum_{s=0}^{\infty} (\beta\xi_p)^s \left[\left(p_t^*(f) - P_{t+s} mc_{t+s}\right) \left(\frac{p_t^*(f)}{P_{t+s}}\right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s} - P_{t+s} mc_{t+s} \Phi \right].$$

We obtain the aggregate price law of motion as follows:

$$P_t = \left[(1-\xi_p)\left(p_t^*(f)\right)^{-\frac{1}{\lambda_{p,t}}} + \xi_p \left(\left(\frac{P_{t=1}}{P_{t=1}}\right)^{\gamma_p} p_{t-1}(f) \right)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}. \tag{13}$$

2.3. Fiscal and Monetary Authorities

2.3.1. Fiscal Policy

The fiscal authority purchases final goods G_t, issues bonds B_t and levies a consumption, a labor income and a capital income tax at rates τ_t^c, τ_t^n and τ_t^k, respectively. The fiscal authority's real flow budget constraint is expressed as follows:

$$G_t + \frac{B_{t-1}}{P_t} = \tau_t^c C_t + \tau_t^n w_t L_t + \tau_t^k r_t^k z_t K_{t-1} + \tau_t^k \frac{D_t}{P_t} + \frac{B_t}{R_t P_t}. \tag{14}$$

This budget constraint needs fiscal rules to stabilize their budget, especially, bonds. I consider three feedback rules for each tax a government spending rule in log-linearlized form following Forni et al (2009) and Iwata (2009). These feedback rules are expressed as:

$$\hat{\tau}_t^c = \rho_{tc} \hat{\tau}_{t-1}^c + (1-\rho_{tc})\left(\hat{b}_{t-1} - \hat{Y}_{t-1}\right) + \eta_t^{tc}, \tag{15}$$

$$\hat{\tau}_t^n = \rho_{tn}\hat{\tau}_{t-1}^n + (1-\rho_{tn})(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{tn}, \tag{16}$$

$$\hat{\tau}_t^k = \rho_{tk}\hat{\tau}_{t-1}^k + (1-\rho_{tk})(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{tk}, \tag{17}$$

$$\hat{G}_t = \rho_g\hat{G}_{t-1} + (1-\rho_g)(\hat{b}_{t-1} - \hat{Y}_{t-1}) + \eta_t^{g}. \tag{18}$$

where the hats above variables denote log-deviations from steady states. $b_t \equiv B_t/P_t$ denotes government bond in real terms. $\eta_t^{tc}, \eta_t^{tn}, \eta_t^{tk}$ and η_t^g are i.i.d.-normal errors. It should be noted that the fiscal policy rules described here allow partial debt finance, while the debt is to be repaid through tax revenue over time. The speed of repayment is determined by a combination of the coefficients on the debt-to-output ratio of the tax and government expenditure rules, namely by the set of parameters $\rho_i, \phi_i (i = tc, tn, tk, g)$.

2.3.2. Monetary policy

The monetary authority sets nominal interest rates according to a simple feedback rule in log-linearized form:

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\phi_{r\pi}\hat{\pi}_{t-1} + (1-\rho_r)\phi_{ry}\hat{Y}_{t-1} + \eta_t^R. \tag{19}$$

where $\pi_{t-1} \equiv \log(P_{t-1}/P_{t-2})$ denotes inflation rate. An i.i.d.-normal shock η_t^R to the interest rate is assumed.

2.4. Aggregation and Market Clearing

Aggregate consumption C_t and labor hour L_t in per-capita term are given by a weighted average of the corresponding variables for each consumer type:

$$C_t = (1-\omega)C_t^R(i) + \omega C_t^{NR}(j), \tag{20}$$

$$L_t = (1-\omega)L_t^R(i) + \omega L_t^{NR}(j),$$

Since I assume all households supply the same amount of labor, aggregate labor hour is given by:

$$L_t = L_t^R(i) = L_t^{NR}(j).$$

Aggregate government bonds B_t, investment I_t, physical capital K_t

and dividends D_t are expressed as:

$$B_t = (1-\omega)B_t^R(i),$$

$$I_t = (1-\omega)I_t^R(i),$$

$$K_t = (1-\omega)K_t^R(i),$$

$$D_t = (1-\omega)D_t^R(i).$$

Finally, aggregate production function and the final-goods market clearing conditions are given by:

$$Y_t = \varepsilon_t^a z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \Phi, \tag{21}$$

$$Y_t = C_t + I_t + G_t + \Psi(z_t)K_{t-1}. \tag{22}$$

2.5. Log-Linearized Model

2.5.1. Ricardian Households

2.5.1.1. Consumption Euler Equation

From Eq. (3) and (4), we obtain the following equations:

$$\begin{aligned}\hat{C}_t^R = {} & \frac{h}{1+h}\hat{C}_{t-1}^R + \frac{1}{1+h}E_t\hat{C}_{t+1}^R - \frac{1-h}{(1+h)\sigma_C}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \frac{1-h}{(1+h)\sigma_C}\left(\hat{\varepsilon}_t^b - E_t\hat{\varepsilon}_{t+1}^b\right) \\ & - \frac{1-h}{(1+h)\sigma_C}\frac{\bar{\tau}^C}{1+\bar{\tau}^C}\left(\hat{\tau}_t^C - E_t\hat{\tau}_{t+1}^C\right),\end{aligned} \tag{23}$$

where

$$\hat{\varepsilon}_t^b = \rho_b\hat{\varepsilon}_{t-1}^b + \eta_t^b. \tag{24}$$

2.5.1.2. Investment Euler Equation

From Eq. (5) and (4):

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1} + \frac{\varsigma}{1+\beta}\hat{Q}_t - \frac{\beta E_t\hat{\varepsilon}_{t+1}^i - \hat{\varepsilon}_t^i}{1+\beta}, \tag{25}$$

where $\varsigma \equiv 1/S''(1)$ and

$$\hat{\varepsilon}_t^i = \rho_i\hat{\varepsilon}_{t-1}^i + \eta_t^i. \tag{26}$$

2.5.1.3. Q Equation

From Eq. (6) and (4):

$$\hat{Q}_t = -\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \frac{1-\delta}{1-\delta+\left(1-\bar{\tau}^k\right)\bar{r}^k} E_t\hat{Q}_{t+1} + \frac{\left(1-\bar{\tau}^k\right)\bar{r}^k}{1-\delta+\left(1-\bar{\tau}^k\right)\bar{r}^k} E_t\hat{r}^k_{t+1} - \frac{\bar{r}^k\bar{\tau}^k}{1-\delta+\left(1-\bar{\tau}^k\right)\bar{r}^k} E_t\hat{\tau}^k_{t+1} + \hat{\eta}^q_t. \tag{27}$$

2.5.1.4. Capital Utilization Decision Equation

From Eq. (7):

$$\hat{z}_t = \psi\left[\hat{r}^k_t - \frac{\bar{\tau}^k}{1-\bar{\tau}^k}\hat{\tau}^k_t\right], \tag{28}$$

where $\psi \equiv \Psi'(1)/\Psi''(1)$.

2.5.1.5. Capital Law of Motion

From Eq. (2):

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t. \tag{29}$$

2.5.1.6. Real Wage Law of Motion

From Eq. (9):

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\gamma_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \times\left[\hat{w}_t - \sigma_l\hat{L}_t - \frac{\sigma_C}{1-h}\left(\hat{C}^R_t - h\hat{C}^R_{t-1}\right) - \hat{\varepsilon}^l_t - \eta^w_t - \frac{\bar{\tau}^n}{1-\bar{\tau}^n}\hat{\tau}^n_t + \frac{\bar{\tau}^c}{1+\bar{\tau}^c}\hat{\tau}^c_t\right], \tag{30}$$

where

$$\hat{\varepsilon}^l_t = \rho_l\hat{\varepsilon}^l_{t-1} + \eta^l_t. \tag{31}$$

2.5.2. Non-Ricardian Households

From Eq. (8):

$$\frac{\bar{C}^{NR}}{\bar{Y}}\left[\hat{C}^{NR}_t\left(1+\bar{\tau}^c\right) + \bar{\tau}^c\hat{\tau}^c_t\right] = \bar{w}\frac{\bar{L}}{\bar{Y}}\left[\left(1-\tau^n_t\right)\left(\hat{w}_t + \hat{L}_t\right) - \bar{\tau}^n\hat{\tau}^n_t\right] \tag{32}$$

2.5.3. Firms

2.5.3.1. Marginal Cost

From Eq. (10):

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{\varepsilon}_t^a. \tag{33}$$

2.5.3.2. Labor Demand

From Eq. (11):

$$\hat{L}_t = -\hat{w}_t + \hat{r}_t^k + \hat{z}_t + \hat{K}_{t-1}. \tag{34}$$

2.5.3.3. Profit Payment

From Eq. (12):

$$\frac{\overline{D}}{\overline{P}\,\overline{Y}}\hat{d}_t = (1-\overline{mc})\hat{Y}_t - \overline{mc}\,\varphi\hat{mc}_t. \tag{35}$$

2.5.3.4. Inflation Law of Motion

From Eq. (13):

$$\begin{aligned}\hat{\pi}_t &= \frac{\beta}{1+\beta\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} \\ &+ \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p\right],\end{aligned} \tag{36}$$

where

$$\hat{\varepsilon}_t^a = \rho_a\hat{\varepsilon}_{t-1}^a + \eta_t^a. \tag{37}$$

2.5.4. Fiscal and Monetary Authorities

2.5.4.1. Fiscal Policy Rules

From Eq. (14)-(18):

$$\begin{aligned}\frac{\overline{G}}{\overline{Y}}\hat{G}_t + \frac{\overline{B}}{\overline{P}\,\overline{Y}}\left(\hat{b}_{t-1} - \hat{\pi}_t\right) &= \overline{\tau}^c\frac{\overline{C}}{\overline{Y}}\left(\hat{\tau}_t^c + \hat{C}_t\right) + \overline{\tau}^n\overline{w}\frac{\overline{L}}{\overline{Y}}\left(\hat{\tau}_t^n + \hat{w}_t + \hat{L}_t\right) \\ &+ \overline{\tau}^k\overline{r}^k\frac{\overline{K}}{\overline{Y}}\left(\hat{\tau}_t^n + \hat{r}_t^k + \hat{z}_t + \hat{K}_{t-1}\right) + \overline{\tau}^k\frac{\overline{D}}{\overline{P}\,\overline{Y}}\left(\hat{r}_t^k + \hat{d}_t\right) + \beta\frac{\overline{B}}{\overline{P}\,\overline{Y}}\left(\hat{b}_t - \hat{R}_t\right),\end{aligned} \tag{38}$$

$$\hat{\tau}_t^c = \rho_{tc}\hat{\tau}_{t-1}^c + (1-\rho_{tc})\phi_{tcb}\left(\hat{b}_{t-1} - \hat{Y}_{t-1}\right) + \eta_t^{tc}, \tag{39}$$

$$\hat{\tau}_t^n = \rho_{tn}\hat{\tau}_{t-1}^n + (1-\rho_{tn})\phi_{tnb}\left(\hat{b}_{t-1} - \hat{Y}_{t-1}\right) + \eta_t^{tn}, \tag{40}$$

$$\hat{\tau}_t^k = \rho_{tk}\hat{\tau}_{t-1}^k + (1-\rho_{tk})\phi_{tkb}\left(\hat{b}_{t-1} - \hat{Y}_{t-1}\right) + \eta_t^{tk}, \tag{41}$$

$$\hat{G}_t = \rho_g\hat{G}_{t-1} + (1-\rho_g)\phi_{gy}\left(\hat{b}_{t-1} - \hat{Y}_{t-1}\right) + \eta_t^{g}. \tag{42}$$

2.5.4.2. Monetary Policy Rule

From Eq. (19):

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\phi_{r\pi}\hat{\pi}_{t-1} + (1-\rho_r)\phi_{ry}\hat{Y}_{t-1} + \eta_t^{R}. \tag{43}$$

2.5.5. Aggregation and Market Clearing

2.5.5.1. Goods Market Equilibrium Condition

From Eq. (20):

$$\frac{\overline{C}}{\overline{Y}}\hat{C}_t = (1-\omega)\frac{\overline{C}^R}{\overline{Y}}\hat{C}_t^R + \omega\frac{\overline{C}^{RN}}{\overline{Y}}\hat{C}_t^{NR}. \tag{44}$$

From Eq. (22):

$$\hat{Y}_t = \frac{\overline{C}}{\overline{Y}}\hat{C}_t + \delta\frac{\overline{K}}{\overline{Y}}\hat{I}_t + \frac{\overline{G}}{\overline{Y}}\hat{G}_t + \left(1-\tau_t^k\right)\overline{\tau}^k\frac{\overline{K}}{\overline{Y}}\hat{z}_t. \tag{45}$$

2.5.5.2. Aggregate Production Equation

$$\hat{Y}_t = \varphi\left(\hat{\varepsilon}_t^a + \alpha\hat{z}_t + \alpha\hat{K}_{t-1} + (1-\alpha)\hat{L}_t\right), \tag{46}$$

where $\varphi \equiv 1 + \Phi/\overline{Y}$.

3. Calibration

I investigate how much tax revenue elasticity to output when positive productivity shock occurs, because I want to investigate the magnitude of tax revenue elasticity. To investigate it, I analyze the positive productivity shock using the above model and a parameter and steady-state values estimated by previous literatures. Basically, I quote the parameters of Iwata (2009), and then frequency of this paper is quarterly. Table 1 summarizes these parameter values.

Table 1: The values of parameters

α	0.3	ρ_{r}	0.934
β	0.99	$\phi_{r\pi}$	1.533
δ	0.06	ϕ_{ry}	0.254
λ_{w}	0.5	ρ_{g}	0.736
$\bar{\tau}^{c}$	0.08	ϕ_{gy}	0.068
$\bar{\tau}^{n}$	0.32	ρ_{tc}	0.507
$\bar{\tau}^{k}$	0.61	ϕ_{tcb}	0.013
h	0.465	ρ_{tn}	0.568
σ_{c}	1.62	ϕ_{tnb}	0.005
σ_{l}	2.113	ρ_{tk}	0.655
φ	1.904	ϕ_{tkb}	0.123
ψ	0.416	ρ_{a}	0.518
ξ_{w}	0.824	$\bar{K}/\bar{Y}$	2.2
ξ_{p}	0.432	$\bar{B}/\bar{P}\bar{Y}$	0.6
γ_{w}	0.211	$\bar{C}/\bar{Y}$	0.56
γ_{p}	0.595	$\bar{G}/\bar{Y}$	0.16
ω	0.248		

Note: $\frac{\bar{L}}{\bar{Y}}, \bar{w}, \bar{r}^{k}$ and $\overline{mc}$ are all set to be consistent with the steady state conditions implied by the model.

3.1. Parameter and Steady-state values setting

I use the parameter and steady-state values of Iwata (2009), which uses the same model of this paper. Iwata (2009) calibrates several parameters (e.g. depreciation rate) and steady-state values (e.g. government bonds to output), and estimates structural parameter Bayesian inference using a Markov Chain Monte Carlo (MCMC) method. I choose the calibrated and estimated posterior mean values of Iwata (2009).

3.2. Calculating tax revenue elasticity to output

First, I calculate the tax revenue elasticity to output when positive productivity shock occurs shown in the following AR (1) process:

$$\hat{\varepsilon}_{t}^{a} = \rho_{a}\hat{\varepsilon}_{t-1}^{a} + \eta_{t}^{a}.$$

I define (marginal) tax elasticity to output at period t in DSGE model as follows:

Consumption tax elasticity $$\kappa_{c,t} = \frac{\hat{\tau}_t^c + \hat{C}_t}{\hat{Y}_t},$$

Labor income tax elasticity $$\kappa_{n,t} = \frac{\hat{\tau}_t^n + \hat{w}_t + \hat{L}_t}{\hat{Y}_t},$$

Capital income tax elasticity $$\kappa_{k,t} = \frac{\tau^k r^k K}{\bar{Y}} \frac{\hat{\tau}_t^k + \hat{r}_t^k + \hat{z}_t + \hat{K}_{t-1}}{\hat{Y}_t} + \frac{\tau^k d}{\bar{Y}} \frac{\hat{\tau}_t^k + \hat{d}_t}{\hat{Y}_t},$$

Total tax elasticity $$\kappa_{t,t} = \frac{\bar{\tau}^c \bar{C}}{\bar{Y}} \kappa_{tc} + \frac{\bar{\tau}^n \bar{w} \bar{L}}{\bar{Y}} \kappa_{tn} + \kappa_{tk}.$$

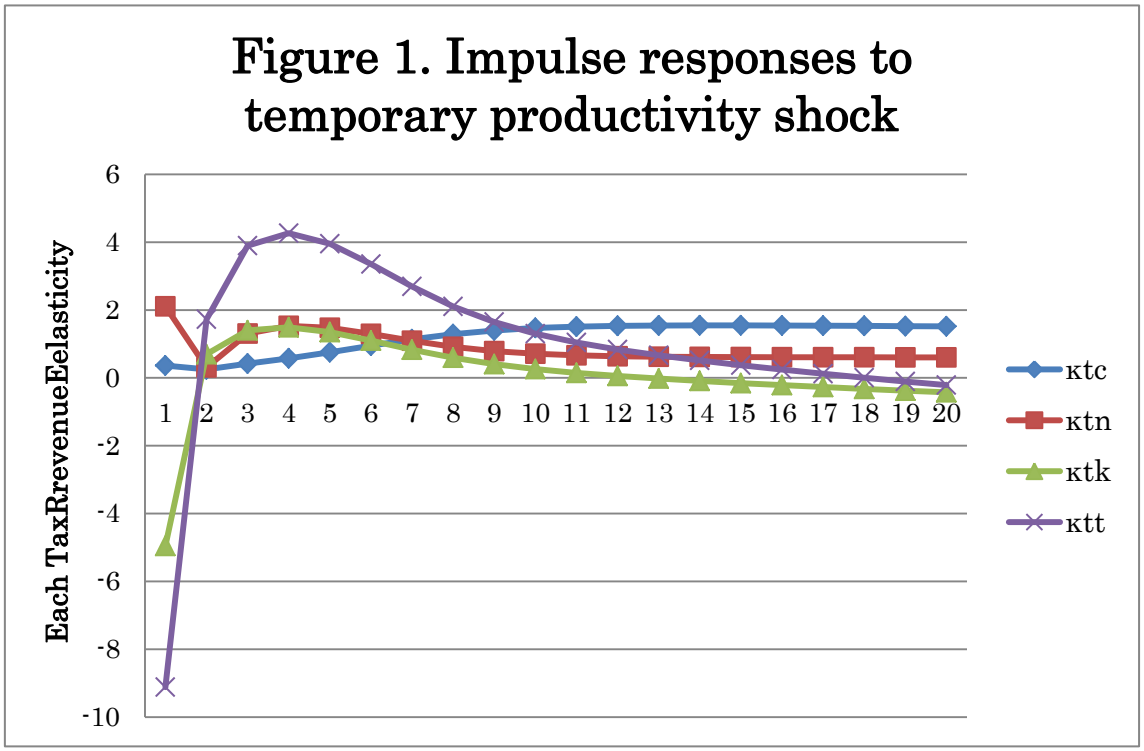
The numerator of consumption and labor and capital tax revenue elasticity means the deviation rate of each tax revenues, and the denominator of them means the deviation rate of output. Total tax revenue elasticity consists of elasticity which weights their steady state values. I calculate the tax revenue elasticity using these measures in next subsection.

3.3. Calculation Results

I show the tax revenue elasticity to output when the temporary and permanent positive productivity shock. I show the impulse responses of each macroeconomics variables in Appendix A.

3.3.1. Temporary productivity shock

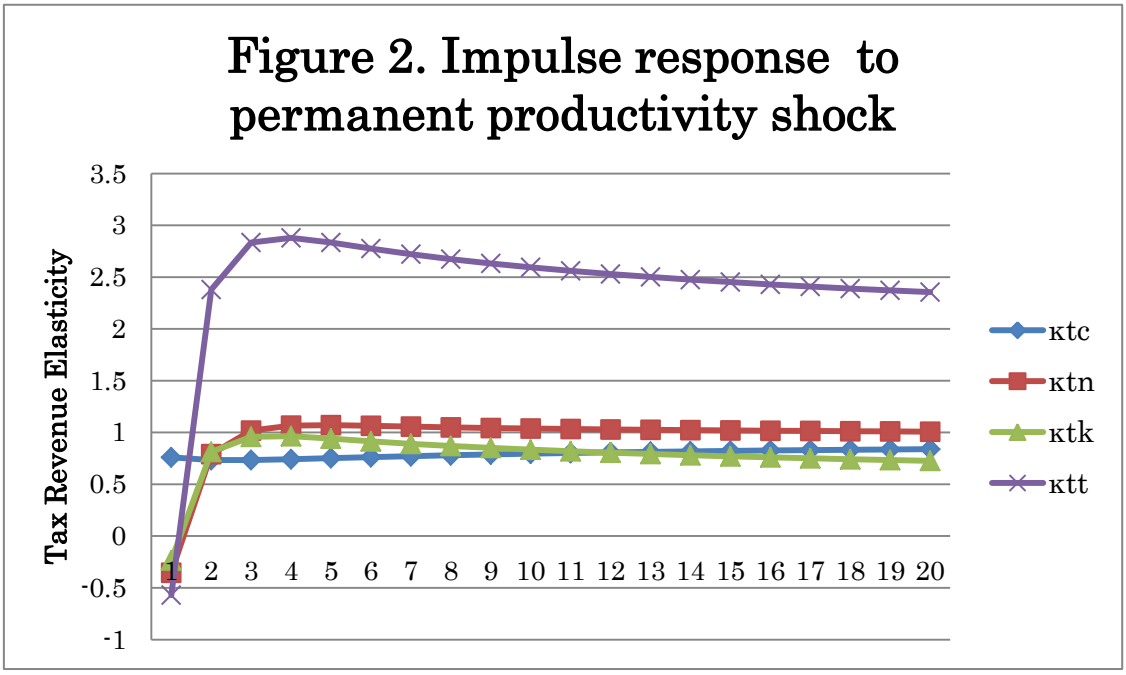
Figure 1 shows the impulse response of tax revenue elasticity to output under temporary positive productivity shock. Figure 1 has several interesting features. First, short-run elasticity (i.e. all impulse responses in period 1) is negative. This is because temporary positive productivity shock decreases labor supply and consumption to increase future output. Second, the perk of elasticity comes nearly period 4 (1 year). The perk value of total tax revenue elasticity is 4.26. Third, the values of these elasticity decreases in long-run. This is because the tax rules work the power to decreasing tax rate via increasing output.



3.3.2. Permanent productivity shock

To analyze the permanent positive productivity shock, I analyze the transition path from the steady states of pre-shock to the of post-shock using the relaxation algorithm[4]. Figure 2 shows the impulse response of tax revenue elasticity to output under permanent positive productivity shock. There are two features which can compare with the results of temporary shock. First, although the magnitude is smaller, the short-run impulses of all elasticity are negative, similar to them of temporary shock. Second, on the other hand, the medium-and-long-run responses are smoother than them of temporary shock and the range of these values hold around 2.

[4] Trinborn, Koch and Steger (2008) detail the relaxation algorithm and provide MATLAB programs for its algorithm.



4. Concluding Remarks

In this paper, I have a contribution: I show the tax revenue elasticity to output using DSGE model. There are two strengths which do not exist in previous literatures. First, this paper can calculate the tax revenue elasticity to structural shock, that is, this paper enables to identify the contribution to economic and tax revenue growth. Second, this paper enables to investigate the dynamic response of the tax revenue elasticity. I obtain two policy implications. First, economic growth to permanent positive productivity shock increases tax revenue much in medium-and-long-run, but decreases it well in short-run. Second, economic growth to temporary positive productivity shock increases tax revenue much in medium-run, but decreases it well in short-and-long-run. These differences of temporary productivity shock come from the initial behavior of Ricardian households and tax policy rules. In the case in temporary shock, Ricardian households decrease labor supply and consumption, and then the total tax revenue. In the long-run, since large output growth induces downward-shifting of tax rate, the total tax revenue decreases in the long-run.

There is a problem which must improve. Standard DSGE model, which includes this paper, cannot analyze the fiscal reconstruction and sovereign default that many politician and researchers are interested in

without some additive (and some of ad-hoc) assumptions[5]. I will try to investigate the tax revenue elasticity to output using DSGE model that can analyze the sovereign default simultaneously.

[5] For example, Uribe (2006) and Bi and Traum (2012) analyzes the (ad-hoc) sovereign default using DSGE framework.

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Appendix A: Impulse responses of main macroeconomic variables to temporary and permanent positive productivity shock