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The Macroeconomic Effects of Individual Commodity Tax

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Abstract
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The Macroeconomic Effects of Individual Commodity Tax

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Abstract[3]

This paper investigates the condition which raising individual consumption tax raises output and private consumption in general equilibrium model. In many of the neoclassical papers, the government expenditure decreases consumption because the government expenditure causes the negative wealth shock. On the other hand, the many empirical results are contrary. This contradiction is called "Fiscal Policy Puzzle". But, if the substitution effect of levying on additive ad-valorem commodity tax of a good is larger than the negative wealth effect of taxing and expanding government expenditure, individual tax finance can increase macroeconomic effect, in contrast to lump-sum and general commodity tax.

Key words: Individual Commodity tax, Substitute effects, Fiscal Policy Puzzle
JEL Classificatuin Numbers: E21, E62, H25.

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1. Introduction

Many developed and several developing countries adopt the multi-commodity (or individual) tax system. Table 1 show that the general commodity tax systems in representative developed countries. Moreover, although several countries, such as United States (U.S.) and Japan, do not accept reduced tax system, they levy different rate specific or ad-varolem tax system, such as alcohol, cigarette and cars etc. On the other hand, we have ever considered the commodity tax as unity in the macroeconomic model, including both non-micro-founded and micro-founded model. We investigate whether the government spending increases (or crowd-in) total private consumption. In other words, multiplier effect of output is larger than 1.

There are a lot of papers which indicate that private consumption rises in response to an increase in government spending such as Fatas and Mihov (2001), Blanchard and Perotti (2002), Gali, Lopez-Salido and Valles (2007), and Monacelli and Perotti (2008), using vector autoregressive (hence force VAR) or structural VAR estimation. In contrast to this fact, Barro and King (1984) show new classical growth model cannot replicate this feature, i.e. private consumption reduces in response to an increase in government spending, because increasing government expenditure occurs negative wealth effect which reduces private consumption. This contradiction is called "Fiscal policy puzzle"[4]. Moreover, this puzzle shows not only short-run fluctuation, but also long-run relation between government expenditure and private consumption.

The inspiration of this paper is originated by Ogaki (1990). Ogaki (1990) shows the substitution or complement effect is divided direct substitution (or complement) effect and indirect one in multi-commodity (more than 3 goods) economy. Figure 1 shows the direct and indirect substitution relationship with respect to three goods (tea, coffee and cream (Samuelson (1974))). In this case, if the indirect substitution effect between coffee and cream is larger than the direct complement one, reducing the price of coffee may decrease the demand of tea via decreasing the demand of tea which is substitute for coffee and complement for cream. We quote this essence and resolve the fiscal policy puzzle. That is, we take advantage of substitution effect of raising individual commodity tax rate. If substitution effect is sufficiently large, expanding government spending financed by individual commodity tax may increase total consumption.

There are several works which try to resolve this puzzle. Especially, we can classify these works into forth types. First is adding assumption to production technology. For

[4] Although Ramey and Shapiro (1998), Edelberg, Eichenbaum and Fischer (1999), Burnside, Eichenbaum and Fischer (2004) and Ramey (2010) insist that government spending does not affect private consumption positively, it is worthwhile to discuss and solve this puzzle.

example, Baxter and King (1993) add the assumption which public capital has productivity externality, such as Aschauer (1989) and show that public investment crowds in public investment and consumption when the production externality is sufficiently high. Productive public investment shows the long-run crowd in effect of private consumption, but cannot prove the short-run. Second is about the assumptions of preferences. For example, Bailey (1971), Barro (1981), Aschauer (1985) Ganelli and Tervasa (2009) and Eguchi and Hosoya (2010) consider direct substitution relations between private consumption and government spending, and show that private consumption rises in response to an increase in government spending when their complementarity is strong enough. Linnemann (2006) and Bilbiie (2008) show the fact satisfies if private consumption is complement with labor. However, their condition satisfies if private consumption is inferior good. Monacelli and Preotti (2008) show the government expenditure crowds in private consumption short-run and long-run, when we introduce Greenwood, Hercowitz and Huffman (1988) type preference and sticky price. Third, Eggertsson (2010) and Christiano, Eichenbaum and Rebelo (2011) show positive response of private consumption occurs when the economy faces zero-lower bound on the nominal interest rate binds. Last is adding the assumption of rule-of-thumb (facing liquidity constraint) household. Gali, Lopez-Salido and Valles (2007) introduces this assumption and the ratio of rule-of-thumb household is large enough (in their paper, more than 0.3), private consumption rises in response to an increase in government spending. But, these papers analyze the model not introducing distortionary tax, but lump-sum tax, and assume that the consumption good is one. Indeed, most of developed countries, they accept consumption tax, besides these countries also accept multiple tax system to deal with regression of consumption tax. In contrast to this fact, however, many researchers doubt the efficacy of reduced tax system[5]. For example, Crawford, Keen and Smith (2008, 2009)[6] show the fact that necessary goods are more complement with luxury ones. This result shows the government should tax heavily on necessary good, if we follow Corlett and Hague (1953). But these do not apply macroeconomic frameworks well and then they focus on efficiency rather than impact to consumption or output. Therefore, we analyze these problems using two-consumption goods general equilibrium model with different commodity tax rate system. And we check the condition whether increasing government spending crowds in (increases) total consumption.

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[5] There are a lot of researches which deal with optimal commodity tax theory: such as Ramsey (1927), Diamond and Mirrlees (1971a, b), Hatta (1986) etc.

[6] Crawford, Keen and Smith (2009) is chapter 4 of "The Mirrlees Review". Crawford, Keen and Smith (2009) clarifies that the necessary goods are more complemented goods than luxury ones empirically.

The rest of this paper is organized as follows. Section 2 investigates the static model in general form, analytical form and numerical simulation. Section 3 discusses several remarkable features and Section 4 expands the model from statics to dynamics and calculates the dynamic response to government expenditure shock. Finally, we notice the conclusion in Section 5.

2. The model

We inspect the condition whether fiscal policy financed by individual consumption tax may increase private consumption or not, using static (without capital and investment) two consumption goods (hence after, we distinct these goods and called them as good C and good X) general equilibrium model.

2.1. General Form

First of all, we analyze the model using general preference and linealized production technology. (Representative) households consume good C, X and earn wages ($W_i(i=1,2)$ is the nominal wage of produced good C(i=1) or X(i=2)) to work N_1 and N_2. For simplicity, we assume that government levies tax rate τ on only good C, disutility of working firm C which produces good C is equal to good X and the pre-tax price of good C is numeraire, the relative price of good X is p.

2.1.1. The Economic Structure

Households face on the following problem:

$$\max_{C,X,N_1,N_2} \quad U(C,X,N_1+N_2) \tag{1}$$

$$\text{s.t.} \quad (1+\tau)C + pX = W_1N_1 + W_1N_2. \tag{2}$$

$$where \quad U_i > 0, U_{ii} < 0 (i=C,X), U_{N_i} < 0, U_{N_iN_j} < 0.$$

We assume that each firm technology is identical (i.e. $Y_i = F(N_i) = N_i$). Using this assumption, wage is equal one because of arbitrage condition (i.e. $W_1 = W_2 = W = 1$) and then relative price p is also equal one and it does not change. Therefore we can rewrite

above problem as following form:

$$\max_{C,X,N} \quad U(C,X,N) \tag{3}$$

$$\text{s.t.} \quad (1+\tau)C + X = N. \tag{4}$$

where $N = N_1 + N_2$.

The government supplies the government expenditure G which is wasteful, or improves welfare, but not affects optimal condition financed by taxing on good C.

$$\tau C = G. \tag{5}$$

We solve the model and show these problems as following equilibrium conditions:

$$U_C = (1+\tau)U_X. \tag{6}$$

$$-U_X = U_N. \tag{7}$$

$$\tau C = G. \tag{8}$$

$$C + X + G = N. \tag{9}$$

We can obtain the value of endogenous variables(C, X, N, G) to specify the preference except for the case which Eq (6) and (7) are linear dependence.

2.1.2. Linearization the equilibrium condition

We linearize the conditions around the each endogenous variable;

$$[U_{CC} - (1+\tau)U_{XC}]dC + [U_{CX} - (1+\tau)U_{XX}]dX + [U_{CN} - (1+\tau)U_{XN}]dN - U_X d\tau = 0, \tag{10}$$

$$(U_{XC} + U_{NC})dC + (U_{XX} + U_{NX})dX + (U_{XN} + U_{NN})dN = 0, \tag{11}$$

$$\tau dC + C d\tau = dG, \tag{12}$$

$$dC + dX - dN = -dG. \tag{13}$$

We represent these equations to matrix,

$$\begin{bmatrix} U_{CC} - (1+\tau)U_{XC} & U_{CX} - (1+\tau)U_{XX} & U_{CN} - (1+\tau)U_{XN} & U_X \\ U_{XC} + U_{NC} & U_{XX} + U_{NX} & U_{XN} + U_{NN} & 0 \\ \tau & 0 & 0 & C \\ 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} dC \\ dX \\ dN \\ d\tau \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}. \tag{14}$$

If the coefficient matrix of left-hand of (14) is non-singular, we can also rewrite Eq (14) to the following reduced form to represent the inverse matrix;

$$\begin{bmatrix} dC \\ dX \\ dN \\ d\tau \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} dG. \tag{15}$$

We obtain the following relations between government spending and consumption of each goods:

$$\frac{dC}{dG} = a_{13} - a_{14},$$
$$\frac{dX}{dG} = a_{23} - a_{24}.$$

Therefore, the government expenditure crowds in the total consumption, if satisfying the following relation: $a_{13} - a_{14} + a_{23} - a_{24} \geq 0$, where:

$$a_{11} = (U_{XX} + U_{XN} + U_{NX} + U_{NN}) \left\{ \begin{array}{l} U_{CC} + \dfrac{\tau U_X}{C} - (1+\tau) U_{XC} \\ - [U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})](U_{XC} - U_{CN} + U_{NC} - U_{NX}) \end{array} \right\}^{-1},$$

$$a_{12} = \left[\begin{array}{l} U_{XC} + U_{XN} + U_{NC} + U_{NN} \\ + (U_{XX} + U_{XN} + U_{NX} + U_{NN}) \dfrac{U_{CC} + U_{CN} + \dfrac{\tau U_X}{C} - (1+\tau)(U_{XC} + U_{XN})}{(1+\tau)(U_{XX} + U_{XN}) - U_{CX} - U_{CN}} \end{array} \right]^{-1},$$

$$a_{13} = \left\{ \tau + \frac{C}{U_X} \left\{ \begin{array}{l} U_{CC} + U_{CN} - (1+\tau)(U_{XC} + U_{XN}) \\ - [U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})] \dfrac{U_{XC} + U_{XN} + U_{NC} + U_{NN}}{U_{XX} + U_{XN} + U_{NX} + U_{NN}} \end{array} \right\} \right\}^{-1},$$

$$a_{14} = \left\{ \begin{array}{l} U_{CC} + U_{CN} - (1+\tau)(U_{XC} + U_{XX}) \\ - [U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})] \dfrac{U_{XC} - U_{XN} + U_{NC} - U_{NN}}{U_{XX} - U_{XN} + U_{NX} - U_{NN}} \end{array} \right\}^{-1}, \\ \cdot \left[U_{CN} - (1+\tau) U_{XN} + \dfrac{U_{XX} + U_{NN}}{U_{XX} + U_{XN} + U_{NX} + U_{NN}} \right]$$

$$a_{21} = -(U_{XX} + U_{XN} + U_{NX} + U_{NN}) \left\{ \begin{array}{l} \left[U_{CC} + \dfrac{\tau U_X}{C} - (1+\tau) U_{XC} \right] \\ - [U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})](U_{XC} - U_{CN} + U_{NC} - U_{NX}) \end{array} \right\}^{-1},$$

$$a_{22}=\left[\begin{array}{l}U_{XX}+U_{XN}+U_{NX}+U_{NN}\\+\left(U_{XC}+U_{XN}+U_{NC}+U_{NN}\right)\dfrac{(1+\tau)\left(U_{XX}+U_{XN}\right)-U_{CX}-U_{CN}}{U_{CC}+U_{CN}+\dfrac{\tau U_X}{C}-(1+\tau)\left(U_{XC}+U_{XN}\right)}\end{array}\right]^{-1},$$

$$a_{23}=\left\{\begin{array}{l}\left[U_{CX}+U_{CN}-(1+\tau)\left(U_{XX}+U_{XN}\right)\right]\dfrac{C}{U_X}\\+\left\{\tau+\dfrac{C}{U_X}\left[U_{CC}+U_{CN}-(1+\tau)\left(U_{XC}+U_{XN}\right)\right]\dfrac{U_{XC}+U_{XN}+U_{NX}+U_{NN}}{U_{XC}+U_{XN}+U_{NC}+U_{NN}}\right\}\end{array}\right\}^{-1},$$

$$a_{24}=\left\{\begin{array}{l}U_{CX}+U_{CN}-(1+\tau)\left(U_{XX}+U_{XN}\right)\\-\left[U_{CC}-U_{CN}-(1+\tau)\left(U_{XC}-U_{XX}\right)\right]\dfrac{U_{XX}-U_{XN}+U_{NX}-U_{NN}}{U_{XC}-U_{XN}+U_{NC}-U_{NN}}\end{array}\right\}^{-1}$$
$$\cdot\left[U_{CN}-(1+\tau)U_{XN}+\frac{U_{XX}+U_{NN}}{U_{XX}+U_{XN}+U_{NX}+U_{NN}}\right],$$

$$a_{31}=\left\{U_{CX}+U_{CN}-(1+\tau)\left(U_{XX}+U_{XN}\right)-\left[U_{CC}+\frac{\tau U_X}{C}-(1+\tau)U_{XC}\right]\left(U_{XC}-U_{CN}+U_{NC}-U_{NX}\right)\right\}^{-1},$$

$$a_{32}=\left[1+\frac{U_{CC}+U_{CN}+\dfrac{\tau U_X}{C}-(1+\tau)\left(U_{XC}+U_{XN}\right)}{(1+\tau)\left(U_{XX}+U_{XN}\right)-U_{CX}-U_{CN}}\right]$$
$$\cdot\left[\begin{array}{l}U_{XC}+U_{XN}+U_{NC}+U_{NN}\\+\left(U_{XX}+U_{XN}+U_{NX}+U_{NN}\right)\dfrac{U_{CC}+U_{CN}+\dfrac{\tau U_X}{C}-(1+\tau)\left(U_{XC}+U_{XN}\right)}{(1+\tau)\left(U_{XX}+U_{XN}\right)-U_{CX}-U_{CN}}\end{array}\right]^{-1},$$

$$a_{33}=\left(1-\frac{U_{XC}+U_{XN}+U_{NX}+U_{NN}}{U_{XC}+U_{XN}+U_{NC}+U_{NN}}\right)$$
$$\left\{\tau+\frac{C}{U_X}\left\{\begin{array}{l}U_{CC}+U_{CN}-(1+\tau)\left(U_{XC}+U_{XN}\right)\\-\left[U_{CX}+U_{CN}-(1+\tau)\left(U_{XX}+U_{XN}\right)\right]\dfrac{U_{XC}+U_{XN}+U_{NC}+U_{NN}}{U_{XX}+U_{XN}+U_{NX}+U_{NN}}\end{array}\right\}\right\}^{-1},$$

$$a_{34}=\left\{U_{CX}+U_{CN}-(1+\tau)\left(U_{XX}+U_{XN}\right)-\left[U_{CC}-U_{CN}-(1+\tau)\left(U_{XC}-U_{XX}\right)\right]\frac{U_{XX}-U_{XN}+U_{NX}-U_{NN}}{U_{XC}-U_{XN}+U_{NC}-U_{NN}}\right\}^{-1}$$
$$\left[U_{CN}-(1+\tau)U_{XN}+\frac{U_{XX}+U_{NN}}{U_{XX}+U_{XN}+U_{NX}+U_{NN}}\right]-\frac{U_{XX}+U_{NX}}{U_{XX}+U_{XN}+U_{NX}+U_{NN}},$$

$$a_{41} = -\frac{\tau}{C}\left(U_{XX} + U_{XN} + U_{NX} + U_{NN}\right)$$
$$\left\{U_{CC} + \frac{\tau U_X}{C} - (1+\tau)U_{XC} - \left[U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})\right](U_{XC} - U_{CN} + U_{NC} - U_{NX})\right\}^{-1},$$

$$a_{42} = -\frac{\tau}{C}\left[\begin{array}{l} U_{XC} + U_{XN} + U_{NC} + U_{NN} \\ + (U_{XX} + U_{XN} + U_{NX} + U_{NN})\dfrac{U_{CC} + U_{CN} + \dfrac{\tau U_X}{C} - (1+\tau)(U_{XC} + U_{XN})}{(1+\tau)(U_{XX} + U_{XN}) - U_{CX} - U_{CN}} \end{array}\right]^{-1},$$

$$a_{43} = \left\{C + \tau U_X \left\{\begin{array}{l} U_{CC} + U_{CN} - (1+\tau)(U_{XC} + U_{XN}) \\ -\left[U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})\right]\dfrac{U_{XC} + U_{XN} + U_{NC} + U_{NN}}{U_{XX} + U_{XN} + U_{NX} + U_{NN}} \end{array}\right\}\right\}^{-1},$$

$$a_{44} = -\frac{\tau}{C}\left\{\begin{array}{l} U_{CC} + U_{CN} - (1+\tau)(U_{XC} + U_{XX}) \\ -\left[U_{CX} + U_{CN} - (1+\tau)(U_{XX} + U_{XN})\right]\dfrac{U_{XC} - U_{XN} + U_{NC} - U_{NN}}{U_{XX} - U_{XN} + U_{NX} - U_{NN}} \end{array}\right\}^{-1}$$
$$\cdot\left[U_{CN} - (1+\tau)U_{XN} + \frac{U_{XX} + U_{NN}}{U_{XX} + U_{XN} + U_{NX} + U_{NN}}\right],$$

We can obtain the sign condition of the consumption with respect to the government expenditure shock using the above equations. But, we cannot clarify the explicit condition in this economy. Therefore, we specify the preference the next subsection.

2.2.Specific Preference

In previous subsection, we solve the crowd-in condition of expanding government spending under general preference. In this section, we analyze the model analytically and numerically using specific preference. For simplicity, we use the following additive-separable type preference:

$$\begin{aligned} U(C, X, N) &= \frac{C^{1-\gamma}}{1-\gamma} + \theta\frac{X^{1-\gamma}}{1-\gamma} - \mu\frac{N^{1+\lambda}}{1+\lambda}, \quad if \quad \gamma \neq 1 \\ &= \ln C + \theta \ln X - \mu\frac{N^{1+\lambda}}{1+\lambda}, \quad if \quad \gamma = 1 \end{aligned}, \tag{16}$$

where γ is the curvature of each goods preferences[7], λ is the inverse of the labor elasticity to wage, θ is the relative weight parameter of consumption, and μ is the disutility parameter of labor.

The specification of Eq (16) usually uses Real Business Cycle model and Dynamic

[7] We assume the curvatures of both goods are equal, because we solve the model analytically. Of course, the districts of these curvature are important to analyze the different elasticity of labor (or leisure), but we cannot solve the model analytically, nor may numerically.

Stochastic General Equilibrium model, but this specification cannot solve the fiscal policy puzzle without sufficiently large of rule-of thumb consumers. We introduce new method to solve the puzzle using common preference.

Solving the model, we can write the following equilibrium conditions:

$$X^{\gamma} = (1+\tau)\theta C^{\gamma}. \tag{17}$$

$$\theta X^{-\gamma} = \mu N^{\lambda}. \tag{18}$$

$$\tau C = G. \tag{19}$$

$$C + X + G = N. \tag{20}$$

Using Eq (17), (18), (19) and (20), we can solve the each endogenous variable(C, X, N, G) analytically:

$$C = \mu^{-\frac{1}{\gamma+\lambda}} \theta^{\frac{1}{\gamma+\lambda}-\frac{1}{\gamma}} (1+\tau)^{-\frac{1}{\gamma}} \left[1 + (1+\tau)^{1-\frac{1}{\gamma}} \theta^{-\frac{1}{\gamma}} \right]^{-\frac{\lambda}{\gamma+\lambda}}, \tag{21}$$

$$X = \mu^{-\frac{1}{\gamma+\lambda}} \theta^{\frac{1}{\gamma}} \left[1 + (1+\tau)^{1-\frac{1}{\gamma}} \theta^{-\frac{1}{\gamma}} \right]^{-\frac{\lambda}{\gamma+\lambda}}, \tag{22}$$

$$N = \theta^{\frac{1}{\lambda}} \mu^{-\frac{1}{\lambda}} X = \mu^{-\frac{1}{\gamma+\lambda}-\frac{1}{\lambda}} \theta^{\frac{2}{\lambda}} \left[1 + (1+\tau)^{\frac{1}{\gamma}-1} \theta^{\frac{1}{\gamma}} \right]^{-\frac{\lambda}{\gamma+\lambda}}, \tag{23}$$

$$G = \tau C = \mu^{-\frac{1}{\gamma+\lambda}} \theta^{\frac{1}{\gamma+\lambda}-\frac{1}{\gamma}} \tau (1+\tau)^{-\frac{1}{\gamma}} \left[1 + (1+\tau)^{1-\frac{1}{\gamma}} \theta^{-\frac{1}{\gamma}} \right]^{-\frac{\lambda}{\gamma+\lambda}}. \tag{24}$$

We can obtain the relationship among consumption and government expenditure using from Eq (21) to (24). We differentiate from Eq (21) to (24) in C, X, N, τ and G, and solve the multipliers of the variables to government expenditure shock. Therefore, we obtain the following proposition:

Proposition:

The government expenditure crowd in total consumption, if

$$\begin{aligned} &\frac{d(C+X)}{d\tau} = \frac{\theta^{\frac{1}{\gamma}}}{\gamma} + \left(1 - \frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda} > 0 \\ &and \quad \frac{dG}{d\tau} = \left[\frac{1}{\gamma(1+\tau)} - \frac{1}{\tau}\right]\left[1 + (1+\tau)^{1-\frac{1}{\gamma}} \theta^{-\frac{1}{\gamma}}\right] + \left(1 - \frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}} \theta^{-\frac{1}{\gamma}} > 0, \end{aligned} \tag{P.1}$$

or

$$\frac{d(C+X)}{d\tau}=\frac{\theta^{\frac{1}{\gamma}}}{\gamma}+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}<0$$

$$and \quad \frac{dG}{d\tau}=\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}<0. \tag{P.2}$$

The proof of this proposition and the other multipliers describe Appendix. First equation is the portion of $\frac{d(C+X)}{d\tau}$ that is tax multiplier. Second equation represents $\frac{dG}{d\tau}$. Therefore, we can obtain the total consumption multiplier $\frac{d(C+X)}{dG}$ calculated by $\frac{d(C+X)}{d\tau}\Big/\frac{dG}{d\tau}$, if $\frac{dG}{d\tau}$ is not zero. This proposition is satisfied when both the tax multiplier to consumption and government spending (equal to tax revenue) are positive or negative. We can interpret the situation which both multipliers are positive as a substitution effect of X is sufficiently large to dominate a negative wealth effect and self substitute effect of good C. And we can interpret the situation which both multipliers are negative as a right hand side of Laffer curve. We can draw a single-peaked Laffer curve with respect to tax revenue and tax rate of good C. Therefore the government reduces tax rate to raise tax base and tax revenue, and total consumption increases because of reduction of tax rate. However, we cannot clarify the condition explicitly. Then we investigate the multiplier numerically in next subsection.

2.3.Numerical Example

We numerically investigate whether the government expenditure crowds in consumption or not. We set the parameters θ =1, τ =0.05.

The multipliers of each variables in each λ and γ are shown Table 2, 3, 4 and 5. Table 2 shows the multiplier of good C, Table 3 shows one of good X, Table 4 shows one of Labor Supply (equal Output) and Table 5 shows one of Total Consumption. We know that the multipliers are larger when λ is larger, and γ is smaller. Especially, the government expenditure crowd in total consumption when λ is equal or larger than 1.5 and γ is equal or smaller than 1.

Figure 1 shows these relations when we tax on tax on good C when λ =1.5 and γ =1

(i.e. the case which increasing good X is larger than decreasing good C). Focusing on substitution effect, taxing on good C reduces consumption on good C, increases consumption on good X and labor supply (output) because of substitution effect of relative price between good C and good X change. If the substitution effect is sufficiently larger than negative wealth effect (which reduces consumption of both goods and output of good X and increases output of good C), raising tax rate on good 1 (and additive government expenditure on good C) let total consumption increase. In this model, λ affects the magnitude of substitution effect to labor supply, and γ affects the one to labor supply and non-taxed good (good X). The larger λ is ,the larger substitution effect of labor supply is, and the smaller γ is, the larger substitution effect of labor supply and good X.

Table 6 shows the relation between tax rate and government expenditure, and Figure 7 shows the marginal effect of labor supply to tax. We can interpret an interesting feature of the model using these tables. That is, there are some extremely cases which reduction of tax rate is increased tax revenue. When the parameters λ is equal or larger than 2 and γ is equal or smaller than 0.5, we deal with the case in right side of Laffer curve and satisfies Eq (P.2) in Proposition. Moreover, Table 6 shows that reduction of tax rate quite increases labor supply.

3. Discussion

In this section, we discuss several remarkable points: the relation between government expenditure and labor supply, multiplier effect of individual commodity tax and alternative financing methods. Case in alternative finance methods: lump-sum tax and general commodity tax. We investigate whether alternative finance method can also increase consumption and multiplier is larger than 1.

3.1.Does the government expenditure really increase labor supply?

It is very important for showing crowd in the consumption to increase the labor supply in this model. Therefore, we investigate the relation between government expenditure and labor supply. In many Dynamic Stochastic General Equilibrium (DSGE) (including Real Business Cycle) papers, the government expenditure expands labor supply (or creates employment) in their economy. Especially, Christiano and Eichenbaum (1992) showed the fact comparing with model and data[8]. Moreover, Forni, Monteforte and

[8]Concretely, Christiano and Eichenbaum (1992) compares with model and data using generalized

Sessa (2009) also clarified this relation in Bayesian-DSGE model with distortionary tax system and Rule-of-thumb household a la Gali, Lopez-Salido and Valles (2007). We think therefore that the government expenditure increases labor supply.

3.2. Financed by lump-sum tax

We rewrite household and resource constraints and government budget constraint:

$$C + X = N - T, \tag{25}$$

$$C + X + G = N, \tag{26}$$

$$G = T, \tag{27}$$

where T is lump-sum tax..

and others remain. Therefore, we rewrite equilibrium:

$$\theta C^{\gamma} X^{-\gamma} = 1, \tag{28}$$

$$\mu C^{\gamma} N^{\lambda} = 1, \tag{29}$$

$$C + X + G = N, \tag{30}$$

We rewrite these conditions and solve the each consumption and labor functions with respect to government expenditure:

$$\left(1 + \theta^{\frac{1}{\gamma}}\right) C - \mu^{-1} C^{-\frac{\gamma}{\lambda}} = -G, \tag{31}$$

$$\left(1 + \theta^{\frac{1}{\gamma}}\right) \theta^{-\frac{1}{\gamma}} X - \mu^{-1} \theta^{\frac{1}{\lambda}} X^{-\frac{\gamma}{\lambda}} = -G, \tag{32}$$

$$\left(1 + \theta^{\frac{1}{\gamma}}\right) \mu^{-\frac{1}{\gamma}} N^{-\frac{\lambda}{\gamma}} - N = -G, \tag{33}$$

Linealized from Eq (31) to (33) in C, X, N and G, we can obtain the following equations:

$$\frac{dC}{dG} = -\left[\frac{\gamma}{\lambda\mu} C^{-\frac{\gamma+\lambda}{\gamma}} + 1 + \theta^{\frac{1}{\gamma}}\right]^{-1} < 0, \tag{34}$$

$$\frac{dX}{dG} = \theta^{\frac{1}{\gamma}} \frac{dC}{dG} = -\theta^{\frac{1}{\gamma}} \left[\frac{\gamma}{\lambda\mu} C^{-\frac{\gamma+\lambda}{\gamma}} + 1 + \theta^{\frac{1}{\gamma}}\right]^{-1} < 0, \tag{35}$$

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method of moment (GMM).

$$\frac{dN}{dG}=\frac{dC}{dG}+\frac{X}{dG}+1=1-\left(1+\theta^{\frac{1}{\gamma}}\right)\left[\frac{\gamma}{\lambda\mu}C^{-\frac{\gamma+\lambda}{\gamma}}+1+\theta^{\frac{1}{\gamma}}\right]^{-1} \tag{36}$$
$$\Leftrightarrow 0<\frac{dN}{dG}<1.$$

We can see that total consumption is decreased. Figure 2 shows these relations. In this case, the negative wealth effect only works, and then the both consumptions are decreased.

3.3. Financed by uniform commodity tax

In this case, we do not need to show this result analytically. This result seems to be a one good economy with consumption tax. We clarify the negative wealth effect decreases total consumption and moreover self-substitute effect also decreases the household's consumption behavior. Figure 3 shows the relation which government decreases consumption because of negative wealth effect and self-substitute effect.

4. Application for Dynamic General Equilibrium Model

We expand the static model to dynamic general equilibrium model to show that the "fiscal policy puzzle" may resolve under individual commodity tax system. We change the previous section to infinite-lived agent model with capital, and the government levies only good C on consumption tax (that tax rate is $\tau_{C,t}$)[9].

4.1. The dynamic model

We assume that a firm produces output and transforms output into consumption good C or X using linear transform technologies. Concretely, we set the environment that one unit of output changes one unit consumption good C or X. Therefore, we represent a firm's production technology as a following Cobb-Douglas functional form (α is capital share of output):

$$F(K_t,N_t)=K_t^{\alpha}N_t^{1-\alpha}.$$

For simplicity, we assume that there is no growth component (e.g. population growth

[9] In this section, we analyze only individual commodity tax, but we can also replicate the positive response of government response of total consumption to the government expenditure shock.

and technology progress) in this economy and relative price is 1+ τ (i.e. both good's (pre-tax) prices are one).

The (representative) household has following lifetime utility:

$$\sum_{t=o}^{\infty} \beta^t \left(\ln C_t + \theta \ln X_t - \mu \frac{N_t^{1+\lambda}}{1+\lambda} \right), \tag{37}$$

and she has a period budget constraint:

$$(1+\tau_{C,t})C_t + X_t + K_{t+1} = w_t N_t + r_t K_t + (1-\delta)K_t. \tag{38}$$

The government budget constraint is balanced at each period:

$$\tau_{C,t} C_t = G_t, \tag{39}$$

The government expenditure follows the AR (1) process (where ρ is the persistency parameter of government expenditure, G is the steady state value and ε is white noise):

$$G_{t+1} = \rho G_t + (1-\rho)G + \varepsilon_{t+1}, \quad 0 \le \rho \le 1. \tag{40}$$

Markets are clearing the following relations (where δ is depreciation rate of capital):

$$C_t + X_t + G_t + K_{t+1} = K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t, \tag{41}$$

4.2.The Equilibrium Conditions

Solving the above model, we can obtain the following equilibrium conditions:

$$\frac{C_{t+1}}{C_t} = \beta\left(1 + \alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} - \delta\right)\frac{1+\tau_{C,t+1}}{1+\tau_{C,t}}, \tag{42}$$

$$X_t = (1+\tau_{C,t})\theta C_t, \tag{42}$$

$$\theta X_t^{-1} = \mu N_t^{\lambda}. \tag{43}$$

$$\tau_{C,t} C_t = G_t. \tag{44}$$

$$C_t + X_t + G_t + K_{t+1} = K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t, \tag{45}$$

$$G_{t+1} = \rho G_t + (1-\rho)G + \varepsilon_{t+1}, \quad 0 < \rho < 1. \tag{46}$$

4.3.The numerical example

In this subsection, we simulate the dynamic response of each macroeconomic variable to government expenditure shock financed by tax on good C. We set the parameters α =0.3, β=0.99, δ=0.01, λ=1.5, θ=1, μ=2, τ=0.05 and ρ=0.95. We solve the steady state values of macroeconomic variables (i.e. C, X,K,N and G) and linearized Eq(42), (43), (44), (45) and (46) by each endogenous variables around the steady state. After log-linearization, we solve the policy function and analyze the dynamic responses of each macroeconomic variable to the government expenditure shock.

Figure 4 shows the result under our model and parameter values. In this case, the government expenditure crowds out consumption C, but crowds in consumption X and total consumption (i.e. the increase of X is larger than the decrease of C). On the other hand, investment does not vary and labor supply absorbs all output adjustment of government expenditure shock in this case. Therefore, real wage decreases in this situation.

5. Conclusion

In this paper, we can show the condition whether the government expenditure increases total consumption and multiplier effect is larger than 1 using separable two-goods general equilibrium model. And we may also show that the relatively actual tax system (i.e. introducing commodity tax (and implicitly labor income tax because of equivalency between commodity tax and linear income tax)) can explain the more than 1 multiplier and positive correlation between government expenditure and total consumption using the equivalency of consumption and labor income taxes. Moreover, we clarify this feature under the dynamic general equilibrium model. To be concluded, in this paper we emphasize the importance of introducing the detailed tax systems and the substitution and complementarity of multiple goods and labor (or leisure), when we analyze a policy using both static and dynamic general equilibrium model.

There are several remaining problems. First, we do not discuss about relative price for simplicity in static analysis. We need to analyze this effect. Second, we need to consider the other puzzle, that is, response of real wage to the government spending shock. But, we can modify the problem using additive assumption, such as productive government

spending, or rule-of-thumb consumers. Third is the application to DSGE model in this paper's concept. We have to analyze more practical model and estimate the some important parameters using structural estimation, such as Bayesian DSGE model.

Appendix: The analytical solutions of the multiplier effects of expanding government expenditure.

First, differentiating Eq (22), (23), (24) and (25):

$$\begin{aligned}\frac{dX}{d\tau} &= -\frac{\lambda}{\gamma+\lambda}(1+\tau)\theta^{-\frac{1}{\gamma}}\left(1-\frac{1}{\gamma}\right)\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}X \\ &= \frac{\lambda}{\gamma+\lambda}\mu^{-\frac{1}{\gamma+\lambda}}\theta^{\frac{2}{\gamma}}\left(1-\frac{1}{\gamma}\right)(1+\tau)\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-\frac{\lambda}{\gamma+\lambda}-1}\end{aligned}, \tag{A.1}$$

$$\frac{dC}{d\tau} = -\frac{1}{\gamma}(1+\tau)^{-\frac{1}{\gamma}-1}\theta^{-\frac{1}{\gamma}}X - (1+\tau)^{-\frac{1}{\gamma}-1}\theta^{-\frac{1}{\gamma}}\frac{dX}{d\tau}, \tag{A.2}$$

$$\frac{dN}{d\tau} = -\mu^{-\frac{1}{\lambda}}\theta^{\frac{1}{\lambda}}X^{-\frac{\gamma}{\lambda}}\frac{dX}{d\tau}, \tag{A.3}$$

$$\frac{dG}{d\tau} = C + \tau\frac{dC}{d\tau} = (1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}X + \tau\frac{dC}{d\tau}. \tag{A.4}$$

where $X = \mu^{-\frac{1}{\gamma+\lambda}}\theta^{\frac{1}{\gamma}}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-\frac{\lambda}{\gamma+\lambda}}$,

If (A.4) is not equal zero, we can obtain the multiplier effects of government expenditure:

$$\begin{aligned}\frac{dC}{dG} &= \frac{\frac{dC}{d\tau}}{\frac{dG}{d\tau}} = \frac{\frac{dC}{d\tau}}{(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}X + \tau\frac{dC}{d\tau}} \\ &= \frac{1}{\tau - \left\{\frac{1}{\lambda} + \frac{\lambda}{\gamma+\lambda}\theta^{-\frac{1}{\gamma}}(1+\tau)^{-\frac{1}{\gamma}}\left(1-\frac{1}{\gamma}\right)\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]\right\}^{-1}},\end{aligned} \tag{A.5}$$

$$\begin{aligned}\frac{dX}{dG} &= \frac{\frac{dX}{d\tau}}{\frac{dG}{d\tau}} = \frac{\frac{dX}{d\tau}}{(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}X + \tau\frac{dC}{d\tau}} \\ &= (1+\tau)^{\frac{1}{\gamma}}\theta^{\frac{1}{\gamma}}\left\{1-\left[1-\frac{\tau}{\gamma(1+\tau)}\right]\frac{\gamma+\lambda}{\lambda}\left(1-\frac{1}{\gamma}\right)(1+\tau)^{\frac{1}{\gamma}}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]\right\}^{-1},\end{aligned} \tag{A.6}$$

$$\frac{dC}{dG}+\frac{dX}{dG}=\frac{C\left\{1+\frac{1}{\gamma+\lambda}\left\{\lambda-1-\frac{(1-\gamma)\lambda}{\gamma}\left[(1+\tau)^{1-\frac{1}{\gamma}}+1\right]^{-1}\right\}\right\}}{C\left\{1+\frac{\tau}{(\gamma+\lambda)(1+\tau)}\left\{\lambda-1-\frac{(1-\gamma)\lambda}{\gamma}\left[(1+\tau)^{1-\frac{1}{\gamma}}+1\right]^{-1}\right\}\right\}}$$

$$=1+\left\{\tau-\left\{\frac{1}{\lambda}+\frac{\lambda}{\gamma+\lambda}\theta^{-\frac{1}{\gamma}}(1+\tau)^{-\frac{1}{\gamma}}\left(1-\frac{1}{\gamma}\right)\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]\right\}^{-1}\right\}^{-1}$$

$$+(1+\tau)^{\frac{1}{\gamma}}\theta^{\frac{1}{\gamma}}\left\{1-\left[1-\frac{\tau}{\gamma(1+\tau)}\right]\frac{\gamma+\lambda}{\lambda}\left(1-\frac{1}{\gamma}\right)(1+\tau)^{\frac{1}{\gamma}}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]\right\}^{-1}$$

$$=\frac{(1+\tau)^{-\frac{1}{\gamma}-1}\left[\frac{\theta^{\frac{1}{\gamma}}}{\gamma}+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}\right]}{\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}} \tag{A.7}$$

$$\frac{dN}{dG}=\frac{dC}{dG}+\frac{dX}{dG}+1=$$

$$=1+\frac{(1+\tau)^{-\frac{1}{\gamma}-1}\left[\frac{\theta^{\frac{1}{\gamma}}}{\gamma}+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}\right]}{\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}}. \tag{A.8}$$

Therefore, the government expenditure crowds in total consumption in the following cases:

$$\frac{\theta^{\frac{1}{\gamma}}}{\gamma}+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}>0$$

$$\textit{and}\quad \left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}>0,$$

or

$$\frac{\theta^{\frac{1}{\gamma}}}{\gamma}+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}<0$$

$$and \quad \left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]+\left(1-\frac{1}{\gamma}\right)\frac{\lambda}{\gamma+\lambda}(1+\tau)^{-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}<0.$$

Therefore, we can only obtain the relation about λ analytically using the above equations:

$$\lambda<-\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1} \quad if \quad 1<\gamma+\theta^{\frac{1}{\gamma}}, \quad and \quad \gamma<\frac{\tau}{1+\tau}.$$

$$or \quad \lambda<\frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}}$$

$$-\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1}<\lambda<\frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}},$$

$$if \quad 1>\gamma+\theta^{\frac{1}{\gamma}}, \gamma<\frac{\tau}{1+\tau}$$

$$and \frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}}+\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1}>0.$$

.

$$\frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}}<\lambda<-\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1}$$

$$if \quad 1<\gamma+\theta^{\frac{1}{\gamma}}, \gamma<\frac{\tau}{1+\tau}$$

$$and \frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}}+\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1}<0.$$

$$\lambda > -\left\{1+(1-\alpha)(1+\tau)^{-\frac{1}{\gamma}}\gamma^{-1}\theta^{\frac{1}{\gamma}}\left[\frac{1}{\gamma(1+\tau)}-\frac{1}{\tau}\right]^{-1}\left[1+(1+\tau)^{1-\frac{1}{\gamma}}\theta^{-\frac{1}{\gamma}}\right]^{-1}\right\}^{-1} \quad ,or \quad \lambda > \frac{\gamma\theta^{\frac{1}{\gamma}}}{1-\gamma-\theta^{\frac{1}{\gamma}}},$$

$$if \quad 1 > \gamma + \theta^{\frac{1}{\gamma}} and \quad \gamma > \frac{\tau}{1+\tau}.$$

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Table 1: Standard and Reduced Commodity Tax rate of EU Countries

Countries	Standard Tax Rate	Reduced Tax Rate	Target Items
U.K	17.5%	0%	Foods, Waters, Newspapers, Magazines, Medicines, etc.
		5%	Lighting and Heating expenses, Rebuilding, etc.
Germany	16%	7%	Foods, Waters, Newspapers, Magazines, Internal Transportations, etc.
France	19.6%	5.5%	Foods, Waters, Magazines, etc.
		2.2%	Newspapers, Medicines, etc.
Netherland	19%	6%	Foods, Medicines, Books, Farm products, etc.
Italy	20%	10%	Groceries, Broadcasting, Movies, Hotels, etc.
		4%	Farm products, Groceries, Magazines, etc.
Spain	16%	7%	Farm products, Waters, Transportations, etc.
		4%	Groceries (except for luxuries), Books, Medicines, etc.

Table 2: The multiplier effect of good C (i.e. $\frac{dC}{dG}$) in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	−18.7394	−20	−20.1687	−20.1938	−20.1757	−20.1284	−20.073
0.5	−1.07905	−2.22222	−2.43163	−2.47503	−2.46386	−2.40689	−2.33095
1	0.297107	−1.05263	−1.35481	−1.4333	−1.4349	−1.35836	−1.23966
1.5	0.767029	−0.68966	−1.05566	−1.16461	−1.18471	−1.10066	−0.94978
2	1.008365	−0.51282	−0.92496	−1.05947	−1.09944	−1.01529	−0.84064
3	1.255977	−0.33898	−0.81319	−0.98718	−1.06485	−0.99217	−0.7879
5	1.45993	−0.20202	−0.74369	−0.96777	−1.10496	−1.07029	−0.84796

Table 3: The multiplier effect of good X (i.e. $\frac{dX}{dG}$) in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	0.086914	1.05	−0.10588	−0.05056	−0.02477	−0.01228	−0.00543
0.5	0.22523	1.05	−0.61155	−0.25219	−0.11972	−0.05956	−0.02669
1	0.28116	1.05	−1.51747	−0.50281	−0.2298	−0.11484	−0.05221
1.5	0.306534	1.05	−2.99767	−0.75188	−0.33138	−0.16629	−0.07664
2	0.321019	1.05	−5.85158	−0.99941	−0.42539	−0.21428	−0.10005
3	0.336941	1.05	−122.02	−1.48993	−0.59388	−0.30122	−0.14405
5	0.350863	1.05	8.19917	−2.45314	−0.86933	−0.44597	−0.22223

Table 4: The multiplier effect of Labor supple (i.e. $\frac{dN}{dG}$) in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	−17.6525	−17.95	−19.2746	−19.2443	−19.2004	−19.1407	−19.0785
0.5	0.146183	−0.17222	−2.04318	−1.72721	−1.58358	−1.46645	−1.35764
1	1.578267	0.997368	−1.87228	−0.9361	−0.6647	−0.47321	−0.29187
1.5	2.073563	1.360345	−3.05333	−0.91649	−0.51609	−0.26695	−0.02642
2	2.329384	1.537179	−5.77654	−1.05889	−0.52483	−0.22957	0.059305
3	2.592918	1.711017	−121.834	−1.4771	−0.65873	−0.29339	0.068044
5	2.810793	1.84798	8.455483	−2.42091	−0.97429	−0.51626	−0.07019

Table 5: The multiplier effect of Labor supple (i.e. $\frac{d(C+X)}{dG}$) in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	−18.6525	−18.95	−20.2746	−20.2443	−20.2004	−20.1407	−20.0785
0.5	−0.85382	−1.17222	−3.04318	−2.72721	−2.58358	−2.46645	−2.35764
1	0.578267	−0.00263	−2.87228	−1.9361	−1.6647	−1.47321	−1.29187
1.5	1.073563	0.360345	−4.05333	−1.91649	−1.51609	−1.26695	−1.02642
2	1.329384	0.537179	−6.77654	−2.05889	−1.52483	−1.22957	−0.9407
3	1.592918	0.711017	−122.834	−2.4771	−1.65873	−1.29339	−0.93196
5	1.810793	0.84798	7.455483	−3.42091	−1.97429	−1.51626	−1.07019

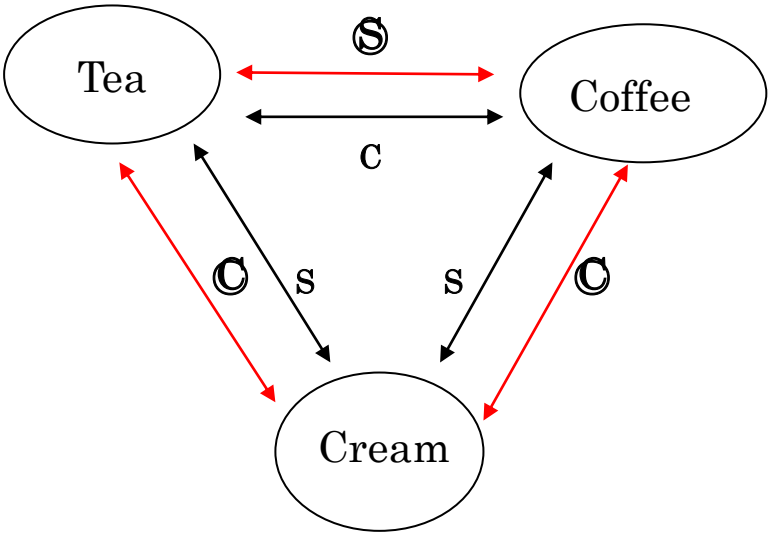
Table 6: The value of $\frac{d\tau}{dG}$ in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	19.73943	21	21.16874	21.19376	21.17566	21.1284	21.07302
0.5	2.079046	3.222222	3.431625	3.475028	3.463865	3.406892	3.330947
1	0.702893	2.052632	2.354805	2.433297	2.434899	2.358364	2.239663
1.5	0.232971	1.689655	2.05566	2.164615	2.184711	2.100659	1.949778
2	−0.00837	1.512821	1.924962	2.059475	2.099439	2.015292	1.840642
3	−0.25598	1.338983	1.813194	1.987176	2.06485	1.992168	1.787904
5	−0.45993	1.20202	1.743687	1.967773	2.10496	2.070288	1.847956

Table 7: The value of $\frac{dN}{d\tau}$ in each γ and λ.

$\lambda \diagdown \gamma$	0.5	1	1.5	2	3	5	10
0.1	−0.89428	−0.85476	−0.91052	−0.90802	−0.90672	−0.90592	−0.90535
0.5	0.070313	−0.05345	−0.5954	−0.49704	−0.45717	−0.43044	−0.40758
1	2.245388	0.485897	−0.79509	−0.38471	−0.27299	−0.20065	−0.13032
1.5	8.900527	0.805102	−1.48533	−0.4234	−0.23623	−0.12708	−0.01355
2	−278.456	1.016102	−3.00086	−0.51415	−0.24999	−0.11392	0.03222
3	−10.1295	1.277848	−67.1928	−0.74332	−0.31902	−0.14727	0.038058
5	−6.11135	1.537395	4.849199	−1.23028	−0.46286	−0.24936	−0.03798

Figure 1: Direct and Indirect Substitution Effect of Tea, Coffee and Cream.



Note:

: Direct effect.

: Indirect effect

Ⓢ: Direct substitution relation

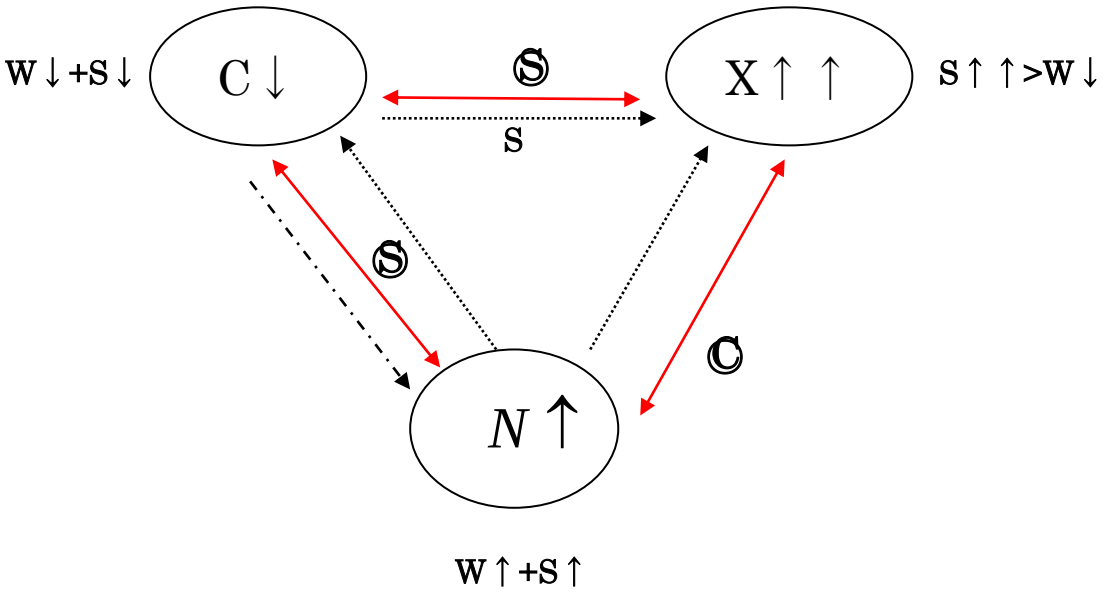
Ⓒ: Direct complement relation

S: Indirect substitution relation

C: Indirect complement relation

Suppose that the price of coffee reduces. The demand of cream decreases, if the indirect substitute effects between coffee and cream is larger than the direct complement one.

Figure 2: Changes of each variable when expanding government expenditure financed by tax rate of good 1 raised (the case which substitution effect increases labor supply).



Note:

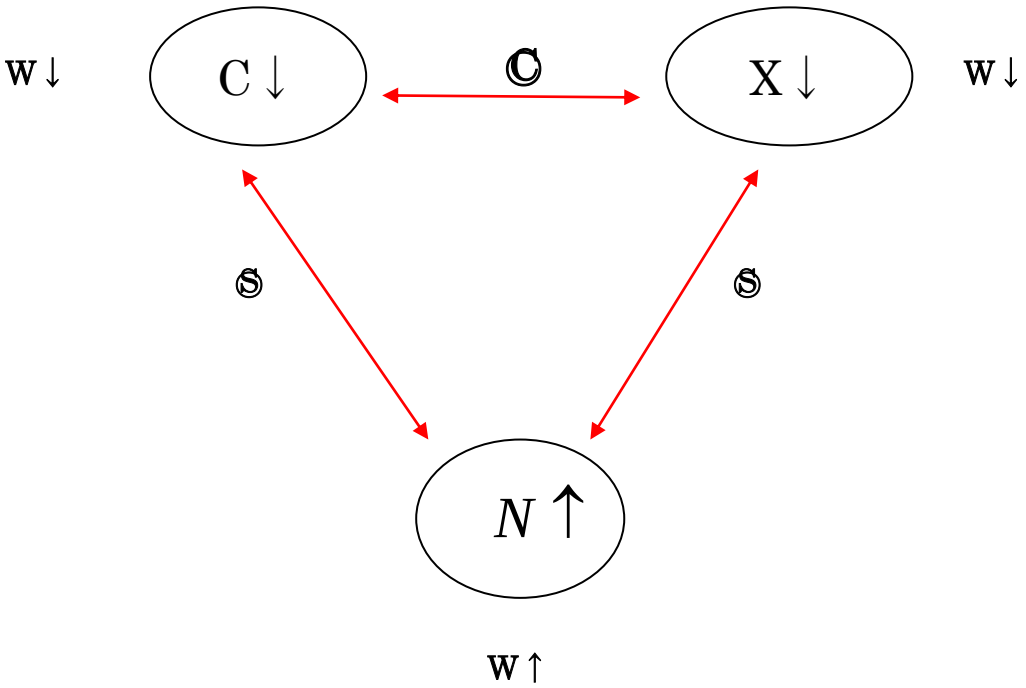
⋯▸ : Indirect positive wealth effect (which is caused by substitution effect to labor supply) which increases both consumptions.

-·-·▸ : Direct substitution effect which increases labor supply

↔ : Total relation of each other (gross substitute Ⓢ or gross complement Ⓒ)

In this case, X↑ ↑>C↓, then total consumption is positive response.

Figure 3: Changes of each variable when expanding government expenditure financed by lump-sum tax on good 1.



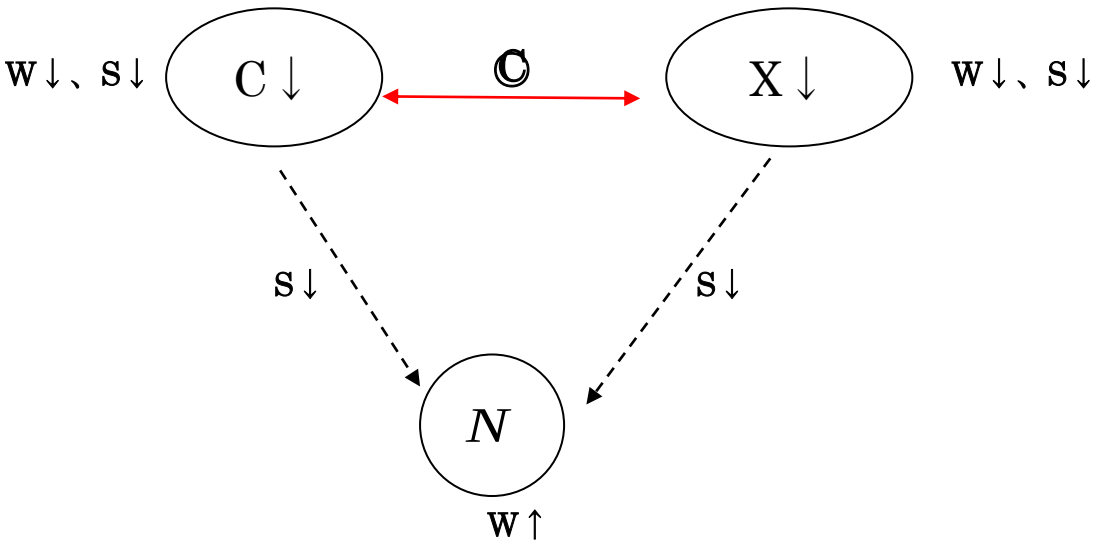
Note:

W: Negative wealth effect of government expenditure of good 1

Ⓒ: Gross complement relation

Ⓢ: Gross substitute relation

Figure 4: Changes of each variable when expanding government expenditure financed by general commodity tax.



Note:

W: Negative wealth effect of government expenditure of good 1

©: Gross complement relation

Ⓢ: Gross substitute relation

The sign of N is not obvious (it depends on the magnitude of negative wealth shock and substitution effects).

Figure 5: The dynamic response of output, labor supply and each consumption defviated from steady state to government expenditure shock with commodity tax on good C (time=50).