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Sharing water resources among cities far from a river basin

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Sharing water resources among cities far from a river basin

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Abstract:
This paper considers welfare allocation rules when cities far from a river basin share water resources. The underlying situation is that the cities share a pipe from the water source and they are willing to distribute the welfare produced by using water. I propose two welfare allocation rule: the Shapley value rule and the nucleolus rule. The outcomes of these rules correspond to the Shapley value and the nucleolus, which embody criteria of fairness in the corresponding game respectively. For each rule, I provide an axiomatic characterization under a doctrine called Unlimited Territorial Integrity advocated in international dispute. Moreover, each rule gives a simple expression for computation of the outcome.

Keywords: Water resources; Unlimited territorial integrity; Shapley value; Nucleolus

JEL classification: C71; D61; D62; Q25

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1 Introduction

Water shortages have been raising big environmental problems on the earth. For example, Financial Times published on August 22, 2006 reported that "*A third of the world's population is suffering from a shortage of water, raising the prospect of "water crises" in countries such as China, India and the US.*" According to this journal, scientists had forecast in 2000 that one in three would face water shortages by 2025, but water experts have been shocked to find that this threshold has already been crossed. This raises an important problem: How can we overcome the growing water scarcity crisis in the 21st century?"

A practical solution to the above problem may be management of water resources under a set of common rules widely accepted all over the world. In fact, two doctrines advocated in international disputes about water resources are well known: Absolute Territorial Sovereignty (or ATS for short) and Unlimited Territorial Integrity (or UTI for short). While ATS says that a country has absolute sovereignty over the area of any river basin on its territory, UTI does that a country should not alter the natural conditions on its own territory to the disadvantage of a neighboring country. However, there are few cases of practical international disputes based on ATS. Therefore management of water resources under a set of rules based on UTI may be useful in order to overcome water scarcity crisis.

This study provide a rationale for management of water resources under a set of rules based on UTI from the view of game theory. I consider welfare allocation rules when cities far from a river basin share water resources. Imagine a line which is divided into n segments indexed in a given order $i = 1, \cdots, n$ from the foot of a mountain to the top of it. There are n cities linearly ordered along the line. City 1 is located at the foot of the mountain, hence only city 1 is near a river basin. Consider a straight pipe from city 1 to city n. Each city except city 1 cannot take water from the river basin without using this pipe. Note that my model is applicable to various welfare sharing problems, in which agents share natural resources such as oil fields or natural gas ones. Under the above setting, I investigate how the cities should distribute the total welfare produced by using water. I propose two welfare allocation rule: the Shapley value rule and the nucleolus rule. The outcomes of these rules correspond to the Shapley value and the nucleolus, which embody criteria of fairness in the corresponding game respectively. For each rule, I provide an axiomatic characterization under UTI. This implies

that management of water resources under a set of rules (axioms) based on UTI leads to fair outcomes.

Cooperative solutions such as the *Shapley value* and the *nucleolus* have been employed as normative measures of distributive outcomes in economic and social problems,e.g., Legros (1987) and Peleg and Sudhölter (2003). The definitions of the Shapley value and the nucleolus embody criteria of fairness respectively. Note that the Shapley value and the nucleolus of a water resources sharing game defined in the present study is in the core, the definition of which embodies a pattern of social stability. Therefore the Shapley rule and the nucleolus rule attain core allocations.

Lastly I state the relationship between the literature and my study. There can be found the literature analyzing cooperative solutions in water resources problems, e.g., Rogers (1969), Suzuki and Nakayama (1978), Ni and Wang (2007) and Ambec and Ehlers (2008)[1]. Rogers (1969) and Ambec and Ehlers (2008) analyzed cores of welfare sharing games derived from international river basins respectively. Suzuki and Nakayama (1978) considered the nucleolus of a cost sharing game derived from a project of water resources development in Kanagawa prefecture, Japan. Ni and Wang (2007) considered the Shapley value of a cost sharing game of a polluted river and presented an axiomatic characterization. As regards methodology in the present paper, my study is concerned with Ambec and Sprumont (2002) and Ni and Wang (2007). While Ambec and Sprumont (2002) interpreted ATS and UTI in terms of rights in a welfare allocation problem of an international river basin, Ni and Wang (2007) did ATS and UTI in terms of responsibilities in a pollution cost sharing problem of it. The present paper interprets UTI in terms of rights in the welfare allocation problem and presents axiomatic characterizations of the Shapley value rule and the nucleolus one based on UTI.

2 A model

Consider a line which is divided into n segments indexed in a given order $i = 1, \cdots, n$ from the foot of the mountain to the top of it. There are n cities located along the line. Each of the cities is one of these segments according

[1] As regards non-cooperative analysis of water resources problems, see Onuma and Sakaue (2007).

to the above order. Only city 1 is near a river basin, hence city 1 has the unique source of water among the cities. Consider a straight pipe from city 1 to city n. Each city except city 1 cannot take water from the river basin without using this pipe. The welfare of city i is b_i if it takes water.

From the viewpoint of rights, in the present study, the welfare sharing problem is considered as how to split the total welfare $\sum_{i\in N} b_i$ among the n cities. Let $N = \{1, \cdots, n\}$ be a finite set of cities and let $B = (b_1, \cdots, b_n) \in \mathbb{R}^n_+$ be a welfare vector. A welfare sharing problem is a pair (N, B). When N is fixed, we simply call vector B a problem. Let $\mathcal{B}$ be the set of welfare sharing problems. A *welfare allocation rule* is a mapping φ that assigns to each problem $B \in \mathcal{B}$ a vector $\varphi(B) \in \mathbb{R}^n_+$, whose component $\varphi_i(B)$ is the welfare assigned to agent $i \in N$. A solution to a problem $B \in \mathcal{B}$ is a vector $\varphi \in \mathbb{R}^n_+$ such that $\sum_{i\in N} \varphi_i = \sum_{i\in N} b_i$.

In order to provide a game theoretic analysis on these rules, introduce a coalitional game with transferable utilities (for short a coalitional TU game). Let $S \subseteq N$ be a coalition. Let v be a characteristic function as a real valued function on 2^N satisfying $v(\emptyset) = 0$. I define a *water resources sharing game* (N, v) satisfying

$$v(S) = \begin{cases} 0 & \text{if} \quad 1 \notin S \\ \sum_{j=1}^{\tilde{i}-1} b_j & \text{if} \quad 1 \in S \subsetneq N \\ \sum_{j=1}^{n} b_j & \text{if} \quad S = N. \end{cases}$$

where

$$\tilde{i} = \min\{i | i \in N \setminus S\}.$$

The above characteristic function $v(S)$ denotes the welfare that S can get by itself. This characteristic function is based on the follows. First, every coalition S not including city 1 cannot get anything because of the lack of the source of water. Second, a pipe available for taking water is through a sequence of cities from city 1, hence the welfare of S is $\sum_{j=1}^{\tilde{i}-1} b_j$ such that $\tilde{i} = \min\{i | i \in N \setminus S\}$ if S including city 1 is not N.

Remark 1 *In the the water resources sharing game the costs of the pipe is not explicitly incorporated. Under a given cost structure, however, we can deal with the water resources sharing game. Consider that a line which is equally divided into n segments indexed in a given order $i = 1, \cdots, n$. Moreover consider that each city has homogeneous technology of constructing the pipe.*

Let c be the cost of the pipe per one segment. I assume $c \leq \min\{b_i : i \in N\}$. Then the net welfare of S can be described as the following characteristic function:

$$w(S) = \begin{cases} 0 & \text{if} \quad 1 \notin S \\ \sum_{j=1}^{\tilde{i}-1}(b_j - c) & \text{if} \quad 1 \in S \subsetneq N \\ \sum_{j=1}^{n}(b_j - c) & \text{if} \quad S = N. \end{cases}$$

where $\tilde{i} = \min\{i | i \in N \setminus S\}$. This game (N, w) is the same as the water resources sharing game (N, v). Therefore we focus on only the game (N, v) for simplicity of our analysis.

3 The Shapley value and the nucleolus

In this section, I will give the definitions of the Shapley value and the nucleolus.

First, I will give a definition of the Shapley value. An idea behind the Shapley value is a pattern of egalitarianism: the aim is to distribute the gains from trade equally.

An n-dimensional vector x is a payoff vector if it satisfies $\sum_{i \in N} x_i = v(N)$. A *value* is a function that assigns a unique payoff vector to every coalitional TU game. A value ϕ satisfies the balanced contributions property if for every coalitional TU game (N, v) we have

$$\phi_i(N, v) - \phi_i(N \backslash \{j\}, v) = \phi_j(N, v) - \phi_j(N \backslash \{i\}, v)$$

whenever $i \in N$ and $j \in N$. Note that given $S \subset N$, I denote by (S, v) the TU game obtained by restricting $v(\cdot)$ to the subsets of S. The balanced contributions property says that what player i gets out of the presence of player j is the same as player j gets out of the presence of player i.

The *Shapley value* $\phi(N, v)$ is defined as the unique value that satisfies the balanced contributions property[2]. The Shapley value $\phi(N, v)$ will be abbreviated to $\phi(v)$. Therefore the Shapley value $\phi(v)$ is an egalitarian solution.

[2]For more extensive recent accounts of the Shapley value, see Osborn and Rubinstein (1994) and Mas-Colell *et al.* (1995).

Next, the nucleolus is defined as the imputation which minimizes the greatest dissatisfaction of any coalition in the following sense.

A payoff vector x is called an imputation if x satisfies the individual rationality conditions $x_i \geq v(\{i\})$ for all $i \in N$. Let X be the set of all imputations. Given an imputation $x \in X$, the *excess* of a coalition S with respect to x is defined as the number $v(S) - \sum_{i \in S} x_i$. The excess of a coalition S with respect to x is a measure of the dissatisfaction that the coalition S feels when x is proposed. Let $e(x)$ be the $(2^n - 2)$ dimensional vector, the components of which are the excesses of every non-empty coalition $S \neq N$ with respect to x, arranged in the non-increasing order.

The *nucleolus* of v, $\mu(v)$ is defined as a set of imputations such that the vector $e(x)$ is *lexicographically minimal* over X. The definition of the nucleolus emphasizes a normative principal of egalitarianism among coalitions. This normative principal is in a certain respect a notion of justice à la Rawls (1971). It is well known that the nucleolus $\mu(v)$ is never empty and a singleton (Schmeidler,1969).

4 Characterization of a welfare allocation rule: the Shapley value case

In this section, I will provide an axiomatic characterization of the Shapley value rule. By the Shapley value rule, I mean the welfare allocation rule that yields the Shapley value.

I now introduce the following axioms.

Additivity: For any $B^1 = (b_1^1, \cdots, b_n^1) \in \mathcal{B}$ and $B^2 = (b_1^2, \cdots, b_n^2) \in \mathcal{B}$, we have

$$\varphi_j(B^1 + B^2) = \varphi_j(B^1) + \varphi_j(B^2) \quad \text{for all } j \in N.$$

Imagine that each firm i has two divisions with welfare b_i^1 and b_i^2 respectively and located along two different lines (pipes). Then, Additivity says that it is same either we allocate the overall total welfare vector or we allocate the two separate vectors and then add them together.

Independence of Downward Welfare: Fix any $i \in N$. For any $B,\ B' \in \mathcal{B}$ such that $b_l = b'_l$ for all $l > i$, we have $\varphi_j(B) = \varphi_j(B')$ for all $j > i$.

This axiom says that an agent's welfare share only depends on her own welfare as well as all upward welfare, but not on downward welfare for which she has no right.

Downward Symmetry: For any $i \in N$, for all $j,\ k \leq i$, we have

$$\varphi_j(0, \cdots, 0, b_i, 0, \cdots, 0) = \varphi_k(0, \cdots, 0, b_i, 0, \cdots, 0).$$

This axiom requires that all downward cities have equal right for a given upward welfare. This is because I assume that it is hard to tell exactly how much each downward agent contributes to the given upward welfare.

Efficiency: For any $B \in \mathcal{B},\ \sum_{j=1}^{n} \varphi_j(B) = \sum_{j=1}^{n} b_j$.

Additivity and Efficiency are natural axioms. Independence of Downward Welfare and Downward Symmetry are derived from UTI.

Proposition 1 *For any $B \in \mathcal{B}$, the only welfare allocation rule satisfying Additivity, Independence of Downward Welfare, Downward Symmetry and Efficiency is given by $\tilde{\varphi}(B)$ satisfying*

$$\tilde{\varphi}_i(B) = \frac{1}{i} b_i + \frac{1}{i+1} b_{i+1} + \cdots + \frac{1}{n} b_n, \quad \textit{for } i = 1, \cdots, n.$$

Proof. Omitted. The proof is the same as Ni and Wang (2007). ■

Now we have the following theorem.

Theorem 1 *The Shapley value rule is the only welfare allocation rule satisfying Additivity, Independence of Downward Welfare, Downward Symmetry and Efficiency.*

Before the proof of this theorem, it is useful to introduce the basic properties of dual games.

Let $\mathcal{G}$ be the class of all TU games with player set N. Given a game $v \in \mathcal{G}$, the dual of $v,\ v^*$ is defined by

$$v^*(S) = v(N) - v(N \backslash S) \quad \text{for all } S \subseteq N.$$

The value $v^*(S)$ is the amount that the complement $N\backslash S$ cannot prevent S from obtaining. The dual game is considered as a cost game when the original game is a saving game, e.g., Peleg and Sudhölter (2003).

It is well known that (1) $v \in \mathcal{G}$ is convex if and only if $v^* \in \mathcal{G}$ is concave and (2) the relationship between the Shapley value of this v and one of it's anti-dual v^* is as follows:

$$\phi(v) = \phi(v^*).$$

Note that a game $v \in \mathcal{G}$ is convex if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$ for all $S, T \subseteq N$; a game $v' \in \mathcal{G}$ is concave if $v'(S) + v'(T) \geq v'(S \cup T) + v'(S \cap T)$ for all $S, T \subseteq N$. While convexity deals with positive bandwagon effect in economics, concavity does with negative one. Moreover, it is well known that the Shapley value and the nucleolus of convex games are in the core. The same holds in concave games. For various economic applications of these games, see Peleg and Sudhölter (2003).

Now we are in a position to prove Theorem 1.

Proof. By simple calculation, we have the dual of the water resources sharing game (N, v) as the game (N, v^*) satisfying $v^*(S) = \sum_{j=\min S}^{n} b_j$ for all $S \subseteq N$ with $v^*(\emptyset) = 0$. Note $\min S = \min\{i | i \in S\}$. Let b_i be the pollutant-cleaning costs of city i. Then (N, v^*) is the cost sharing game of polluted rivers defined by Ni and Wang (2007). In Ni and Wang (2007), the Shapley value of (N, v^*), $\phi(v^*)$ is determined as $\phi(v^*) = \frac{1}{i} b_i + \frac{1}{i+1} b_{i+1} + \cdots + \frac{1}{n} b_n$. By Proposition 1 and the property of the dual, namely $\phi(v) = \phi(v^*)$, the proof is completed. ■

Remark 2 *Ni and Wang (2007) demonstrated that the cost sharing game of polluted rivers is concave. Also, by the proof of our theorem 1, (N, v^*) is the cost sharing game in Ni and Wan (2007). These facts imply that the water resources sharing game (N, v) is convex, hence the outcome of the Shapley value rule is a core allocation.*

5 Characterization of a welfare allocation rule: the nucleolus case

In this section, I will provide an axiomatic characterization of the nucleolus rule. By the nucleolus rule, I mean the welfare allocation rule that yields the

nucleolus.

I now introduce the following axioms.

Scale Invariance: This consists of (1) and (2) as follows.
(1) For any $B \in \mathcal{B}$ and $\alpha > 0$, $\varphi(\alpha B) = \alpha\varphi(B)$.
(2) Whenever $B, B' \in \mathcal{B}$ satisfy $b_j = b'_j$ for $j \in N\backslash\{1\}$ and $b'_1 = b_1 + \beta$ with $\beta > 0$, $\varphi(B') = \varphi(B) + \beta \cdot (1, 0, \cdots, 0)$.

This axiom says the following conditions: If the two problems B, $B' \in \mathcal{B}$ such that $\alpha > 0$, $b'_j = \alpha b_j$ for $j \in N\backslash\{1\}$ and $b'_1 = \alpha b_1 + \beta$ with $\beta > 0$, then their solutions are related by the same transformation on the utilities of the cities.

Equal Treatment Property: A city $k \in N$ and a city $l \in N$ are substitutes with respect to $B \in \mathcal{B}$ if $\sum_{j=1}^{\min(S\cup\{k\})^C - 1} b_j = \sum_{j=1}^{\min(S\cup\{l\})^C - 1} b_j$ for all $S \subseteq N\backslash\{k, l\}$. Then a rule φ has the equal treatment property if the following condition is satisfied: If $B \in \mathcal{B}$ and $k, l \in N$ are substitutes, then $\varphi_k(B) = \varphi_l(B)$. Note that $\min S = \min\{i | i \in S\}$ and S^C is the complement of S.

This axiom says the following condition: If two cities are substitutes with respect to $B \in \mathcal{B}$, then their allocations are equal.

Reduced Flow Consistency: For $B \in \mathcal{B}$ and a consecutive set $T = \{1, 2, \cdots, k\} \subset N$, the reduced flow sharing problem with respect to T is denoted by B_T, where B_T is the projection of B into $\mathbb{R}_+^{|T|}$. This sharing problem B_T is considered as how to split the total welfare $\sum_{i\in N} b_i - \sum_{i=k+1}^{n} \varphi_i(B)$ among the $|T|$ cities. Then a rule φ has reduced flow consistency if the following condition is satisfied: For $B \in \mathcal{B}$ and any consecutive set T, $\varphi_T(B) = \varphi(B_T)$, where φ_T is the projection of $\varphi(B)$ into $\mathbb{R}_+^{|T|}$.

This axiom says the solution of the reduced flow sharing problem B_T is consistent with the solution of the original sharing problem B. The reduced flow sharing problem B_T such that $T = \{1, 2, \cdots, k\}$ means a welfare sharing problem caused by a straight pipe from city 1 to city k after each city $i = k + 1, \cdots, n$ gets $\varphi_i(B)$.

Scale Invariance is a natural axiom. Equal Treatment Property and Reduced Flow Consistency are derived from UTI.

Now we are in a position to prove Theorem 2.

Theorem 2 *The nucleolus rule is the only welfare allocation satisfying Scale Invariance, Equal Treatment Property and Reduced Flow Consistency.*

Proof. Let Γ_U be the set of all games whose set of players is contained in N. It is well known that the prenucleolus on Γ_U satisfies covariance, equal treatment property and reduced game consistency. Hence the nucleolus allocation rule satisfies our properties. Let φ be a mapping $\mathcal{B} \to \mathbb{R}^n_+$ that satisfies our properties. Then φ satisfies efficiency $\sum_{j=1}^n \varphi_j(B) = \sum_{j=1}^n b_j$ by Scale Invariance and Reduced Flow Consistency. The proof of it is the same as one of Lemma 6.2.11 in Peleg and Sudhölter (2003). Moreover, it is an element of the prekernel. This is because Peleg and Sudhölter (2003) demonstrated that this assertion in a general framework holds. The prekernel is a singleton, hence coincides with the nucleolus, in this case the corresponding water resources sharing game under consideration are convex. ■

Remark 3 *Potters and Sudhölter (1999) presented an axiomatic characterization of the nucleolus rule in their airport problem. Note that the water resources sharing game is different from the airport game. Although the underlying idea of our axioms is the same as that of Potters and Sudhölter (1999), our axioms such as Equal Treatment Property and Reduced Flow Consistency are characterized by the specific structure of welfare sharing problems.*

Remark 4 *The nucleolus is in the core since the water resources sharing game is convex.*

Lastly, I will give in the nucleolus rule, as in the Shapley value rule, a simple expression for computation of the outcome.

Proposition 2 *Let $\varphi^*(B)$ be the nucleolus rule of the welfare sharing problem $B \in \mathcal{B}$. Let $b_i > 0$ and $M_i = \sum_{j=i}^n b_j$ for each $i \in N$. Then φ_i^* is given by $\varphi_i^* = \Lambda_t^*$ and $\varphi_1^* = b_1 + \Lambda_{t'}^*$ for $m_{t-1}^* - k_t^* + 2 \leq i \leq m_{t-1}^*$ and $1 \leq t \leq t'$ such that $m_{t'}^* = 1$, where Λ_t^*, k_t^* and m_t^* are defined inductively by*

$$\Lambda_t^* = \min_{k=2,\cdots,m_{t-1}^*} \left\{ \frac{M_{m_{t-1}^*-k+2} - \sum_{i=0}^{t-1}(k_i^* - 1)\Lambda_i^*}{k} \right\},$$

k_t^* denotes the largest value of k for which the above expression attains its minimum and $m_t^* = m_{t-1}^* - k_t^* + 1$ (beginning with $m_0^* \equiv n,\ \Lambda_0^* \equiv 0$ and continuing for $t = 1, \cdots, t'$ such that $m_{t'}^* = 1$).

The anti-duality defined by Oishi and Nakayama (forthcoming) is highly useful for the proof of this theorem. First, I will introduce the anti-dual as follows:

Definition 1 *(Oishi and Nakayama, forthcoming) The anti-dual of v, $(-v)^*$ is defined by the dual of $-v$; that is,*

$$(-v)^*(S) = (-v)(N) - (-v)(N \setminus S) = -v(N) + v(N \setminus S), \quad \textit{for all } S \subseteq N.$$

An economic interpretation of the anti-dual is that it is a cost game when we regard the original game as a profit game. For more accounts of the anti-dual, see Oishi and Nakayama (forthcoming).

Theorem 2 in the Oishi and Nakayama's paper implies the following remark:

Remark 5 *Let $v \in \mathcal{G}$ be a convex game and let $\mu(v)$ be the nucleolus of v. Then, $\mu((-v)^*) = -\mu(v)$.*

This remark is useful to prove Preposition 2. Now we are in a position to prove Preposition 2.

Proof. The anti-dual of our water resources game, $(-v)^*(S)$ is as follows:

$$(-v)^*(S) = \begin{cases} -\sum_{j=1}^{n} b_j & \text{if} \quad 1 \in S \\ -\sum_{j=\min S}^{n} b_j & \text{if} \quad 1 \notin S. \end{cases}$$

Let $c_i = \sum_{j=i}^{n} b_j$. Then the above anti-dual is regarded as a well-known airport game as follows:

$$(-v)^*(S) = -\{c_i : i = \min S\}.$$

Note that c_i is the construction cost of the runway for type i aircraft. The underlying situation of the game $(-v)^*$ is that the cost is increasing in the length of the runway, so that

$$c_1 > c_2 > \cdots > c_n > 0.$$

By using Remark 5, the nucleolus of our water resources game is given by that of the airport game. The nucleolus of the airport game $(-v)^*(S)$ is given by Littlechild (1974). Therefore we can give the nucleolus of the water resources game as mentioned in this proposition, which completes the proof. ■

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