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Effects of Exit on Growth in an Imperfectly Competitive Economy

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Abstract

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Effects of Exit on Growth in an Imperfectly Competitive Economy*

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Abstract

This paper develops a tractable model for the simultaneous investigation of the exit of firms and economic growth. Using this model, it shows that the existence of exit of firms weakens the positive effects of an intensified product market competition on the growth rate. A more intense product market competition encourages surviving firms to innovate by raising the marginal gain from innovation, while it discourages them to innovate due to raising the possibility of firms to exit the market. This implies that an increase of intensity of competition may reduce the growth rate if it causes too many firms to exit the market. The result contrasts with that of standard creative-destruction models.

JEL classifications: L1, O3, O4
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1 Introduction

The relationship between innovation and competition is considered as one of the interesting issues on economics and is investigated in recent theoretical studies. In standard R&D-based endogenous growth models, i.e., Romer (1990), Grossman

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and Helpman (1991), and Aghion and Howitt (1992), this relationship is predicted to be negative since the increased intensity of competition reduces the monopoly rent, which discourages the incentives of outside innovators to innovate. However, the prediction derived from these models is inconsistent with the empirical evidence[1], which motivated recent theoretical studies to construct models to predict a positive relationship between innovation and competition. For example, Aghion, Harris and Vickers (1997) and Aghion, Harris, Howitt and Vickers (2001) construct models of step-by-step innovation and emphasize that an increase in the intensity of competition encourages neck-and-neck firms to innovate by raising the incremental profits of their innovation.[2] Contrary to these models, the model in this paper emphasizes the role of exit of firms on the growth rate and shows that the existence of exit mitigates the incentives of surviving firms to innovate. Therefore, it implies that competition is harmful for growth if it causes too many firms to exit. This result contrasts with that of standard creative-destruction models (e.g. Grossman and Helpman (1991), and Aghion and Howitt (1992)), where exit of firms has a positive effect on economic growth since it is occured as a simply side-effect of entry of newcomers with the state-of-the-art technology, the source of economic growth.

One notable theoretical literature that focuses on the entry-exit behavior of firms in dynamic frameworks is Hopenhayn (1992)[3] . He constructs a competitive industry equilibrium model and determines the exit rate of firms in a steady state. His framework is applied to various studies. For example, Hopenhayn and Rogerson (1993) apply a general equilibrium model version of the model to study the effects of the changes in firing costs on total employment and welfare. Das and Das (1997) analyze the effect of entry adjustment costs on the transition path to the stationary state. Melitz (2003) constructs a general equilibrium version of the model to analyze the structure of international trade. More recently, Asplund and Nocke (2006) apply the model to investigate the relationship between firm turnover and market size. On the contrary, the model in this paper is an R&D-based endogenous growth model, which incorporates the framework of Hopenhayn (1992) to investigate the relationship between the growth and exit of firms.

Originally, the model in this paper is an extension of fully endogenous growth models developed by Dinopoulos and Thompson (1998), Peretto (1998), Howitt

[1]See, for example, Scherer (1967), Nickell (1996), Blundell, Griffith and Reenen (1995), etc.

[2]More recently, using a similar model, Aghion, Bloom, Blundell, Griffith and Howitt (2005) showed that there is an inverted-U relationship between innovation and competition. In their model, the result is derived by the fact that, on the one hand, competition raises the incentives of firms in the neck-and-neck state to innovate; however, on the other hand, it reduces the fraction of firms in the neck-and-neck state, which discourages innovation by technological leaders.

[3]The other seminal work in this field is Jovanovic (1982), who firstly introduced a stochastic industry dynamic model with heterogeneous firms.

(1999), and Segerstrom (2000), which incorporate both quality improvements and variety expansion so that the R&D subsidy policy affects the growth rate without the scale effects on growth. However, the model in this paper is different from these models in that innovation is executed by the surviving firms in the market, instead of outsiders and there exist stochastic production shocks, which lead to the involuntary exit of firms from the market.

The remaining paper is organized as follows. Section 2 sets up a model without exit to define the effects of exit on growth later. The main result of this section is Theorem 1, which proves that the effects of competition on growth can be decomposed into two parts: one is the "escape-competition effect,"[4], in which an increase in the intensity of competition raises the growth rate due to an increase in the marginal gain from innovation. The other is the "entry-reducing" effect, in which an increase in the intensity of competition raises growth due to a reduction in the number of entrants in the market. Section 3, which sets up the model with exit, is the main part of this paper. In this section, the rates of growth and exit of firms in the steady state are determined, and the effect of the exit of firms on growth and policy implications are investigated. In particular, Theorem 3 shows that the existence of exit introduces an additional effect to Theorem 1. Therefore, this implies that an increase in the intensity of competition reduces the growth rate if it forces too many firms to exit. Finally, Section 4 concludes the paper.

2 Model without Exit

There are two sectors in the economy: the final goods sector and the intermediate goods sector. Firms in the final goods sector provide their goods to households in a perfectly competitive market by using intermediate goods. In the intermediate goods sector, there is a continuum of firms, which have their own brands, and thus, they monopolistically compete with each other. They provide intermediate goods with their own brands to firms in the final goods sector by using a certain amount of labor. Moreover, firms in the intermediate goods sector improve their brands of goods by including a certain amount of final goods in research to sustain their monopoly power. To enter the intermediate goods sector, newcomers have to pay an entry cost, which is measured by final goods. Finally, each household provides labor to firms in the intermediate goods sector, and consumes the final good. The population is assumed to increase at a constant rate.

[4]This effect is referred to in Aghion et al. (2001) and Aghion et al. (2005),

2.1 Final Goods Sector

Firms in the final goods sector competitively produce goods, given as numeráire, by using intermediate goods. In this paper, it is assumed that the production technology of final good takes the constant elasticity of substitution (CES)[Editor2] form as follows:

$$Y_t = N_t^{\frac{1}{1-\sigma}} \left(\int_{S_t} A_{it}^{1-\frac{1}{\sigma}} x_{it}^{1-\frac{1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Here, Y_t is the amount of final goods; N_t, which is multiplied in the production function just to ensure that the production function is linear with respect to N_t, is the total mass of the surviving firms producing intermediate goods in period t; A_{it} is the quality level of the intermediate goods produced by firm i during the stage t; x_{it} is the amount of intermediate goods produced by firm i; S_t, the Lebesgue measure of which is N_t, is the index set of surviving firms producing intermediate goods, and σ is the elasticity of substitution among intermediate goods, which is assumed to be greater than 1. Hence, the maximization of the firm's profit derives the following condition:

$$p_{it} = A_{it}^{1-\frac{1}{\sigma}} N_t^{-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}} x_{it}^{-\frac{1}{\sigma}}. \tag{2}$$

2.2 Intermediate Goods Sector

Here, we describe the behavior of firms in the intermediate goods sector. In the intermediate goods sector, each firm determines the amount of intermediate goods produced and research expenditure in order to maximize their value.

2.2.1 Production

In the intermediate goods sector, firms employ labor, monopolistically compete with each other, and supply intermediate goods with their brands to firms that produce final goods. To produce intermediate goods, each firm incurs an amount of fixed cost, which is assumed in this paper to be measured as the form of labor. Specifically, firms are assumed to have an affine production technology with respect to labor, and thus, the gross profit of surviving firm i, Π_{it} is written as

$$\Pi_{it} = p_{it} x_{it} - w_t \left(x_{it} + f_{it} \right), \tag{3}$$

where w_t denotes the real rate of wage; and f_{it} is the minimum amount of labor required for production in firm i in period t. To maximize (3), it must hold that

$$x_{it} = \left(1 - \frac{1}{\sigma} \right)^{\sigma} A_{it}^{\sigma-1} N_t^{-1} Y_t w_t^{-\sigma}. \tag{4}$$

By using (2) and the above equation, (3) can be rewritten as

$$\Pi_{it} = \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^{\sigma-1} A_{it}^{\sigma-1} N_t^{-1} Y_t w_t^{1-\sigma} - f_{it} w_t. \tag{5}$$

This section makes the following assumption.

Assumption 1. $f_{it} = f$ for all $i \in S_t$ and $t \in \mathbb{R}_+$.

2.2.2 Research and Development

Each firm in the intermediate goods sector improves the quality of intermediate goods. The quality depends on its effort to encourage R&D, measured by the amount of final goods. Specifically, the quality is assumed to follow the form that

$$A_{it+1} = \gamma z_{Rit}, \tag{6}$$

where z_{Rit} is the expenditure of firm i for R&D in period t.

The objective of firm i is to maximize its value, which is written as

$$V_{it}(A_{it}) = \max\left[\Pi_{it} - z_{Rit} + \frac{1}{1+r_t} V_{it+1}(A_{it+1}), 0\right], \tag{7}$$

where $V_{it}(A_{it})$ denotes the value of firm i with quality A_{it} in period t. Considering (5) and (6), the maximization of the value of the firm satisfies the following condition:[5]

$$\frac{1+r_t}{\gamma} = \left(1-\frac{1}{\sigma}\right)^{\sigma} A_{it+1}^{\sigma-2} N_{t+1}^{-1} Y_{t+1} w_{t+1}^{1-\sigma}. \tag{8}$$

2.3 Households

This section characterizes the behavior of households. Each household consumes final goods, and it is endowed with a unit amount of labor provided to firms in the intermediate goods sector for production. The population is assumed to grow at a constant rate n. Specifically, this paper assumes that its overall utility is additive separable among periods, and the instantaneous utility takes the logarithmic form. Specifically, the overall utility is given as

$$U = \sum_{t=0}^{\infty} \beta^t (1+n)^t \ln c_t, \tag{9}$$

[5]For the derivation of this equation, see Appendix A.

where $\beta \in (0,1)$ is the subjective discount factor, n is the growth rate of population, and c_t is the amount of final goods consumed in period t. Moreover, its budget constraint in period t is given as

$$W_{t+1} = \frac{1+r_t}{1+n} W_t + w_t - c_t, \tag{10}$$

where W_t is the amount of per capita asset holdings in period t. Therefore, the aim of households is to maximize (9) under the budget constraints (10). The Euler equation of this problem can be written as

$$\frac{c_{t+1}}{c_t} = \beta (1+r_t). \tag{11}$$

2.4 Free Entry Condition in the Intermediate Goods Sector

To enter the intermediate goods sector, it is assumed that each entrepreneur has to employ δA_t units of fixed cost. Since each entrepreneur enters the intermediate goods sector as far as it expects to earn a positive net value, the free entry condition is written as

$$\frac{V_{it+1}(A_{it+1})}{1+r_t} = \delta A_t. \tag{12}$$

2.5 Steady-State Equilibria

This subsection is devoted to characterize the steady-state equilibria. Henceforth, subscriptions i and t are eliminated, wherever variables have the same values across firms and over periods, respectively; $\chi_{it} = \chi_t$ and $\chi_t = \chi$ for any variable χ.

First, I introduce the following lemma.

Lemma 1. *Considering Assumption 1 and equations (1), (4), (5), and (6), equations (7), (8), (11), and (12) are respectively rewritten as*

$$v_t = \frac{x_t}{\sigma} - \left(1 - \frac{1}{\sigma}\right) f - \hat{z}_{Rt} + \frac{v_{t+1}}{1+r_t} \gamma \hat{z}_{Rt} \tag{13}$$

$$1 + r_t = \gamma \left(1 - \frac{1}{\sigma}\right) x_{t+1}, \tag{14}$$

$$\frac{\hat{c}_{t+1}}{\hat{c}_t} \gamma \hat{z}_{Rt} = \beta (1+r_t), \tag{15}$$

and

$$\frac{v_{t+1}}{1+r_t} \gamma \hat{z}_{Rt} = \delta, \tag{16}$$

where $v_t = V_t(A_t)/A_t$, $\hat{c}_t = c_t/A_t$, *and* $\hat{z}_{Rt} = z_{Rt}/A_t$.

The proof of this lemma is relegated to Appendix B.

These four equations, (13)-(16), together with the market clearing conditions, characterize the equilibrium of the economy[6].

I further present the system of equations that characterize the steady-state equilibria. Throughout this paper, the following assumption is made in order for firms to have positive values in a steady-state equilibrium.

Assumption 2.

$$\sigma < 1 + \frac{1}{\beta}.$$

Henceforth, subscription t is eliminated whenever variables are constant in the steady state.

Proposition 1. *In the steady state, where $\hat{c}_t$ and $\hat{z}_{Rt}$ are constant for all $t \in \mathbb{R}_+$, (13)-(16) are reduced to the following two equations:*

$$\hat{z}_R = \beta \left(1 - \frac{1}{\sigma}\right) x, \tag{17}$$

and

$$(1-\beta)v = \frac{x}{\sigma} - \left(1 - \frac{1}{\sigma}\right) f - \hat{z}_R, \tag{18}$$

where v denotes the free-entry level of the firm's value.

$$v = \frac{\delta}{\beta}. \tag{19}$$

The proof of the Proposition is given in the Appendix C.

Here, (17) and (18) characterize the steady state of the economy and show the arbitrage condition for innovation and the free-entry condition. In (17), which is obtained by substituting (15) into (14), the left-hand side represents the marginal cost of improving their qualities of brands and the right-hand side represents the marginal gain. It is represented by line RD in Figure 1. Line RD is upward sloping

[6]The market-clearing conditions for the goods and labor markets are respectively written as follows.

$$L_t = (x_t + f) N_t,$$

and

$$Y_t = c_t L_t + z_{Rt} N_t + \delta (N_{t+1} - N_t) A_t.$$

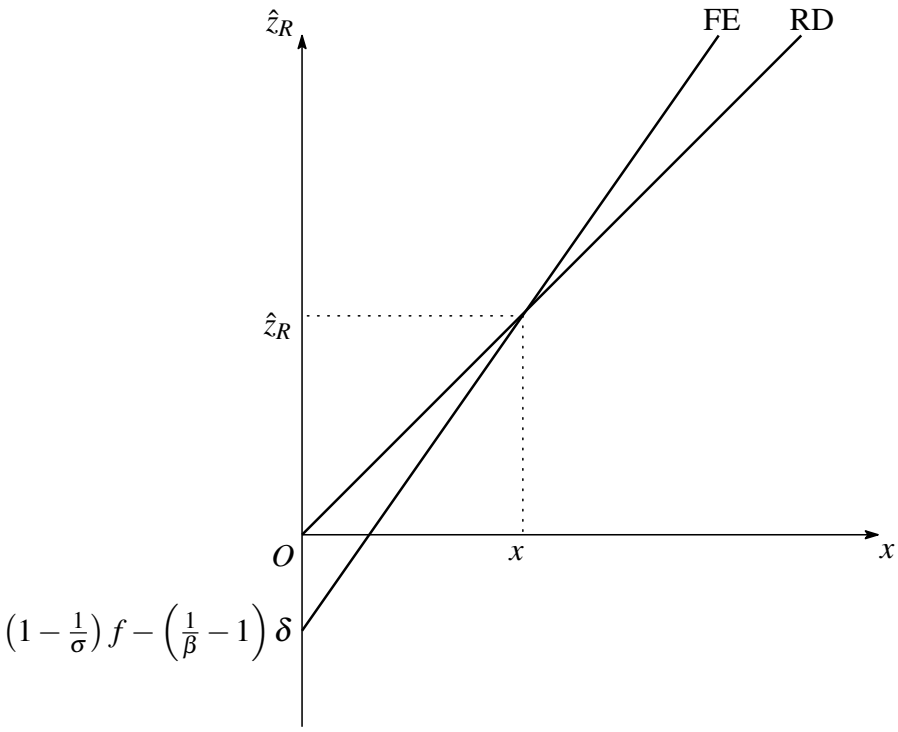

Figure 1: Steady state equilibrium

since a larger firm size implies a higher marginal gain from innovation.

Equation (18) is obtained by substituting (13) and (15) into (18) in order to eliminate r and v. It shows the relationship between x and z to attain the firm's value determined by the free-entry condition (19). It is represented by line FE in Figure 1. Line FE is also upward sloping since a higher expenditure for innovation requires a higher firm size to sustain the free-entry level of firm's value. It should be noted that the gradient of line FE is larger than that of line RD by Assumption 2.

Next, by combining (17) and (18) to eliminate x, the following proposition is obtained.

Proposition 2. *Under Assumptions 1 and Assumption 2, the per capita gross rates of growth in the steady state, g, are given by*

$$g_{NE} = \gamma \hat{z}_R^*,$$

where

$$\hat{z}_R^* = \left[\frac{1}{(\sigma-1)\beta} - 1\right]^{-1} \left[\left(1 - \frac{1}{\sigma}\right) f + \left(\frac{1}{\beta} - 1\right) \delta\right]. \tag{20}$$

I further consider the effects of competition on the growth rates. On the basis of the increase in σ, the two curves drawn from (17) and (18) move from RD_0 and FE_0 to RD_1 and FE_1, respectively, in Figure 2. The curve drawn from (17) rotates anticlockwise, reflecting the fact that an increase in the intensity of product market competition, σ, raises the marginal gain from innovation, thereby enhancing the incumbent firms' incentives to innovate. On the contrary, the curve drawn from (18) moves downward, reflecting the fact that an increase in the intensity of product market competition reduces the level of profit in equilibrium and admits lower expenditure for innovation to hold a free-entry level of firm's value. These changes of (17) and (18) increase the amounts of $\hat{z}_R$ and x from $\hat{z}_0$ and x_0 to $\hat{z}_1$ and x_1, respectively.

The effects of the increase in σ on $\hat{z}_R$ can be decomposed into two parts. One is the escape-competition effect[7], represented by $\hat{z}' - \hat{z}_0$, that is, under a constant x, an increase in the intensity of product market competition enhances the incentives for surviving firms to innovate due to the increase in the marginal gain from innovation. The other factor is the entry-reducing effect, in which an increase in the intensity of product market competition discourages the entry of firms due to the reducing profits of incumbent firms, thereby providing more resources to incumbent firms, represented by $\hat{z}_1 - \hat{z}'$. Since both the effects are positive on $\hat{z}_R$,

[7]This effect is referred to as in Aghion et al. (2001).

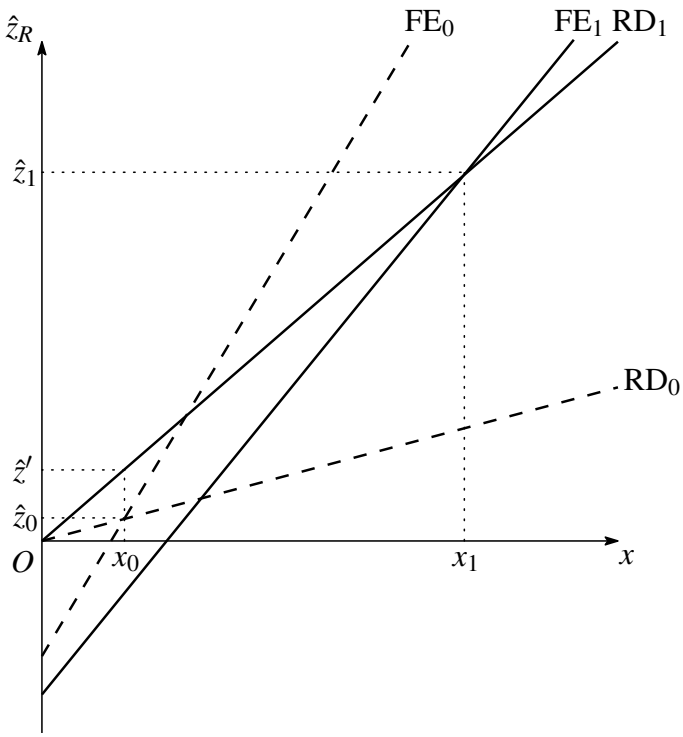

Figure 2: The effects of an increase in σ

there is a positive relationship between the intensity of product market competition and the growth rates in the case that there is no exit. It should also be noted that the firm size, x, has a positive correlation with the intensity of product market competition. Therefore, the following theorem summarizes the results.

Theorem 1. *Let $\hat{z}_R$ and x be the solution of the system of equations, (17) and (18). Then, under Assumption 2, the differentiation of $\hat{z}_R$ with respect to σ can be written as*

$$\frac{\mathrm{d}\hat{z}_R^*}{\mathrm{d}\sigma} = (\textit{escape-competition effect}) + (\textit{entry-reducing effect}) > 0, \tag{21}$$

where

$$(\textit{escape-competition effect}) = \frac{\partial \hat{z}_R^*}{\partial \sigma} > 0 \tag{22}$$

and

$$(\textit{entry-reducing effect}) = \beta \left(1 - \frac{1}{\sigma}\right) \frac{\mathrm{d}x}{\mathrm{d}\sigma} > 0. \tag{23}$$

Proof. Partially differentiatiating (17) with respect to σ gives

$$\frac{\partial \hat{z}_R^*}{\partial \sigma} = \frac{\beta}{\sigma^2} > 0$$

Moreover, substituting (17) into (18) to eliminate $\hat{z}_R$ yields

$$x = \frac{(\sigma - 1)f + \sigma\left(\frac{1}{\beta} - 1\right)\delta}{1 - (\sigma - 1)\beta}.$$

Therefore, differentiating the above equation with respect to σ yields

$$\frac{\mathrm{d}x}{\mathrm{d}\sigma} = \frac{f + (1+\beta)\left(\frac{1}{\beta} - 1\right)\delta}{[1 - (\sigma - 1)\beta]^2},$$

where the right-hand side is positive under Assumption 2. Finally, differentiating (17) with respect to σ yields (21). □

3 Model with Exit

3.1 Introducing Disturbances

This section introduces production disturbances in the model. The important assumption here is that the fixed labor required for the production of intermediate

goods is stochastically determined due to firm-specific reasons. The disturbances represent, for example, unanticipated breakdowns of machines, fires in plants, expenditure for the quality control of newly developed technology, etc. Owing to the existence of this stochastically required fixed cost, firms are faced with the possibility of involuntary exit from the market of the intermediate goods sector. When a firm faces a higher fixed cost, its value may be negative, forcing it to exit the market. Specifically, instead of Assumption 1, the following is assumed throughout this section.

Assumption 3. *f_{it} is drawn from an independent and identically distribution, which is represented by a differentiable distribution function, $G(\cdot)$, satisfying*

$$G(0) = 0 \tag{24}$$

and

$$G(f) < 1 \quad \textit{for all } f \in \mathbb{R}_+. \tag{25}$$

Here, (24) and (25) are imposed to guarantee the existence of the exit of firms and the differentiability is merely for tractability. According to these specifications, the expected value of firm i in the intermediate goods sector satisfies the following Bellman equation.

$$V_{it}\left(A_{it}\right) = \max\left[\Pi_{it} - z_{Rit} + \frac{1}{1+r_t}\mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right)\right], 0\right]. \tag{26}$$

Therefore, considering (5) and (6), the maximization of the value of the firm satisfies the following condition:[8]

$$\frac{1+r_t}{\gamma} = \left(1-\frac{1}{\sigma}\right)^{\sigma} A_{it+1}^{\sigma-2} N_{t+1}^{-1} Y_{t+1} w_{t+1}^{1-\sigma} G\left(\bar{f}_{it+1}\right), \tag{27}$$

where $\bar{f}_{it}$ denotes the maximum fixed cost at which firms can survive in the intermediate goods sector. Moreover, a fraction of firms are faced with a higher fixed cost and experience the negative firms' values. Without liquidity constraints, the firm would exit when its value is negative since no creditor invests in a firm with a negative value. Hence, it follows from (26), and the shutdown condition for firm i can be written as

$$0 = \frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)^{\sigma-1} A_{it}^{\sigma-1} N_t^{-1} Y_t w_t^{1-\sigma} - w_t \bar{f}_{it} - z_{Rit} + \frac{\mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right)\right]}{1+r_t}. \tag{28}$$

[8]For derivation of this equation, see Appendix A.

Multiplying the above equation by $G\left(\bar{f}_{it}\right)$ and subtracting it from the expectation of (26), we obtain

$$\mathrm{E}_{t-1}\left[V_{it}\left(A_{it}\right)\right] = \Gamma\left(\bar{f}_{it}\right) w_t. \tag{29}$$

Here, $\Gamma\left(\bar{f}_{it}\right)$ is the second-order distribution function of the fixed cost, representing a truncated expectation of difference between the upper bound and the actual fixed cost, which can be written as

$$\Gamma\left(\bar{f}_{it}\right) = \left(\bar{f}_{it} - \psi\left(\bar{f}_{it}\right)\right) G\left(\bar{f}_{it}\right) = \int_{\bar{f}_{it} > f} G(f)\mathrm{d}f,$$

where $\psi\left(\bar{f}_{it}\right)$ denotes the average productivity when firm i survives.

$$\psi\left(\bar{f}_{it}\right) = \frac{\int_{\bar{f}_{it} > f} f \mathrm{d}G(f)}{G\left(\bar{f}_{it}\right)}.$$

Therefore, it shows the average productivity among surviving firms if $\bar{f}_{it}$ is the same for all $i \in S_t$.

Finally, the free-entry condition in the intermediate goods sector, (12), is replaced by

$$\frac{\mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right)\right]}{1+r_t} = \delta A_t. \tag{30}$$

3.2 Steady-State Equilibria

First, the following lemma is given.

Lemma 2. *Suppose that Assumption 2, Assumption 3, and the symmetric equilibria, where $\bar{f}_{it} = \bar{f}_t$ for all $i \in S_t$. Then, considering equations (1), (4), (5), and (6), equations (15), (26), (27), (29), and (30) are respectively rewritten as*

$$\frac{\hat{c}_{t+1}}{\hat{c}_t}\gamma\hat{z}_{Rt} = \beta\left(1+r_t\right), \tag{15}$$

$$\bar{v}_t = \left[\frac{x_t}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\psi\left(\bar{f}_t\right) - \hat{z}_{Rt} + \frac{\bar{v}_{t+1}}{1+r_t}\gamma\hat{z}_{Rt}\right] G\left(\bar{f}_t\right) \tag{31}$$

$$1 + r_t = \gamma\left(1 - \frac{1}{\sigma}\right) x_{t+1} G\left(\bar{f}_{t+1}\right), \tag{32}$$

$$\bar{v}_t = \left(1 - \frac{1}{\sigma}\right)\Gamma\left(\bar{f}_t\right) \tag{33}$$

and

$$\frac{\bar{v}_{t+1}}{1+r_t}\gamma\hat{z}_{Rt} = \delta, \tag{34}$$

where $\bar{v}_t$ denotes the productivity-adjusted expected value of a firm in the intermediate goods sector in period t; $\bar{v}_t = \mathrm{E}_{t-1}\left[V_t\left(A_t\right)\right]/A_t$.

The proof of this lemma is relegated to Appendix D.

Here, the system of equations, (15) and (31)-(34), together with the market-clearing conditions, characterizes the symmetric equilibria.[9]

Next, I consider the steady-state equilibria in the case with disturbances.

Proposition 3. *In a steady-state symmetric equilibrium, where $\hat{c}_t$, $\hat{z}_{Rt}$, and $\bar{f}$ are constant for all $t \in \mathbb{R}_+$, (15), and (31)-(34) are reduced to the following three equations:*

$$\left(1-\frac{1}{\sigma}\right)\Gamma\left(\bar{f}\right) = \frac{\delta}{\beta}, \tag{35}$$

$$\hat{z}_R = \beta\left(1-\frac{1}{\sigma}\right)xG\left(\bar{f}\right), \tag{36}$$

and

$$\left(1-\beta\right)v = \frac{x}{\sigma} - \left(1-\frac{1}{\sigma}\right)\psi\left(\bar{f}\right) - \hat{z}_R, \tag{37}$$

where v is given by (19).

The proof of the Proposition is given in the Appendix C.

In (35), the left-hand side represents the firm's value that satisfies the shutdown condition, and the right-hand side represents the firm's value that satisfies the free-entry condition. Since the left-hand side of (35) is increasing in $\bar{f}$, it uniquely determines $\bar{f}$. Equations (36) and (37) correspond to (17) and (18), respectively. Comparing (17) and (18), the right-hand side of 36) is multiplied by $G(\bar{f})$, and the second term on the right-hand side of (36) is replaced by the average productivity of surviving firms, $\psi\left(\bar{f}\right)$, times $(1-1/\sigma)$.

In Figure 3, lines RD and FE represent (36) and (37), respectively It should be

[9]The market-clearing conditions for the goods and labor markets are also given as

$$L_t = \left(x_t + \psi\left(\bar{f}_t\right)G\left(\bar{f}\right)\right)N_t,$$

and

$$Y_t = \hat{c}_t A_t L_t + \hat{z}_{Rt} A_t N_t + \delta\left(\frac{N_{t+1}}{G\left(\bar{f}\right)} - N_t\right)A_t,$$

respectively.

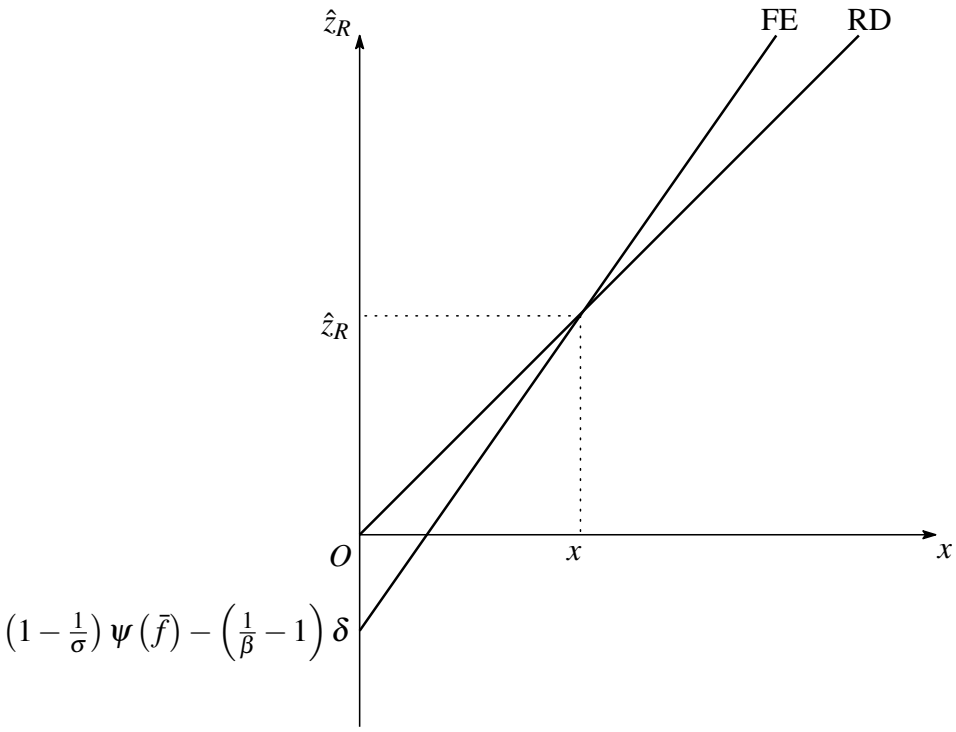

Figure 3: Steady state equilibrium

noted that lines RD and FE intersect under Assumption 2. By combining (36) and (37) to eliminate x, the following proposition is obtained.

Proposition 4. *Let $\hat{z}_R$ and $\bar{f}$ be the solution of the system of equations, . Under Assumption 2 and 3, the per capita gross rates of growth in the steady state, g_E, are given by*

$$g_E = \gamma \hat{z}_R^{**},$$

where

$$\hat{z}_R^{**} = \left[\frac{1}{(\sigma-1)\beta G\left(\bar{f}\right)} - 1 \right]^{-1} \left[\left(1 - \frac{1}{\sigma}\right) \psi\left(\bar{f}\right) + \left(\frac{1}{\beta} - 1\right) \delta \right] \tag{38}$$

and $\bar{f}$ satisfies (35).

It should be noted that the difference between (38) and (20) is the fact that the denominator of the first term in the first brackets of (38) includes $G\left(\bar{f}\right)$ and that f on the first term in the second brackets of (20) is replaced by $\psi\left(\bar{f}\right)$ in (38).

3.3 Properties of the Steady State

This subsection is devoted to analyzing the effects of an increase in the intensity of product market competition, σ, on the exit rate of firms and the growth rate. First, the effects on the exit rate are considered. The following theorem is easily obtained.

Theorem 2. *Let $\bar{f}$ be the solution of (35). Then, the exit rate of firms in the intermediate goods sector in the steady-state equilibrium, $1 - G(\bar{f})$, is increasing in the degree of the intensity of product market competition, σ, where $\bar{f}$ is given by (35).*

Proof. Differentiating (35) with respect to σ yields

$$\frac{\mathrm{d}\bar{f}}{\mathrm{d}\sigma} = -\frac{\Gamma\left(\bar{f}\right)}{\sigma(\sigma-1)G\left(\bar{f}\right)} < 0.$$

Therefore,

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\left(1 - G\left(\bar{f}\right)\right) = -G'(\bar{f})\frac{\mathrm{d}\bar{f}}{\mathrm{d}\sigma} > 0.$$

□

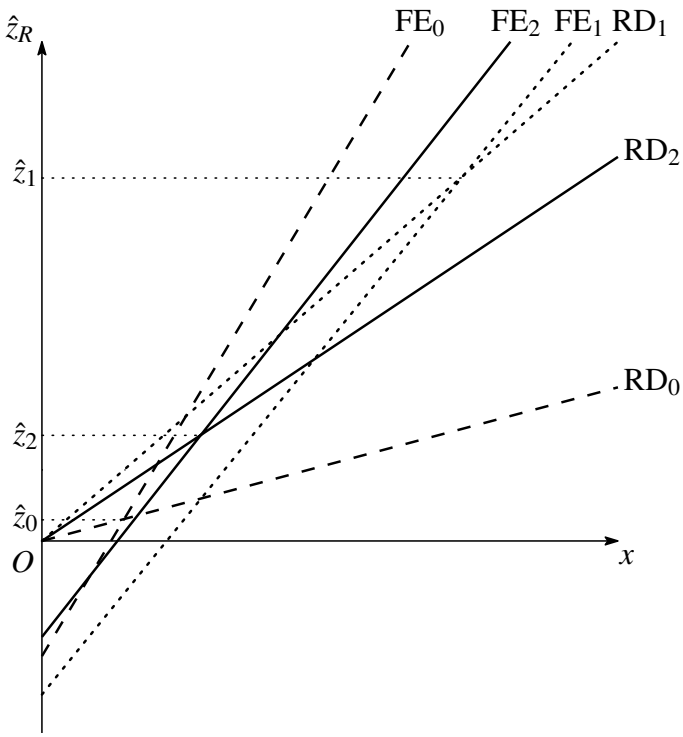

Figure 4: The effects of an increase in σ

Intuitively, Theorem 2 follows from the fact that an increase in the degree of competition reduces the monopoly rent, which weakens the firms considering the risk incurring high fixed cost shocks.

Next, I consider the effects of competition on the growth rates. Figure 4 illustrates the effects of competition on the growth rate, which is shown by the shift of the two lines, RD_0 and FE_0, drawn from (36) and (37) to RD_2 and FE_2, respectively, according to an increase in σ. Here, lines RD_1 and FE_1 represent the effects of competition on $\hat{z}_R$ taking $\bar{f}$ as given, corresponding to that in Figure 2. In other words, it is said that shifting the two lines from RD_0 and FE_0 to RD_1 and FE_1 represents the sum of the escape-competition and entry-reducing effects, which raises the amount of $\hat{z}_R$ in the steady-state equilibrium. On the other hand, shifting them from RD_1 and FE_1 to RD_2 and FE_2 reflects the effects of the increase in exit on $\hat{z}_R$. The effects can be decomposed into two factors. One is the marginal effect, which is represented by the clockwise rotation of line RD from RD_1 to RD_2, implying that the increase in the exit rate decreases the expected marginal gain from R&D, discouraging the R&D efforts, $\hat{z}_R$. The other is the level effect, which is represented by the upward shift of line FE from FE_1 to FE_2, implying that an increase in the exit rate raises the expected value of firms due to an increase in the average productivity of surviving firms, attracting newcomers and discouraging the R&D efforts. In summary, the increase in the exit rate tends to reduce $\hat{z}_R$. Consequently, the total effects of an increase in the intensity of product market competition depend on the effect that dominates, the effects of an increase in the exit, or other effects. The following theorem summarizes these results.

Theorem 3. *Let $\hat{z}_R$, x, and $\bar{f}$ be the solution of the system of equations, (35)-(37). Then, under Assumptions 2 and Assumption 3, the differentiation of $\hat{z}_R$ with respect to σ can be written as*

$$\frac{\mathrm{d}\hat{z}_R^{**}}{\mathrm{d}\sigma} = (\textit{escape-competition effect}) + (\textit{entry-reducing effect}) + (\textit{exit effect}) \tag{39}$$

where

$$(\textit{escape-competition effect}) = \frac{\partial \hat{z}_R^{**}}{\partial \sigma} > 0, \tag{40}$$

$$(\textit{entry-reducing effect}) = \beta \left(1 - \frac{1}{\sigma}\right) \frac{\partial x}{\partial \sigma} > 0. \tag{41}$$

and

$$(\textit{exit effect}) = -\frac{\partial x}{\partial G} G'\left(\bar{f}\right) < 0. \tag{42}$$

Proof. The partial differentiation of (36) with respect to σ yields

$$\frac{\partial \hat{z}_R^{**}}{\partial \sigma} = \frac{\beta}{\sigma^2} G\left(\bar{f}\right) > 0$$

Substituting (36) into (37) to eliminate $\hat{z}_R$ yields

$$x = \frac{(\sigma-1)\psi\left(\bar{f}\right) + \sigma\left(\frac{1}{\beta}-1\right)\delta}{1-(\sigma-1)\beta G\left(\bar{f}\right)}.$$

Hence, differentiating the above equation with respect to σ and $G\left(\bar{f}\right)$ yields

$$\frac{\partial x}{\partial \sigma} = \frac{f + \left(1+\beta G\left(\bar{f}\right)\right)\left(\frac{1}{\beta}-1\right)\delta}{\left[1-(\sigma-1)\beta G\left(\bar{f}\right)\right]^2} > 0 \tag{43}$$

and

$$\frac{\partial x}{\partial G} = \frac{\sigma-1}{1-(\sigma-1)\beta G\left(\bar{f}\right)}\left(\frac{\Gamma\left(\bar{f}\right)}{G\left(\bar{f}\right)^2} + \beta x\right) > 0, \tag{44}$$

where (43) and (44) are positive under Assumption 2. Therefore, considering the proof of Theorem 2, the differentiation of (36) with respect to σ yields (39) – (42). □

From Theorem 3, it is evident that if $G'\left(\bar{f}\right)$ is sufficiently high, $\hat{z}_R^{**}$ may be negative. This suggests that if the increased intensity of competition forces too many firms to exit the market, it should reduce the growth rate.

3.3.1 Effects of other policies

■ The effects of reduction of entry cost

Proposition 5 (The effects of reduction of entry cost)**.** *Let $\hat{z}_R$ and $\bar{f}$ be the solution of the system of equations, (35)-(37). Then, under Assumptions 2 and Assumption 3, an increase in δ reduces the exit rate, $1-G\left(\bar{f}\right)$, and raises $\hat{z}_R^{**}$.*

Proof. To find the effects of the changes in the entry cost, differentiating (35) with respect to δ, yields

$$\frac{\partial \bar{f}}{\partial \delta} = \frac{\sigma}{\beta(\sigma-1)G\left(\bar{f}\right)} > 0. \tag{45}$$

This shows that an increase in δ reduces $1-G(\bar{f})$.

It follows from Proposition 4 that $\hat{z}_R^{**}$ in the steady-state equilibrium is given by (38). Therefore, considering (45) and differentiating (38) with respect to δ, we obtain

$$\frac{\partial \hat{z}_R^{**}}{\partial \delta} = \left(\frac{1}{\sigma-1} - \beta G\left(\bar{f}\right)\right)^{-1} \left(1 + \frac{\bar{f} - \frac{\sigma}{\sigma-1}\delta}{1-\beta(\sigma-1)G\left(\bar{f}\right)} \frac{G'\left(\bar{f}\right)}{G\left(\bar{f}\right)}\right) > 0,$$

Therefore, a reduction in the entry cost reduces the balanced growth rate. □

Therefore, Proposition 5 suggests that a reduction in the entry cost, δ, raises the exit rate, $1-G(\bar{f})$, resulting in a higher average and less dispersion of productivity, and reduces the balanced growth rate, $\gamma\hat{z}_R^{**}$.

■ **The effects of R&D subsidy** Incorporating the subsidy to R&D expenditure, (36) and (37) are respectively rewritten as

$$(1-s_R)\hat{z}_R^{**} = \beta\left(1-\frac{1}{\sigma}\right) xG\left(\bar{f}\right), \tag{46}$$

and

$$(1-\beta)\,v = \frac{x}{\sigma} - \left(1-\frac{1}{\sigma}\right)\psi\left(\bar{f}\right) - (1-s_R)\hat{z}_R^{**}, \tag{47}$$

where v is given by (19), and s_R denotes the subsidy rate to R&D expenditure.

Proposition 6 (The effects of R&D subsidy)**.** *Let $\hat{z}_R$ and $\bar{f}$ be the solution of the system of equations, (35), (46), and (47). Then, under Assumptions 2 and Assumption 3, an increase in s_R has no effect on the exit rate, $1-G\left(\bar{f}\right)$, but raises the growth rate, $\gamma\hat{z}_R^{**}$.*

Proof. Since there is no change in (35), $\bar{f}$ is independent of s_R.

Combining (46) and (47) yields

$$\hat{z}_R^{**} = (1-s_R)^{-1}\left(\frac{1}{\beta(\sigma-1)G\left(\bar{f}\right)} - 1\right)^{-1}\left[\left(1-\frac{1}{\sigma}\right)\psi\left(\bar{f}\right) + \left(\frac{1}{\beta}-1\right)\delta\right].$$

Therefore, the above equation shows that $\hat{z}_R^{**}$ is increasing in s_R. □

■ **The effects of fixed cost subsidy** More interestingly, the effects of the subsidy to fixed cost are counterintuitive. Incorporating the lump sum subsidy to fixed cost into the model, (37) is rewritten as

$$(1-\beta)\,v = \frac{x}{\sigma} - \left(1-\frac{1}{\sigma}\right)\left(\psi\left(\bar{f}\right)-F\right)-\hat{z}_R^{**}, \tag{48}$$

where v is given by (19), and F denotes a lump sum subsidy to fixed cost.

Proposition 7 (The effects of fixed cost subsidy)**.** *Let $\hat{z}_R^{**}$ and x be the solution of the system of equations, (35), (36), and (48). Then, under Assumption 2, an increase in F has no effect on the exit rate, $1-G\left(\bar{f}\right)$, but reduces the growth rate, $\gamma_{\hat{z}_R^{**}}$.*

Proof. Since there is no change in (35), $\bar{f}$ is independent of s_R.

Combining (46) and (47) yields

$$\hat{z}_R^{**} = \frac{\left(1-\frac{1}{\sigma}\right)\left(\psi\left(\bar{f}\right)-F\right)+\left(\frac{1}{\beta}-1\right)\delta}{\frac{1}{\beta(\sigma-1)G\left(\bar{f}\right)}-1}.$$

Therefore, the above equation shows that $\hat{z}_R^{**}$ is decreasing in F. □

4 Conclusion

This paper presented a model wherein the interrelationship between the exit and growth rates is analyzed. In particular, this paper focused on the effects of the market production competition on the growth rate. First, it analyzed the effects of competition on growth in the case without the exit of firms and showed that there are two positive factors of effects of the market production competition on the growth rate: one is the escape-competition effect, in which the increased intensity of competition raises the growth rate, since it directly raises the marginal gain for innovation, and the other is the entry-reducing effect, in which the number of entrants decreases since the value of firms producing intermediate goods increases. In the case of the exit of firms, the growth rate depends on the distribution of productivities. In particular, it implies that the existence of exit has a negative impact on the growth rate as the intensity of competition increases, since it reduces the expected values of surviving firms due to an increase in the exit rate. This implies that an increase in the intensity of competition is harmful for growth when it forces too many firms to exit the market. Moreover, some other policy implications were investigated.

Appendix

A The Maximization Condition for R&D Expenditure

The first-order and envelope conditions for the Bellman equation (26) derive the following equations:

$$(1-s_R)\frac{w_t}{\gamma \bar{A}_t} = \frac{1}{1+r_t}\mathrm{E}_t\left[V'_{it+1}\left(A_{it+1}\right)\right] \tag{49}$$

and

$$V'_{it}\left(A_{it}\right) = \begin{cases} \left(1-\frac{1}{\sigma}\right)^{\sigma} A_{it}^{\sigma-2} Y_t w_t^{1-\sigma} & \text{if } f_{it} \leq \bar{f}_{it} \\ 0 & \text{if } f_{it} > \bar{f}_{it}. \end{cases} \tag{50}$$

To combine the above equations, note that the following conditions are satisfied:

$$\mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right)\right] = \mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right) \mid f_{it+1} \leq \bar{f}_{it+1}\right] G\left(\bar{f}_{it}\right). \tag{51}$$

The above equation follows from the fact that $\mathrm{E}_t\left[V_{it+1}\left(A_{it+1}\right) \mid f_{it+1} > \bar{f}_{it+1}\right] = 0$. Therefore, the derivative of (51) can be written as

$$\begin{aligned} \mathrm{E}_t\left[V'_{it+1}\left(A_{it+1}\right)\right] &= \mathrm{E}_t\left[V'_{it+1}\left(A_{it+1}\right) \mid f_{it+1} \leq \bar{f}_{it+1}\right] G\left(\bar{f}_{it}\right) \\ &= \left(1-\frac{1}{\sigma}\right)^{\sigma} A_{it}^{\sigma-2} Y_t w_t^{1-\sigma} G\left(\bar{f}_{it}\right), \end{aligned} \tag{52}$$

where we used the result of (50) in the last line of the above expressions. Therefore, substitution of (52) into (49) and a little manipulation yield (27).

B Proof of Lemma 1

Here, the proof of Lemma 1 is given.

Proof. It follows from (8) that $A_t = A_{it}$, and thus, from (4) and (6) that $x_{it} = x_t$ and $z_{Rit} = z_{Rt}$ for all $i \in S_t$ and $t \in \mathbb{R}_+$. Then, (1) gives

$$Y_t = x_t A_t N_t. \tag{53}$$

Substituting the above equation into (4) yields

$$w_t = \left(1-\frac{1}{\sigma}\right) A_t. \tag{54}$$

Thus, considering Assumption 1 and substituting (56) and (57) into (5) give

$$\Pi_{it} = \left[\frac{1}{\sigma}x_t - \left(1 - \frac{1}{\sigma}\right) f\right] A_t, \tag{55}$$

for all $i \in S_t$ and $t \in \mathbb{R}_+$.

Therefore, substituting (56)-(58) into (7), (8), (11), and (12) and considering (6), we obtain (13)-(16). □

C Proof of Proposition 1

This section gives the proof of Proposition 1.

Proof. Since $\hat{c}_t$ and $\hat{z}_{Rt}$ are constant, substituting (15) into (14) shows that x_t is constant for all t and gives (17). Therefore, substituting (16) into (13) shows that v_t is constant for all t and gives (18). □

D Proof of Lemma 2

Here, the proof of Lemma 2 is given.

Proof. It follows from (27) that $A_t = A_{it}$ in symmetric equilibrium, and thus, from (4) and (6) that $x_{it} = x_t$ and $z_{Rit} = z_{Rt}$ for all $i \in S_t$ and $t \in \mathbb{R}_+$. Next, (1) gives

$$Y_t = x_t A_t N_t. \tag{56}$$

Substituting the above equation into (4) yields

$$w_t = \left(1 - \frac{1}{\sigma}\right) A_t. \tag{57}$$

Thus, considering Assumption 1 and substituting (56) and (57) into (5) give

$$\Pi_{it} = \left[\frac{1}{\sigma}x_t - \left(1 - \frac{1}{\sigma}\right) f_{it}\right] A_t, \tag{58}$$

for all $i \in S_t$ and $t \in \mathbb{R}_+$.

Therefore, substituting (56)-(58) into (26), (27), (11), and (30) and considering (6), we obtain (31)-(34). □

E Proof of Proposition 3

This section gives the proof of Proposition 3.

Proof. Since $\hat{c}_t$, $\hat{z}_{Rt}$, and $\bar{f}_t$ are constant, substituting (15) into (32) shows that x_t is constant for all t and gives (36). Therefore, substituting (34) into (31) shows that v_t is constant for all t and gives (37). Moreover, substituting (15) into (34) gives $\bar{v} = \delta/\beta$. Therefore, substituting it into (33) gives (35). □

References

Aghion, Philippe and Peter Howitt (1992) "A Model of Growth through Creative Destruction," *Econometrica*, Vol. 60(2), pp. 323–351.

Aghion, P., C. Harris, and J. Vickers (1997) "Competition and growth with step-by-step innovation: An example," *European Economic Review*, Vol. 41, No. 3-5, pp. 771–782.

Aghion, P., C. Harris, P. Howitt, and J. Vickers (2001) "Competition, Imitation and Growth with Step-by-Step Innovation," *Review of Economic Studies*, Vol. 68, No. 3, pp. 467–492.

Aghion, P., N. Bloom, R. Blundell, R. Griffith, and P. Howitt (2005) "Competition and Innovation: An Inverted-U Relationship," *The Quarterly Journal of Economics*, Vol. 120, No. 2, pp. 701–728.

Asplund, M. and V. Nocke (2006) "Firm Turnover in Imperfectly Competitive Markets," *Review of Economic Studies*, Vol. 73, No. 2, pp. 295–327.

Blundell, R., R. Griffith, and J. V. Reenen (1995) "Dynamic Count Data Models of Technological Innovation," *The Economic Journal*, Vol. 105, No. 429, pp. 333–344.

Das, S. and S. P. Das (1997) "Dynamics of entry and exit of firms in the presence of entry adjustment costs," *International Journal of Industrial Organization*, Vol. 15, No. 2, pp. 217–241.

Dinopoulos, E. and P. Thompson (1998) "Schumpeterian Growth without Scale Effects," *Journal of Economic Growth*, Vol. 3, No. 4, pp. 313–335.

Grossman, G. M. and E. Helpman (1991) "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, Vol. 58, No. 1, pp. 43–61.

Hopenhayn, H. A. (1992) "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, Vol. 60, No. 5, pp. 1127–1150.

Hopenhayn, H. and R. Rogerson (1993) "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, Vol. 101, No. 5, p. 915.

Howitt, P. (1999) "Steady Endogenous Growth with Population and R& D Inputs Growing," *Journal of Political Economy*, Vol. 107, No. 4, p. 715.

Jovanovic, B. (1982) "Selection and the Evolution of Industry," *Econometrica*, Vol. 50, No. 3, pp. 649–670.

Melitz, M. J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, Vol. 71, No. 6, pp. 1695–1725.

Nickell, S. J. (1996) "Competition and Corporate Performance," *The Journal of Political Economy*, Vol. 104, No. 4, pp. 724–746.

Peretto, P. F. (1998) "Technological Change and Population Growth," *Journal of Economic Growth*, Vol. 3, No. 4, pp. 283–311.

Romer, P. M. (1990) "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, No. S5, p. 71.

Scherer, F. M. (1967) "Market Structure and the Employment of Scientists and Engineers," *The American Economic Review*, Vol. 57, No. 3, pp. 524–531.

Segerstrom, P. S. (2000) "The Long-Run Growth Effects of R&D Subsidies," *Journal of Economic Growth*, Vol. 5, No. 3, pp. 277–305.