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Economic Growth, Unemployment, and Business Cycles

Katsuhiko Hori*

Abstract
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*Katsuhiko Hori
Institute of Economic Research, Kyoto University

KEIO/KYOTO MARKET QUALITY RESEARCH PROJECT
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Graduate School of Economics and Graduate School of Business and Commerce,
Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345 Japan

Kyoto Institute of Economics,
Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501 Japan
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Abstract

This paper explores relationships among economic growth, unemployment, and business cycles by constructing a model, which views the process of creative destruction as a major source of business cycles, as well as of economic growth. The main results are as follows: first, the long-run growth rate has a negative relationship with the amplitude of business cycles. Second, the growth rate has a negative connection with the frequency of slump. Third, a permanent shock causes unemployment, but a transitory shock leads to full employment.

KEYWORDS: business cycles, economic growth, unemployment.
JEL classifications: E24, E32, O33
1 Introduction

Traditionally, economic growth, unemployment, and business cycles are more or less treated as separated issues in the theoretical analyses. The model presented here tries to integrate these issues and to explore relationships among economic growth, unemployment, and business cycles. The main results are as follows. In the first place, this paper shows that there is a negative correlation between growth and the amplitude of business cycles. The relationship between growth and business cycles has been studied in recent literature. Aghion and Howitt [1992] construct an open economy model with exogenous demand shocks. They show a positive relationship between the long-run rate of growth and the amplitude of firm-level fluctuation when productivity shocks in the economy have negative effects on current production, even though the amplitude of aggregate fluctuation is irrelevant to growth. However, literature finds the evidence that is not consistent with their result. For example, Ramey and Ramey [1995] and Martin and Rogers [2000] find that there is a strong negative relationship between volatility and growth. To be consistent with these empirical works, the model here displays a negative correlation between growth and the amplitude of business cycles but a negative correlation between permanent productivity shocks and employment.¹

Second, we also show that the frequency of slump has a negative relationship with the expected rate of long-run growth. Many recent works examine the relationship between long-run growth and unemployment. For example, Pissarides [2000, ch. 2] presents a model in which higher growth rates lead to lower natural rates of unemployment. Aghion and Howitt [1994] construct an endogenous growth model with a matching process and emphasize that a higher growth rate would raise the unemployment rate in some cases. Along the line of these studies, our model shows a similar effect to that of Aghion and Howitt [1994]. Nevertheless, it particularly emphasizes the frequency of slump, not only does unemployment.

Finally, we show that positive permanent shocks reduce employment, while positive transitory shocks raise it. One well-known paradox of the competitive-equilibrium business cycle models is that they display a strong and highly positive relationship between shocks and employment. In competitive-equilibrium business cycle models, productivity shocks raise labor demand, increasing the employment level. This result of competitive-equilibrium business cycle models is inconsistent with empirical evidence and much of literature tries to reconcile this puzzle by introducing various kinds of transitory shocks. (e.g. Benhabib, Rogers-⁰¹

¹Martin and Rogers [2000] also present a simple theoretical model in which human capital is accumulated through the learning-by-doing processes, and derive a negative relationship between growth and the amplitude of business cycles. However, in their model exogenous productivity shocks have a positive relationship with employment since it is competitive-equilibrium one.
son, and Wright [1991], Hansen and Wright [1992], Christiano and Eichenbaum [1992], etc.) These modified versions of the models necessarily lead us to conclude that positive permanent shocks raise the employment level, while positive transitory shocks reduce it. In their assumption, transitory shocks reduce labor demand, thereby reducing the employment level. However, recent empirical literature suggests evidence to the contrary. (e.g. Galí [1999], Francis and Ramey [2005], Basu, Fernald, and Kimball [2006], Marchetti and Nucci [2005], etc.)

The paper is organized as follows. Section 2 specifies the behavior of agents in the economy and determines the equilibrium of the model. Section 3 considers the properties of perfect foresight equilibria. Section 4 concludes the paper.

2 Model

We first specify the behavior of households and firms in the intermediate good and research sectors. Next, we determine the wage rate in each sector and derive the fraction of research labor, which characterizes the equilibrium of the economy.

2.1 Behavior of Households

Each household, which is distributed over the unit interval, is endowed with one unit of labor and land, and consumes an amount of the final good. It earns income by rent of land, wage of labor devoted to production of the intermediate good, and dividend payments from the monopolist in the intermediate good sector. Each household has identical risk-neutral preferences with a positive and constant subjective discount rate, denoted by \( r \), which is equal to the interest rate. There is no instantaneous disutility from supplying labor. Moreover, this paper introduces a simple search process into the labor market, where each worker seeks a job with an arrival rate and it breaks up at each instance.

2.2 Behavior of Firms

Here we specify the behavior of firm in each sector: the final good, intermediate good, and research sectors. In the final good sector, firms competitively provide goods for household. In the intermediate good sector, the monopolist provides its output for firms in the final good sector by employing an amount of labor and the current technology. During the adjustment period, where the monopolist incurs an extra cost, the demand of labor, and thus, the employment rate and the amount of final good decrease, as will be shown later. Finally, in the research sector,

2Some works challenge these results: see Chang and Hong [2003] and Christiano, Eichenbaum, and Vigfusson [2004].
firms stochastically develop an advanced technology, which produces the more productive intermediate good.

### 2.2.1 Final Good Sector

Firms in the final good sector are assumed to have identical neoclassical technology. They input the intermediate good and a unit of the fixed resource, such as land, to produce an amount of final good. In this paper, the final good serves as numéraire.

Firm \( i \) in the final good sector solves the following decision problem:

\[
\max_{y_{it}} y_{it} - p_t x_{it} \\
\text{s.t.} \quad y_{it} = A_{s(t)} x_{it}^\alpha,
\]

where \( y_{it} \) is the amount of the final good produced by firm \( i \) at time \( t \); \( p_t \) is the price of the intermediate good; \( x_{it} \) is the amount of the intermediate good employed by firm \( i \); and \( A_{s(t)} \) is the productivity indicator of the intermediate good with the \( s(t) \)-th technology, where \( s(t) \) denotes a stochastic stage of technology at time \( t \) (following a Poisson process as shown later)\(^3\).

Solving the profit-maximizing problem, one finds the inverse demand function of firm \( i \) for the intermediate good:

\[
p_t = \alpha A_{s(t)} x_{it}^{\alpha-1}. \tag{1}
\]

Since the marginal productivity of the final good, the right-hand side of (1), is monotonically decreasing, demand for the intermediate good of any firm \( i \), \( x_{it} \), is identically determined for all firms. Therefore, the aggregate demand for the intermediate good, denoted by \( x_t \), is equal to \( x_{it} \) for all \( i \).

### 2.2.2 Intermediate Good Sector

The intermediate good with state-of-the-art technology is provided by a monopolist, which hires workers without recruiting cost at each instance. Since this paper assumes the drastic innovation, the existing technology becomes obsolete when a succeeding innovation occurs. Specifically, the \( k \)th technology is assumed to survive during \( [t_k, t_{k+1}) \), where \( t_k \) denotes the stochastic instance of time at which the \( k \)th technology is developed; that is, \( t_k = \inf \{ s^{-1}(k) \} \).

A unit of the intermediate good is assumed to be produced by a unit of labor. Moreover, the monopolist incurs \( A_k \eta \) units of an adaptation cost per unit of labor during a certain adjustment period to produce the intermediate good with the new technology.

\(^3\)See Figure 1.
technology. The adjustment period is terminated with a transitory shock or the arrival of a successful innovation. After the adjustment period, the monopolist releases from the adaptation cost.

Specifically, the profit of the monopolist, \( \pi_t \), is given as

\[
\pi_t = \begin{cases} 
\pi_{Rt} = \alpha A_s(t) x_t^\alpha - (w_t + A_s(t) \eta) x_t & \text{during the adjustment period} \\
\pi_{Bt} = \alpha A_s(t) x_t^\alpha - w_t x_t & \text{after the adjustment period},
\end{cases}
\]

(2)

where \( w_t \) denotes the real wage rate at time \( t \). Therefore, the present-discounted value of expected profit during and after the adjustment period, denoted by \( V_{Bt} \) and \( V_{Rt} \) respectively, satisfy the following Bellman equations.

\[
rV_{Rt} = \pi_{Rt} + \dot{V}_{Rt} - \mu_t V_{Rt} + \xi (V_{Bt} - V_{Rt}),
\]

(3)

and

\[
rV_{Bt} = \pi_{Bt} + \dot{V}_{Bt} - \mu_t V_{Bt}
\]

(4)

where \( \mu_t \) denotes the arrival rate of succeeding innovation at time \( t \).

2.2.3 Innovation Processes

Research firms stochastically develop new technologies, by employing an amount of final good, which is paid through issuing shares. If a research firm succeeds in innovation, it becomes a monopolist. Each research firm assumes to innovate independently of the other research firms. Specifically, the arrival rate that a research firm succeeds in innovation at time \( t \), \( \mu_t \), is assumed that

\[
\mu_t = \lambda \hat{z}_t.
\]

Here \( \lambda \) does an indicator of the frequency of innovations and \( \hat{z}_t \) denotes the productivity-adjusted expenditure for research of firms at time \( t \), that is \( \hat{z}_t = z_t / A_s(t) \), where \( z_t \) is expenditure for research of firms at time \( t \). Here, the expenditure for innovation is divided by productivity \( A_s(t) \) since the development of more productive technology will be more challenging. Thus, by denoting the expected value of the monopolist by \( V_t \), the zero-profit condition for the research firms is written as

\[
\lambda \hat{z}_t V_{Rt} = z_t,
\]

(5)

where the left-hand side represents the expected current value of research activities and the right-hand side does the cost.
3 Perfect-Foresight Equilibria

The remaining of this paper only concerns the perfect-foresight equilibria, where productivity-adjusted expenditure for research, \( \hat{z}_t \) is constant over time, and thus, \( \dot{V}_{Bi} = \dot{V}_{Ri} = 0 \) in each stage of the technology. Hereafter, subscription \( i \) is eliminated whenever variables are constant over time.

3.1 Wage Rate

The monopolist and each matched worker determine the wage rate by Nash bargaining. Formally, the wage is determined to maximize the Nash products:

\[
\max_{w_t} w_t^\beta \pi_t^{1-\beta}
\]

where \( \beta \) is the identical bargaining power of each worker. Solving problem (6) yields

\[
w_t = \begin{cases} 
\beta \left( \alpha x_t^{\alpha-1} - \eta \right) A_{s(t)} & \text{during the adjustment period} \\
\beta \alpha x_t^{\alpha-1} A_{s(t)} & \text{after the adjustment period.}
\end{cases}
\]

Taking the above wage rates into account, the monopolist maximizes its profit subject to the resource constraint for labor; \( x_t \leq 1 \). Substituting (7) into (2), we get

\[
\pi_{Ri} = (1 - \beta) \left( \alpha x_t^{\alpha} - \eta x_t \right) A_{s(t)}
\]

\[
\pi_{Bi} = (1 - \beta) \alpha x_t^{\alpha-1} A_{s(t)}
\]

Therefore, the first-order condition for the maximization yields

\[
x_t = \begin{cases} 
x_R = \left( \frac{\alpha^{2}}{\eta} \right)^{\frac{1}{1-\alpha}} & \text{during the adjustment period} \\
1 & \text{after the adjustment period,}
\end{cases}
\]

where we assume that

\[\eta > \alpha^2\]

to be that \( x_t < 1 \) in the adjustment period on the perfect foresight equilibria. Finally, substituting (10) back into (8) and (9) yields

\[
\frac{\pi_t}{A_{s(t)}} = \begin{cases} 
\hat{\pi}_R = (1 - \beta) \left( \alpha x_R^{\alpha} - \eta x_R \right) & \text{during the adjustment period} \\
\hat{\pi}_B = (1 - \beta) \alpha & \text{after the adjustment period.}
\end{cases}
\]
3.2 Research Expenditure

From (4) and (3), the following values are obtained in perfect-foresight equilibria.

\[ v_B = \frac{V_{Bt}}{A_{s(t)}} = \frac{\hat{\pi}_B}{r + \lambda \hat{z}} \]

and

\[ v_R = \frac{V_{Rt}}{A_{s(t)}} = \frac{(r + \lambda \hat{z}) \hat{\pi}_R + \xi \hat{\pi}_B}{(r + \lambda \hat{z})(r + \lambda \hat{z} + \xi)}. \]

Therefore, (5) is rewritten as

\[ \lambda q v_R = 1 \]

or

\[ \lambda q \left( \hat{\pi}_R + \frac{\xi \hat{\pi}_B}{r + \lambda \hat{z}} \right) = r + \lambda \hat{z} + \xi. \]  

(12)

Since the left-hand side of the above equation is increasing and the right-hand side is decreasing with respect to \( \hat{z} \), it uniquely determines the level of research efforts.

Therefore, factors raising \( \lambda \hat{z} \) are increases in the size of innovation, \( q \), and the frequency of innovation, \( \lambda \), and decreases in the bargaining power for workers, \( \beta \), and adaptation cost, \( \eta \).
3.3 Properties of Perfect Foresight Equilibria

This section is devoted to showing the main results: the relationships between economic growth and the amplitude of business cycles, business cycles and shocks, and growth and the frequency of slump.

3.3.1 Economic Growth and Business Cycles

Here we specify the expected growth rate, the frequencies of boom and slump, the amplitude of business cycles, and the expected unemployment rate, and show that any factor that raises the amplitude of business cycles has a non-positive relationship with the expected growth rate.

□ The Expected Rate of Economic Growth

The expected growth rate takes the same form as in Aghion and Howitt [1992].

\[ g = \lambda \hat{z} \ln q. \]  

(13)

By substituting the fraction of research labor in perfect foresight equilibria into (13), one finds that the expected growth rate is increasing in any factor that raises the research expenditure, \( \hat{z} \), except \( q \).

□ The Frequencies of Booms and Slumps

The frequencies of booms and slumps are defined by the ratios of the expected boom and slump periods to the expected periods of business cycles, respectively. The slump period is defined by a successive series of preliminary periods. This definition of slump is intuitive in the sense that the economy experiences unemployment during these preliminary periods. On the other hand, a boom period is defined by a period outside the preliminary periods, and thus, the economy attains full employment in this period.

We first calculate the expected periods of boom and slump periods and of business cycle, and then find the frequencies of boom and slump. The expected boom period, denoted by \( B \), can be obtained by straightforward calculation. By noting that the probability density of the length of the arrival time of innovation, \( t \), is \( \lambda \hat{z} e^{-\lambda \hat{z} t} \), it is obtained as

\[ B = \int_{0}^{\infty} \lambda \hat{z} e^{-\lambda \hat{z} t} dT = \frac{1}{\lambda \hat{z}}. \]  

(14)

\[ ^{4} \text{For calculation in detail, see appendix A.} \]
Since transitory shocks are independent of innovation, the expected slump period, $S$, can be calculated as

$$S = \int_0^\infty \xi e^{-\xi t} dt = \frac{1}{\xi}. \quad (15)$$

Therefore, it follows from (14) and (15) that the expected length of a business cycle, $C$, can be written as

$$C = B + S = \frac{1}{\lambda \hat{z}} + \frac{1}{\xi}. \quad (16)$$

Finally, dividing the expected boom and slump periods by (16), we have the frequencies of boom and slump, respectively.

$$b = \frac{B}{C} = \frac{\xi}{\lambda \hat{z} + \xi}, \quad (17)$$

and

$$s = \frac{S}{C} = \frac{\lambda \hat{z}}{\lambda \hat{z} + \xi}, \quad (18)$$

where $b$ and $s$ denote the frequencies of booms and slumps, respectively.

Combining the results of the comparative statics of the research efforts stated in below (12), with (14) to (18), one finds that increases of the frequency of innovation, $\lambda$, and the size of innovation, $q$, and decreases of the bargaining power of wokers, $\beta$, and the adaptation cost, $\eta$, have no effects on the expected slump period, $S$, raises its frequency, $s$, and reduces the expected boom period, $B$, the frequency of boom, $b$, and the expected period of business cycles, $C$.\footnote{The occurrence of boom and slump hinges on the one industry structure in the intermediate good sector, and thus, the results seem to hold only on the extreme case. However, it would still hold in the multi-industry version of this model when shocks are positively correlated among industries.}

\section*{The Amplitude of Business Cycles}

This paper defines the amplitude of business cycles, denoted by $a$, as

$$a = 1 - x_R. \quad (19)$$

The first term on the right-hand side of (19) shows the employment level in boom and the rest of the term does its level in slump. Therefore, the amplitude of business cycles represents the difference between the employment rates in boom and slump.
Unemployment  Combining the frequency of slump, (18), and unemployment rate in slump, which is equal to the amplitude of business cycles, (19), the expected unemployment rate, $\psi$, can be written as

$$\psi = sa.$$  

The comparative statics are as follow. Increases of $q$ and $\lambda$ and a decrease of $\beta$ have no effects on the amplitude of business cycles but raises the expected unemployment rate; an increase of $\eta$ raises the amplitude of business cycles but has ambiguous effects on the expected unemployment rate.

Economic Growth and the Amplitude of Business Cycles  Combining the results of the comparative statics stated in the previous paragraph with (13), we find that an increase of adaptation cost, $\eta$, which is a factor that raises the amplitude of business cycles, has a negative impact on the expected growth rate.

Economic Growth and the Frequency of Slump  From (13) and (18) we find that the expected growth rate has a positive relationship with the frequency of slump: any factor that raises the expected growth rate raises the frequency of slump, and any factor that raises the frequency of slump has non-negative impact on the expected growth rate. Intuitively, permanent shocks raise the unemployment rate since a permanent shock accompanies the adaptation cost, a factor reducing the demand for labor in the intermediate good sector.

3.3.2 Business Cycles and Shocks

The model also shows that permanent productivity shocks reduce the employment rate while transitory productivity shocks raise it. In the first place, we show a positive relationship between the occurrence of transitory shocks and changes in employment. From the production function in the final good sector, (2), the changes in the logarithmic amount of output is written as

$$d\ln y_t = d\ln A_{s(t)} + \alpha d\ln x_t,$$

where the first and second terms on the right-hand side represent the contributions of permanent productivity improvements and of increases of employment on the changes in output, respectively. Therefore, the covariance between the changes in the logarithmic amount of output and employment is decomposed as

$$\text{Cov}[d\ln y_t, d\ln x_t]$$

$$= \text{Cov}[d\ln A_{s(t)}, d\ln x_t] + \alpha \text{Var}[d\ln x_t],$$  (20)
where the first and second terms on the right-hand side give the relations of permanent and transitory productivity shocks to the changes in employment, respectively. Immediately from (20), we find a positive relationship between transitory shocks and the changes in employment.

On the contrary, there is a negative relationship between permanent shocks and the changes in employment. The changes in productivity can be expressed as

\[ \text{d} \ln A_{s(t)} = \ln q ds(t), \]

where \( s(t) \) follows a Poisson process as

\[ ds(t) = \begin{cases} 1 & \text{with probability } \lambda \ddt, \\ 0 & \text{with probability } 1 - \lambda \ddt. \end{cases} \quad (21) \]

Likewise, the processes of log employment is written as

\[ \text{d} \ln x_t = \begin{cases} \ln \frac{1}{x_R} d\phi(t) & \text{in slump,} \\ -\ln \frac{1}{x_R} ds(t) & \text{in boom,} \end{cases} \quad (22) \]

where \( \phi(t) \) denotes the number of transitory shocks following

\[ d\phi(t) = \begin{cases} 1 & \text{with probability } \xi \ddt, \\ 0 & \text{with probability } 1 - \xi \ddt. \end{cases} \quad (23) \]

The first term on the right-hand side of (22) represents the effects of transitory productivity shocks on employment in the intermediate good sector in slump: the employment at time \( t \) turns from \( x_R \) to \( 1 \) if \( d\phi = 1 \) that shows the occurrence of a transitory shock and stays \( x_R \) otherwise. The second term represents the effects of permanent productivity shocks on employment in boom: the employment at time \( t \) turns from \( 1 \) to \( x_R \) if \( ds(t) = 1 \) that shows the development of a succeeding technology and stays \( 1 \) otherwise. Therefore, the covariance between permanent shocks and the changes in log employment is calculated as

\[
\begin{align*}
\text{Cov} \left[ \text{d} \ln A_{s(t)}, \text{d} \ln x_t \right] & \quad \frac{\text{d} t}{t_R} \\
& = \frac{\text{d} t}{t_R} \text{Cov} \left[ \ln q ds(t), \ln \frac{1}{x_R} d\phi(t) \ | \ x_t = x_R \right] \\
& + t_B \text{Cov} \left[ \ln q ds(t), -\ln x_R ds(t) \ | \ x(t) = 1 \right] \\
& = -t_B \ln q \ln \frac{1}{x_R} \frac{\text{Var} \left[ ds(t) \right]}{\text{d} t} \\
& = -t_B \lambda \xi \ln q \ln \frac{1}{x_R} < 0. \quad (24)
\end{align*}
\]
Since the sign of (24) is negative, permanent productivity shocks and the changes in log employment correlate negatively. This negative relationship due to the fact that growth is driven by creative destruction, which accompanies the adaptation costs.

4 Conclusion

This paper investigated the relationship among economic growth, business cycles, and unemployment, driven by the existence of the adaptation cost. First, the expected rate of growth has a negative relationship with the amplitude of business cycles. Second, the expected rate of growth has a negative connection with the frequency of slump. Finally, a permanent shock reduces the fraction of employment, while the transitory shock raises the fraction of employment.

Appendix

A The Expected Rate of Economic Growth

The expected amount of the output at time $t (> t_0)$ conditioned by time $t_0$ can be written as

$$E_{t_0} \left[ \ln A_s(t) x^\alpha_t \right] = E_{t_0} \left[ \ln A_s(t) \right] + \alpha E_{t_0} \left[ \ln x_t \right]$$

$$= \sum_{i=0}^{\infty} e^{-\hat{\lambda}(t-t_0)} \frac{[\hat{\lambda}(t-t_0)]^i}{i!} \ln q^i A_s(t_0) + \alpha E_{t_0} \left[ \ln x_t \right]$$

$$= \hat{\lambda}(t-t_0) \ln q + \ln A_s(t_0) + \alpha E_{t_0} \left[ \ln x_t \right].$$

Therefore, the expected growth rate is written as

$$g = \frac{E \left[ d \ln A_s(t) x^\alpha_t \right]}{dt}$$

$$= \lim_{\Delta t \to 0} \frac{E_{t_0} \left[ \ln A_s(t+\Delta t) x^\alpha_{t+\Delta t} \right] - E_{t_0} \left[ \ln A_s(t) x^\alpha_t \right]}{\Delta t}$$

$$= \hat{\lambda} \hat{z} \ln q$$

References

Figure 1: Innovation Processes


