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CORRUPTION IN ENTRY REGULATION
A GAME-THEORETIC ANALYSIS WITH A TRACK OF BUREAUCRATS

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In this paper, we analyze the corruption in entry regulation that involves an entrepreneur and a track of bureaucrats. Instead of formulating a game in extensive form to analyze the sequential nature of the process involved in the application for a permit, we focus on the corruption in entry regulations that involves both entrepreneur and multiple bureaucrats to negotiate simultaneously for bribes from the mechanism design perspectives. Our results are the following: First, because of the asymmetry of information, the entrepreneur might not obtain the required permit, although collectively as a group, the joint net payoff of the entrepreneur and the bureaucrats is positive. Second, the entrepreneur might pay the bribes without getting the permit.

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1. INTRODUCTION

Governments often regulate the entry into markets by means of permits or licenses. In some countries, the bureaucratic procedures involved in such a process are extremely burdensome. Red tape, which includes the process of the acquisition of official permits, licenses, or approvals through multiple ministries, represents considerable expenses for applicants in those countries. When an applicant applies for a government-regulated permit, he often experiences time-consuming and painful procedures associated with the process. To obtain a permit for starting a new business, an entrepreneur often has to complete a number of required procedures and pay numerous fees at various stages of the process. In this paper, we formalize and analyze a game played by an entrepreneur who wants to enter a market and several corrupted bureaucrats who are in charge of regulating the entry of new firms into an industry. The game is formulated from the perspective of mechanism design.

The relationship between economic efficiency and bureaucratic red tape has attracted much attention from scholars working in various disciplines, such as political science, sociology, and economics. A path-breaking study in this subject is The Other Path of De Soto (1990), who presented a chronological review of the economic history of Peru, and methodically and analyzed the deficiency of the property law and bureaucratic structure of this country. According to this researcher, to obtain the allocation for a minibus route, an entrepreneur must complete 4 procedures that take him approximately 26 months.¹ De Soto also estimated the cost of acquiring formal status of economic

¹ For a description of the hypothetical access procedure needed to obtain a minibus route, interest readers can consult De Soto’s The Other Path, p. 149.
activities and compares the choice of applying for formal status and the decision to operate in the informal sector. This researcher concludes that most of the regulations enforced by the Peruvian government are so restrictive to make the development of the market mechanism impossible.

The work of De Soto has inspired several recent empirical studies on the cost of entry generated by administrative barriers. Djankov et al. (2002) described the required procedures of entering a new business, the time, and the cost needed to complete these procedures in 85 countries. These researchers documented that to meet government requirements for starting a business in Bolivia, an entrepreneur must complete 20 different procedures, pay US$2,696 in fees to the government and wait at least 82 business days to acquire the necessary permits. In contrast, in Canada the process requires only 2 procedures; takes roughly 2 days; and costs the entrepreneur US$280. Their empirical analysis indicates that the cost of entry is extremely high in most countries. They have also shown that countries with more regulations on entry have higher level of corruption, wider underground economies, and highly inefficient markets. Based on their empirical analysis, these researchers concluded that reality is better described by the grabbing hand than the helping hand.

Morisset and Neso (2002) used a new database to present the effect of administrative barriers in 32 countries. Their database includes several new elements, such as land access, site development, etc. According to their analysis, the level of administrative costs is positively correlated with corruption and negatively correlated with the quality of government, degree of country openness, and public wages. Classens et al. (1999) analyzed the ultimate ownership structure for the 2980 corporations in 9 East Asian
countries. In the context of entry regulation, they have also found that heavier regulation is correlated with higher corruption and thus less competition. Using cross-country data, Lambsdorff (2003) verified the negative relationship between corruption and the ratio of investment to GDP. From the results of the recent empirical studies on the negative correlation between productivity and corruption, this author attempted to investigate why corruption may cause a negative effect by a variety of government failures. The conclusion this researcher drew from his empirical analysis is that a country’s law and order tradition is a key sub-component affecting capital inflows, such as FDI (foreign direct investment).

There are two opposing views on entry regulation by the government that are related to our conceptual research core. Advocates of entry regulation argue that the government must screen new entrants so that they meet minimum standards on health, safety, or the environment, and in the literature on corruption, this argument is known as the helping hand theory (Pigou (1938)). The grabbing-hand view, on the other hand, sees the government as less benign and regulation as a means used by the government to extract bribes from businesses and consumers. As support for this view, proponents of the grabbing-hand theory often point to many regulation procedures that no rational economic criteria can justify (De Sote, op cit., Shleifer and Vishny (1998)). The game that we formulate in our paper reflects the view of the grabbing hand.

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2 The grabbing-hand view is also called tollbooth theory, which is analogous to tollbooths collecting the tolls on the road. “The independent monopolists solution means that different towns through which the road passes independently erect their own tollbooths and charge their own tolls.” (Shleifer and Vishny (1998), p.100). In the case of administration area, government officials pursue their own benefit instead of maximizing the social welfare.
According to the grabbing hand view, corruption can be *centralized* or *decentralized* (Bardhan (1997, 2006), Mookherjee (2006)). An example of centralized corruption was the Communist Party in Russia, which centralized the bribe collection system and monitored (with the help of KGB) the deviation of the corrupt agreement. On the other hand, in decentralized corruption, corrupt agents act independently so that there is uncertainty about the number of bribing procedures as well as the number of corrupt agents involved in the bribing game. Post-communism Russia is a good example of decentralized corruption. There are several features that differentiate centralized from decentralized corruption. In centralized corruption, the centralized bribe takers consider the adverse effect of the multiple bribing on total bribe collection. In that sense, the consequences of the centralized corruption might be less harmful than those of decentralized corruption. Another distinguishing feature that characterizes decentralized corruption is that a corrupt agent – after taking the bribe – might breach the agreement with briber because of weak law enforcement power, lack property right, and instability of government, etc. In this paper, we share the idea of the decentralized corruption view and construct a theoretical model of entry regulation with multiple agents.

In comparison to the empirical research, the theoretical literature on bureaucratic procedures and corruption is relatively small. There exist several theoretical studies on the relationship between administrative corruption and economic efficiency. Lien (1990) formulated a bribe-bidding procurement game between a corrupt official and two private firms. Under asymmetric information about the manufacturing cost structure and the degree of favor attitude toward firms (called discrimination), each firm chooses a bribery level that corresponds to its own cost level. The analysis of the model indicates that in the presence of discrimination, the corrupt official selects the inefficient
allocation, and, as a result, the inefficient allocation may increase the level of discrimination.

Waller et al. (2002) built a principal-agent model of government corruption under several settings to provide a theoretical answer to the question of Bardhan – which is better centralized or decentralized corruption? These researchers suggest that under the regime of a government with high monopoly power and lower public wages, centralized corruption may worsen the level of corruption. According to the model, high public-sector wage help to reduce allocation inefficiency since lower-tier bureaucrats are highly motivated by high wages. Mookherjee and Png (1992) formulated a principal-agent model under the asymmetric information to analyze the optimal compensation policy for a corrupt inspector, who is charged with monitoring pollution from a factory. In their model, the regulator (principal) can neither directly control the inspector's monitoring effort, nor hinder a factory (agent) from bribing an inspector. If bribing is leaked out, the regulator can penalize both the inspector (supervisor) and the factory. In the same spirit as Mookherjee and Png, Guriev (2003, 2004) set up a principal-bureaucrat-agent three-tier model to provide some answers to the questions asked by the former researchers. In the model of Guriev, an agent applies to the government for a license. The government has its own ideas concerning which type of applicant deserves to have a license, and uses red tape to sort out the bad type (the one who does not deserve a license). If the applicant is of the bad type, he will be willing to pay a large bribe to the bureaucrat to obtain the license. Guriev differentiated two types of corruption – ex ante and ex post corruption. In ex ante corruption, the agent pays speed money to reduce the amount of red tape before the information is produced. In contrast, in ex post corruption the bureaucrat colludes with the agent to conceal the
information after the information has been revealed through red tape. To extract bribe from an applicant, a corrupt bureaucrat will generate more red tape to discover the bad type. Thus, the effect of ex post corruption on red tape (more of it) is opposite to that of ex ante corruption (less of it). Guriev showed that the impact on red tape generated by ex post corruption dominates that of ex ante corruption: the equilibrium level of red tape is socially excessive.

Although the literature on corruption is vast, the game-theoretic subset of this literature is rather restrained, and within this subset, the only contribution that deals with multiple bureaucrats is Lambert-Mogilansky et al. (2007). These authors first formalized a game-theoretic model for a single period in which entrepreneurs may apply sequentially to a track of bureaucrats for approval of their projects. A project can only be approved if it is approved by each of the bureaucrats in the track, and each bureaucrat in the track demands a bribe for approval. The single-period game is then repeated indefinitely, with an entrepreneur arriving to face the same track of bureaucrats in each period. To obtain the structure of a repeated game, the values of the projects – assumed to be private information – of the successive entrepreneurs are assumed to be independent and identically distributed random variable. For the single-period game, these researchers showed that there exists no Bayesian Nash equilibrium under which a project is approved with positive probability. When the single-period game is repeated indefinitely the super-game has multiple equilibria in trigger strategies.

We eschew the repeated game approach and choose to formulate our model under the framework of mechanism design. The model we formalize can be considered as a voluntary trading institution that regulates the trade between a buyer (the entrepreneur)
and several sellers (the bureaucrats). As such, the mechanism can represent the bargaining process between the entrepreneur and the bureaucrats who can make offers and counteroffers to each other. The mechanism can also represent an arbitration process in which all parties – entrepreneur and bureaucrats – tell their types to a third party, who will propose whether trade should take place at which prices. Our modeling strategy has several advantages over the repeated game approach. First, it allows us to apply the elegant results of the research on mechanism design developed in the last two decades. Second – and in contrast with the sequential nature of the repeated game approach of Lambert-Mogilansky et al. – our approach allows for a more globally oriented strategy of the entrepreneur. In the single-period model of the researchers just mentioned, the entrepreneur approaches the bureaucrats sequentially, and at any point in time in this process, the entrepreneur will forfeit all the bribes that he has made to the previous bureaucrats if the present bureaucrat refuses to cooperate. Hence it might be in the interest of the entrepreneur to secure a global agreement with all the bureaucrats involved in the process since there is the uncertainty that every track of bureaucrats agreed with the bribery offered by entrepreneur. Entrepreneur prefers the global negotiation with the collected entity which is seemed as all bureaucrats agreed the total amount of bribery to the sequential negotiation with each of bureaucrats because the global strategy can reduce the cost of negotiation. In that sense, this global strategy is captured in our model.

In our model settings, an entrepreneur applies for a permit, and the process evolves sequentially on a track of two or more bureaucrats in order to acquire the permits. Since the bureaucrats’ non-monetary losses are unknown to the entrepreneur, and only the entrepreneur knows the true realized value of the project, there is an incentive for each
player not to reveal his type truthfully to an intermediary. Proposition 1 describes a set of necessary and sufficient conditions for a social-choice function to be Bayesian incentive compatible. Next, we use the theoretical approach of Myerson and Satterthwaite (1983), and prove in Proposition 2 that there is no Bayesian incentive compatible social-choice function that is Bayesian incentive compatible; satisfies the interim individual rationality constraints; and is ex post efficient. We find a second-best mechanism in dominant strategies that maximizes the expected total gains from trade subject to the incentive compatibility constraint, the interim individual rationality constraints, and the expected balanced budget constraint. This result is stated as Proposition 3. Now in the dominant-strategy game, the budget constraint is only balanced in the expected sense. There is no reason to expect that it is actually balanced under all circumstances. Furthermore, because corruption is an illegal act, and is not conducive to raise social welfare, the actual mismatch between bribe payments and bribe receipts in the dominant-strategy game cannot be solved by appealing to an outside source of financing, say a subsidy. Hence in order for trade to take place a mechanism in which the actual budget is balanced must be found. Such a mechanism is presented in Proposition 4, which describes another game with a Bayesian Nash equilibrium under which actual bribe payments by the entrepreneur are equal to the sum of actual bribes received by the track of bureaucrats for each possible realization of their types. Because the agreement involving bribe payments and bribe receipts constitutes an implicit contract between the entrepreneur and the track of bureaucrats, there is the possibility that the bureaucrats will take the bribes without delivering the permit. Such a breach of contract is not formalized in our model.
This paper is organized as follows. In Section 2, we present the entry regulation model, which is a formalization of the interactions between an entrepreneur and a track of bureaucrats in the setting of a direct-revelation mechanism. In section 3, the solution of and a characterization of the mechanism is presented. Section 4 contains some concluding remarks and possible future research avenues.

2. THE MODEL

2.1. Preferences and Payoffs

An entrepreneur – also called player 0 – has a project whose value is \( \theta_0 \) if it is realized. The value \( \theta_0 \) is a random variable drawn from a distribution function \( \Phi_0(\theta_0) \), which is defined on an interval \( \Theta_0 = [\theta_0, \bar{\theta}_0] \) of the real line, where \( 0 \leq \theta_0 < \bar{\theta}_0 \). The distribution function \( \Phi_0(\theta_0) \) is assumed to be common knowledge and the value of \( \theta_0 \) is assumed to be the entrepreneur’s private information.

In order to realize the project, the entrepreneur must apply for a permit, and the process evolves sequentially on a track of \( N \) bureaucrats. The entrepreneur must apply to each bureaucrat in the track in a prescribed order, and the project is approved if and only if it is approved by each bureaucrat. In what follows, bureaucrats are indexed by \( i, i = 1, \ldots, N \), and the \( ith \) bureaucrat will also be referred to as player \( i \).

Let \( \omega_i \) be the salary of bureaucrat \( i, i = 1, \ldots, N \). If a bureaucrat, say bureaucrat \( i \), does not accept bribes, then his payoff is simply \( \omega_i \). If he is offered a bribe, say \( b_i \),
and chooses to accept it, then his payoff will be $\omega_i + b_i$ if he manages to escape detection. On the other hand, if he accepts the bribe and gets caught, then he can expect to lose his job and the bribe to be confiscated. Because bribery is universally shameful, there are also non-monetary losses, such as loss of reputation, loss of family good name, moral costs associated with loss of virtue and integrity. We capture these non-monetary losses by a parameter, say $\Theta_i$. The value of $\Theta_i$ is assumed to be drawn from a distribution function $\Phi_i(\theta_i)$, which is defined on an interval $\Theta_i = [\theta_i^l, \theta_i^u]$ of the real line, where $0 \leq \theta_i^l < \theta_i^u$. The distribution function $\Phi_i(\theta_i)$ is assumed to be common knowledge, and the value of $\Theta_i$ is assumed to be the private information of bureaucrat $i$. Also, the distribution functions $\Phi_0, \Phi_1, ..., \Phi_N$ are assumed to be independent. If the bureaucrat does accept the bribe, then his payoff is given by

$$ (1) \quad (1 - p)(\omega_i + b_i) - p\Theta_i, \quad (i = 1, ..., N). $$

In (1), $p$ denotes the probability that the bureaucrat gets caught in the bribery act. The probability $p$ is assumed to be given, and applies to each bureaucrat. The exact value of $p$ reflects the efforts that the law and order authorities expend in fighting corruption, and is not our main concern in this paper.

As for the entrepreneur, if he does not apply for a permit, then his payoff is obviously 0. His payoff is also 0 if he does not obtain the permit. On the other hand, if he pays an

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3 For instance, in 1975, corruption was rampant in the Philippine’s Bureau of Internal Revenue (BIR). Parts of the BIR employers were embezzling money from tax payer. Justice Efren Plana had responsibilities to reform the corrupted BIR organizations. One of the anti-corruption strategies which Plana did was to convey the information to the mass media if the corrupted BIR official got a criminal charge. Since one’s family name is the most sacred in Philippine, his strategy was very effective to curb the corruption in tax correction. These case studies are referred from Klitgaard (1985).
amount $-b_0$ to bribe all the bureaucrats in the track, and obtains the permit needed for the realization of the project, then his payoff is

$$\theta_0 + b_0.$$  \hfill (2)

The one-track model of corruption – one entrepreneur and two or more bureaucrats in the track – can be formulated as a single-period game in extensive form, and the single-period game can then be repeated indefinitely, as in Lambert-Mogilansky et al (2007). However, we eschew this approach and choose to formulate this game under the framework of mechanism design. Our approach allows for a much richer modelling of the behaviour of bureaucrats than that found in the work of the preceding researchers.

A profile of types for the group made up of the entrepreneur and the $N$ bureaucrats is a list $\theta = (\theta_i)_{i=0}^N$, with $\theta_i \in \Theta_i$ for $i = 0, 1, \ldots, N$. An alternative for the group is a list $a = (q_0, q_1, \ldots, q_N, b_0, b_1, \ldots, b_N)$, with the following interpretations. First, $q_0$ is the probability that the entrepreneur chooses to apply for the permit, and $q_i, i = 1, \ldots, N$, is the probability that bureaucrat $i$ approves the project. Second, $b_i$ is a real number representing the transfer received by player $i, i = 0, 1, \ldots, N$, conditioned on the event that the entrepreneur applies for a permit and the application is approved by each bureaucrat in the track. Here we follow the convention that $b_i > 0$ indicates a receipt and $b_i < 0$ indicates a payment. In our model, for each $i = 1, \ldots, N, b_i$ is non-negative, and represents the bribe that the entrepreneur offers to bureaucrat $i$, while $-b_0$ is non-negative, and represents the total amount that the entrepreneur spends in bribing the $N$ bureaucrat. In equilibrium, we should have $b_0 + b_1 + \ldots + b_N = 0$. Presumably, if the application is rejected by one of the bureaucrats in the track, then none of them will be offered any bribe. Also, it is obvious that if the entrepreneur chooses not to apply for
a permit, then he will not offer any bribe to any bureaucrat in the track. The set of alternatives is denoted by $A$.

The expected payoff of the $ith$ bureaucrat – as a function of the alternative chosen $a = (q_0, q_1, \ldots, q_N, b_0, b_1, \ldots, b_N)$ – is given by

$$u_i(q_0, \ldots, q_N, b_0, \ldots, b_N, \theta) = \left[ -\left( \prod_{j=0}^{N} q_j \right) p \theta_i \right] + \left[ \left( \prod_{j=0}^{N} q_j \right) (1-p) b_j \right] + \left[ 1 - \left( \prod_{j=0}^{N} q_j \right) p \right] \theta_j .$$

Note that on the third line of (3) the expression inside the first pair of grand square brackets represents the non-monetary loss for the bureaucrat if he is caught in accepting a bribe; the expression inside the second pair of grand square brackets represents the expected bribe received by the bureaucrat; and the expression inside the third pair of grand square brackets represent the expected labor income that the bureaucrat might enjoy after accounting for the expected income loss if he engages in corrupt behaviour.

As for the entrepreneur, his payoff – as a function of the alternative chosen $a = (q_0, q_1, \ldots, q_N, b_0, b_1, \ldots, b_N)$ – is given by

$$u_0(q_0, \ldots, q_N, b_0, \ldots, b_N, \theta) = \left[ \left( \prod_{i=0}^{N} q_i \right) (1-p) \right] \left[ \theta_0 + b_0 \right] .$$

2.2. Mechanism Design

In this sub-section, we formalize the collective-choice problem for the group constituted by the entrepreneur and the bureaucrats in the track from whom he seeks approval for his project.
2.2.1. Direct Revelation Mechanism

**DEFINITION 1:** A *social choice function* for the group made up of the entrepreneur and the bureaucrats in the track is a map \( f : \prod_{i=0}^{N} \Theta_i \rightarrow A \), where the image of each profile of types \( \theta = (\theta_0, \theta_1, ..., \theta_N) \in \prod_{i=0}^{N} \Theta_i \) is given by
\[
(5) \quad f(\theta) = (q_0(\theta), q_1(\theta), ..., q_N(\theta), b_0(\theta), b_1(\theta), ..., b_N(\theta)).
\]

**DEFINITION 2:** A *direct revelation mechanism* is a list \( \Gamma = (\Theta_0, \Theta_1, ..., \Theta_N, f) \), where \( f : \prod_{i=0}^{N} \Theta_i \rightarrow A \) is a social choice function. Under the mechanism \( \Gamma \), a mediator asks each of the players to reveal his type. If the announced profile of types is \( \theta = (\theta_0, \theta_1, ..., \theta_N) \), then the mediator will implement the alternative \( f(\theta) \).

The mechanism \( \Gamma \) induces a game of incomplete information in the following manner. For each \( i \in \{0,1, ..., N\} \), a strategy for the \( ith \) player is a map \( s_i : \Theta_i \rightarrow \Theta_i \), where \( s_i(\theta_i) \) is the type announced by this player when his type is \( \theta_i \). If \( s_i(\theta_i) = \theta_i \) for each \( \theta_i \in \Theta_i \), then \( s_i \) is called *truth-telling*.

**DEFINITION 3:** A combination of strategies \( (s^*_i)_{i=0}^{N} \) is a *Bayesian Nash equilibrium* for the mechanism \( \Gamma \) if for each \( i \in \{0,1,...,N\} \) and each \( \theta_i \in \Theta_i \), the following condition is satisfied:
\[
(6) \quad \int u_i(f(s^*_i(\theta_i), s^*_j(\theta_j), \theta_j) d\Phi_{-i}(\theta_j | \theta_i) \geq \int u_i(f(\hat{\theta}_i, s^*_j(\theta_j), \theta_j) d\Phi_{-i}(\theta_j | \theta_i)
\]
for all \( \hat{\theta}_i \in \Theta_i \).
Note that in (6) we have use the following convention: 
\[ \theta_{-i} = (\theta_0, \theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_N), \] 
and \( \Phi_i(\theta_{-i}) \) is the distribution function of \( \theta_{-i} \).

Also, \( s_{-i} = (s_0, s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N) \) represents the lists of strategies of bureaucrats other than the \( ith \) bureaucrat.

2.2.2. The Bayesian Incentive Compatible Constraint and the Interim Individual Rationality Constraints

It is well known that without some appropriate restrictions on \( f \) the direct-revelation mechanism \( \Gamma = (\Theta_0, \Theta_1, \ldots, \Theta_N, f) \) will not work. First, there is no a priori reason to believe that a player will reveal truthfully her type. Under such a scenario, the profile of types, say \( \theta \), announced by the players \( 0, 1, \ldots, N \) might not be the true profile of types, and the alternative \( f(\theta) \) will not be the alternative the mediator wishes to implement. Second, a player will not participate in the process unless he obtains at least his reservation payoff. In the parlance of mechanism design, the mediator can only successfully implement the social choice function \( f(\theta) \) if this function is Bayesian incentive compatible and satisfies the interim rationality constraint for each of the players.

**DEFINITION 4:** The social choice function \( f \) is **truthfully implementable in Bayesian Nash equilibrium** (or Bayesian incentive compatible) if the combination of strategies \( (s_i^*)_{i=0}^N \), where \( s_i^* \) is truth telling for each \( i \in \{0, 1, \ldots, N\} \), is a Bayesian Nash equilibrium for the mechanism \( \Gamma = (\Theta_0, \Theta_1, \ldots, \Theta_N, f) \).
Now consider a direct revelation mechanism  \( \Gamma = (\Theta_0, \Theta_1, \ldots, \Theta_N, f) \), where

\[
f(\theta) = (q_0(\theta), q_1(\theta), \ldots, q_N(\theta), b_0(\theta), b_1(\theta), \ldots, b_N(\theta))
\]

is the alternative implemented when \( \theta \) is the profile of types of the entrepreneur and the \( N \) bureaucrats in the track. Under this mechanism, the payoff of bureaucrat \( i \) is given by

\[
u_i(f(\theta), \theta) = -pQ(\theta)\theta_i + (1 - p)Q(\theta)b_i(\theta) + [1 - pQ(\theta)]\omega_i,
\]

where we have let

\[
Q(\theta) = \prod_{j=0}^{N} q_j(\theta)
\]

denote the probability that the entrepreneur applies for the permit and his application is approved by each of the bureaucrats. As for the entrepreneur, his payoff is given by

\[
u_0(f(\theta), \theta_0) = (1 - p)Q(\theta)[\theta_0 + b_0(\theta)].
\]

Under the mechanism \( \Gamma \), the expected payoff of the \( ith \) bureaucrat, given that he announces \( \hat{\theta}_i \) as his type and all the other players reveal truthfully their types, is then given by

\[
\int u_i(f(\hat{\theta}, \theta), \theta) d\Phi_{-i}(\theta) = -p\bar{Q}_i(\hat{\theta})\theta_i + \bar{b}_i(\hat{\theta}) + [1 - p\bar{Q}(\hat{\theta})]\omega_i, \quad (i = 1, \ldots, N).
\]

In (10), we have let

\[
\bar{Q}_i(\hat{\theta}) = \int Q(\hat{\theta}, \theta) d\Phi_{-i}(\theta), \quad (i = 0,1,\ldots, N),
\]

and

\[
\bar{b}_i(\hat{\theta}) = (1 - p)\int Q(\hat{\theta}, \theta) b_i(\hat{\theta}, \theta) d\Phi_{-i}(\theta), \quad (i = 1, \ldots, N).
\]

As defined, \( \bar{Q}_i(\hat{\theta}) \) represents the probability that the entrepreneur applies for the permit and the application is approved under the mechanism \( \Gamma \), given that bureaucrat \( i \) announces \( \hat{\theta}_i \) as his type while all the other players reveal their types truthfully. Also,
\( b_i(\hat{\theta}_i) \) is the expected bribe received by bureaucrat \( i \), given that this bureaucrat announces \( \hat{\theta}_i \) as his type while all the other players reveal their types truthfully. If the type of the \( ith \) bureaucrat is \( \theta_i \), and if all the players reveal their types truthfully, then the expected payoff of this bureaucrat is given by

\[
U_i(\theta_i) = -p\overline{Q}(\theta_i)\theta_i + \overline{b}_i(\theta_i) + [1 - p\overline{Q}(\theta_i)]\omega_i,
\]

As for the entrepreneur, his expected payoff, given that he announces \( \hat{\theta}_0 \) as its type and all the other players reveal truthfully their types, is given by

\[
\int u_0(f(\hat{\theta}_0, \theta_{-0}), \theta_0) d\Phi_{-0}(\theta_{-0}) = (1 - p)\overline{Q}_0(\hat{\theta}_0)\theta_0 + \overline{b}_0(\hat{\theta}_0),
\]

where we have let

\[
\overline{Q}_0(\hat{\theta}_0) = \int Q(\hat{\theta}_0, \theta_{-0}) d\Phi_{-0}(\theta_{-0}),
\]

and

\[
\overline{b}_0(\hat{\theta}_0) = (1 - p)\int [Q(\hat{\theta}_0, \theta_{-0})b_0(\hat{\theta}_0, \theta_{-0})] d\Phi_{-0}(\theta_{-0}).
\]

As defined, \( \overline{Q}_0(\hat{\theta}_0) \) represents the probability that the entrepreneur applies for the permit and the application is approved under the mechanism \( \Gamma \), given that the entrepreneur announces \( \hat{\theta}_0 \) as his type while all the other players reveal their types truthfully. Also, \( -\overline{b}_0(\hat{\theta}_0) \) is the total expected bribe paid by the entrepreneur, given that he announces \( \hat{\theta}_0 \) as his type while all the bureaucrats reveal their types truthfully. If the type of the entrepreneur is \( \theta_0 \), and if all the bureaucrats reveal their types truthfully, then the expected payoff of the entrepreneur is given by

\[
U_0(\theta_0) = (1 - p)\overline{Q}_0(\hat{\theta}_0)\theta_0 + \overline{b}_0(\theta_0).
\]
PROPOSITION 1: Let \( f : \Theta \rightarrow f(\Theta) = (q_0(\Theta), q_1(\Theta), \ldots, q_N(\Theta), b_0(\Theta), b_1(\Theta), \ldots, b_N(\Theta)) \) be a social choice function. Then \( f \) is Bayesian incentive compatible if and only if the following conditions are satisfied:

(a) For each bureaucrat \( i \in \{1, \ldots, N\} \),

(a.1) \( \overline{Q}_i : \Theta \rightarrow \overline{Q}_i(\Theta) \) is non-increasing, and

(a.2) \( U_i(\Theta) = U_i(\overline{\Theta}) + p \int_{\overline{\Theta}}^{\Theta} \overline{Q}_i(t) \, dt \) for all \( \Theta \in \Theta_i \).

(b) For the entrepreneur

(b.1) \( \overline{Q}_0 : \Theta_0 \rightarrow \overline{Q}_0(\Theta_0) \) is non-decreasing, and

(b.2) \( U_0(\Theta_0) = U_0(\overline{\Theta}_0) + (1 - p) \int_{\overline{\Theta}_0}^{\Theta_0} \overline{Q}_0(t) \, dt \) for all \( \Theta_0 \in \Theta_0 \).

PROOF:

It is a restatement of the Proposition 23.D.2 in Mas-Collel, Whinston, and Green (1995).

Let \( f \) be a social choice function, and consider the direct revelation mechanism \( \Gamma = (\Theta_0, \Theta_1, \ldots, \Theta_N, f) \). Suppose that \( \Theta_i \) is the private information of player \( i, i = 0, 1, \ldots, N \). Let \( \overline{\pi}_i(\Theta) \) denote the expected payoff obtained by this player if he refuses to participate in the mechanism \( \Gamma \). Then we have

\[
\overline{\pi}_i(\Theta_0) = 0 \quad \text{for all} \quad \Theta_0 \in \Theta_0,
\]

and

\[
\overline{\pi}_i(\Theta_i) = \omega_i \quad \text{for all} \quad \Theta_i \in \Theta_i, \quad (i = 1, \ldots, N).
\]

Next, let
(20) \[ U_i(\theta | f) = \int u_i(f(\theta_i, \theta_{-i}), \theta_i) d\Phi_{-i}(\theta_{-i}), \quad (i = 0, 1, \ldots, N), \]

denote the \textit{interim expected profit} of player \( i \) if he participates in the mechanism \( \Gamma \).

\textsc{Definition 5:} The mechanism \( \Gamma = (\Theta_0, \Theta_1, \ldots, \Theta_N, f) \) is said to satisfy the \textit{interim individual rationality constraints} if for each \( i \in \{0, 1, \ldots, N\} \) the following \textit{interim participation constraint} is satisfied: \( U_i(\theta | f) \geq \bar{\pi}_i(\theta) \) for all \( \theta \in \Theta_i \).

Let \( f : \theta \to f(\theta) = (q_0(\theta), q_1(\theta), \ldots, q_N(\theta), b_0(\theta), b_1(\theta), \ldots, b_N(\theta)) \) be a social choice function that is Bayesian incentive compatible. If trade does not take place, then no money changes hands. If trade takes place, then the difference between the amount that the entrepreneur pays the \( N \) bureaucrats and the sum of the bribes is \( -b_0(\theta) - \sum_{i=1}^{N} b_i(\theta) \). Hence for any profile of types \( \theta \), the expected budget under \( f \), namely the expected value of the difference between what the entrepreneur is required to pay and the sum of the bribes received by the track of bureaucrats is given by

\[ V(f) = -\int \bar{b}_0(\theta_0) d\Phi_0(\theta_0) - \sum_{i=1}^{N} \int \bar{b}_i(\theta_i) d\Phi_i(\theta_i). \]

Now for each \( i = 1, \ldots, N \), using (13), we can write the expected bribe received by bureaucrat \( i \) as follows:

\[ \int \bar{b}_i(\theta_i) d\Phi_i(\theta_i) = U_i(\bar{\theta}) + \left[ \int p\bar{\theta}_{i}(\theta_i) d\Phi_i(\theta_i) - \int [1 - p\bar{\theta}_{i}(\theta_i)] \omega_i \right] d\Phi_i(\theta_i) \]

(22)

Using integration by parts, we can rewrite the integral on the last line of (41) as follows:

\[ \int p\int_{\theta_i}^{\bar{\theta}_i} \bar{q}_i(t) dt d\Phi_i(\theta_i) = \int_{\theta_i}^{\bar{\theta}_i} \left[ p\bar{q}_i(\theta_i) \right] d\Phi_i(\theta_i) d\theta_i. \]

(23)
Using (23) in (22), we obtain the following expression for the expected bribe received by bureaucrat \( i, i = 1, \ldots, N \),

\[
\int b_i(\theta) d\Phi_i(\theta) = U_i(\theta) + \int \left[ p \bar{Q}(\alpha) \theta - [1 - p \bar{Q}(\alpha)] \alpha_0 \right] d\Phi_i(\theta) + \int p \bar{Q}(\alpha) \Phi_i(\theta) d\theta_i
\]

\[= U_i(\theta) - \alpha_0 + \int \cdots \int p Q(\theta) \left[ \alpha_0 + \theta_i + \frac{\Phi_i(\theta)}{\bar{Q}(\alpha)} \right] \left[ \prod_{j=0}^{N} \phi_j(\theta_j) \right] d\theta_0 \cdots d\theta_N.\]

For the entrepreneur, the expected total bribe he offers to the \( N \) bureaucrats is

\[
\int [-\bar{b}_0(\theta_0)] d\Phi_0(\theta_0) =
\]

\[= -U_0(\theta_0) - (1 - p) \left\{ \int \cdots \int \bar{Q}_0(t) \phi_0(\theta_0) d\theta_0 - \frac{N}{\bar{Q}_0(\theta_0) \theta_0} \phi_0(\theta_0) d\theta_0 \right\}.\]

Now using integration by parts, we can evaluate the first integral on the equation (44) as follows:

\[
\int \cdots \int \bar{Q}_0(t) \phi_0(\theta_0) d\theta_0 = \frac{N}{\bar{Q}_0(\theta_0) \theta_0} \phi_0(\theta_0) d\theta_0.
\]

Using (26) in (25), we obtain

\[
\int [-\bar{b}_0(\theta_0)] d\Phi_0(\theta_0) = -U_0(\theta_0) + (1 - p) \frac{N}{\bar{Q}_0(\theta_0) \theta_0} \left[ \theta_0 - \frac{1 - \Phi_0(\theta_0)}{\phi_0(\theta_0)} \right] \phi_0(\theta_0) d\theta_0.
\]

Using (24) and (26), we can rewrite (21) as follows:

\[
V(f) = -U_0(\theta_0) - \sum_{i=1}^{N} \left[ U_i(\theta_i) - \alpha_i \right]
\]

\[+ \int \cdots \int Q(\theta) \left\{ (1 - p) \left[ \theta_0 - \frac{1 - \Phi_0(\theta_0)}{\phi_0(\theta_0)} \right] \right\} \left[ \prod_{j=0}^{N} \phi_j(\theta_j) \right] d\theta_0 \cdots d\theta_N.
\]
Now observe that if \( V(f) = 0 \), then the expected budget is balanced under \( f \), and this is one of the conditions for the social-choice function \( f \) to be ex post efficient.

Next, note that if the entrepreneur applies for the permit and his application is approved by all the bureaucrats, then the total expected gain—as a function of \( \theta \)—of all these agents is given by

\[
\theta_0 + \sum_{i=1}^{N} \omega_i - p \sum_{i=1}^{N} \theta_i - \sum_{i=1}^{N} \omega_i = (1-p)\theta_0 - p \sum_{i=1}^{N} (\omega_i + \theta_i).
\]

For a Bayesian incentive compatible social-choice function \( f \) to be ex post efficient (29) must be non-negative, in addition to the requirement that the budget be balanced.

Myerson and Satterthwaite (1983) showed – in a model of bilateral trade in which the buyer and the seller are risk-neutral and their valuations for the object of the trade are private information – that there is no Bayesian incentive compatible social-choice function that is ex post efficient and satisfies the interim individual rationality constraints. This impossibility result also holds in our model that we state formally as follows:

**PROPOSITION 2:** Suppose that \( \theta_0 < \frac{p}{1-p} \sum_{i=1}^{N} (\omega_i + \theta_i) \) and \( \frac{p}{1-p} \sum_{i=1}^{N} (\omega_i + \bar{\theta}) < \bar{\theta}_0 \).

Then there is no social-choice function that is Bayesian incentive compatible; satisfies the interim individual rationality constraints; and is ex post efficient.

**PROOF:** Note that the first inequality ensures that there is a positive probability that the project will not be carried out, and this event occurs when the value of the project is
small. As for the second inequality, it ensures that no matter what the types of the \( N \) bureaucrats, there is a positive probability that the project will be carried out if the value of the project is high. We will prove the proposition for the case of \( N = 2 \) bureaucrats; the proof is by reductio ad absurdum.

Suppose then that there exists a social-choice function \( f \) that is Bayesian incentive compatible; satisfies the interim individual rationality constraints; and is ex post efficient. Using the balanced-budget constraint and (28), we can write

\[
\begin{align*}
\bar{\pi}_i \cdots \bar{\pi}_N \int_{\bar{x}_0}^{\bar{x}_N} \int_{x_0}^{x_N} Q(\theta) & \left\{ (1 - p) \left[ \theta_0 - \frac{1 - \Phi_0(\theta_0)}{\phi_0(\theta_0)} \right] \prod_{j=0}^{N} \phi_j(\theta_j) \right\} d\theta_0 \cdots d\theta_N \\
& = U_0(\theta_0) + \sum_{i=1}^{N} [U_i(\bar{\theta}_i) - \omega_i] \geq 0.
\end{align*}
\]

Note that the inequality in (30) has been obtained by using the interim individual rationality constraints: \( U_0(\theta_0) \geq 0 \), and \( U_i(\bar{\theta}_i) - \omega_i \geq 0 \), \( i = 1, \ldots, N \). Furthermore, because \( f \) is ex post efficient, we must have \( Q(\theta) = 1 \) when (29) is non-negative, and \( Q(\theta) = 0 \) when (29) is negative, and these results allow us to rewrite the multiple integral in (30) as follows
where \( I \) and \( J \) are defined as follows:

\[
I = \frac{p}{1-p} \sum_{i=1}^{N} (\omega_i + \theta_i) \left[ 1 - \Phi_0 \left( \frac{p}{1-p} \sum_{i=1}^{N} (\omega_i + \theta_i) \right) \right],
\]

and

\[
J = \frac{p}{1-p} \sum_{i=1}^{N} \left( \omega_i + \theta_i \right) \left[ 1 - \phi_i \left( \frac{p}{1-p} \sum_{i=1}^{N} (\omega_i + \theta_i) \right) \right] \int \prod_{j=1}^{N} \phi_j(\theta_j) d\theta_j \ldots d\theta_N.
\]
Using (33) and (34) in (32), we obtain

\[ (35) \]

Using (35), we can rewrite (31) as follows

\[ (36) \]

Clearly, (36) contradicts (30).
3. THE MECHANISM THAT MAXIMIZES EXPECTED TOTAL GAINS FROM TRADE

According to Proposition 2, ex post efficiency is unattainable. Hence it is natural to search for a mechanism that maximizes the expected total gains from trade subject to the incentive compatible constraint, the interim individual rationality constraints, and the balanced budget constraint. Following the technique developed by Myerson and Satterthwaite, op cit., for a bilateral trading model, we now find the second-best mechanism for the model of corruption; the argument proceeds in several stages.

3.1. The Probability that the Entrepreneur Applies for the Permit and his Application is Approved by All the Bureaucrats in the Track

First, we find a function \( \theta \rightarrow Q(\theta), \theta \in \Theta, \) where \( Q(\theta) \) is the probability that the entrepreneur applies for the permit and his application is approved by every bureaucrat in the track, to solve the expected total gains from trade

\[
\max_{Q(\theta)} \int (1-p)\theta_0 - p \sum_{i=1}^{N} (\theta_i + \theta) \, Q(\theta) \, d\theta
\]

subject to the following constraint\(^4\)

\[
\int_{\Theta_0} \cdots \int_{\Theta_N} Q(\theta) \left( (1-p) \left[ \theta_0 - \frac{1}{\phi_i(\theta_i)} \right] - p \sum_{i=1}^{N} \left[ \theta_i + \theta \right] \right) \, d\theta_0 \cdots d\theta_N \geq 0.
\]

\(^4\) The constraint will be shown to be binding for the solution of this maximization problem, and the integral in (38) will be shown to be the expected budget generated by the optimal solution.
To solve the preceding maximization problem, define for each $0 \leq \alpha \leq 1$, the following functions:

\[(39) \quad c_0(\theta_0, \alpha) = (1 - p) \left[ \theta_0 - \alpha \frac{1 - \Phi(\theta_0)}{\phi(\theta_0)} \right],\]

and

\[(40) \quad c_i(\theta_i, \alpha) = p \left[ \omega_i + \theta_i + \alpha \frac{\Phi(\theta_i)}{\phi(\theta_i)} \right],\]

\((i = 1, \ldots, N)\).

Next, define

\[(41) \quad \Theta(\alpha) = \left\{ \theta \in \Theta : c_0(\theta_0, \alpha) \geq \sum_{i=1}^{N} c_i(\theta_i, \alpha) \right\},\]

then let

\[(42) \quad Q(\theta \mid \alpha) = \begin{cases} 1 & \text{if } \theta \in \Theta(\alpha), \\ 0 & \text{otherwise}. \end{cases}\]

Because $c_0(\theta_0, \alpha) - \sum_{i=1}^{N} c_i(\theta_i, \alpha)$ is decreasing in $\alpha$, for any given $\theta$, the probability of trade $Q(\theta \mid \alpha)$ is also decreasing in $\alpha$. Next, observe that $Q(\theta \mid 0)$ is the ex post efficient probability that trade takes place between the entrepreneur and the track of bureaucrats, i.e., the entrepreneur applies for the permit, and his application is approved by all the bureaucrats in the track whenever $(1 - p)\theta_0 - p \sum_{i=1}^{N} (\omega_i + \theta_i) \geq 0$.

As for $Q(\theta \mid 1)$, it maximizes the integral in (28).

Now consider the following function of $\alpha$:
We claim that $G(\alpha)$ is increasing in $\alpha$. To establish this result, write

$$G(\alpha) = \int_{\theta_0}^{\theta_N} \cdots \int_{\theta_0}^{\theta_N} Q(\theta | \alpha) \left\{ (1-p) \left[ \frac{\theta_0 - 1 - \Phi_0(\theta_0)}{\phi_0(\theta_0)} \right] - p \sum_{i=1}^{N} \omega_i + \theta_i + \frac{\Phi_i(\theta_i)}{\phi_i(\theta_i)} \right\} \prod_{j=0}^{N} \phi_j(\theta_j) \, d\theta_0 \cdots d\theta_N.$$  

(43)

Note that the expression under the integral sign on the last line of (44) is negative.

Furthermore, the set $\Theta(\alpha)$ shrinks as $\alpha$ rises. Hence $G(\alpha)$ is an increasing function of $\alpha$.

Also, we claim that $G(\alpha)$ is a continuous function of $\alpha$. Indeed, if $\alpha' < \alpha$, then $\Theta(\alpha') \subset \Theta(\alpha)$, and

$$G(\alpha') - G(\alpha) = \int_{\theta_0}^{\theta_N} \cdots \int_{\theta_0}^{\theta_N} Q(\theta | \alpha') \left\{ (1-p) \left[ \frac{\theta_0 - 1 - \Phi_0(\theta_0)}{\phi_0(\theta_0)} \right] - p \sum_{i=1}^{N} \omega_i + \theta_i + \frac{\Phi_i(\theta_i)}{\phi_i(\theta_i)} \right\} \prod_{j=0}^{N} \phi_j(\theta_j) \, d\theta_0 \cdots d\theta_N.$$  

(45)

Furthermore, when $\alpha' \uparrow \alpha$, the set difference $\Theta(\alpha') - \Theta(\alpha)$ shrinks to the empty set, and this implies that $G(\alpha') - G(\alpha)$ tends to 0 as $\alpha' \uparrow \alpha$. In the same manner, we can show that $G(\alpha') - G(\alpha)$ tends to 0 as $\alpha' \downarrow \alpha$.  

27
Because $Q(\theta | 1) > 0$ only when $c_o(\theta_o,1) - \sum_{i=1}^{N} c_i(\theta_i,1) \geq 0$, we must have $G(1) \geq 0$.

Also, according to (36), we have $G(0) < 0$. Hence by continuity there exists a value of $\alpha$, say $\alpha = \alpha^*$, $0 < \alpha^* \leq 1$, such that

\begin{equation}
G(\alpha^*) = \int \ldots \int Q(\theta | \alpha^*) \left\{ (1 - p) \left[ \theta_o - \frac{1 - \Phi_o(\theta_o)}{\phi(\theta_o)} \right] - p \sum_{i=1}^{N} \left[ \phi_i(\theta_i) \right] d\theta_o \ldots d\theta_N \right\} = 0.
\end{equation}

Because $G(\alpha)$ is increasing in $\alpha$, the value of $\alpha^*$ is unique.

We now show that $\theta \rightarrow Q(\theta | \alpha^*)$ is the solution of the maximization problem constituted by (37) and (38). To this end, let $\lambda^*$ be such that $\lambda^*/(1 + \lambda^*) = \alpha^*$.

Next, consider an arbitrary function $\theta \rightarrow Q(\theta)$ that satisfies the constraint (38). We have

\begin{equation}
\int \left[ (1 - p)\theta_o - p \sum_{i=1}^{N} (\omega_i + \theta_i) \right] Q(\theta) d\theta \leq \int \left[ (1 - p)\theta_o - p \sum_{i=1}^{N} (\omega_i + \theta_i) \right] Q(\theta) d\theta
\end{equation}

\begin{equation}
\quad \quad + \lambda^* \int \ldots \int Q(\theta) \left\{ (1 - p) \left[ \theta_o - \frac{1 - \Phi_o(\theta_o)}{\phi(\theta_o)} \right] - p \sum_{i=1}^{N} \left[ \phi_i(\theta_i) \right] d\theta_o \ldots d\theta_N \right\} = (1 + \lambda^*) \int \ldots \int Q(\theta) \left( c_o(\theta_o,\alpha^*) - \sum_{i=1}^{N} [c_i(\theta_i,\alpha^*)] \right) d\theta_o \ldots d\theta_N.
\end{equation}

When $\theta \rightarrow Q(\theta | \alpha^*)$ is used as a candidate to solve the maximization problem constituted by (37) and (38), it yields the following value for the objective function:
Because $Q(\theta | \alpha^*)$ is equal to 1 when $c_0(\theta_0, \alpha^*) - \sum_{i=1}^{N} c_i(\theta_i, \alpha^*) \geq 0$, and is equal to 0 otherwise, the last line of (48) must be at least equal to the last line of (47), showing that $\theta \rightarrow Q(\theta | \alpha^*)$ is the solution of the maximization problem constituted by (37) and (38).

### 3.2. The Second-Best Mechanism with Expected Balanced Budget Constraint

To find a second-best mechanism which satisfies the expected balanced budget constraint, we need to set up some preliminary machinery.

For any profile of types $\theta \in \Theta$, let the transfer to the entrepreneur be defined by

\begin{equation}
(49)
    b^*_i(\theta) = \begin{cases} 
        0 & \text{if } Q(\theta | \alpha^*) = 0, \\
        -\min\{ t_o \in \Theta_0 | Q(t_o, \theta_o | \alpha^*) = 1 \} & \text{if } Q(\theta | \alpha^*) = 1.
    \end{cases}
\end{equation}

Next, note that for a bureaucrat, say $i$, with type $\theta_i$, the bribe offered him must be at least $p(\omega_i + \theta_i)/(1-p)$ to make him corrupt. Thus, we define the transfer to bureaucrat $i, i=1,\ldots,N$, as follows:
(50) \[ b_i^*(\theta) = \begin{cases} 0 & \text{if } Q(\theta | \alpha^*) = 0, \\ \max \left\{ \frac{p}{1 - p} (\omega_i + t_i) I_i \in \Theta_i, Q(t_i, \theta_{-i} | \alpha^*) = 1 \right\} & \text{if } Q(\theta | \alpha^*) = 1. \end{cases} \]

(i = 1, ..., N).

Note that when \( Q(\theta | \alpha^*) = 1 \), we have \( b_i^*(\theta) \leq \theta_0 \). Furthermore, when \( Q(\theta_0, \theta_{0*} | \alpha^*) = 1 \), we have \( b_i^*(\theta_0, \theta_{0*}) = \theta_0 \). Also, when \( Q(\theta | \alpha^*) = 1 \), we have \( b_i^*(\theta) \geq p(\omega_i + \theta_i) / (1 - p) \). Furthermore, we have \( b_i^*(\theta_{-i}) = p(\omega_i + \theta_i) / (1 - p) \), when \( Q(\theta_{-i}, \theta_{-i} | \alpha^*) = 1 \).

Now consider the following modified second-price sealed-bid auction. The arbitrator asks each agent – entrepreneur as well as bureaucrats – to reveal his type. If \( \theta \) is the announced profile of types, then the arbitrator will allow trade to take place with probability \( Q(\theta | \alpha^*) \). When \( Q(\theta | \alpha^*) = 0 \), trade does not take place, and no money will change hands. When \( Q(\theta | \alpha^*) = 1 \), the entrepreneur will only be asked to pay a total bribe of \(-b_i^*(\theta)\) for the permit and bureaucrat \( i \) will receive the bribe \( b_i^*(\theta) \).

We make the following claims:

CLAIM 1: Revealing truthfully his type is a weakly dominant strategy for the entrepreneur and gives this agent a non-negative net payoff.

PROOF: See Appendix 1.
CLAIM 2: Revealing truthfully his type is a weakly dominant strategy for each bureaucrat and yields a payoff of at least \( \omega_i \).

PROOF: See Appendix 2.

The mechanism just described is truthfully implementable in dominant strategies, and satisfies the interim individual rationality constraints. Furthermore, the expected payments by the entrepreneur minus the expected total bribes under this mechanism is given by

\[
U^*_0(\theta_0) - \sum_{i=1}^{N} [U^*_i(\bar{\theta}_i) - \omega_i] + \int_{\theta_0}^{\theta_{\bar{\theta}_*}} \prod_{j=0}^{N} \Phi_j(\theta_j) d\theta_0 \ldots d\theta_N
\]

In (51), \( U^*_0(\theta_0) \) is the expected net payoff of the entrepreneur, given that his type is \( \theta_0 \), and \( U^*_i(\bar{\theta}_i) \) is the expected payoff of bureaucrat \( i \), given that \( \bar{\theta}_i \) is his type – both under the mechanism \( \Gamma^* \). Note that the last line in (51) has been obtained by using the result that the integral in (51) is equal to 0. Furthermore, recall that \( \theta^*_0 = \theta_0 \) when \( \theta_0 = \theta_0 \), which implies that \( U^*_0(\theta_0) = 0 \). Recall also that for each \( i = 1, \ldots, N \),
if \( \theta_i = \overline{\theta}_i \), then \( \theta_i^* = \overline{\theta}_i \), which implies that \( U_i^*(\overline{\theta}_i) = \omega_i \). Hence the expected budget is balanced under the mechanism.

Now let

\[
(52) \quad f^*: \theta \to f^*(\theta) = (q_0^*(\theta), q_1^*(\theta), ..., q_n^*(\theta), b_0^*(\theta), b_1^*(\theta), ..., b_n^*(\theta))
\]

be the social-choice function defined in the following manner, where

\[
(53) \quad q_0^*(\theta) = q_1^*(\theta) = ... = q_n^*(\theta) = Q(\theta | \alpha^*).
\]

To show that \( f^* \) maximizes the expected total gains from trade among Bayesian incentive compatible social-choice functions that satisfy the interim individual rationality constraints and the expected balanced budget constraint, let \( f \) be another social-choice function that satisfies all these requirements. Then (30) must hold for \( f \).

Hence \( f \) is a feasible candidate for the constrained maximization problem constituted by (37) and (38), and thus the expected total gains from trade under \( f \) cannot exceed that generated under \( f^* \). We summarize the results just obtained in the following proposition.

**Proposition 3:** The social-choice function \( f^* \) is truthfully implementable in dominant strategies. Furthermore, it maximizes the expected total gains from trade among the class of social-choice functions that are (i) Bayesian incentive compatible; (ii) satisfy the interim individual rationality constraints; and (iii) satisfy the expected balanced budget constraint.
In what follows, we shall refer to the game induced by the social-choice function $f^*$ as the dominant-strategy game and denote it by $\Gamma^*$. In the game $\Gamma^*$, the expected bribe payments – as a function of his own type – made by the entrepreneur are given by

$$\bar{b}_0^*(\theta_0) = \int (1 - p) Q(\theta_0, \theta_0) b_0^*(\theta_0, \theta_0) \Phi(\theta_0) d\theta_0.$$  

(54)

It follows directly from the definition of $b_0^*(\theta)$ that $\bar{b}_0^*(\theta_0)$ is increasing in $\theta_0$. As for the expected bribe received by bureaucrat $i$ in the game $\Gamma^*$, it is given by

$$\bar{b}_i^*(\theta_i) = \int (1 - p) Q(\theta_i, \theta_i) b_i^*(\theta_i, \theta_i) \Phi(\theta_i) d\theta_i, \quad (i = 1, \ldots, N).$$  

(55)

It follows directly from the definition of $b_i^*(\theta)$ that $\bar{b}_i^*(\theta_i)$ is decreasing in $\theta_i$.

3.3. The Second-Best Mechanism with Actual Balanced Budget Constraint

Under the mechanism described by Proposition 3, the expected bribes paid by the entrepreneur are equal to the expected value of the sum of the bribes that the $N$ bureaucrats receive. There is no guarantee that actual payments by the entrepreneur are equal to the sum of actual bribes received by the track of bureaucrats for each possible realization of $\theta$. Thus we must go further to find a mechanism that fulfils the requirement that actual payments by the entrepreneur are equal to the sum of actual bribes received by the track of bureaucrats. To accomplish this task, we create another game with a Bayesian Nash equilibrium that is the same as the equilibrium of the game with dominant strategies described in the preceding sub-section.$^5$

---

$^5$ The analysis used to find this mechanism draws on the technique developed by Bulow and Roberts (1989) for the bilateral trading model of Myerson and Satterthwaite (1983).
To this end, let us consider the following game with incomplete information. In this game, that we call \( \Gamma^a \), each agent is asked to reveal his type. If \( \theta = (\theta_0, \theta_1, \ldots, \theta_N) \) is the announced profile of types, then the arbitrator allows trade to take place with probability \( Q(\theta | \alpha^*) \), as in the game with dominant strategies analyzed in the preceding sub-section. However, in contrast with the dominant-strategy game, the following transfers are made, whether trade takes place or not:

For the entrepreneur, the transfer he receives in the new game is given by

\[
\sum_{i=1}^{N} b_{i}^{0}(\theta) = \sum_{i=1}^{N} \bar{b}_{i}^{0}(\theta) + \sum_{i=1}^{N} \int \bar{F}_{i}^{*}(\theta) \Phi_{0}(\theta_{0}) d\Phi_{0}(\theta_{0}).
\]

For bureaucrat \( i \), the transfer he receives is

\[
b_{i}^{\#}(\theta) = \bar{b}_{i}^{\#}(\theta) - \frac{1}{N} \left[ \bar{b}_{i}^{0}(\theta_{0}) - \int \bar{F}_{i}^{*}(\theta_{0}) \Phi_{0}(\theta_{0}) d\Phi_{0}(\theta_{0}) \right], \quad (i = 1, \ldots, N).
\]

Now if \( \theta = (\theta_0, \ldots, \theta_i, \ldots, \theta_N) \) is the true type profiles of the agents, and if bureaucrat \( i \) chooses to announce \( \theta_i^* \) as his type while all the other agents reveal their types truthfully, then his payoff in the game \( \Gamma^a \) will be

\[
-p \theta_i + (1-p) b_{i}^{\#}(\theta_0, \ldots, \theta_i^*, \ldots, \theta_N) + (1-p) \omega_i
\]

\[
= -p \theta_i + (1-p) \left[ \bar{b}_{i}^{*}(\theta_i) - \frac{1}{N} \left[ \bar{b}_{i}^{0}(\theta_{0}) - \int \bar{F}_{i}^{*}(\theta_{0}) \Phi_{0}(\theta_{0}) d\Phi_{0}(\theta_{0}) \right] \right] + (1-p) \omega_i,
\]

which yields the following expected payoff

\[
\int \left[ -p \theta_i + (1-p) \left[ \bar{b}_{i}^{*}(\theta_i) - \frac{1}{N} \left[ \bar{b}_{i}^{0}(\theta_{0}) - \int \bar{F}_{i}^{*}(\theta_{0}) \Phi_{0}(\theta_{0}) d\Phi_{0}(\theta_{0}) \right] \right] + (1-p) \omega_i \right] d\Phi_{0}(\theta_{0}) 
\leq -p \theta_i + (1-p) \bar{b}_{i}^{*}(\theta_i) + (1-p) \omega_i.
\]

Note that the expression on the second line of (59) is the expected payoff for this bureaucrat in the dominant-strategy game if he announces \( \theta_i^* \) as his type. Also, the
expression on the last line of (59) represents the expected payoff he receives in the
dominant-strategy game by revealing his type truthfully. Hence in the game \( \Gamma_i \) being
truthful – when all the other agents are truthful – is a best response for bureaucrat \( i \).

To show that in the game \( \Gamma_i \) being truthful is also a best response for the entrepreneur
– when all the other agents are truthful – let \( \theta' \) be the type he chooses to announce. If
\( Q(\theta', \theta_1, ..., \theta_N) = 0 \), then his payoff consists only of the transfer he receives, namely
\( b^g(\theta', \theta_1, ..., \theta_N) \). On the other hand, if \( Q(\theta', \theta_1, ..., \theta_N) = 1 \), then the arbitrator allows
trade to take place, and the payoff he obtain – in addition to the transfer he receives –
also includes the benefit yielded by the project if it comes to fruition without the bribery
act being detected. That is, the expected payoff obtained by the entrepreneur if he
announces \( \theta' \) as his type is given by

\[
\int_{\{\theta' \}} b^g(\theta', \theta_1, ..., \theta_N) d\Phi_{\theta_0}(\theta_1, ..., \theta_N) \\
+ \int_{\{\theta' \}} [(1 - p)\theta - b^g(\theta', \theta_1, ..., \theta_N)] d\Phi_{\theta_0}(\theta_1, ..., \theta_N)
\]

\( (60) \)

Note that the third line in (60) represents the expected net payoff in the
dominant-strategy game that the entrepreneur obtains if he announces \( \theta' \) as his type. If
he is truthful in the game \( \Gamma_i \), then his expected net payoff he obtains will be given by
the last line in (60), which is exactly the net expected payoff he receives in the
dominant-strategy game. We have just shown that the strategy profile in which each
agent reveals his type truthfully constitutes a Bayesian Nash equilibrium of the game $\Gamma^\theta$. Under this Bayesian Nash equilibrium, the sum of the transfers to all the agents – for any type profile $\theta$ – is given by

$$
\left. b^{\theta}_0(\theta) + \sum_{i=1}^N b^{\theta}_i(\theta) - \sum_{i=1}^N \bar{b}^{\theta}_i(\theta) \right. 
= \sum_{i=1}^N \left[ \int \bar{b}^{\theta}_i(\theta) d\Phi_i(\theta) - \int \bar{b}^{\theta}_0(\theta) d\Phi_0(\theta) \right] = 0.
$$

Note that the last equality in (61) follows from the fact that the expected budget is balanced in the dominant-strategy game. Hence in the game $\Gamma^\theta$, regardless of the profiles of types, the actual budget is always balanced. We summarize the results just obtained in the following proposition:

**Proposition 4:** In the game $\Gamma^\theta$, the strategy profile in which all agents – entrepreneur and bureaucrats – reveal their types truthfully constitutes a Bayesian Nash equilibrium. Under this Bayesian Nash equilibrium, the actual budget is balanced for each type profile announced. Furthermore, the probability that trade takes place is under the game $\Gamma^\theta$ is the same as the probability that trade takes place under the dominant-strategy game. Also, the expected payoffs of each agent are the same under both games. Finally, in contrast with the dominant-strategy game – in which no money changes hands if trade does not take place – transfers from the entrepreneur to the bureaucrats do take place even if trade does not occur.

A rather unpleasant feature of the game $\Gamma^\theta$ is that bribes are paid even when trade does not occur. Also, the equilibrium now is a Bayesian Nash equilibrium, not an equilibrium...
in dominant strategies. These are the prices we pay for achieving the actual balanced budget constraint, in addition to the participation constraint. In spite of this unpleasant feature, the payoff structure in the game $\Gamma^e$ does have some intuitive properties. First, note that the higher is the type of the entrepreneur, ceteris paribus, the higher is the total expected bribe payments he makes under the game $\Gamma^e$. Second, note that the higher is the type of a bureaucrat, ceteris paribus, the lower is the expected bribe he receives under the game $\Gamma^e$. Loosely speaking, these two results together imply that every other thing equal, the entrepreneur pays more when his type is higher, and a bureaucrat receives a higher bribe when his moral cost is lower.

4. CONCLUSION

In this paper, we formulate and analyze a model of entry regulation that involves an entrepreneur and a track of corruptible bureaucrats. The model is formulated under the framework of mechanism design, and is based on the work on bilateral trading under asymmetric information of Myerson and Satterthwaite (1983). The bargaining between the entrepreneur and the track of bureaucrats is modelled by introducing a fictitious arbitrator who asks each agent – entrepreneur and bureaucrats – to reveal his type, and makes the transfers according to the profile of types announced.

Under our original game settings, we discover that there is no social choice function that is Bayesian incentive compatible; satisfies the interim individual rationality constraints; and is ex post efficient. Next, we construct a second-best mechanism that maximizes the total gain from trade subject to the constraints that (i) it is Bayesian incentive
compatible; (ii) satisfies the interim individual rationality constraints; and satisfies the balanced budget constraint. The mechanism we find is truthfully implementable in dominant strategies and satisfies the interim individual rationality constraints. Under this second-best mechanism, no money changes hands if trade does not take place. However, the mechanism only satisfies the expected balanced budget constraint, not the actual balanced budget constraint. Now because bribery is an illegal act, it is not possible to achieve the actual balanced budget constraint by appealing to outside sources of financing, say a subsidy from a third party, and thus a mechanism that satisfies the actual balanced budget constraint must be found.

To solve the problem of actual balanced budget constraint, we create another mechanism that yields the same probability of trade and the same expected payoff for each agent as the dominant-strategy game. The payoffs structured under the new mechanism we find is such that (i) each agent finds it in his interest to reveal his type truthfully, if all the other agents choose to reveal their types truthfully; (ii) each agent is willing to participate in the mechanism; and (iii) the actual bribe payments by the entrepreneur are equal to the actual total bribes receipts. The actual balanced budget constraint has been obtained by weakening the mechanism that is truthfully implementable in dominant strategies. The equilibrium in the new mechanism is now only a Bayesian Nash equilibrium, not an equilibrium in dominant strategies. In the new second-best the entrepreneur has to pay even if trade does not take place.

The mechanism we propose does not touch on the possibility of the breach of commitments; that is, the entrepreneur pays the bribes, but does not get the permit from
the track of bureaucrats, since the commitments constitute an implicit contract that has no enforcement power. We believe that our model can be used to analyze problems of entry regulation in developing countries. Our model can be extended in several directions. First, the model can be extended to include more than one entrepreneur competing for the same project. In this case, the game will be a tournament bidding game. It would be the entrepreneur that has the highest willingness to pay for entrance win the game. In that sense, the bribery bidding is more intense than that of our model. Second, our model can be extended into a dynamic setting – as a repeated game – with a sequence of entrepreneurs applying to the same track of bureaucrats for a permit. These are the subjects for our future research.

APPENDIX 1

PROOF OF CLAIM 1: Let $\theta_0$ be the real type of the entrepreneur. If the entrepreneur reveals his type truthfully, then his payoff can be computed as follows. Let $(\tilde{\theta}_1, ..., \tilde{\theta}_N)$ be the announced profile of types of the bureaucrats. If $Q(\theta_0, \tilde{\theta}_1, ..., \tilde{\theta}_N | \alpha^*) = 0$, then his payoff will be 0 because trade does not occur. On the other hand, if $Q(\theta_0, \tilde{\theta}_1, ..., \tilde{\theta}_N | \alpha^*) = 1$, then his payoff will be $\theta_0 + b_0^*(\theta_0, \tilde{\theta}_1, ..., \tilde{\theta}_N) \geq 0$. Thus revealing his type truthfully will yield the entrepreneur a non-negative net payoff, regardless of the announced profile of types, and this means that for the entrepreneur truth telling satisfies the interim rationality constraint.

To show that truth telling is weakly dominant for the entrepreneur, suppose that he understates his type by announcing his type to be $\theta'_0 < \theta_0$. If $Q(\theta_0, \tilde{\theta}_1, ..., \tilde{\theta}_N | \alpha^*) = 0$,......
then we still have \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 0 \), and trade still does not take place: understating his type does not improve his payoff under this scenario. If \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 1 \), then as long as the understatement is not substantial, we still have \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 1 \), and the transfer is still given by \( b_0^*(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N) \), yielding the same net payoff \( \theta_0 + b_0^*(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N) \geq 0 \) that he can obtain by telling the truth. On the other hand, if the understatement is substantial, we will have \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 0 \), which implies that trade will not take place, with the ensuing zero net payoff for the entrepreneur. We have just shown that relative to truth telling, understating his type does not improve the net payoff for the entrepreneur.

Could the entrepreneur improve his net payoff by overstating his type? To answer this question, suppose that he announces a type \( \theta'_0 > \theta_0 \). Consider first the scenario \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 0 \). If the overstatement is not substantial, then we still have \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 0 \), and trade still does not take place: overstating his type does not improve his payoff under this scenario. On the other hand, if the overstatement is excessive, we will have \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 1 \), and trade will take place. However, in this case, the transfer to the entrepreneur will be \( b_0^*(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N) \) – with \( \theta_0 < -b_0^*(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N) \), according to (68) – yielding the entrepreneur a net payoff of \( \theta_0 + b_0^*(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N) < 0 \). Next, consider the scenario \( Q(\theta_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N | \alpha^*) = 1 \). Under this scenario, trade will take place if the entrepreneur tells the truth, and continues to
take place if the entrepreneur overstates his type. Therefore, relative to truth telling, overstating does not improve the net payoff of the entrepreneur.

APPENDIX 2

PROOF OF CLAIM 2: Consider a bureaucrat, say , whose type is . If the bureaucrat reveals his type truthfully, then his payoff can be computed as follows. Let be the announced profile of types of the other agents. If , then his payoff will be because trade does not occur. On the other hand, if , then the transfer that he receives will be , which, according to (69), satisfies the following inequality:

To show that truth telling is weakly dominant for the bureaucrat, suppose that he overstates his type by announcing his type to be . If , then we still have , and trade still does not take place: overstating his type does not improve his payoff under this scenario. If , then as long as the overstatement is not substantial, we still have , and the transfer is still given by , yielding the same net payoff , that he can obtain by telling the truth. On the other hand, if the overstatement is substantial, we will have , which implies that trade will not take place, with the
ensuing net payoff of $\omega_l$ for the bureaucrat. We have just shown that relative to truth telling, over stating his type does not improve the net payoff for the bureaucrat.

Could the bureaucrat improve his net payoff by understating his type? To answer this question, suppose that he announces a type $\hat{\theta}_i < \theta_i$. Under the scenario $Q(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N | \alpha^*) = 0$, if the understatement is not substantial, then we still have $Q(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N | \alpha^*) = 0$, and trade still does not take place: understating his type does not improve the bureaucrat’s payoff under this scenario. On the other hand, if the understatement is substantial, then we will have $Q(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N | \alpha^*) = 1$, and trade will take place. However, in this case, the transfer to the bureaucrat will be $b_i^*\left(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N \right)$ which satisfies $-p\theta_i + (1-p)(\omega_i + b_i^*\left(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N \right)) = \omega_i$. His net payoff of is then given by

$$-p\theta_i + (1-p)(\omega_i + b_i^*\left(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N \right)) < -p\theta_i + (1-p)(\omega_i + b_i^*\left(\theta_i, (\tilde{\theta})_{j=0,j \neq i}^N \right)) = \omega_i.$$

Thus, relative to truth telling, understating his type does not improve the net payoff of the bureaucrat, either.

\[\blacksquare\]
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