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Measuring the Intertemporal Elasticity of Substitution in Japan*

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Abstract

We employ the C-CAPM (Consumption-Capital Asset Pricing Model) on measuring the Intertemporal Elasticity of Substitution (IES) after the collapse of bubble economy in Japan. When we use "nondurable goods and services (Benchmark year is 2000)" as the consumption data and long term government bond as the asset, we find that the Hansen's J-test does not reject the model and the obtained values of the IES are in the range from 0.2531 to 0.3857.

JEL classification: D91; E21; E44; G12
Keywords: Intertemporal Elasticity of Substitution; Generalized Method of Moments; Consumption-CAPM

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1 Introduction

In general, a household changes the intertemporal consumption allocation when the real asset return changes. This relationship can be explained by the intertemporal elasticity of substitution (IES). Therefore, measuring the representative consumer's IES is one of the most important problems. In particular, we must specify the value of IES when we attempt to evaluate the effect of policies on consumer welfare.

Measuring the IES with Generalized Method of Moments (GMM) is often based on consumption-based capital asset pricing model (C-CAPM). However, it is well known that the standard C-CAPM with constant relative risk aversion (CRRA) utility function fails empirically in U.S. As a result, many studies attempt to improve the empirical performance of C-CAPM. For example, Epstein and Zin (1991) apply the type of Kreps-Porteus utility function to estimate the IES, Ferson and Constantinides (1991) use habit persistence and durability when they attempt to estimate the IES, and Ogaki and Reinhart (1998) use a cointegration approach to estimate the IES.

On the other hand, there is no consensus over the validity of standard C-CAPM in Japan. For example, Hamori (1992a,b, 1996) concludes that empirical results using Generalized Method of Moments (GMM) estimation support standard C-CAPM, while Nakano and Saito (1998) reject standard C-CAPM from a point of view that it can not explain the actual variation of asset prices in Japan. There is, however, a serious problem of mismatch between data and model in their study. Although their consumption data is measured in terms of "total", "nondurable goods and services" should be used. This is because they assume the model with time separable preference.

Baba (2000), in contrast, uses "nondurable goods and services" as consumption data which is in the range from 1980Q3 to 1998Q4 when he measures the IES by using standard C-CAPM and habit formation model. He concludes that standard C-CAPM performs as well as Hamori (1992a,b, 1996) and obtains about 1.4 as the value of IES. However, it seems that it is against our intuition. This is because it is well known that there is the strong liquidity constraint in the Japanese financial market. Therefore, Japanese household tend to prolong the borrowing period through their life-cycle. It means that Japanese household prefer to obtain fixed level of utility regardless of their income pattern and therefore IES should be (strictly) smaller than 1.

In order to improve the measured value of IES in standard C-CAPM, we keep the following in our mind. First, we should take into account the fact that the Japanese government bonds market is not liberalized until October 19, 1985. Therefore, we should only use data after the liberalization, provided we use the long-term government bonds as the asset. Secondly, we should only use consumption data after the collapse of bubble economy. This is because Japanese household might have drastically changed their preference before as well as after the collapse of bubble economy. Then we can use "nondurable goods and services (Benchmark year is 2000)" as the consumption data which is in the range from 1994Q2 to 2006Q3 when we attempt to measure the IES.

In section 2, we describe the model and empirical method. In Section 3, we explain the data that this study uses. In Section 4, we show the empirical result. We conclude in Section 5.

2 Model and Empirical Method

In this section, we explain the model and empirical method. The representative consumer at time 0 chooses his/her life-time consumption and asset holding to maximize the expected utility subject to the budget constraint. The consumer's optimization problem is summarized as follows:

$$\text{Max } E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\gamma}-1}{1-\gamma}\right], \quad 0<\beta<1, \quad 0<\gamma, \tag{1}$$

$$\text{s.t. } C_t+\sum_{i=1}^{N}p_{it}A_{it}=\sum_{i=1}^{N}[p_{it}+d_{it}]A_{it-1}+Y_t, \quad i=1,2,\dots N, \tag{2}$$

where C_t is the real per capita consumption at time t, p_{it} is the price of the ith asset at time t, d_{it} is the dividend of the ith asset at time t, A_{it} is the amount of the ith per capita asset holding at time t, Y_t is the real per capita labor income at time t, β is the subjective discount rate, γ is the relative risk aversion (RRA), and $E_t[\cdot]$ is the expectation operator conditional on information available at time t. In equation (1), we assume that utility function is of constant relative risk aversion (CRRA) class. In CRRA utility functions, the IES is the reciprocal of the coefficient of RRA.

By solving the above utility maximization problem, we can derive the following Euler equation:

$$E_t[\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1+r_{it+1})-1]=0, \quad i=1,2,\dots N, \tag{3}$$

where r_{it+1} is the real return of asset at time $t+1$, which is defined as

$$r_{it+1}=\frac{p_{it+1}+d_{it+1}}{p_{it}}-1, \quad i=1,2,\dots N.$$

In order to estimate the parameters in the Euler equation, we adopt GMM estimator by Hansen (1982). Define the error term vector $\mathbf{u}_{t+1}(\theta)$ as

$$\mathbf{u}_{t+1}(\theta)=\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(\mathbf{1}+\mathbf{r}_{t+1})-\mathbf{1},$$

where $\theta=(\beta,\gamma)'$, $\mathbf{r}_{t+1}=(r_{1t+1},\dots,r_{Nt+1})'$ and $\mathbf{1}$ is an $N\times 1$ unit vector.

Let $\mathbf{z}_t$ be an $R\times 1$ vector of instrumental variables known at time t. Define an $N\times R$ vector $\mathbf{g}_t(\theta)$ as

$$\mathbf{g}_t(\theta)=\mathbf{u}_{t+1}(\theta)\otimes\mathbf{z}_t, \tag{4}$$

where $\otimes$ is the Kronecker Product. Then, the Euler equation implies

$$E[\mathbf{g}_t(\theta)]=\mathbf{0}, \tag{5}$$

where $E[\cdot]$ is the unconditional expectations operator. Moreover, if we define $\overline{\mathbf{g}_T(\theta)}$ as

$$\overline{\mathbf{g}_T(\theta)}\equiv\frac{1}{T}\sum_{t=1}^{T}\mathbf{g}_t(\theta), \tag{6}$$

where T is the sample size, then the GMM estimator of θ, $\hat{\theta}_{GMM}$, minimizes the quadratic form:

$$\hat{\theta}_{GMM} = \arg\min_{\theta} \overline{\mathbf{g}_T(\theta)}' \mathbf{W}_T \overline{\mathbf{g}_T(\theta)}, \tag{7}$$

where $\mathbf{W}_T$ is an $(N \times R) \times (N \times R)$ weighting matrix, which is supposed to be postive definite for any finite T.

We can obtain the most efficient GMM estimator by choosing the weighting matrix $\mathbf{W}_T = \mathbf{S}^{-1}$, where $\mathbf{S}^{-1}$ is the inverse of the asymptotic covariance matrix of $T^{1/2}\overline{\mathbf{g}_T(\theta)}$. However, because we cannot observe the true value of θ, we cannot know $\mathbf{S}^{-1}$ neither. Therefore, we need to adopt two-step GMM (See, for example, Hayashi (2000)). In estimation of $\mathbf{S}$, we use the estimator proposed by Newey and West (1987). As the lag selection criteria of Newey-West estimator, we use $int(4(T/100)^{2/9})$, where $int(\cdot)$ is the integer part of the argument, following Newey and West (1994).

Finally, in order to analyze the goodness-of-fit of the model, we adopt Hansen's J test (overidentifying restrictions test) developed by Hansen (1982). Under the null hypothesis that equation (5) is true, T times the minimized value of equation (7) is asymptotically distributed as $\chi^2_{N\times R-K}$, where K is the number of parameters (i.e. 2 in our case).

3 Data

We construct 3 datasets, "Dataset 1", "Dataset 2" and "Dataset 3" in this paper. Table 1 shows the description of the three datasets.

(Table 1 around here)

The description of the "Dataset 1" in detail is as follows. We consider the following relatively risk-free asset as the asset in equation (2): the return of long-term government bond prepared by Ibbotson Associates. For a per capita and seasonally adjusted consumption, we use "nondurable goods and services (Benchmark year is 2000)" divided by population which is reported in the *Annual Report on National Accounts* in Japan. We use the consumer's price index (CPI) which is published in the *Annual Report on the Consumer's Price Index.* As instrumental variables, we use lagged variables of the return of risk-free asset data, the consumption growth data, and the growth rate of CPI. The asset return and the consumption series are deflated by CPI. Sample period is from 1994Q2 to 2006Q3 and the frequency is quarterly.

The description of the "Dataset 2" in detail is as follows. "Dataset 2" is exactly the same with "Dataset 1" except that "Dataset 2" uses the return of stock in the Tokyo stock exchange (first section) as relatively risky asset instead of the risk-free asset in "Dataset 1", which is prepared by Ibbotson Associates. Sample period is same as "Dataset 1".

The description of the "Dataset 3" in detail is as follows. "Dataset 3" is exactly the same with "Dataset 1" and "Dataset 2" except that "Dataset 3" uses the return of long-term government bond and stock in the Tokyo stock and exchange (first section) as the assets, which is prepared by Ibbotson Associates. Sample period is same as "Dataset 1" and "Dataset 2".

In GMM, all variables that appear in the moment condition equation should satisfy stationarity. To see if the variables satisfy stationarity, we use Augmented Dickey Fuller (ADF) test. Table 2 shows descriptive statistics and ADF test statistics of our datasets.

(Table 2 around here)

It is found that ADF test rejects the null that the data contain unit roots at a conventional significant level.

4 Empirical Result

Table 3 shows the empirical results for three datasets using “nondurable goods and services” [1].

(Table 3 around here)

Panels “Dataset 1” shows that β and γ are estimated statistically significant at a conventional level when we only use “LGB (Long-term Government Bonds)” as the asset. The estimated values of β are in the range from 0.9923 to 0.9972, which is economically realistic; the estimated values of γ are in the range of 2.5924 to 3.9519, implying IES being in the range from 0.2531 to 0.3857. Moreover, p values are large enough that we cannot reject the null that the moment condition holds.

Our study shows that the estimated value of IES (RRA) appears to be smaller (larger) than that estimated by earlier studies. We can consider that this is because the uncertainty of Japanese economy is risen by long depression through 1990s, a lost decade. Actually, many studies show it (see, for example, Hayashi and Prescott (2002)). And it also means that Japanese representative household may prefer more to obtain fixed level of utility regardless of their income pattern than before.

On the other hand, Panel “Dataset 2” and “Dataset 3” show the result when we use “TSE1 (Tokyo Stock Exchange (First Section))” or “LGB and TSE1” as the asset(s). We can see that (i) p values are large enough that we cannot reject the null that the moment condition holds, (ii) the estimated value of β are larger than one for the most case, (iii) the estimated value of γ unnaturally high or negative for the most case (Recall that RRA being near 10 implies that the representative household is too risk aversive). It means that Japanese representative household does not respond much to the variation of “TSE1”. Therefore, we can consider that the most part of the Japanese household does not hold the risky asset but the most part of their assets is risk-free asset. In practice, many studies point out that Japanese household tend to prefer risk-free asset to risky asset (see, for example, Hayashi (1997)).

Thus, we insist that the C-CAPM performs empirically well when we only use “LGB (Long-term Government Bonds)” as the asset; the estimated value of β and γ is valid which is assumed by economic theory.

5 Conclusion

This paper has examined the Intertemporal Elasticity of Substitution (IES) after the collapse of bubble economy in Japan. At the same time, we only use asset return after

[1] The reported estimation results are based on the following initial values for parameters: $\beta = 0.99$, $\gamma = 1.5$. We have confirmed that our estimation results are robust in the change of the initial values.

the liberalization of Japanese financial market when we measure the value of IES. Then we have only specified "LGB (Long-term Government Bonds)" as the asset that appear in the C-CAPM and estimated the Euler equation by GMM. As a result, we find that the model performs well when we use the datasets with sample period from 1994Q2 to 2006Q3 and the estimated value of RRA was in the range of 2.5924 to 3.9519, implying that IES being in the range from 0.2531 to 0.3857.

Moreover, the estimated value of IES (RRA) appears to be smaller (larger) than that estimated by earlier studies. Then we can show that Japanese household drastically changes their preference after the collapse of bubble economy. Therefore, it indicate that Japanese representative household may prefer more to obtain fixed level of utility regardless of their income pattern than before.

We can also conclude that the measured IES is economically more valid, unlike the earlier studies. This is because our obtains value is similar to that obtained in U.S. For example, Ogaki and Reinhart (1998) conclude that the estimated value of IES is in the range from 0.329 to 0.447. Moreover, our empirical results justify the value of IES which is used when we attempt to evaluate the effect of policies on consumer welfare (See, for example, Altig et al. (2001) and Okamoto (2005)).

However, it is still of interest to consider more general model. This is because a household changes intertemporal consumption and leisure allocation when the real asset return changes. This remains to be our future research.

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Table 1: Description of Database

	Consumption	Inflation Rate	Asset Return	Sample Period	Sample Size
Dataset 1	ND+S	CPI	LGB	94/2-06/3	50
Dataset 2	ND+S	CPI	TSE1	94/2-06/3	50
Dataset 3	ND+S	CPI	LGB/TSE1	94/2-06/3	50

ND+S =Nondurable plus Services.
CPI =Consumer Price Index.
LGB =Rate of Return for Long-term Government Bonds.
TSE1 =Rate of Return for Stock in Tokyo Stock Exchange (First Section).

Table 2: Description of Basic Statistics

Dataset 1 (94/2-06/3, Sample size=50, Consumption : ND+S)						
	Mean	SD	Max	Min	ADF	Lag length
CG_t	1.0018	0.0080	1.0194	0.9738	-7.9203***	2
R_{1t}	0.0111	0.0301	0.0801	-0.1052	-7.8337***	1
π_t	1.0001	0.0052	1.0216	0.9899	-9.0947***	2
Dataset 2 (94/2-06/3, Sample size=50, Consumption : ND+S)						
	Mean	SD	Max	Min	ADF	Lag length
CG_t	1.0018	0.0080	1.0194	0.9738	-7.9203***	2
R_{2t}	0.0079	0.1012	0.2060	-0.2075	-8.9430***	1
π_t	1.0001	0.0052	1.0216	0.9899	-9.0947***	2
Dataset 3 (94/2-06/3, Sample size=50, Consumption : ND+S)						
	Mean	SD	Max	Min	ADF	Lag length
CG_t	1.0018	0.0080	1.0194	0.9738	-7.9203***	2
R_{1t}	0.0111	0.0301	0.0801	-0.1052	-7.8337***	1
R_{2t}	0.0079	0.1012	0.2060	-0.2075	-8.9430***	1
π_t	1.0001	0.0052	1.0216	0.9899	-9.0947***	2

CG_t denotes gross real per capita consumption growth C_t/C_{t-1}, R_{1t} denotes net risk free asset return based on term, R_{2t} denotes net risk asset return based on term, and π_t denotes growth rate of CPI. "SD" denotes standard deviation. "ADF" denotes ADF test statistics. We assume a model with time trend and constant in ADF test. *** indicates that the null hypothesis that each variable has a unit root is rejected at 1% significant level. Lag length indicates optimal lag length for ADF test statistics by the Schwarz criterion. The statistical softwares that we use to compute ADF test statistics and others are Eviews version 6 and TSP version 5.0, respectively.

Table 3: Empirical Results of GMM

Dataset 1 (94/2-06/3, Sample size=50, Rate of Return : LGB)							
Lag	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	$\hat{\gamma}$	$\widehat{SE}(\hat{\gamma})$	J-Statistic	p-value	DF
1	0.9941	0.0055	3.1264	1.4912	0.0341	0.9831	2
2	0.9972	0.0059	3.9519	1.3260	5.1940	0.3927	5
3	0.9924	0.0041	2.5924	1.0751	7.2800	0.5067	8
4	0.9923	0.0030	3.1330	0.7128	7.3258	0.7721	11
Dataset 2 (94/2-06/3, Sample size=50, Rate of Return : Stock)							
Lag	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	$\hat{\gamma}$	$\widehat{SE}(\hat{\gamma})$	J-Statistic	p-value	DF
1	1.0050	0.0170	9.3350	5.7995	4.2791	0.1177	2
2	1.0046	0.0156	9.4049	4.5699	6.2476	0.2829	5
3	1.0014	0.0156	8.7060	3.5709	8.0180	0.4317	8
4	0.9949	0.0120	6.4412	2.5336	9.2791	0.5961	11
Dataset 3 (94/2-06/3, Sample size=50, Rate of Return : LGB and Stock)							
Lag	$\hat{\beta}$	$\widehat{SE}(\hat{\beta})$	$\hat{\gamma}$	$\widehat{SE}(\hat{\gamma})$	J-Statistic	p-value	DF
1	1.0065	0.0087	9.8487	2.6833	7.7629	0.4570	8
2	1.0043	0.0086	11.5073	2.0129	9.7009	0.8818	16
3	1.0079	0.0075	13.1509	1.5456	10.9541	0.9893	24
4	0.9851	0.0008	-2.0358	0.1726	11.5734	0.9997	32

$\hat{\beta}$ denotes the estimated value of subjective discount rate, $\hat{\gamma}$ denotes the estimated value of relative risk aversion (RRA), "$\widehat{SE}(\cdot)$" denotes the Newey-West adjusted standard error of $\hat{\beta}$ or $\hat{\gamma}$, "J-Statistic" denotes overidentifying restriction test statistic by Hansen (1982), and "DF" denotes the Degree of Freedom. "Lag" denotes the lag of instrument variables used. For example, Lag=2 denotes that the following variables are used: constant term, one-lagged variables of consumption growth, return rate on the asset(s), and inflation rate; and two-lagged variables of consumption growth, return rate on the asset(s), and inflation rate. The statistical softwares that we use to implement GMM estimation is R version 2.6.1. In this study, we assume that utility function is of constant relative risk aversion (CRRA) class. Therefore, the intertemporal elasticity of substitution (IES) is the reciprocal of the coefficient of relative risk aversion (RRA).