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This paper uses a variety expansion model of endogenous growth to examine the effect of intellectual property rights (IPR) protection on economic growth in a closed economy. Most studies in the literature show that, in closed economies, enhancing the protection of IPR increases the expected duration of monopoly and the associated incentive to innovate. A large incentive to innovate enhances the growth rate. However, allowing for technological sophistication that is driven by the cumulative experience in producing a final good, enhanced protection can have a negative effect on growth by increasing the share of monopolized sectors. Because the scale of production falls as a result of monopoly pricing, the experience accumulation declines with stronger protection of IPR. As a result, stronger IPR decreases the productivity of the final sector, the associated demand for innovation, and economic growth. This paper shows that, if the latter dominates the former, IPR protection is not growth enhancing.

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1 Introduction

In recent decades, the broader issue of intellectual property rights (IPR) protection has been at the center of public policy debates. A major argument in favor of stronger IPR is that it stimulates economic growth by protecting innovators from imitation, thereby encouraging innovation. In fact, many countries have strengthened their protection of IPR by reforming their patent systems. However, although this view is widely accepted, recent work, both theoretical and empirical, indicates that, in a closed economy, the relationship between IPR protection and economic growth is actually not so clear. [1]

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[1] See Gould and Gruden (1996), Koléda (2004), and Horii and Iwaisako (2005).

This paper shows, in line with the recent work, that IPR protection may not enhance economic growth in an endogenous growth model with costless imitation: that is, "stronger is always better" is incorrect.

The crucial assumption in the model is that the accumulation of experience gained by using intermediate machinery improves the productivity of final good firms that make use of this machinery as an input, through the *learning by experience* effect.[2] Although the literature has studied the effects of IPR protection on economic growth and the economic implications of learning by experience separately,[3] this paper finds an important link between them as described in the following.

A common perception is that tightening the protection of IPR increases the expected duration of monopoly and the associated incentive to innovate. A large incentive to innovate enhances the growth rate. However, allowing for technological sophistication that is driven by the cumulative experience of using machinery, tightened protection can have a negative effect on growth. Because tightening IPR increases the proportion of monopolized sectors and monopolistic pricing reduces the level of production, the accumulation of experience is reduced, producing a decline in final sector productivity. Less productive final output firms imply smaller demand for intermediate machinery because intermediates are bought only by final firms in the model presented below. Finally, this reduced demand in turn weakens the incentive to innovate new machinery as a source of economic growth. If the latter dominates the former, IPR protection is not growth enhancing. More specifically, the long-run rate of innovation has an inverse-U shape as a function of the rate of imitation, which is an inverse measure of IPR protection, when the intensity of accumulated experience in the final sector is sufficiently high. The conclusion of the present paper emphasizes a role for relaxing IPR protection as a growth-enhancing policy.

The issues of IPR protection often, as mentioned above, are raised in current discussions on development of countries around the world. Consequently, the result of this paper can help to understand the pros and cons of IPR and inform policy discussions about IPR:[4] the inverted U-shaped relationship between innovation and IPR protection suggests that too strong as well as too weak protection hurts the incentive to innovate: rather a balanced approach is required for innovation and growth.

The result presented here is related to Aghion et al. (2005) who find strong evidence of an inverted-U relationship between competition and innovation by using panel data. Since a strengthening of IPR implies reduced competition in the current model, the finding of the current paper is consistent with the evidence by Aghion et al. (2005).

[2]Some of the literature uses the term "learning by doing." Although, in general, these terms are used almost interchangeably, here I adopt the term "learning by experience" to emphasize the accumulation of *experience*.

[3]The learning by experience is only one departure from the existing literature on IPR protection and growth: the model presented below differs from that of Kwan and Lai (2003) essentially in that the learning by experience is assumed, and the Kwan and Lai model had differed from the "lab-equipment" model used by Rivera-Batiz and Romer (1991) only in assuming costless imitation.

[4]The result of this paper would not contribute to understanding the interregional impact of a strengthening of IPR protection across the world, since the economy presented below is closed. Rather, this closed economy model might be a good benchmark for the US economy. Hence, the paper would inform discussions on the changes in the US legal environment to strengthen IPR protection during the 1980s, as reflected, for example, in the establishment of a centralized appellate court, the Court of Appeals for the Federal Circuit (CAFC) in 1982.

Also related is recent empirical literature on institutions and growth, which investigates the relationship between initial institutional quality and growth rates (Hall and Jones 1999, Acemoglu et al. 2001).

Many papers have examined the effects of IPR protection on economic growth and social welfare in endogenous growth models. Helpman (1993) investigates the effects of IPR protection on welfare and growth by allowing for exogenous imitation in a variety expansion model, with an innovative country and an imitative country. Helpman shows that strengthening IPR in an imitative country need not stimulate innovations in innovative country in the long run.[5]

Following Helpman (1993), there have been many studies of how IPR protection affects economic growth in *closed economy models*. Received wisdom in the existing literature is that in a closed economy model, tightening IPR necessarily enhances innovation and economic growth. For example, Kwan and Lai (2003) incorporate the exogenous imitation rate into a lab-equipment version of variety expansion models to examine how IPR protection affects welfare and growth. Iwaisako and Futagami (2003) show that extending patent length enhances economic growth in the variety expansion model of Romer (1990). These closed economy models conclude that strengthening IPR always enhances economic growth.

However, as I have already mentioned, most recent papers have indicated that the protection of IPR does not necessarily enhance growth *even* in a closed economy model, as in the current paper. Horii and Iwaisako (2005) use the average growth rate from 1966 to 2000 to indicate that it is difficult to find a positive relationship between IPR protection and the growth rate, and Gould and Gruden (1996) show a positive but "weak" relationship between them. To explain this fact, Horii and Iwaisako (2005) construct a quality ladder model where strengthened protection can depress the incentive to innovate. Koléda (2004) shows that the effect of patent novelty requirements on growth can be inverse U-shaped, which implies that tightening the IPR protection dampens economic growth for a range of stronger novelty requirements. The current paper presents a mechanism through which stronger IPR protection depresses economic growth by focusing on learning by experience, which has not been stressed in the literature on IPR and growth.

The present model is close in spirit to that of Romer (1986), the seminal paper in the endogenous growth literature. Romer (1986) assumes that knowledge creation is a by-product of investment, using Arrow's (1962) setup. The common feature is that technological sophistication arises from the externalities resulting from learning by experience from using intermediate products (machines). Whereas Romer (1986) notes temporary experience, the present model focuses on *cumulative* experience, as in most models in the literature on learning by experience (e.g., Arrow 1962, Fudenberg and Tirole 1983, Dasgupta and Stiglitz 1988). Specifically, I assume that final sector productivity increases with a weighted sum of the amount of intermediate goods used up to the present period. The formulation used here is much closer to Arrow's (1962) original setup of learning by doing, which assumes that production cost decreases with *cumulative* gross investment up to the present.

[5]Although his result is the same as that found by the current paper, it depends on assuming that there is no migration between the two regions, whereas the current paper constructs a closed economy model, so that people can freely migrate.

Also related is a paper by Bessen and Maskin (2000), which shows that patent protection may reduce overall innovation and welfare if innovation is sequential and complementary.[6] Although the implications in Bessen and Maskin's paper are similar to the one presented here, their purpose is different from mine in that they are not interested in the effects of strengthening IPR on the macroeconomy.

Other contributions of this paper are associated with transitional dynamics. One is related to the temporary effects of stronger IPR on the transitional path: a strengthening of IPR increases the growth rate of consumption temporarily, because of the increased expected duration of monopoly. Therefore, in the model, the effects of IPR in the long run and in the short run can be quite opposite. A strengthening of IPR raises the rate of consumption growth initially, but it may eventually fall below the previous level. It is also pointed out that, in this case, IPR policy changes can destabilize the economy. Another contribution is that the transitional path can be oscillatory due to interactions between innovation and the accumulation of experience. The fraction of monopolized sectors increases if the rate of innovation is high enough to exceed the exogenous rate of imitation. Applying the same logic mentioned above, this depresses the accumulation of experience and productivity growth in the final sector, leading to a fall in demand for innovation. As a consequence, the rate of innovation declines gradually when it is at higher levels.

The rest of this paper is organized as follows. The next section presents a basic framework. Section 3 investigates determinants of the long-run growth, and Section 4 verifies the properties of the transitional dynamics. Section 5 concludes.

2 The Basic Framework

The basic framework differs from the "lab-equipment" model of Rivera-Batiz and Romer (1991) only in the following two aspects. First, to analyze the effects of patent policy on growth, I assume the existence of costless imitation. Second, I allow for the accumulation of experience from using intermediate machines, and assume that the accumulated experience has a positive external effect on the productivity of the final sector. As I already mentioned, the present model differs from Kwan and Lai (2003) only in that I allow for this learning-by-experience process.

Time is continuous and extends from zero to infinity. Households supply N units of labor inelastically and consume the single final good, which is taken as the numeraire. The single final good, which can be used for consumption, for production of intermediate machines, and for R&D needed to invent new types of intermediates, is produced by competitive firms.

2.1 Final Production

Competitive firms share the identical constant returns to scale production function,

$$Y_t = A_t^{\beta} N^{1-\alpha} \int_0^{n_t} x_t(j)^{\alpha} dj \, , \quad \alpha \in (0,1) \, , \tag{1}$$

[6] See, for example, Scotchmer (1996) for theoretical implications of sequential innovations.

where Y_t is the quantity of final good, A_t is the productivity level, which increases with the accumulation of experience gained from using intermediates,[7] $x_t(j)$ is the variety of perishable, differentiated intermediate machines indexed by j used, and n_t is the number of varieties. α is the share of intermediate machines and $\beta > 0$ is a parameter that reflects the contribution of A_t on productivity. As mentioned above, final goods include consumption and capital goods: that is, the goods can be consumed, used for the production of intermediates, and used for R&D. At every moment a subset of differentiated intermediates, $[0, n_t]$, is available in the economy. Initially, the economy inherits a given size of available varieties, n_0, and expands over time through the entry process into R&D activity.

As in Romer (1990), the market for intermediate goods is monopolistically competitive. Final good firms buy these intermediates from the innovators, each of which invents a blueprint of a new good and supplies it exclusively. However, departing from the Romer model, while I will allow for the imitation process, the monopoly power can disappear in this model.

2.2 Imitation Process and the Protection of IPR

To analyze a tightening of IPR, I introduce the costless imitation process into the model, and, following Helpman (1993), I specify the process as follows.

$$\dot{n_t^c} = \mu (n_t - n_t^c) , \quad \mu > 0 , \tag{2}$$

where n_t^c is the number of intermediates that have already been imitated. I assume that once an intermediate is imitated, competition pushes the price down to marginal cost. Therefore, while an imitated intermediate with index $j,\ j \in [0, n_t^c]$ is supplied by competitive firms, an intermediate with index $j,\ j \in (n_t^c, n_t]$, which has not been imitated yet, is supplied by a single monopolistic firm.

Because μ denotes the hazard rate at which every product that has not been imitated yet is imitated at the next date, we can interpret this model's strengthening of the protection of IPR as a decline of μ, as in the model of Helpman (1993). The tighter are legal protection and its administrative enforcement, the slower is the process of imitation.[8] This way of specifying IPR protection has, of course, a critical limitation because it does not capture how patent policy affects the rate of imitation (the hazard rate of being imitated) or how to design patent policy. Nevertheless, because it is the simplest way, the associated tractability enables us to clarify the effects of tightening IPR on macroeconomic issues, such as the determinants of economic growth in a dynamic general equilibrium model.

Without the protection of IPR, the rate of imitation is still restricted by the fundamentals, technologies, systems, and institutions of the economy. It should therefore be assumed that the imitation rate has an upper bound: $\mu \leq \mu_f$ where μ_f represents the potential (or natural) rate of imitation, and is a constant exogenously determined by the factors listed above. No IPR protection implies that $\mu = \mu_f$, and full IPR protection implies $\mu = 0$, while μ is an inverse measure of IPR protection.

[7] Detailed specification appears later.

[8] A strengthening of IPR comes from the patent policy concerning patent length and breadth, copyright policy, and the monitoring and enforcement of the rights guaranteed by these policies.

2.3 Intermediate Sector

The imitation process introduced in the previous section divides intermediate machines into two types: the imitated, $[0, n_t^c]$, and the monopolized, $(n_t^c, n_t]$. The imitated machines are competitively priced, whereas the monopolized ones have monopolistic pricing. With the Cobb–Douglas specification, demand for each intermediate has constant price elasticity, $1/(1-\alpha)$. Producing a unit of each intermediate requires a unit of final good (the one-for-one technology) for all intermediate producers, so that the marginal cost is constant and unity. Hence, each intermediate good producer hence sets the price equal to

$$p_t(j) = \begin{cases} 1 & j \in [0, n_t^c] \\ \frac{1}{\alpha} & \in [n_t^c, n_t] \end{cases} . \tag{3}$$

The demand functions for the two types are

$$x_t(j) = \begin{cases} x_t^c = A_t^{\frac{\beta}{1-\alpha}} N \alpha^{\frac{1}{1-\alpha}} \ , & j \in [0, n_t^c] \\ x_t^m = A_t^{\frac{\beta}{1-\alpha}} N \alpha^{\frac{2}{1-\alpha}} \ , & j \in (n_t^c, n] \end{cases} . \tag{4}$$

Note that $x_t^c > x_t^m$ always holds, reflecting the standard distortion of monopoly. Let the aggregate demand for intermediates be

$$X_t \equiv \int_0^n x_t(j) dj = n_t x_t^c \left[\frac{n_t^c}{n_t} (1 - \alpha^{\frac{1}{1-\alpha}}) + \alpha^{\frac{1}{1-\alpha}} \right] . \tag{5}$$

For a given size of differentiated varieties, it is clear that the aggregate demand, X_t, increases with the fraction of imitated varieties, n_t^c / n_t. This implies that tightening IPR reduces the amount of intermediates used, X_t, through an increase in monopolized varieties for a given variety size.

2.4 Innovation

I assume that developing a new variety requires b units of final goods. Innovators issue ownership shares to finance the entry cost in the stock market. In the next time interval of length dt, innovators that have not previously imitated face the probability μdt of being imitated, and thus the arbitrage condition for asset markets and the free entry condition for the R&D market are expressed as

$$r_t V_t = \pi_t + \dot{V}_t - \mu V_t \ , \tag{6}$$

$$V_t \leq b \ , \quad \dot{n}_t (b - V_t) = 0 \ , \tag{7}$$

where r_t is the interest rate, V_t is the value of the innovator, and π_t is the monopolistic profits in the differentiated intermediates market. The arbitrage condition relates the stock market value of the innovator to the interest rate on a riskless bond, and free entry ensures that the value of the innovator does not exceed the entry cost for R&D

activity. Since the profit function is expressed as $\pi_t = \frac{1-\alpha}{\alpha} x_t^m$, these conditions give us the following expression.

$$r_t = r_t^m - \mu \ , \quad \dot{n}_t > 0 \ , \tag{8}$$

$$r_t^m \equiv \frac{\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)}{b} A_t^{\frac{\beta}{1-\alpha}} N \ , \tag{9}$$

where r_t^m is the instantaneous rate of return for innovation. Note that the rate of return for innovation increases with the accumulation of experience. As my interest centers on innovations, I henceforth focus on the economy where innovations occur: $\dot{n}_t > 0$ is assumed.

2.5 The Accumulation of Experience

To formulate the notions mentioned in the Introduction, the final sector productivity at date t, A_t, is defined by a weighted sum of the past per-variety average amount of intermediate goods (machines) used up to date t:

$$A_t = \hat{\theta} \int_{-\infty}^{t} e^{-\theta(t-\tau)} \frac{X_\tau}{n_\tau} d\tau \ , \tag{10}$$

where X_τ / n_τ is the average per-variety level of the amount of intermediate goods used in the final production, θ is the depreciation (forgetting) rate for the past experience, and $\hat{\theta} > 0$ is a constant.

In equation (10), I make two assumptions about the final sector productivity, A_t. First, I assume that learning by experience works through each final firm's investment, as in the literature (Arrow 1962, Romer 1986).[9] When final firms use a variety of machines more, they become much better at using them and, as a result, the productivity of final firms increases. Hence, the experience that a particular final firm receives at date τ increases with the amount of intermediate goods (machines) that that firm uses at date τ; $x_\tau(j)$, $\forall j \in [0, n_\tau]$.[10] Here, I take the per-variety average of machines that a final firm uses as inputs, $\int_0^{n_\tau} x_\tau(j) dj / n_\tau = X_\tau / n_\tau$, as an index of *instantaneous* experience, which the firm receives at date τ.[11]

Romer (1986) focuses only on a temporary effect of the learning by experience. However, learning by experience is essentially a dynamic effect, as considered by the literature on learning by experience.[12] Therefore, I assume in (10) that the productivity level in a given period is a function of *cumulative* experience up to that period (i.e., a weighted sum of the past instantaneous experience), not of *instantaneous* experience. Hence, today's experience hence boosts future productivity in this model. The view

[9] Learning by experience has been observed in many industries: examples of empirical evidence are Wright (1936), Rapping (1965), Argote and Epple (1990), and Lester and McCabe (1993).

[10] Note that the number (measure) of final firms is normalized to be one in (1).

[11] I believe that the results of the current paper do not depend on this specification qualitatively, just on the property that final firm's instantaneous experience increases with $x(j)$, $\forall j \in [0, n]$.

[12] See Arrow (1962), Fudenberg and Tirole (1983), and Dasgupta and Stiglitz (1988). Empirical evidence also stresses the *intertemporal* effects of experience.

that the *cumulative* experience determines productivity growth is much more akin to Arrow's (1962) original formulation on the learning by doing. [13]

Second, following Romer (1986), I assume that knowledge creation is a by-product of investment (i.e., the use of intermediate goods). Due to the nonrival character of knowledge, each firm's experience/knowledge is a public good in the sense that any other firm can freely access it. Thus each firm's technology depends on the economy's aggregate experience, X/n. In other words, each firm's experience has a positive external effect on final firms' productivity at every point in the future (dynamic externality).

Taking into account the fact that A_0 is given as an initial condition, I can rewrite (10) as follows:

$$A_t = e^{-\theta t} A_0 + \theta \int_0^t e^{-\theta(t-\tau)} \frac{X_\tau}{n_\tau} d\tau \,, \tag{11}$$

where I normalize $\hat{\theta} = \theta$ to make the analysis more tractable. Without this normalization, the condition of local stability could be somewhat changed, but the results and implications of this paper do not change: "stronger is always better" is incorrect.

By differentiating (11) with respect to time, I have the law of motion in experience accumulation as

$$\dot{A}_t = \theta \left[\frac{X_t}{n_t} - A_t \right] \,. \tag{12}$$

This equation, with its two underlying assumptions, is the key element of the model. To understand the importance of it, I discuss here how the dynamic process of learning by experience affects the relationship between IPR protection and growth. A tightening of IPR protection decreases the fraction of competitive sectors, n^c/n, and thereby reduces the aggregate use of intermediate goods, X.[14] This reduces the instantaneous experience of the final firms and, due to the dynamic externality of learning by experience, it discourages the *evolution* of productivity, $\dot{A}$. It follows that tightening IPR leads to a slowdown in the growth of productivity. Since the incentive to innovate a new variety depends on the future demand of the final firms for an intermediate good, reductions in the future productivity of the final firms, which are caused by a slowdown in the growth of A, can decrease innovation, thus reducing economic growth. This is the key mechanism in this paper, through which IPR protection can have a negative effect on growth. As is apparent from the above, the two assumptions behind (10)–(12), learning by experience and its dynamic externality, play an essential role in determining the relationship between IPR and growth.

From (4) and the definition of aggregate demand for intermediates, X_t, I have the following:

$$X_t = A_t^{\frac{\beta}{1-\alpha}} Nn \left[\frac{n_t^c}{n_t} (\alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}) + \alpha^{\frac{2}{1-\alpha}} \right] \,. \tag{13}$$

[13]Equation (10) also states that the impact of the current experience, X_t/n_t, on productivity at every point in the future, $A_\tau,\ \tau \in [t, +\infty)$, decreases at a rate of θ over time. The depreciation rate θ reflects how quickly firms' production experience depreciates (or is forgotten) over time. Then, the larger θ is, the less weight is given to the past experience.

[14]Note that, for a given size of varieties, X_t increases with the fraction of imitated (competitive) sectors, n_t^c/n_t, as shown in the previous subsection.

Substituting (13) for (12), I have the equilibrium law of motion in A_t as

$$\frac{\dot{A}_t}{A_t} = \theta \left[A_t^{-\frac{1-\alpha-\beta}{1-\alpha}} N \left(\frac{n_t^c}{n_t} (\alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}) + \alpha^{\frac{2}{1-\alpha}} \right) - 1 \right] . \tag{14}$$

I assume that the external effect is not so large, i.e., $\alpha + \beta < 1$.

2.6 Market Clearing for Final Goods

The final good is used for consumption, for production of intermediate machines, and for innovation. Thus, the market clearing condition is expressed as:

$$Y_t = C_t + X_t + b\dot{n}_t \ , \tag{15}$$

where C_t is aggregate consumption. From (1) and (4), the aggregate production is expressed as:

$$Y_t = A_t^{\frac{\beta}{1-\alpha}} Nn \left[\frac{n_t^c}{n_t} (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{2\alpha}{1-\alpha}}) + \alpha^{\frac{2\alpha}{1-\alpha}} \right] . \tag{16}$$

Substituting (13) and (16) for (15), the following differential equation is derived.

$$\frac{\dot{n}_t}{n_t} = \frac{A^{\frac{\beta}{1-\alpha}} N}{b} \left[\frac{n_t^c}{n_t} (\gamma_1 - \gamma_2) + \gamma_3 \right] - \frac{C_t}{bn_t} \ , \tag{17}$$

where $\gamma_1 \equiv \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{2\alpha}{1-\alpha}} > 0$, $\gamma_2 \equiv \alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} > 0$, and $\gamma_3 \equiv \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}} > 0$. Clearly, $\gamma_1 - \gamma_2 > 0$.

2.7 Consumer

The representative consumer, who owns the firms, maximizes a lifetime utility function by choosing a consumption path. The dynamic maximization problem for the representative consumer is given as

$$\begin{array}{rl} \max_{\{C_t\}} & U = \int_0^\infty e^{-\rho t} \ln C_t dt \ , \\ \text{subject to} & \dot{S}_t = r_t S_t + w_t N - C_t \ , \end{array} \tag{18}$$

where $\rho > 0$ is the subjective discount rate, S_t is the value of asset holdings, and w_t is the wage rate. Using equation (8), the solution for this problem is

$$\frac{\dot{C}_t}{C_t} = r_t^m - \mu - \rho \ . \tag{19}$$

Although tightening IPR (a decrease in μ) quickens the consumption growth *directly*, the present model has another channel of influence. From (9), r_t^m is increasing in accumulated experience, A_t, and, if the IPR protection reduces this accumulation, tightening IPR can have a negative impact on consumption growth *indirectly*. These two opposite effects determine the relationship between IPR protection and the equilibrium growth rate of this economy. When the latter effect dominates, IPR protection may not enhance growth.

As is well known, the solution to this dynamic optimization problem also requires that the transversality condition holds:

$$\lim_{\tau\to+\infty} e^{-\int_0^\tau r(t)dt} S_\tau = 0 \; . \tag{20}$$

3 Dynamic Equilibrium

To analyze the dynamic equilibrium, it is useful to describe the dynamics of this economy in the three variables, $l \equiv C/(bn)$, $m \equiv n^c/n$, and A. From (2), (9), (14), (17), and (19), the perfect foresight equilibrium is characterized by the following:

$$\frac{\dot{l}_t}{l_t} = l_t - \frac{A_t^{\frac{\beta}{1-\alpha}} N}{b} \left[m_t(\gamma_1 - \gamma_2) + \gamma_3 - \alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha) \right] - (\mu+\rho) \; , \tag{21}$$

$$\frac{\dot{m}_t}{m_t} = l_t + \mu(m_t^{-1} - 1) - \frac{A_t^{\frac{\beta}{1-\alpha}} N}{b} \left[m_t(\gamma_1 - \gamma_2) + \gamma_3 \right] \; , \tag{22}$$

$$\frac{\dot{A}_t}{A_t} = \theta \left[A_t^{-\frac{1-\alpha-\beta}{1-\alpha}} N \left(m_t \gamma_2 + \alpha^{\frac{2}{1-\alpha}} \right) - 1 \right] \; . \tag{23}$$

Note that, whereas n_0, n_0^c, and A_0 are given as initial conditions, m_t and A_t are state variables and l_t is a jump variable. For any initial values the economy inherits, (m_0, A_0), a market equilibrium for this economy is a path of $\{l_t, m_t, A_t\}$ that satisfies (21)–(23) and the transversality condition (20).

3.1 Local Stability of the System

By assuming the existence of a unique balanced growth path (BGP), I clarify the dynamic properties by log linearizing the system of differential equations, (21)–(23), around a unique BGP (l^*, m^*, A^*), which is defined to satisfy $\dot{l} = \dot{m} = \dot{A} = 0$ and is assumed to be strictly positive. Proposition 1 in the next subsection proposes a condition for the existence of the BGP.

The linear system is expressed as:

$$\dot{\boldsymbol{\xi}} = J\boldsymbol{\xi} \; , \tag{24}$$

where $\boldsymbol{\xi} = {}^t(\ln(l_t/l^*), \ln(m_t/m^*), \ln(A_t/A^*))$ and J is the coefficient matrix in the system (see the Appendix for details of local stability analysis). Since the linear dynamic system involves one jump variable and two non-jump variables, the initial value of one non-jump variable, l_0, is uniquely determined and the system is locally saddle-path stable if and only if the number of stable roots of J is two.

As shown in the Appendix, the determinant of J is always positive, while its trace is negative when the discount rate for the past experience, θ, is sufficiently large. I maintain the assumption that the past experience has less impact on accumulated experience (θ is sufficiently large), so that two of the three eigenvalues are negative real numbers or have a negative real part (if they are complex roots), and the other is a positive real number. Hence, since both the number of stable roots and the number of non-jump variables is two, the transitional path is uniquely determined for an arbitrary initial condition and the dynamic system is saddle-path stable near the BGPs. [15]

3.2 IPR Protection and Economic Growth

Equations (21), (22), and (23) are the three dynamic equilibrium conditions. Combining with $\dot{l} = \dot{m} = \dot{A} = 0$, they simultaneously determine the equilibrium growth rate of the BGP $g \equiv \dot{C}^*/C^* = \dot{n}^*/n^* = \dot{n}^{c*}/n^{c*} > 0$, in which y^* denotes the BGP value of variable y, $y = C, n, n^c$.

As I have already mentioned, there are always at least two forces acting upon the relationship between the protection of IPR and the growth rate of the BGP, both of which are represented in (19). One is captured by the term $-\mu$ in (19). A strengthening of IPR (a reduction of μ) makes the innovators safer from imitation, and thus increases the *expected* instantaneous rate of return for innovation, $r^m - \mu$, directly. However, while r^m is determined by parameters and constant over time in other lab-equipment models (e.g., Rivera-Batiz and Romer, 1991, Kwan and Lai 2003), this paper focuses on the accumulation of experience, so that r^m depends on the state variable A_t and varies over time; see (9) and (14). Another force is hence generated by experience in using machinery, which has an external effect on the productivity of final sector firms. These two forces determine the relationship between IPR and long-run growth, depending on the magnitude of accumulated experience on productivity, β.

The above discussion suggests that there exist several possible relationships (e.g., an inverted-U shape) between IPR and growth, depending on parameters such as β. The following proposition, which is the main result of this paper, states this formally.

Proposition 1 (IPR protection and economic growth) *Define $\hat{N} \equiv N^{\frac{1-\alpha}{1-\alpha-\beta}}/(b\rho)$. The unique BGP exists if $\hat{N} > \alpha_1$ holds. Consider an economy along the BGP where $\hat{N} > \alpha_1$ holds, then the growth rate as a function of the imitation rate, $g(\mu)$, has the following properties.*

1. *If $\beta \geq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, the growth rate along the BGP as a function of the imitation rate $g(\mu)$) always has the inverse U-shaped configuration: there exists the threshold value of μ, $\hat{\mu}$, below which the BGP growth rate g increases with the imitation rate μ.*

2. *If $\beta < \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, the economy with $\hat{N} \in (\alpha_1, \alpha_2]$ has the inverse U-shaped configurations of $g(\mu)$, while g decreases with μ in the economy with $\hat{N} \in$*

[15] I prove here the uniqueness and the local saddle-path stability of the transitional path, but the result does not rule out the case where a unique transitional path is oscillatory. Although my interest is mainly in the relationship between long-run growth and IPR protection, I discuss several properties of transitional dynamics in Section 4.

$(\alpha_2, +\infty)$.[16]

3. *In each case, an increase in the labor supply, N, and a reduction in the entry cost, b, and the discount rate of the representative consumer, ρ, unambiguously quicken the pace of economic growth along the BGP.*

4. $g(\mu) = (>) 0$ *when* $\mu = 0$, *if and only if* $\hat{N} \leq (>) \alpha_1 \alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}$.

Proof
See Appendix. ||

α_1 and α_2, formal definitions of which are shown in footnote 16 and the Appendix, are both threshold values: the former determines whether the BGP exists, and the latter determines whether the graph of $g(\mu)$ is one-peaked or monotonically decreasing. My focus is on the case where innovations occur, and thus I henceforth assume that the parameters satisfy $\hat{N} > \alpha_1$ and $\mu < \bar{\mu}$ to ensure the existence of the BGP.[17] The first assumption, $\hat{N} > \alpha_1$, is weaker than $\hat{N} > \alpha_1 \alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}} > \alpha_1$, which ensures that $g(0) > 0$, as shown in the proposition. Therefore, if the BGP exists without imitation ($g(0) > 0$), which is a standard assumption,[18] then $\hat{N} > \alpha_1$ always holds. The second assumption implies that the imitation rate is not too high: the natural rate of imitation is restricted by the fundamentals of the economy, so that it would also be a natural assumption.

Proposition 1 suggests that relaxing IPR protection may enhance economic growth, which sharply contrasts with the propositions in the existing studies, which examine how the patent policy affects growth and welfare in closed economy models of endogenous growth, such as Kwan and Lai (2003). The result that there is an inverted U-shaped relationship between IPR protection and growth emphasizes that relaxing IPR protection can be a growth-enhancing policy *even* in a closed economy model, not only in open economy models with the innovative North and the imitative South. "Stronger is not always better" holds whenever the intensity of accumulated experience on the final sector firms, β, is high enough to exceed $\alpha^{\frac{1}{1-\alpha}}(1-\alpha)$. However, it can be broken if β does not exceed $\alpha^{\frac{1}{1-\alpha}}(1-\alpha)$: depending on the value of $\hat{N}$, in some circumstances, tighter protection always gives a boost to economic growth. Figure 1 illustrates the function $g(\mu)$ of each case. Note that, when β is smaller than $\alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, IPR protection is always growth enhancing in the economy with the

[16]I here define that $\alpha_1 \equiv \left[\alpha^{\frac{1+\alpha}{1-\alpha}+\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}(1-\alpha)\right]^{-1}$ and

$$\alpha_2 \equiv \alpha_1(1-\alpha-\beta)\alpha^{\frac{(1-\alpha-\beta)-\beta}{(1-\alpha)(1-\alpha-\beta)}}\left[\alpha^{\frac{1}{1-\alpha}}(1-\alpha)-\beta\right]^{-1}.$$

As shown in the Appendix, these definitions always ensure $\alpha_1 < \alpha_2$, if $\beta < \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$.

[17]These two conditions are necessary and sufficient for the existence of the BGP.

[18]For example, Helpman (1993) assumes that, in an endogenous growth model with an innovative North and an imitative South, an isolated North innovates at a positive rate.

higher $\hat{N}$ and the associated higher growth rate, g. Hence, the results of strengthening IPR protection on economic growth, if the intensity of the accumulation of experience is lower, depends crucially on $\hat{N}$, which is increasing in N and decreasing in b and ρ.

Why is there a potential for stronger not always to be better? The economic intuition behind this result is the following. As mentioned above, tightening IPR has two opposite effects on growth, direct and indirect. On the one hand, tightening IPR increases the expected benefit from innovation through reducing the risk on imitation and then raising the interest rate, as is expressed in (8). On the other hand, tighter IPR protection increases the fraction of monopolized sectors, $(n - n^c)/n$, and monopolistic pricing means a smaller amount of intermediate machines used in the final sector, as shown in (13), which leads to less accumulated experience in using machinery, noting (12) and (14) . It reduces the productivity of final firms that are the only users of machinery, A, and then the rate of return for innovation, r^m, as shown in (9). Finally, that in turn weakens the incentive to innovate new machinery as a source of economic growth, which implies a slowdown on economic growth. If the latter dominates the former, there is an inverted U-shaped relationship between IPR protection and growth and thus tightening IPR is not always growth enhancing because of diminishing experience accumulation.

Note again that β represents the intensity of the accumulation of experience drawn out by using intermediates. Thus, since the higher β makes the indirect channel more dominant against the other, too strong protection of IPR must be harmful to economic growth in the economy with the higher intensity β. However, if β is small, such that the direct effect dominates the indirect one, then the "stronger is not always better" assertion may change. Indeed, part (2) of Proposition 1 states that there exists the case where protection is always growth enhancing, which is characterized both by lower β and by larger $\hat{N}$. In other words, if experience accumulation were to have little effect and if the innovation sector were more profitable,[19] it would be better for productivity growth to develop a new machine because an increase in monopolized sectors has very little effect, if any, in hindering final firms from learning by experience, rather than encouraging the accumulation of experience by more active imitation accompanied by increased competitive sectors.

3.3 Role of the Natural Rate of Imitation: An Example

To focus on a role of the natural rate of imitation, which can be interpreted as an upper bound of μ, for growth-enhancing policy in this economy, I set the parameter values in an example.

Example 1 *Let $\alpha = 0.4$, $\beta = 0.2$, $\rho = 0.5$, $\mu_f = 1$, and $N = 30$ be the parameters. Numerical calculations indicate that $\beta > \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, which is the condition for the one-peaked configuration of $g(\mu)$ presented in Proposition 1, holds. Consider two economies, each of which has the different size of entry costs, respectively, $b' = 1$ and $b'' = 0.5$. Each economy has the relationships between the level of IPR protection and economic growth along the BGP that are depicted in Figure 2.*

[19]Equation (9) implies that the profitability of innovators increases with N and decreases with b, and thus the larger $\hat{N}$ is positively related to the profitability of innovators.

Numerical calculations show that in both economies, the BGP growth rate as a function of μ has the inverse U-shaped configuration. In addition, a rise in productivity of the R&D sector (a reduction in b) promotes economic growth. These are consistent with Proposition 1, which asserts this configuration when $\beta \geq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, and suggests a role of IPR protection for any μ that belongs to $[\hat{\mu}, \infty)$.

However, as is apparent in Figure 2, the economy with the lower entry cost $b'' = 0.5$ does not need IPR protection at all: no IPR protection ($\mu = \mu_f$) maximizes the BGP growth rate g in this low entry-cost economy. Clearly, the existence of an upper bound of the imitation rate, μ_f, which can be thought to be restricted by the fundamentals and laws of the economy, brings about this extreme policy implication, that is, no protection is best for BGP growth in the case of higher intensity β, generating the possibility of corner solutions. This contrasts with implications in the case of lower β: if $\beta \leq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, a lower entry cost implies that strengthening IPR must be growth enhancing, as in Proposition 1 and Figure 1. Thus, the effective patent policy that is designed to pursue economic growth depends on β, μ_f, and $\hat{N}$. The following statement summarizes the above argument. Consider an economy where the natural rate of imitation is low and the entry cost is small. *Relaxing* IPR protection can always work as growth-enhancing policy if the intensity of accumulated experience is higher. In contrast, if the economy inherits the lower intensity, *tightening* IPR protection is always growth enhancing.

3.4 Optimal IPR Protection

The inverse U-shaped configuration in Figure 2 implies possibilities for optimal IPR protection. Noting that, in this model, strengthening IPR is captured by a decrease in the rate of imitation, μ. I can consider the optimal IPR protection along the BGP as the welfare-maximizing BGP imitation rate, which is assumed to be denoted by μ^*. Specifically, I define μ^* as follows.

$$\mu^* = \text{argmax}_{\mu \in (0, \mu_f]} U^* \,, \tag{25}$$

where U^* denotes the representative agent's utility of the BGP. While the BGP level of consumption is $C^*(t) = C^*(0)e^{gt} = bn(0)l^*(0)e^{gt}$, the utility of the BGP can be expressed as

$$U^* = \frac{1}{\rho}\left(\ln l^*(\mu) + \frac{g(\mu)}{\rho}\right) \,. \tag{26}$$

Thus, μ^* can be characterized by

$$\mu^* = \text{argmax}_{\mu \in (0, \mu_f]} \ln l^*(\mu) + \frac{g(\mu)}{\rho} \,. \tag{27}$$

While this problem is too complex, by using numerical examples for Example 1, I investigate the optimal IPR protection. Consider again the two economies in Example 1, both of which satisfy $\beta > \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$ and then have one-peaked graphs of $g(\mu)$.

Numerical calculations indicate that, in both economies, the graph of the optimal utility as a function of the imitation rate $U^*(\mu)$ is inverse U-shaped, just like $g(\mu)$. However, the optimal patent policy of each economy can be different from other, depending on levels of the natural rate of imitation, μ_f. The same logic as in the previous subsection implies that no IPR protection can be optimal (maximize the utility along the BGP) in spite of the inverse U-shaped configuration of $U^*(\mu)$. When the natural rate of imitation is low enough, a corner solution, $\mu = \mu_f$, can be optimal IPR policy in the higher β economy. In these cases, policies that are too strong hurt both growth and welfare.

Numerical calculations also suggest that the welfare maximizing IPR is almost the same as the growth maximizing one. In the case where the intensity of experience capital is low enough to satisfy $\beta < \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$ (e.g., $\beta = 0.08$ with other parameters unchanged), full IPR protection ($\mu = 0$) maximizes the rate of innovation (Proposition 1), while the welfare maximizing IPR is characterized by *almost* full protection (μ is very close to zero). Also in the case of the inverted-U shape, I cannot make examples in which these two values are markedly different: they seem to be almost the same in many calculations. In the light of these calculations, it may be stated that the agents gain from higher rates of economic growth.

The intuition is as follows. There are two externalities in this economy: one related to the R&D technology and one related to learning by experience. As is well known, the equilibrium growth rate is likely to be lower than the optimal level in the economy with such externalities. This is because each firm (each innovator) does not recognize that each firm's increase in production (each innovation) adds to the aggregate production (the total number of machinery) and, hence, contributes the accumulation of experience (a reduction in the relative cost of R&D). It follows that the growth rate is positively related to the welfare of the BGP (i.e., higher is better) if these externalities dominate other forces (e.g., the force related to the price distortion because of increased monopoly power). The numerical calculations considered above imply that the impacts of these externalities are large enough to dominate in this model.

3.5 Discussion

The analysis so far has highlighted a mechanism of learning by experience, through which a tightening of IPR protection negatively affects an incentive to innovate and then growth. To clarify the main issue, I kept the analysis as simple as possible. I briefly mention a possible modification. The issue to note is that the model presented here exhibits a scale effect in the sense that the growth rate increases as a result of an increase in population or labor supply. In recent years, it has been shown that this scale effect is hardly supported by the facts (Jones 1995), and instead there have been studies that present growth models without scale effects (see Jones 1995, Young 1998, and Howitt 1999). The current model is therefore at odds with the facts. However, it can be shown that the mechanism incorporated in the current model would remain in a modified model that omits the scale effect.

In such a modified model, analyses of the transitional dynamics become less tractable (e.g., the condition for saddle-path stability will be more complex) and some additional restrictions need to be imposed. Consequently, I choose to retain the scale effect in this paper in order to make the analyses simpler and more tractable.

4 Transitional Dynamics

This section examines some properties of the transitional dynamics by using the log-linearized system (24) with the coefficient matrix J that is given by (30) in the Appendix. In the previous section I mentioned that the system has two stable roots and one unstable root: the system is locally saddle-path stable. There are two possible cases: one where the two stable roots are complex and one where all the roots are real. As is well known, the dynamic system is oscillatory in the former case, while it is monotonic in the latter case.

By using again the parameter set of Example 1, I now show the existence of a cyclical equilibrium. In the economy with small start-up costs in Example 1,[20] I assume that $\theta = 5$ and $\mu = 0.1$. Numerical calculations imply that eigenvalues of the coefficient matrix J include complex ones: $\lambda_1 \simeq 13.6547$, $\lambda_2, \lambda_3 \simeq -4.01671 \pm i0.679459$. Here, λ_1, λ_2, and λ_3 denote the eigenvalues of the coefficient matrix J, as defined in the Appendix. While the unique saddle path displays damped oscillations because of complex roots, this economy experiences growth cycles in converging to the BGP. The configuration of the out-of-BGP growth rate as a function of t is depicted in Figure 3 where $\eta_t \equiv \dot{C}_t/C_t = r_t^m - \mu - \rho$ is the growth rate of consumption off the BGP.[21]

The logic behind the emergence of oscillations is easy to grasp. Suppose that the consumption growth rate, η_t, is lower today. As is apparent from equation (19), this means lower expected instantaneous benefits from innovation, r_t^m, which is related to a smaller incentive to innovate a new intermediate. This, in turn, implies that less innovation occurs ($\dot{n}_t/n_t$ is smaller). If η_t is low enough to ensure that imitation dominates innovation, the fraction of competitive sectors, n^c/n, increases at the next date through the process of imitation with a constant hazard rate, μ. Thus, noting equation (13) and, from equation (9), that lower r_t^m is always accompanied by lower A_t, the aggregate use of intermediate goods, X_t, is greater, which leads to an increase in the efficiency level of final good firms, A_t, through the accumulation of experience. Finally, while an increase in A_t is positively related to r_t^m, the lower is the present consumption growth rate, the higher it becomes at the next date: a negative relationship between present and future growth can emerge in this model.

My remaining interest is concerned with the short-run effects of stronger IPR protection. How does strengthening IPR protection affect transitional paths? To answer this question, by using (19), noting that A_t is a non-jump variable, I have the following expression.

$$\frac{\partial \eta_T}{\partial \mu} = -1 < 0 \,, \tag{28}$$

[20] I computed a large number of numerical examples (one is presented here) in which the dynamic system displays complex roots. Moreover, in the economy with large start-up costs, there also exist numerical examples that exhibit complex roots.

[21] By computing the example, it is easy to show that innovation rate, $\dot{n}/n$, can also fluctuate.

from which, clearly, strengthening IPR protection (a decrease in μ) instantaneously enhances the consumption growth. It follows from Proposition 1 that the short-run and long-run effects on consumption growth can be quite opposite when the relationship between IPR and the long-run growth is an inverted-U shape (the case of $\beta \geq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$). For simplicity, I now restrict myself to the acyclical case. If IPR protection is sufficiently strong (i.e., $\mu < \hat{\mu}$), the consumption growth initially rises as a result of a tightening of IPR, but it declines gradually, and finally falls below the previous level, as shown in Figure 4.

The economic intuition behind this result is as follows. As already mentioned, a tightening of IPR has two opposing effects on the growth rate. The negative effect is related to the accumulation of experience. Consequently, there is no negative effect temporarily (in the short run) because accumulated experience is a nonjump variable. It follows that the positive direct effect, which reduces the risk of being imitated, dominates in the short run.

Figure 4 also shows effects of *repeated* changes in IPR policy on the transitional dynamics of the growth rate of consumption, η_t. Consider a situation in which IPR policy changes repeatedly. There is an economy where relaxing IPR enhances growth, such as the one with small start-up costs in Example 1. At date $T' > 0$, the IPR protection is tightened: μ reduces to $\mu' < \mu$ (tighter), and this change is sustained until date T''. That is, the patent policy that I consider is captured by the following expression.

$$\text{the hazard rate of imitation} = \begin{cases} \mu & t \in [0, T') \\ \mu' < \mu & t \in [T', T'') \\ \mu & t \in [T'', +\infty) \end{cases} . \tag{29}$$

As is apparent from Figure 4, in the economy where strengthening the IPR protection is not growth enhancing, repeated changes in IPR policy can destabilize the economy in the sense that the rate of consumption growth fluctuates.[22]

The next proposition summarizes the discussion in this section.

Proposition 2 *The economy has the following properties on the local transitional dynamics.*

1. *The economy can experience growth cycles: there exist the parameter sets that the local dynamic system displays complex roots, as is depicted in Figure 3.*

2. *The short-run effect of stronger IPR is positive: strengthening IPR protection must be growth enhancing instantaneously, as is depicted in Figure 4.*

[22]One might justify such changes in IPR policy by the following. A policy planner who believes that protection is always growth enhancing would adopt the respective policy schedule. At date T', he or she strengthens protection, pursuing the growth-enhancing effects. As mentioned above, although it enhances the growth rate initially, the growth rate eventually falls below the pre-policy-change level. In response to this, the policy planner might weaken the IPR protection, taking it back to its original level of protection, in order to enhance growth, if he or she still believes there is a positive relationship between IPR protection and economic growth. This causes a temporary decline in the consumption growth rate, and, finally, the growth rate goes back to the initial level, that is, the BGP level corresponding to μ.

5 Conclusion

This paper investigates the effects of IPR protection on economic growth in a variety expansion model of endogenous growth. The crucial assumption is that experiences accumulate on using machinery. This assumption generates a negative relationship between tightening IPR and growth, in contrast to most models in the existing literature on growth and IPR protection. The logic behind this interesting result is as follows. Tightening IPR increases the fraction of monopolized sectors. Experience accumulation decreases with a greater number of monopolized sectors, because production levels are lower in a monopolized sector than in a competitive sector. This implies smaller demand for new intermediate goods, which leads to a lesser incentive to innovate. The result of this paper is that the relationship between IPR protection and innovation has an inverted-U shape when the impact of accumulated experience on productivity is large enough.

This result suggests a role for relaxing IPR protection as a growth-enhancing policy. Due to the inverted U-shaped relationship between IPR protection and growth, both too strong and too weak protection hurts innovation and the resulting growth: instead, a balanced approach is required for growth. This can provide insight into the role of IPR protection in economic growth, a topic often raised in public policy discussions.

This paper also shows that the equilibrium growth rate can be maximized when there is no IPR protection if the natural rate of imitation, which is determined by the economy's fundamentals, is low enough. Therefore, preferred growth-enhancing policies can vary across countries with different potential imitation rates: in countries where imitation hardly takes place, there is no need to protect IPR for innovation and economic growth.

Appendix

Local stability of the log-linearized system.

By log-linearizing the system (21)–(23), I have the coefficient matrix as

$$J = \begin{pmatrix} l^* & -\frac{(\gamma_1-\gamma_2)Nm^*(A^*)^{\frac{\beta}{1-\alpha}}}{b} & -\frac{\beta N(A^*)^{\frac{\beta}{1-\alpha}}\left[\frac{m^*(\gamma_1-\gamma_2)+\gamma_3}{1-\alpha}-\alpha^{\frac{1+\alpha}{1-\alpha}}\right]}{b} \\ l^* & -\frac{(\gamma_1-\gamma_2)Nm^*(A^*)^{\frac{\beta}{1-\alpha}}+b\mu/m^*}{b} & -\frac{\beta N(A^*)^{\frac{\beta}{1-\alpha}}[m^*(\gamma_1-\gamma_2)+\gamma_3]}{b(1-\alpha)} \\ 0 & \frac{\theta\gamma_2 Nm^*}{(A^*)^{\frac{1-\alpha-\beta}{1-\alpha}}} & -\frac{(1-\alpha-\beta)\theta N(m^*\gamma_2+\alpha^{\frac{2}{1-\alpha}})}{(1-\alpha)(A^*)^{\frac{1-\alpha-\beta}{1-\alpha}}} \end{pmatrix} . \quad (30)$$

By using (30), I have the determinant of J as

$$|J| = \frac{\theta Nl^*}{(A^*)^{\frac{1-\alpha-\beta}{1-\alpha}}}\left[\frac{\alpha^{\frac{1+\alpha}{1-\alpha}}\beta\gamma_2 Nm^*(A^*)^{\frac{\beta}{1-\alpha}}}{b} + \frac{\mu(1-\alpha-\beta)(\gamma_2 m^* + \alpha^{\frac{2}{1-\alpha}})}{(1-\alpha)m^*}\right] > 0 . \quad (31)$$

Noting that θ appears only in (23), the dynamic system (21)–(23) implies that the BGP values, l^*, m^*, and A^*, θ, are independent of θ. Therefore, it is obvious that the trace of J is negative when θ is sufficiently large. Let λ_1, λ_2, and λ_3 denote the eigenvalues of the coefficient matrix of the log-linearized system, J. Under the assumption that the discount rate for the past experiences, θ, is large enough to ensure that the trace of J is negative, both

$$\text{determinant of } J = \lambda_1\,\lambda_2\,\lambda_3 > 0 ,$$
$$\text{trace of } J = \lambda_1 + \lambda_2 + \lambda_3 < 0 ,$$

hold. This proves the saddle-path stability of the linear system (21) – (23). ||

Proof of Proposition 1.

It is useful to illustrate the equilibrium conditions graphically. Before the graphical analysis, using $\dot{l} = \dot{m} = \dot{A} = 0$, I reduce the three equilibrium conditions to the following two.

$$M = \alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)b^{-1}N(A^*)^{\frac{\beta}{1-\alpha}} - \rho \qquad \text{from (21) and (22),} \qquad (32)$$

$$M = \gamma_2\mu\left(\frac{(A^*)^{\frac{1-\alpha-\beta}{1-\alpha}}}{N} - \alpha^{\frac{2}{1-\alpha}}\right)^{-1} \qquad \text{from (23),} \qquad (33)$$

where $M \equiv \frac{\mu}{m^*} = \frac{\mu n^*}{n^{c*}}$, which reflects the degree of imitation. From these equations, the relationship of the BGP between M and μ can be expressed as

$$\mu = f(M; b, N, \alpha, \beta, \rho) \equiv \frac{M}{\gamma_2 N^{\frac{1-\alpha}{\beta}}}\left[\frac{b(M+\rho)}{\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)}\right]^{\frac{1-\alpha-\beta}{\beta}} - \frac{\alpha^{\frac{2}{1-\alpha}}}{\gamma_2} , \qquad (34)$$

from which it is obvious that $f(M)$ is continuous on $[0,\infty)$.

The determination of the growth rate of the BGP is closely related to properties of function $f(M)$ because equations (2), (21), (22), $g = \frac{\dot{n}^{c*}}{n^{c*}}$, and $\dot{l} = \dot{m} = 0$ imply

$$g = \mu(\frac{1}{m^*} - 1) = M - \mu \ . \tag{35}$$

Clearly, $g > 0$ exists if and only if $M > \mu$, and a tightening of IPR, which is captured by a reduction in μ, decreases the growth rate of the BGP (i.e., $g_\mu > 0$) if and only if $f_M < 1$, noting that $g_\mu = \frac{\partial M}{\partial \mu} - 1 = \frac{1}{f_M} - 1$ holds.

Next, the following lemma states more detailed properties of the function f in (34).

Lemma 1 *The continuous function f of M satisfies the following four properties: (a) $f(0) = 0$, (b) $f(+\infty) = +\infty$, (c) $\lim_{M\to+\infty} f(M) = +\infty$, and (d) $f(M)'' > 0$.*

Proof

It is straightforward to prove (a) and (b).

The proof of (d) is as follows. Differentiating (34) with respect to M,

$$f'(M) = \frac{1}{\gamma_2}\left\{\left[\frac{b(M+\rho)^{\frac{1-\alpha-2\beta}{1-\alpha-\beta}}}{\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)N^{\frac{1-\alpha}{1-\alpha-\beta}}}\right]^{\frac{1-\alpha-\beta}{\beta}}\left(\frac{1-\alpha}{\beta}M+\rho\right) - \alpha^{\frac{2}{1-\alpha}}\right\} , \tag{36}$$

is derived. By differentiating the equation (36) again, I have the second-order derivative of f as

$$f''(M) = \frac{1-\alpha-\beta}{\beta\gamma_2}\left[\frac{b(M+\rho)^{\frac{1-\alpha-3\beta}{1-\alpha-\beta}}}{\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)N^{\frac{1-\alpha}{1-\alpha-\beta}}}\right]^{\frac{1-\alpha-\beta}{\beta}}\left(\frac{1-\alpha}{\beta}M+2\rho\right) > 0 \ . \tag{37}$$

From (36), it is easy to prove the property (c). ||

Lemma 1 asserts that the function f is convex (f' is increasing in M), which implies that there exist three typical configurations of the function f: one with $f'(0) \geq 1$, one with $f'(0) \in [0,1)$, and one with $f'(0) < 0$. Each case is depicted in Figure 5.

The BGP never exists if $f'(0) \geq 1$ because the BGP growth rate $g = M - \mu$ appears in this figure as the horizontal difference between the function f and 45° line. In addition, even in the economy with $f'(0) < 1$, the BGP growth rate cannot be positive when μ is sufficiently large such as $\mu \geq \bar{\mu}$ where $\bar{\mu}$ satisfies $\bar{\mu} = \bar{M} = f(\bar{M})$: that is, $\bar{\mu}$ is a unique fixed point of the function $f : \boldsymbol{R}_{++} \to \boldsymbol{R}_{++}$.

The graphical analysis also implies that, while $f(M)$ is a convex function, the case with $f'(0) \in [0,1)$ displays the inverse U-shaped configuration of the growth rate $g = M - \mu$ as a function of the imitation rate μ. Thus, in this case, strengthening IPR protection (a reduction in μ) may or may not be growth enhancing, depending on the level of μ: if the IPR protection is too strong, then weakening it, as represented by an increase in μ, enhances economic growth. In the case with $f'(0) < 0$, this configuration depends on the value of $f'(\hat{M})$, in which $\hat{M}$ is defined to satisfy $f(\hat{M}) = 0$, $\hat{M} > 0$. Hence, while the IPR protection is always growth enhancing ($g_\mu < 0$) if and only if

$f'(\hat{M}) > 1$ holds, it is not necessarily growth enhancing if and only if $f'(\hat{M}) < 1$. From equation (34), the larger is N, the smaller is b, and/or the lower is ρ, the smaller is $f(M)$, which implies a downward shift of function f in Figure 5. Therefore, the larger is the labor supply (the smaller the entry costs, or the lower the discount rate), the more $f'(0)$ is likely to be negative. Moreover, they are also positively related to the growth rate along the BGP while, for given μ, a downward shift of $f(M)$ increases M, and then $g = M - \mu$.

The above graphical analysis suggests that a reduction of the imitation rate as a result of patent policy can either increase or decrease the growth rate along the BGP, depending on parameter values such as N, b, and ρ.

Noting Lemma 1, which assures the strict convexity of f, $f'(+\infty) = +\infty$, $f(0) = 0$, and $f(+\infty) = +\infty$, and that the growth rate along the BGP is given as $g = M - \mu$ and assumed to be strictly positive, the graphical analysis in Figure 5 would suffice to prove the following, as mentioned above. (a) the BGP never exists if $f'(0) \geq 1$, (b) if $f'(0) \in [0,1)$, then there exists $\hat{\mu} \in (0, \bar{\mu})$ such as $g(\mu)' > 0\ \forall \mu \in (0, \hat{\mu})$ and $g'(\mu) \leq 0\ \forall \mu \in [\hat{\mu}, +\infty)$, and (c) if $f'(0) < 0$, then the sign of $f'(\hat{M}) - 1$ crucially determines the configuration of $g(\mu)$, that is, for any $\mu > 0$, $g'(\mu) < 0$ when $f'(\hat{M}) > 1$ and such a $\hat{\mu}$ exists when $f'(\hat{M}) < 1$. Hence, it is possible to prove the proposition by calculating $f'(0)$ and $f'(\hat{M})$.

Setting M equal to zero in equation (36), I derive:

$$f'(0) = \frac{1}{\gamma_2}\left\{\left[\frac{b\rho}{\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)N^{\frac{1-\alpha}{1-\alpha-\beta}}}\right]^{\frac{1-\alpha-\beta}{\beta}} - \alpha^{\frac{2}{1-\alpha}}\right\}, \tag{38}$$

from which $f'(0) \geq 1$ implies

$$\hat{N} \equiv \frac{N^{\frac{1-\alpha-\beta}{1-\alpha}}}{b\rho} \leq \frac{1}{\alpha^{\frac{1+\alpha}{1-\alpha}+\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}(1-\alpha)} \equiv \alpha_1 . \tag{39}$$

Because $M - \mu$ is less than zero when $f'(0) \geq 1$, the BGP growth rate cannot be positive if $\hat{N} \leq \alpha_1$. Hence, if $\hat{N} > \alpha_1$, the unique BGP exists. The uniqueness can be easily proven by using Figure 5 and Lemma 1. $\|$

Proof of part (1)

Using (38), $0 < f'(0) \leq 1$ can be rewritten as

$$\hat{N} \in \left[\alpha_1, \alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}\right) . \tag{40}$$

If $\hat{N}$ exceeds $\alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}$, the sign of $f'(\hat{M})$ plays an important role in the determination of the shape of $g(\mu)$.

Assuming $\hat{N} > \alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}$, I now characterize the threshold value, $\hat{M}$, below which $f(M) < 0$ holds. $f(\hat{M}) = 0$ implies

$$\hat{M} = \frac{\alpha^{\frac{1+\alpha}{1-\alpha}+\frac{2\beta}{(1-\alpha)(1-\alpha-\beta)}}(1-\alpha)N^{\frac{1-\alpha}{1-\alpha-\beta}}}{b} - \rho . \tag{41}$$

By substituting (41) into $f(M)$, I characterize the condition, $f'(\hat{M}) \geq 1$, as

$$\hat{N}\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\beta-(1-\alpha-\beta)}{(1-\alpha)(1-\alpha-\beta)}}\left[\frac{\alpha^{\frac{1}{1-\alpha}}(1-\alpha)}{\beta}-1\right] \geq \frac{1-\alpha-\beta}{\beta}\ , \tag{42}$$

which implies that, if $\beta \geq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$, the inequality condition (41) is always broken: $f'(\hat{M}) < 1$ must hold. Thus, $g(\mu)$ has the inverse U-shaped configuration in the case with $\beta \geq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$ for any $\hat{N} > \alpha_1$. Part (1) of the proposition has been proven. ||

Proof of part (2)

I here turn to the lower β case where $\beta < \alpha^{\frac{1}{1-\alpha}}(1-\alpha)$ holds. Two possibilities arise in this case: one $f'(\hat{M}) \geq 1$ and one $f'(\hat{M}) < 1$. Equation $(A4)$ can be easily rewritten as follows.

$$\hat{N} \geq \frac{1-\alpha-\beta}{\alpha^{\frac{1+\alpha}{1-\alpha}+\frac{2\beta-(1-\alpha-\beta)}{(1-\alpha)(1-\alpha-\beta)}}(1-\alpha)\beta\left[\frac{\alpha^{\frac{1}{1-\alpha}}(1-\alpha)}{\beta}-1\right]}\ . \tag{43}$$

While

$$\frac{(1-\alpha-\beta)\alpha^{\frac{1}{1-\alpha}}}{\alpha^{\frac{1}{1-\alpha}}(1-\alpha)-\beta} \tag{44}$$

is always greater than unity, the right-hand side of (43) must be greater than

$$\alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}\ , \tag{45}$$

which appears in equation (40), and is the critical value that determines the sign of $f'(0)$.

On the one hand, if $\hat{N}$ is below this value, $f'(0) \geq 0$ holds, and then there exists $\hat{\mu}$ as defined above. On the other hand, (43) implies that, if $\hat{N}$ is greater than

$$\alpha_2 \equiv \frac{1-\alpha-\beta}{\alpha^{\frac{1+\alpha}{1-\alpha}+\frac{2\beta-(1-\alpha-\beta)}{(1-\alpha)(1-\alpha-\beta)}}(1-\alpha)\beta\left[\frac{\alpha^{\frac{1}{1-\alpha}}(1-\alpha)}{\beta}-1\right]} > \alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}\ , \tag{46}$$

$g(\mu)$ becomes decreasing in μ. These results are summarized in part (2) of Proposition 1.

Clearly, $\alpha_2 > \alpha_1$ holds because α_2 is greater than $\alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}$, which is greater than α_1. ||

Proof of part (3)

From equation (34), $f(M)$ is increasing in N and decreasing in b and ρ. Consequently, an increment in N and/or a reduction in b and ρ lower the graph of f. For any $\mu > 0$, a downward shift of f rises up to $g = M - \mu$. ||

Proof of part (4)

I have already shown that $\alpha_1\alpha^{-\frac{\beta}{(1-\alpha)(1-\alpha-\beta)}}$ is the critical value of $\hat{N}$, below which $f'(0)$ exceeds zero. While $f'(0) > 0$ means $g(\mu) = 0$ when $\mu = 0$, $f'(0) < 0$ means $g(\mu) > 0$ when $\mu = 0$. ||

References

Acemoglu, D., Johnson, S., Robinson, J., 2001. The Colonial Origins of Comparative Development: An Empirical Investigation. American Economic Review 91, 1369–1401.

Aghion, P., Bloom, N., Blundell, R., Griffith, R., Howitt, P., 2005. Competition and Innovation: An Inverted-U Relationship. Quarterly Journal of Economics 120, 701–728.

Argote, L., Epple, D., 1990. Learning Curves in Manufacturing. Science 247, 920–924.

Arrow, K. J., 1962. The Economic Implications of Learning by Doing. Review of Economic Studies 29, 155–173.

Bessen, J., Maskin, E., 2000. Sequential Innovation, Patents, and Imitation. Working Paper, Department of Economics, Massachusetts Institute of Technology.

Dasgupta, P., Stiglitz, J., 1988. Learning-by-Doing, Market Structure and Industrial and Trade Policies. Oxford Economic Papers 40, 246–268.

Fudenberg, D., Tirole, J., 1983. Learning-by-Doing and Market Performance. Bell Journal of Economics 14, 522–530.

Gould, D. M., Gruden, W. C., 1996. The Role of Intellectual Property Rights in Economic Growth. Journal of Development Economics 48, 323–350.

Hall, R., Jones, C. I., 1999. Why Do Some Countries Produce So Much More Output Per Worker Than Others? Quarterly Journal of Economics 114, 83–116.

Helpman, E., 1993. Innovation, Imitation, and Intellectual Property Rights. Econometrica 61, 1247–1280.

Horii, R., Iwaisako, T., 2005. Economic Growth with Imperfect Protection of Intellectual Property Rights. Development and Comp Systems 0508001, Economics Working Paper Archive at WUSTL.

Howitt, P., 1999. Steady Endogenous Growth with Population and R&D Inputs Growing. Journal of Political Economy 107, 715–730.

Iwaisako, T., Futagami, K., 2003. Patent Policy in an Endogenous Growth Model. Journal of Economics 78, 239–258.

Jones, C. I., 1995. R&D–Based Models of Economic Growth. Journal of Political Economy 103, 759–84.

Koléda, G., 2005. Patents' Novelty Requirement and Endogenous Growth. Revue d'économie Politique 114, 201–222.

Kwan, Y. K., Lai, E. L.-C., 2003. Intellectual Property Rights Protection and Endogenous Economic Growth. Journal of Economic Dynamics and Control 27, 853–873.

Lester, R. K., McCabe, M. J., 1993. The Effect of Industrial Structure on Learning by Doing in Nuclear Power Plant Operation. RAND journal of Economics 24, 418–438.

Rapping, L., 1965. Learning and World War II Production Functions. Review of Economics and Statistics 47, 81–86.

Rivera-Batiz, L., Romer, P. M., 1991. Economic Integration and Endogenous Growth. Quarterly Journal of Economics 106, 531–555.

Romer, P. M., 1986. Increasing Returns and Long-Run Growth. Journal of Political

Economy 94, 1002–1037.
Romer, P. M., 1990. Endogenous Technological Change. Journal of Political Economy 98, S71–S102.
Scotchmer, S., 1996. Protecting Early Innovators: Should Second-Generation Products Be Patentable? RAND Journal of Economics 27, 322–331.
Wright, T. P., 1936. Factors Affecting the Cost of Airplanes. Journal of the Aeronautical Sciences 3, 122–128.
Young, A., 1998. Growth without Scale Effects. Journal of Political Economy 106, 41–63.

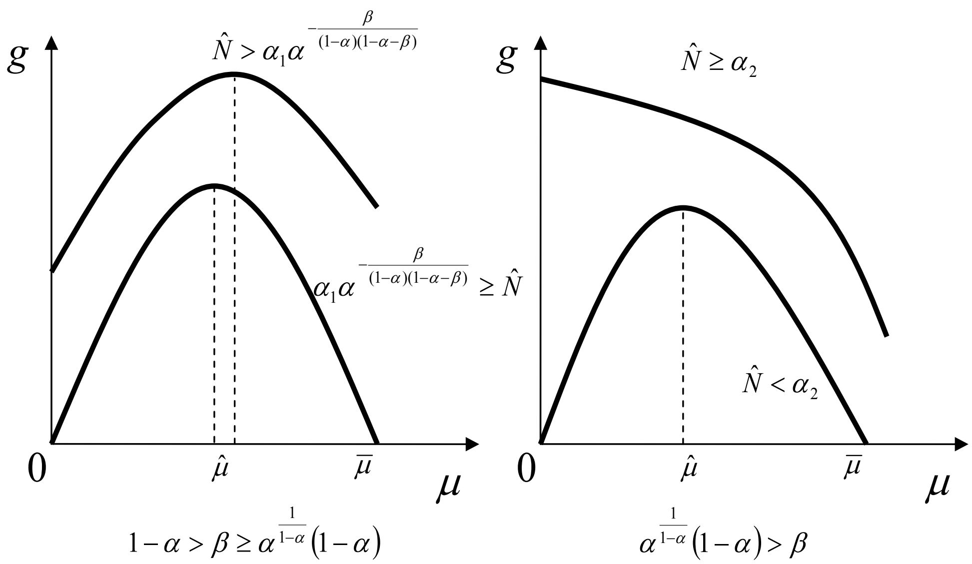

Figure 1: Economic growth and IPR

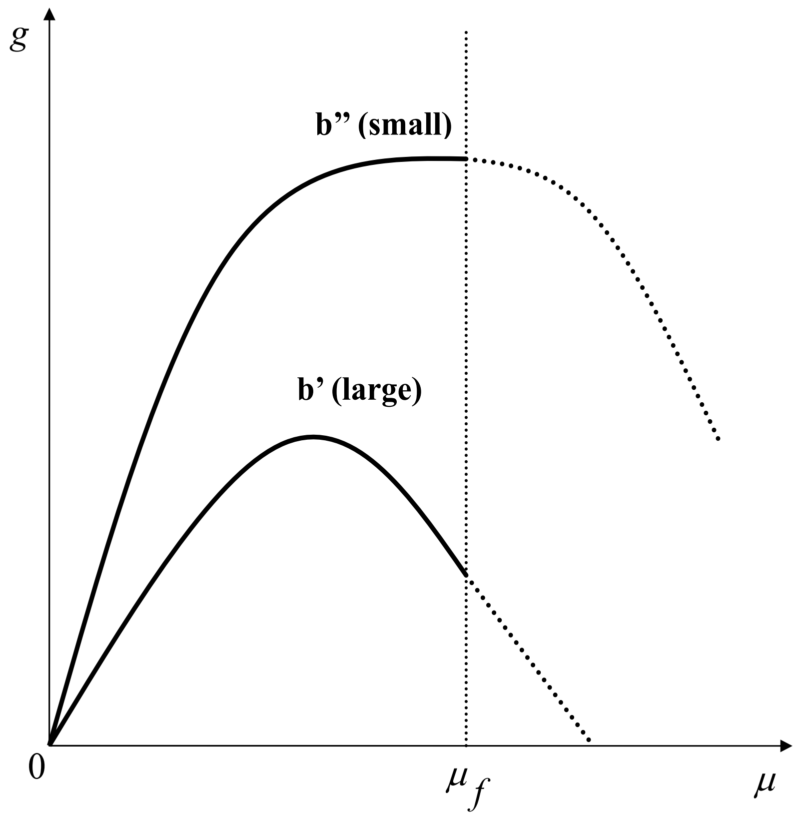

Figure 2: Upper bound of the imitation rate and IPR protection

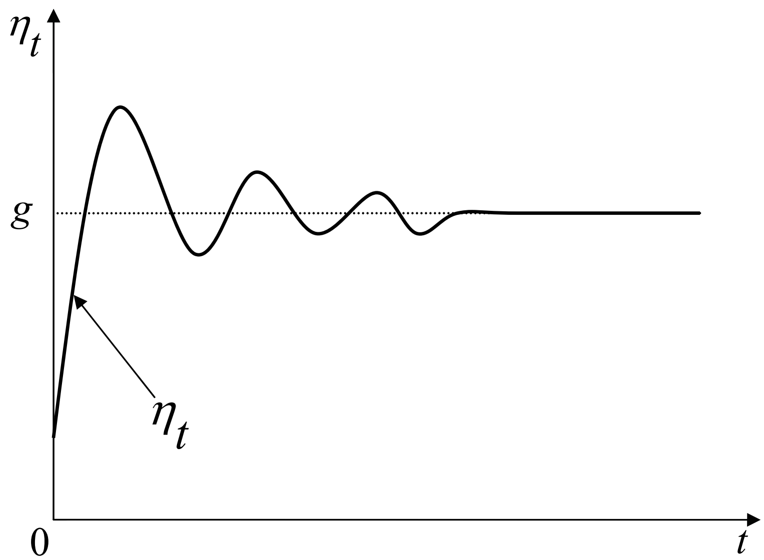

Figure 3: Damped oscillations

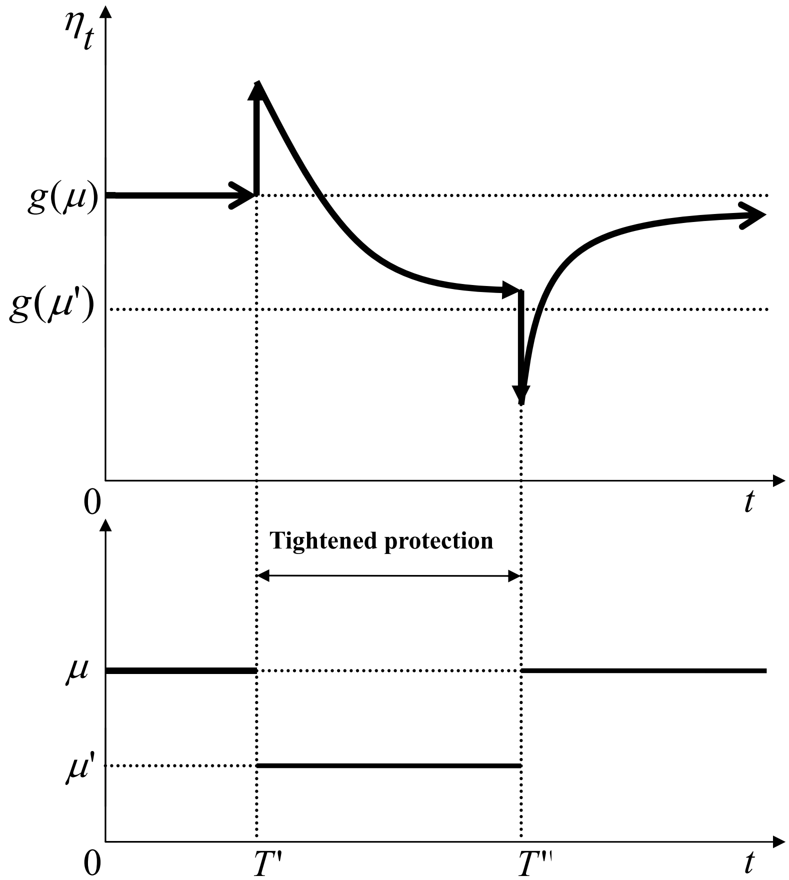

Figure 4: Effects of changes in patent policy on the transitional dynamics

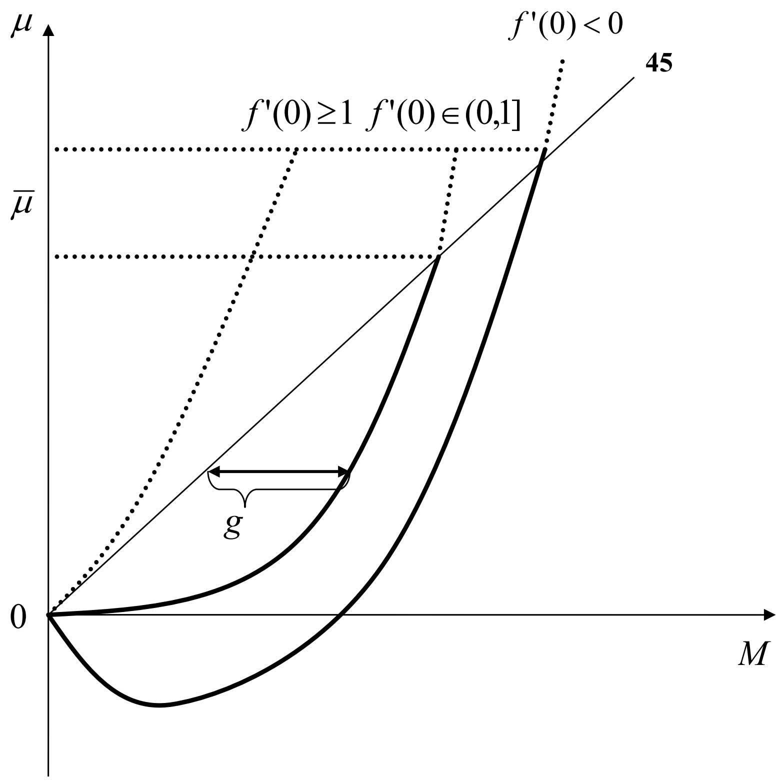

Figure 5: Three cases