KEIO UNIVERSITY
MARKET QUALITY RESEARCH PROJECT
(A 21st Century Center of Excellence Project)

KUMQRP DISCUSSION PAPER SERIES

DP2005-001

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A Price Competition Game under Free Entry

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April 20, 2005

[1]This paper is dedicated to Professor Mukul Majumdar to celebrate his sixtieth birthday. I am deeply indebted to Mukul for the generosity and kindness that I have received throughout my career. I am also grateful to Fumio Dei, Jim Friedman, Koji Ishibashi, Atsushi Kajii, Takashi Kamihigashi, Kunio Kawamata, Kei-ichi Koda, Takashi Negishi, Michihiro Ohyama, Santanu Roy, and Yoshimasa Shirai for useful conversations and correspondence and, in particular, Motonari Kurasawa, who brought Demsetz's work to my attention.

Abstract

This study builds a game of Bertrand-like price competition in a market with free entry. Under the assumption of a standard U-shaped average cost curve, it demonstrates that even if the number of sellers is small, a long-run competitive outcome can be supported as a Nash equilibrium. This game provides unifying treatments to the standard Bertrand equilibrium, the long-run competitive outcome, Demsetz's equilibrium as well as other types of equilibria that have not been known in the existing literature.

1 Introduction

According to Bertrand (1883), even if the number of competing sellers is small, price competition leads to a perfectly competitive outcome in a market for a non-differentiated product. Since Edgeworth (1897), however, Bertrand's conclusion has been criticized for holding only in the case of constant average cost. In the modern literature, as a result, it has come to be viewed as highly limited and is often referred to as the Bertrand paradox.

This study introduces a new game theoretic model of Bertrand-like price competition, which I call a price competition game. It demonstrates that so long as free entry is guaranteed, Bertrand's conclusion holds even in the case of a standard U-shaped average cost curve. Moreover, it demonstrates that various standard equilibria, including the Bertrand equilibrium and the long-run competitive outcome, as well as several new types of equilibria can be supported as a Nash equilibrium in a price competition game.

These results provide contestable market theory (Baumol, Panzar, and Willig, 1982) with a game theoretic foundation, which has been lacking in the existing literature. Demsetz (1968) demonstrates that even a natural monopoly may be forced to set its price equal to the average cost price due to price competition with potential entrants. Baumol, Panzar, and Willig (1982) demonstrate that price competition establishes a long-run competitive outcome even in the case of an identical U-shaped average cost curve in a market with free entry. This study demonstrates that these typical contestable market equilibria can be supported as a Nash equilibrium in a price competition game.

In a different context, Grossman (1981) and Mandy (1991) demonstrate that in a market with free entry, a long-run competitive outcome can be interpreted as a Nash equilibrium even if the number of firms is small.[1] They, however, assume very large strategy spaces by treating a firm's strategy to be to announce a complete pricing schedule relating each of various possible prices to a quantity (or a set of quantities) that the firm is willing to sell at that price. This study adopts a much smaller strategy space by assuming a firm's strategy to be to announce a particular unit price and the selection of quantities that the firm is willing to sell at that unit price.[2]

In the modern literature, the Bertrand-Edgeworth debate has been treated primarily in two-stage games (Kreps and Scheinkman, (1983), Allen and Hellwig (1986), and Davidson and Deneckere (1986)).[3] In this study, in contrast, the debate is treated in a single stage game. In providing a mechanism by which

[1] More broadly, this study is related to Novshek and Sonnenschein (1978) and Novshek (1980), who demonstrate that a long-run competitive outcome may be supported by a Cournot equilibrium in the case with many firms. It is also related to Stahl (1988), Dixon (1992) and Dastidar (1995), who proves a short-run equilibrium can be supported as a Nash equilibrium in a game of Bertrand-like price competition.

[2] This strategy space extends those of quantity-price games, in which a firm chooses a price and a quantity in different stages of a game (see Bennasy (1986) and Friedman (1988), who are concerned with differentiated products).

[3] For more recent literature, see Vives (1999) and Allen, Deneckere, Faith, and Kovenock (2000).

a long-run competitive equilibrium can be established outside of a perfectly competitive economy, this study is related to the literature on decentralization and mechanism design (for recent studies, see Majumdar, 1992).

The rest of this study is organized as follows. I will introduce the price competition game in Section 2 and demonstrate in Section 3 that the standard Bertrand equilibrium is a unique Nash equilibrium in that game. Section 4 is concerned with the description of contestable outcomes as a Nash equilibrium, while Section 5 with the integer problem. Section 6 is for concluding remarks.

2 Price Competition Game

In this section, I will introduce the price competition game. Assume that there are N firms that sell an identical product in a single market. Following the standard literature on perfect competition and contestability, assume that the total cost functions of firms are identical and described by $c(y)$. For the sake of simplicity, assume that there is a continuum of identical buyers. Denote by $u(x)$ an individual buyer's total willingness to pay for x units of the product. Assume $u' \geq 0$ and that $u' > 0$ implies $u'' < 0$. The total mass of buyers is normalized to be equal to 1. Then, the market demand is determined by $u'(x) = p$ in the case in which all firms choose an identical price p. Denoted by $D(p)$ the market demand curve in that case.

Assume that a strategy of a firm is a paring of a unit price and the set of quantities that the firm is indifferent to sell at that unit price; a firm is said to be indifferent between any two quantities that give rise to the same amount of profit. Denote by $\mathcal{Y}(p)$ the set of sets of quantities that a firm is indifferent to sell at price p, that is,

$$\mathcal{Y}(p) = \{Y \in \mathcal{P}(R_+) : \forall y \in Y,\ \forall y' \in Y,\ py - c(y) = py' - c(y')\}, \tag{1}$$

where $\mathcal{P}(R_+)$ is the set of all subsets of R_+. A strategy of firm k, $k = 1, ..., N$, is a pair of a unit price, $p_k \in R_+$, and a set of quantities $Y_k \in \mathcal{Y}(p_k)$. The strategy space of a firm is $\mathcal{S} = \{(p, Y) : p \in R_+,\ Y \in \mathcal{Y}(p)\}$.

By adopting this strategy space, I do not intend to imply that a real-world firm actually announces a combination of a price, p, and a set of quantities, Y, as a strategy. Concerning the strategic choice of a firm, more directly observable variables are a price and a quantity, p and y. The strategic choice of a real-world firm is made with respect to many variables. A price competing firm may be defined as a firm that treats, of those variables, a unit price as its primary strategic variable. If it is observed that such a firm sells a particular amount, y, at a particular unit price, p, it does not imply that the firm is indifferent with respect to the amount that it sells. A profit maximizing firm is, however, indifferent between any two alternatives that give rise to the same profit level as each other. Therefore, if a price competing firm sets its price, it should be indifferent between any two quantities that give rise to the same profit level as each other. This implies that if a price competing firm picks a price p and sells a quantity y, it can be deduced that the firm will be willing to sell at p any other quantity y' that will not affect the profit level, i.e., any y' in $Y \in \mathcal{Y}(p)$ such that $y \in Y$. In short, if what is observable with respect to a firm's strategic choice might be (p, y), it is reasonable to assume that its real strategy is $(p, Y) \in \mathcal{S}$.

This assumption is even more reasonable in dealing with a long-run equilibrium. It is a standard assumption of a long-run model that all firms have identical technologies; behind this assumption, it is implicitly assumed that all firms can freely learn from other firms' technologies in the long run. If, in such circumstances, a particular firm, X, is indifferent between selling one amount y at a price p and selling another amount y' at the same price, and if its competitor observes that firm X sells y at p, the competitor should be able to deduce that X is willing to sell y' at p as well, for all firms adopt the same technology in the long run. In that case, it is natural to assume that the competitor believes that the real strategy that X chooses is $(p, Y) \in \mathcal{S}$ such that $y \in Y$.

Before moving to the next section, note the following notation. The set of players (firms) is denoted as $\mathbb{N} = \{1, ..., N\}$. Let $\mathbf{p} = (p_1, ..., p_N)$ and $\mathbf{Y} = (Y_1, ..., Y_N)$. Denote by $(\mathbf{p}, \mathbf{Y})$ a strategy profile of firms; adopt the convention that $(\mathbf{p}, \mathbf{Y}) \in \mathcal{S}^N$ means $(p_k, Y_k) \in \mathcal{S}$ for each $k \in \mathbb{N}$. Let $\mathbf{y} = (y_1, ..., y_N)$ be the vector of quantities that the firms actually sell. Let $\mathbf{p}_{\sim k}$, $\mathbf{y}_{\sim k}$, and $\mathbf{Y}_{\sim k}$ be those constructed by deleting p_k, y_k, and Y_k from $\mathbf{p}$, $\mathbf{y}$, and $\mathbf{Y}$.

Remark 1: Although strategy space $\mathcal{S}$ might appear unnecessarily complicated, as is shown in the next section, it provides a natural way to formalize Bertrand's notion. The strategy space consisting of pairs of a price and a quantity, (p, y), is not suitable for describing Bertrand's idea for precisely the same reason as that adopted in Edgeworth's criticism, which will be explained in the next section.

Remark 2: Strategy space $\mathcal{S}$ is closely related to, but smaller than, those of Grossman (1981) and Mandy (1991). Grossman assumes that a strategy of a firm is a set-valued function (supply function) $S : R_+ \to \cup_{p \in R_+} \mathcal{Y}(p)$, $Y = S(p)$, associating each price p with a selection of quantities $S(p)$ that a firm is willing to sell at that price. In equilibrium, each firm is assumed to choose a profit maximizing supply function, given the other firms' choices of supply functions. Mandy, in contrast, assumes that it is a set-valued function associating each quantity y with its revenue $r : R_+ \to R_+$, $r = R(y)$.

Although this study is based on a standard process of proportionate rationing, a formal description becomes fairly complicated because an individual firm offers a selection of quantities, Y_k, to sell rather than one particular quantity.[4] As in the standard literature on imperfect competition, assume (i) that sellers take an active role by making their offers. Buyers take a passive role by deciding only whether or not to take offers of sellers. Buyers prefer a lower price. (ii) If a total amount that certain firms offer to sell at a particular price matches that which the buyers desire to purchase, that amount is actually traded at that price. (iii) If the total amount that the buyers desire to purchase at a particular

[4] See Tirole (2000) for the literature on proportionate and efficient rationing in the existing models of Bertrand price competition.

price is not equal to that which firms offer to sell at that price, each agent on the long side get to trade proportionately to the amount that he desire to trade in such a way that the actual total amount of purchase equals to the actual total amount of sale (proportionate rationing). As is explained in the next section, this assumption is adopted so as to describe a rationed demand by a standard Cournot-like residual demand curve. (iv) At each price, the buyers purchase from a combination of firms in such a way that the maximum total amount may be purchased. (v) If there are more than one combinations of firms, the probability with which one such combination is chosen is equal to that with which another such combination is chosen.

This process of rationing is specified inductively by what I call **rationed demand functions** and **rationed purchase functions**. In order to specify these functions, denote by $[\alpha, \beta]$ the closed interval between α and β, by $\max A$ the maximum element in $A \subset R$, and by $\arg\max_x f(x)$ the set of x maximizing $f(x)$. Moreover, let $\mathbf{p}^i$ be the i-th lowest of prices $\mathbf{p} = (p_1, ..., p_N)$ and $\mathbb{K}^i(\mathbf{p})$ be the set of firms that set their respective prices at the i-th lowest price, i.e., $\mathbb{K}^i(\mathbf{p}) = \{k : p_k = \mathbf{p}^i\}$.

The i-th rationed demand function captures the amount that the buyers desire to purchase at the $i+1$-th lowest price, given that the strategy profile chosen by the firms is $(\mathbf{p}, \mathbf{Y})$. It is denoted as $d^{i+1} = d^{i+1}(\mathbf{p}, \mathbf{Y})$. The 0-th rationed market demand function is defined as capturing the amount that the buyers desire to purchase the lowest price, $\mathbf{p}^1$. It is safe to assume that this demand is equal to the unrationed demand $D(p)$ for $p = \mathbf{p}^1$, i.e., that

$$d^1(\mathbf{p}, \mathbf{Y}) = D(\mathbf{p}^1). \tag{2}$$

The i-th purchase function captures the amount that the buyers actually purchase at the i-th lowest price. The i-th purchase function is defined as

$$\delta^i(\mathbf{p}, \mathbf{Y}) = \max_{\mathbb{K} \subset \mathbb{K}^i(\mathbf{p})} \max\left(\sum_{k \in \mathbb{K}} Y_k \cap [0, d^i(\mathbf{p}, \mathbf{Y})]\right) \tag{3}$$

if the maximum is well defined. This implies that at each price level, the buyers purchase the maximum amount that they can purchase. If the maximum in (3) is not well defined, it is assumed that

$$\delta^i(\mathbf{p}, \mathbf{Y}) = 0. \tag{4}$$

This reflects the fact that under the above rationing process, no combination of firms, $\mathbb{K} \subset \mathbb{K}^i(\mathbf{p})$, takes the long side of transactions. In order to demonstrate this fact, suppose that a combination of firms were to take the long side. Then, by assumptions (ii) and (iii), the buyers should be able to purchase $d^i = \sum_{k \in \mathbb{K}} z_k \notin \sum_{k \in \mathbb{K}} Y_k$. However, if so, there would be at least one firm in $\mathbb{K} \subset \mathbb{K}^i(\mathbf{p})$, say k, such that $z_k \notin Y_k$, which firm k would not be willing to sell an amount outside of its quantity offer ($z_k \notin Y_k$). Thus, the amount that the buyers purchase cannot exceed that which they desire to purchase, d^i; in other words, no combination of sellers take the long side. Under assumption (iv), the buyers purchase from firms from which they can purchase the maximum amount.

Since the buyers cannot purchase more than d^i, this amount is described by the upper expression of (3). Moreover, if the maximum in (3) does not exist, by (10), then, the buyers cannot purchase any, because in that case the offer of any combination of firms must exceed the demand, i.e., $\sum_{k\in\mathbb{K}} z_k > d^i(\mathbf{p},\mathbf{Y})$ for any $\sum_{k\in\mathbb{K}} z_k \in \sum_{k\in\mathbb{K}} Y_k$ and $\mathbb{K}\subset\mathbb{K}^i(\mathbf{p})$.

The i-th rationed demand function is defined as

$$d^{i+1}(\mathbf{p},\mathbf{Y}) = \max\{0, D(\mathbf{p}^{i+1}) - \sum_{j=1}^{i} \delta^j(\mathbf{p},\mathbf{Y})\}. \tag{5}$$

This reflects assumption (iii), in which the total amount that is purchased for a price below or equal to $p^i(\mathbf{p})$ is $\delta = \sum_{j=1}^{i} \delta^i(\mathbf{p},\mathbf{Y})$. Under (iii), each agent purchase this amount, his total willingness to pay for d^{i+1} units of the product sold at the $i+1$-th lowest price, $\mathbf{p}^{i+1}$, is $u(d^{i+1}+\delta) - u(\delta)$. Thus, the demand for the product sold at the $i+1$-th lowest price, d^{i+1}, is determined by

$$\begin{cases} u'(d^{i+1}+\delta) = \mathbf{p}^{i+1} & \text{if } u'(\delta) \geq \mathbf{p}^{i+1} \\ d^{i+1} = 0 & \text{if } u'(\delta) < \mathbf{p}^{i+1}. \end{cases} \tag{6}$$

This implies $d^{i+1}+\delta = D(\mathbf{p}^{i+1})$ for $u'(\delta) \geq \mathbf{p}^{i+1}$, which results in (5).

The lemma below shows that if an actual purchase is equal to a rationed demand at a certain price, the rationed demand for the product sold at a higher price is zero.

Lemma 1 *Suppose* $d^i(\mathbf{p},\mathbf{Y}) = \delta^i(\mathbf{p},\mathbf{Y})$. *Then,* $d^{i+1}(\mathbf{p},\mathbf{Y}) = 0$.

Proof. Take the case of $d^i(\mathbf{p},\mathbf{Y}) \neq 0$. Since, by (5), $d^i(\mathbf{p},\mathbf{Y}) = D(\mathbf{p}^i) - \sum_{j=1}^{i-1} \delta^j(\mathbf{p},\mathbf{Y})$, the hypothesis of the lemma implies $D(\mathbf{p}^i) = \sum_{j=1}^{i} \delta^j(\mathbf{p},\mathbf{Y})$. Since $u'(D(\mathbf{p}^i)) = \mathbf{p}^i$ by the definition of $D(p)$, and since $\mathbf{p}^i < \mathbf{p}^{i+1}$, it holds

$$u'(d^{i+1} + D(\mathbf{p}^i)) \leq u'(D(\mathbf{p}^i)) = \mathbf{p}^i < \mathbf{p}^{i+1}. \tag{7}$$

This implies $d^{i+1}(\mathbf{p},\mathbf{Y}) = 0$. Next, take the case of $d^i(\mathbf{p},\mathbf{Y}) = 0$. This implies, by (3), $\delta^i(\mathbf{p},\mathbf{Y}) = 0$ and, by (5) and (6), $u'(\sum_{j=1}^{i-1} \delta^j(\mathbf{p},\mathbf{Y})) \leq \mathbf{p}^i$. These relationships imply (7), again. Thus, $d^{i+1} = 0$. ■

In order to complete the description of the price competition game, define

$$\pi(p_k, Y_k) = p_k y_k - c(y_k),\ y_k \in Y_k \in \mathcal{Y}(p_k); \tag{8}$$

given the structure of $\mathcal{S}$, $\pi : \mathcal{S} \to R$ is a function associating each strategy $s \in \mathcal{S}$ with a particular profit level. Moreover, define $i = i^k(\mathbf{p})$ by $\mathbf{p}^i = p_k$. That is, $i^k(\mathbf{p})$ is the order of firm k's price in the range of offered prices $\mathbf{p}^1, ..., \mathbf{p}^{I(\mathbf{p})}$, where $I(\mathbf{p})$ is the number of different prices in $\mathbf{p}$.

Think of a firm, say k, that chooses the i-th lowest price ($i = i^k(\mathbf{p})$). This firm's offer (p_k, Y_k), in general, may or may not be accepted by the buyers. Factoring this fact in, firm k discounts its profit by the probability with which its offer will be accepted. As is explained below, it is safe to assume that this probability is given by

$$\varphi_k(\mathbf{p}, \mathbf{Y}) = \begin{cases} \frac{\#\{\mathbb{K}\in\mathcal{K}^i(\mathbf{p},\mathbf{Y}):\, k\in\mathbb{K}\}}{\#\mathcal{K}^i(\mathbf{p},\mathbf{Y})} & \text{if } \mathcal{K}^{i^k(\mathbf{p})}(\mathbf{p}, \mathbf{Y}) \neq \phi \\ 0 & \text{if } \mathcal{K}^{i^k(\mathbf{p})}(\mathbf{p}, \mathbf{Y}) = \phi, \end{cases} \tag{9}$$

where $\#A$ the number of elements in set A, and where

$$\mathcal{K}^i(\mathbf{p}, \mathbf{Y}) = \arg\max_{\mathbb{K}\subset\mathbb{K}^i(\mathbf{p})} \max(\sum_{k\in\mathbb{K}} Y_k \cap [0, d^i(\mathbf{p}, \mathbf{Y})]). \tag{10}$$

Under the above rationing process, the group of firms that the buyers actually purchase at the i-th lowest price must be an element of $\mathcal{K}^i(\mathbf{p}, \mathbf{Y})$. Moreover, the probability with which the buyers purchase from the firms in $\mathbb{K} \in \mathcal{K}^i$ is equal to that with which they purchase from those in $\mathbb{K}' \in \mathcal{K}^i$. Thus, from the viewpoint of a firm k that sets the i-th lowest price ($i = i^k(\mathbf{p})$), the probability with which k's offer is accepted is equal to the ratio between the number of elements $\mathbb{K}$ in $\mathcal{K}^i$ that contain k and that of all elements of $\mathcal{K}^i$. This ratio is equal to $\varphi_k(\mathbf{p}, \mathbf{Y})$. The firm maximizes the expected profit, $\varphi_k(\mathbf{p}, \mathbf{Y})\pi(p_k, Y_k)$.

In summary, the price competition game is described by the set of firms (players), $\mathbb{N}$, the space of strategy profiles, $\mathcal{S}^N$, and the return functions on $\mathcal{S}^N$,

$$\Pi_k(\mathbf{p}, \mathbf{Y}) = \varphi_k(\mathbf{p}, \mathbf{Y})\pi(p_k, Y_k),\ k \in \mathbb{N}. \tag{11}$$

A Nash equilibrium in this price competition game is called a price competition equilibrium.

Remark 3: In a price competition equilibrium, $(\mathbf{p}, \mathbf{Y})$, the total supply at each price $\mathbf{p}^i$ is determined by $\delta^i(\mathbf{p}, \mathbf{Y}) \leq d^i(\mathbf{p}, \mathbf{Y})$. In the theorems below, it may be demonstrated that $\delta^{I(\mathbf{p})}(\mathbf{p}, \mathbf{Y}) = d^{I(\mathbf{p})}(\mathbf{p}, \mathbf{Y})$, where $I(\mathbf{p})$ denotes the number of different prices in $\mathbf{p}$. This implies that the demand and the supply are balanced at the highest price in equilibrium.

2.1 Bertrand v.s. Edgeworth

For the sake of explanation, Figure 1 illustrates a standard Bertrand equilibrium, in which two firms, 1 and 2, each sell $y = D(c)/2$ units at the unit price equal to the constant average cost c.

This equilibrium reflects several important features of Bertrand's model. First, firms can charge different prices even though they are selling an identical product. In other words, it is assumed that the law of one price does not hold automatically; this assumption is justified in a market in which transaction costs are too high for anyone to profit from arbitrage. In our setting, this feature is captured by the assumptions that the prices that the firms choose are described by a vector of prices $\mathbf{p} = (p_1, ..., p_N)$ and that the buyers may pay different prices for the same product.

The second feature is that if the price is set at the constant average cost, c, the quantity that the firm desires to sell is not determined uniquely; the set of optimal quantities coincides with the non-negative half line. This fact is captured by the assumption that each firm's strategy is a combination of a price and the set of quantities with respect to which the firm is indifferent to sell at that price, (p_k, Y_k). In particular, under Bertrand's assumption that the marginal cost is constant at p, $p_k = c$ and $(p_k, Y_k) \in \mathcal{S}$ imply $Y_k = R_+$.

The third feature is on the beliefs of the firms when each firm chooses strategy $(p_k, Y_k) = (c, R_+)$. That is, each firm believes (i) that even if it changes its own price, the other firm maintains its current price at c and (ii) that if that firm were to raise its price above c, its customers would be taken away by the other firm, because the other firm is willing to sell any positive quantity. In my model, it may be demonstrates that this standard Bertrand equilibrium can be captured by strategy profile

$$(\mathbf{p}^B, \mathbf{Y}^B) = (p_1^B, p_2^B, Y_1^B, Y_2^B) = (c, c, R_+, R_+) \in \mathcal{S}^2. \tag{12}$$

More precisely, the following results may be demonstrated.

Proposition 1 *Assume that $c(y) = cy$ and that $D(c) < \infty$. Then, strategy profile $(\mathbf{p}^B, \mathbf{Y}^B)$ is a Nash equilibrium in the price competition game. In this equilibrium, the two firms sell in total $D(c)$ units at price c.*

Proposition 2 *Under the hypothesis of the previous theorem, there is no Nash equilibrium other than $(\mathbf{p}^B, \mathbf{Y}^B)$.*

Edgeworth's criticism against Bertrand is constructed on a process of demand rationing that may emerge in the case of a variable average cost. Following Edgeworth's original argument, assume that the maximum that each firm can produce is $\bar{y}$ units. Up to this amount, each firm can produce output at the constant average cost, c. Edgeworth's criticism is that in this case, the state in which each firm sells $\bar{y}$ units at price c cannot be supported as an equilibrium. In the model of this study, this is captured by the fact that a strategy profile

$$(\mathbf{p}^E, \mathbf{Y}^E) = (p_1^E, p_2^E, Y_1^E, Y_2^E) = (c, c, I, I) \in \mathcal{S}^2, \tag{13}$$

where $I = [0, \bar{y}]$, cannot be supported as a price competition equilibrium; note that, in this case, $(c, I) \in \mathcal{S}$ can easily be demonstrated.

If, in this state, firm 1 alone raises its price above c, the buyers will be able to purchase only half of their demand at price c, $\frac{1}{2}D(c) = \bar{y}$. In order to make an additional purchase, therefore, the buyers will be willing to pay a higher price. Thus, by setting a price higher than c, firm 1 can sell a positive amount, from which the firm can obtain a positive profit. Thus, there is no reason why each firm desires to maintain its price at c.

This may be summarized as follows.

Proposition 3 (*Edgeworth's Criticism*) *Let $N = 2$, and $c(y) = cy$ for $0 \le y \le \bar{y}$ and $c(y) = \infty$ for $y > \bar{y}$. If $\bar{y} < D(c) \le 2\bar{y}$, $(\mathbf{p}^E, \mathbf{Y}^E)$ is not a Nash equilibrium.*

This study describes the rationed demand by a Cournot-like residual demand curve, i.e., curve D^R in Figure 1, although it is possible to establish much the same results as those in this study under different specifications of a proportionate rationing process. A Cournot-like representation is justified by the assumption of a continuum of identical buyers who can purchase the same amount as one another. In that case, the amount that each buyer can purchase at price c is equal to $\bar{y}$ unit, for the total mass of buyers is normalized to be 1. Thus, his willingness to pay for an amount additional to $\bar{y}$ is described by the part of demand curve D to the right of the vertical line through $\bar{y}$. This rationed demand curve is described by curve D^R in Figure 1.

3 Intuitive Illustration of the Results

The main concern of this study is price competition in a long run market, in which potential entrants can freely enter in the market. Following the literature on long-run perfect competition, assume that all firms have an identical U-shaped average cost curve and that the demand curve $D(p)$ at least once in the positive orthant of the price-quantity space.

Assumption 1: *Function c satisfies that $c(0) = 0$, $c'(y) > 0$ and $c'''(y) > 0$ for all $y \geq 0$ and that there is* a $y^{LR} > 0$ *such that* $c''(y^{LR}) = 0$.
Assumption 2: *There is at least one (p, y) satisfying $D(p) = y > 0$ and $c(y)/y = p$.*

Let $p^{LR} = c'(y^{LR})$. As is shown in Figures 2 and 3, (p^{LR}, y^{LR}) is the point at the bottom of the average cost curve, AC. Moreover, define $\rho^{LR} = D(c'(y^{LR}))/y^{LR} = \rho(y^{LR})$, which I call the long-run market-to-firm ratio. Let N^{LR} be the largest integer not exceeding ρ^{LR}. Define (p^{AC}, y^{AC}) as the point at the intersection between the average cost curve, AC, and a Cournot-like demand curve,

$$D^R = D(y) - N^{LR}y^{LR}. \tag{14}$$

This demand is illustrated by curve D^R for the case of $N^{LR} = 1$ in Figure 2A and for the case of $N^{LR} = 2$ in Figure 3.

3.1 Contestability: A Game Theoretic Representation

A typical contestable market equilibrium is a "long-run perfectly competitive equilibrium" with a small number of sellers, in which each firm sets its price at the bottom of the average cost curve, and Demsetz's equilibrium, in which a natural monopoly adopts the average cost pricing. As is shown below, these equilibria can be supported as a Nash equilibrium in the price competition game of this study.

What I call a **generalized Bertrand equilibrium** holds if the long-run market-to-firm ratio, $\rho^{LR} = D(p^{LR})/y^{LR}$, is a positive integer and if the number of firms is larger than this ratio ($N > \rho^{LR}$). This equilibrium is characterized by a strategy profile

$$(\mathbf{p}^{GB}, \mathbf{Y}^{GB}) = (p^{LR}, ..., p^{LR}, Y^{LR}, ..., Y^{LR}) = (\mathbf{p}^{LR}, \mathbf{Y}^{LR}) \in \mathcal{S}^N$$

with

$$N > \rho^{LR} = N^{LR} \tag{15}$$

where $Y^{LR} = \{0, y^{LR}\}$ (see Theorem 1 and its proof below).

Figure 2A illustrates a generalized Bertrand equilibrium for the case of $N = 3$ and $\rho^{LR} = 2$. In this equilibrium, two of the firms, say 1 and 2, each sell y^{LR} units at p^{LR}; these firms may be thought of as "incumbents." The third firm, 3, chooses price p^{LR} but sells none; this firm may be interpreted as a "potential entrant." The profit of each firm is zero. The market demand is equal to $D(p^{LR}) = 2y^{LR}$.[5]

In order to explain that $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is a Nash equilibrium, it is important at the outset to note that (p^{LR}, Y^{LR}) is in fact a strategy profile in the present model, i.e., $(p^{LR}, Y^{LR}) \in \mathcal{S}$. This is because (p^{LR}, y^{LR}) is at the bottom of the average cost curve and because, by assumption, the total cost is zero at $y = 0$, i.e., $c(0) = 0$. As a result, if a firm sets its price at p^{LR}, the profit is zero at either $y = 0$ or $y = y^{LR}$ and negative at any other output levels. Thus, by the definition of $\mathcal{Y}(p)$, $Y^{LR} = \mathcal{Y}(p^{LR})$, or, in other words, $(p^{LR}, Y^{LR}) \in \mathcal{S}$.

It follows from this that the profit of each firm, either an incumbent or a potential firm, is zero. That is, since $\pi(p^{LR}, Y^{LR}) = 0$, by (9), (10), and (11),

$$\Pi_k(\mathbf{p}^{GB}, \mathbf{Y}^{GB}) = \pi(p^{LR}, Y^{LR}) = 0. \tag{16}$$

This implies that, in order to demonstrate that $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is an equilibrium strategy profile, no firm can make a positive profit by changing its strategy (p^{LR}, Y^{LR}). In order to make a positive profit, a firm, say k, must set its price above p^{LR}, which is the minimum possible average cost; i.e., it must choose a strategy profile (p_k, Y_k) such that $p_k > p^{LR}$. However, if a firm sets its price above p^{LR}, it will fail to attract customers, as is shown below.

Intuitively, this fact may be illustrated in Figure 2A. For the potential entrant, 3, the market has already been taken by firms 1 and 2. The set of quantities that firms 1 and 2 are willing to sell, $Y_1^{GB} + Y_2^{GC}$, is $\{0, y^{LR}, 2y^{LR}\}$, which is indicated by points P_0, P_1 and P_2. This set contains the demand at p^{LR}, $D^{LR} = D(p^{LR})$. This implies that there is no room for firm 3 to start selling and to make a positive profit. For an incumbent, either 1 or 2, the market would be taken away by the potential entrant if an incumbent raises its price above p^{LR}; if firm 1 were to decide not to supply at P_1 by raising its price, firm 3 would fill that point by selling y^{LR} at p^{LR}. In this case as well, the set of quantities that the buyers can choose from is shown by points P_0, P_1 and P_2. In

[5] In the literature on perfect comptition, strategy profile $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ can be supported as a long-run equilibrium if the firm-to-market ratio is a positive integer, $\rho^{LR} = N^{LR}$. Unlike in a contestable market, this result is based on the implicit assumption that N^{LR} is large.

short, in the state illustrated in Figure 2A, the market is completely captured by two of the three firms in $\mathbb{N} = \{1, 2, 3\}$.

The rationing process of this study is set up in such a way that it reflects this process. For the sake of explanation, denote by $\#\mathbb{K}$ the number of elements in $\mathbb{K}$. The following lemma characterizes the first rationed purchase and demand functions, δ^1 and d^2, for the case in which the lowest price on the market is $\mathbf{p}^1 = p^{LR}$ and in which the firms that set the lowest price adopt (p^{LR}, Y^{LR}).

Lemma 2 *Suppose that* $(\mathbf{p}, \mathbf{Y})$ *satisfies that* $(p_k, Y_k) = (p^{LR}, Y^{LR})$ *for every* $k \in \mathbb{K}^1(\mathbf{p})$. *Let* $\hat{N} = \min\{N^{LR}, \#\mathbb{K}^1(\mathbf{p})\}$. *Then, the following holds:*

$$d^1(\mathbf{p}, \mathbf{Y}) = D(p^{LR}); \tag{17}$$

$$\delta^1(\mathbf{p}, \mathbf{Y}) = \hat{N} y^{LR}; \tag{18}$$

$$d^2(\mathbf{p}, \mathbf{Y}) = \max\{0, D(\mathbf{p}^2) - \hat{N} y^{LR}\}. \tag{19}$$

Proof. Since $\mathbf{p}^1 = p^{LR}$, (17) follows from (2). Under the hypothesis of the lemma, $\mathbb{K} \subset \mathbb{K}^1(\mathbf{p})$ implies that

$$\sum_{k \in \mathbb{K}} K_k = \{0, y^{LR}, 2y^{LR}, ..., K y^{LR}\},$$

where $K = \#\mathbb{K}$. Since $N^{LR} = \max_{K \in \mathbb{N}}\{K \leq \rho^{LR}\}$, by (3),

$$\delta^1(\mathbf{p}, \mathbf{Y}) = \max_{K \leq \#\mathbb{K}^1(\mathbf{p})} \max(\{0, ..., K y^{LR}\} \cap [0, D(p^{LR})]) = \hat{N} y^{LR},$$

which establishes (18). Thus, (19) follows from (5). ■

Now, suppose that firm k changes its strategy to $(p_k, Y_k) \in \mathcal{S}$ with $p_k > p^{LR}$ and that the other firms maintain their strategy (p^{LR}, Y^{LR}). In this case, firm k offers the second lowest (highest) price, and the lowest price is $(p_k, \mathbf{p}^{GB}_{\sim k})^1 = p^{LR}$. Thus, the market demand at the lowest price is, by (2),

$$d^1(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) = D(p^{LR}). \tag{20}$$

The firms offering the lowest price, p^{LR}, are captured by $\mathbb{K}^1(p_k, \mathbf{p}^{GB}_{\sim k}) = \mathbb{N} \backslash \{k\}$, and their number is $\#\mathbb{K}^1(p_{\sim k}, \mathbf{p}^{GB}_{\sim k}) = 2$. Since, by assumption, $\rho^{LR} = N^{LR} = 2$, $\hat{N} = 2$. This implies that the first rationed purchase is, by (18),

$$\delta^1(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) = 2y^{LR}. \tag{21}$$

Since $D(p^{LR}) = 2y^{LR}$, by (20) and (21), Lemma 1 implies that the rationed demand function is

$$d^2(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) = 0 \text{ for } p_k > p^{LR}. \tag{22}$$

This implies that a firm cannot sell any by raising its price above p^{LR}.

In order to prove that $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is a Nash equilibrium, by (16), it suffices to prove $\Pi_k(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) \leq 0$ for any $(p_k, Y_k) \in \mathcal{S}$. Suppose not. Then, there is $(p_k, Y_k) \in \mathcal{S}$ such that $\Pi_k(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) > 0$. This implies, by (8) and (9), $\mathcal{K}^2(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) \neq \phi$ and $\pi(p_k, Y_k) > 0$. The latter implies $p_k > p^{LR}$ and $0 \notin Y_k$. Thus, $\mathbb{K}^2(p_k, \mathbf{p}^{GB}_{\sim k}) = \{k\}$. Thus, by (5),

$$\max_{\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^{GB}_{\sim k})} \max(\sum_{k' \in \mathbb{K}} Y_{k'} \cap [0, d^2(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k})]) = Y_k \cap [0, 0]. \tag{23}$$

However, since $0 \notin Y_k$, this maximum is not well defined. This contracts $\mathcal{K}^2(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) \neq \phi$. Thus, the market illustrated in Figure 2A is in Nash equilibrium.

An important condition for supporting $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ as a Nash equilibrium is that the number of firms, N, is larger than the long-run market-to-firm ratio, ρ^{LR}. If this number is not larger than ρ^{LR}, it may not be supported as a Nash equilibrium. For the case of $N = 2$ and $\rho^{LR} = 2$, this fact can be illustrated in Figure 2A. In this case, it holds that $\#\mathbb{K}^1(p_{\sim k}, \mathbf{p}^{GB}_{\sim k}) = 1$. Since $\hat{N} = 1$ in this case, by (18), it holds

$$d^2(p_k, Y_k, \mathbf{p}^{GB}_{\sim k}, \mathbf{Y}^{GB}_{\sim k}) = \max\{0, D(p_k) - y^{LR}\} \text{ for } p_k > p^{LR}. \tag{24}$$

This shows that, by raising $p_k > p^{LR}$, firm k faces the Cournot-like demand curve, $D^R = D(p) - y^{LR}$, illustrated the dotted demand curve, D^R. Because this curve, D^R, lies above the average cost curve at P, it is possible for firm k to make a positive profit by choosing a price above P. Thus, $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is not a Nash equilibrium if $N = 2$.

What I call a **Demsetz equilibrium** holds if the long-run market-to-firm ratio, ρ^{LR}, is smaller than 1 and if the number of firms is larger than 2 ($N > 2$). Since $\rho^{LR} < 1$, $N^{LR} = 0$. Thus, the Cournot-like demand curve, (14), coincides with the market demand curve, $D^R = D(p)$. It intersects the average cost curve AC on its downward-sloping part, (p^{AC}, y^{AC}). A Demsetz equilibrium is described by a strategy profile

$$(\mathbf{p}^D, \mathbf{Y}^D) = (p^{AC}, ..., p^{AC}, Y^{AC}, ..., Y^{AC})$$

with

$$\rho^{LR} < 1 \text{ and } N > 2, \tag{25}$$

where $Y^{AC} = \{y | c(y) = p^{AC} y\} = \{0, y^{AC}, \hat{y}\}$ (see Theorem 2 and its proof below). As is shown in Figure 2B, $\hat{y}$ is at the other intersection between the average cost curve and the horizontal line through (p^{AC}, y^{AC}).

Figure 2B illustrates a Demsetz equilibrium for the case of $N = 2$. In this case, only one firm, say 1, sells y^{AC} units at p^{AC}; the other firm, 2, sells none. The market demand is equal to $D(p^{AC}) = y^{AC}$. That only one firm sells at the average cost price, p^{AC}, implies that the Demsetz equilibrium, $(\mathbf{p}^D, \mathbf{Y}^D)$, captures the case of a natural monopoly, with which Demsetz (1968) is concerned.

In the Demsetz equilibrium illustrated in Figure 2B, the incumbent complete captures the market by adopting the average cost price, p^{AC}. The set of quantities that the incumbent, 1, is willing to sell is $Y_1^D = \{0, y^{AC}, \hat{y}\}$, which is illustrated by points Q_0, Q_1, and Q_2 in Figure 2B; the set of quantities that 1 and 2 are willing to sell is illustrated by points Q_0 through Q_5. For the potential entrant, 2, the market has already been taken by firm 1 supplying y^{AC} at p^{AC} on point P_1. Thus, the potential entrant would not be able to sell any if it sets its price above p^{AC}. For the incumbent, 1, the market would be taken away by the potential entrant if it raises its price above p^{AC}; if firm 1 were to raise its price, the buyers can purchase from firm 2 at the same price; the set of quantities that 2 is willing to sell is captured by P_0, P_1 and P_2.

It follows from this that $(\mathbf{p}^D, \mathbf{Y}^D)$ is a Nash equilibrium. In order to explain that $(\mathbf{p}^D, \mathbf{Y}^D)$ is a Nash equilibrium, note $(p^{AC}, Y^{AC}) \in \mathcal{S}$ and $\pi(p^{AC}, Y^{AC}) = 0$. In this state, either the incumbent or the potential entrant has no incentive to change its strategy (i.e., the market is in Nash equilibrium). In order to describe this fact in the present model, suppose that firm k, changing its strategy, can make a positive profit, i.e., $\Pi_k(p_k, Y_k, \mathbf{p}^{DP}_{\sim k}, \mathbf{Y}^{DP}_{\sim k}) > 0$. Then, $p_k > p^{AC}$, as Figure 2B illustrates. In this case, $(p_k, \mathbf{p}^D_{\sim k})^1 = p^{AC}$ and $(p_k, \mathbf{p}^D_{\sim k})^2 = p_k$. Thus, by (2),

$$d^1(p_k, Y_k, \mathbf{p}^D_{\sim k}, \mathbf{Y}^D_{\sim k}) = D(p^{AC}). \tag{26}$$

Since the set of firms charging the lowest price is $\mathbb{K}^1(p_k, \mathbf{p}^D_{\sim k}) = \{1, 2\} \backslash \{k\}$, $\mathbb{K} \subset \mathbb{K}^1(p_k, \mathbf{p}^D_{\sim k})$ implies that

$$\sum_{k \in \mathbb{K}} K_k = \{0, y^{AC}, \hat{y}\}. \tag{27}$$

Since $y^{AC} = D(p^{AC})$, by (3), (27) implies

$$\delta^1(p_k, Y_k, \mathbf{p}^D_{\sim k}, \mathbf{Y}^D_{\sim k}) = \max(\{0, y^{AC}, \hat{y}\} \cap [0, D(p^{AC})]) = y^{AC}. \tag{28}$$

Thus, by (26) and (28), Lemma 1 implies

$$d^2(p_k, Y_k, \mathbf{p}^D_{\sim k}, \mathbf{Y}^D_{\sim k}) = 0.$$

By this, it may be demonstrated in much the same way as above that firm k cannot make a positive profit. Thus, $(\mathbf{p}^D, \mathbf{Y}^D)$ is a Nash equilibrium.

3.2 Non-Integer Problem

As Willig (1987) points out, one difficulty in both contestable and long-run perfectly competitive models is that an equilibrium does not exist in the case in which the ratio between the market demand and an individual firm's supply is not an integer (the market-to-firm ratio). One contribution of this study is to demonstrate that the price competition game of the present study does not suffer from this difficulty, which is often referred to as the non-integer problem. In other words, a Nash equilibrium exists in the price competition game even if the firm-to-market ratio is a non-integer.

What I call a **single price equilibrium**, or a single price solution to the non-integer problem, emerges if the market-to-firm ratio at price p^{LR}, ρ^{LR}, is a non-integer larger than 1, if the rationed demand curve, $D^R = D(p) - N^{LR}y^{LR}$, lies below the average cost curve, AC, and if the number of firms is larger than this ratio ($N > \rho^{LR}$). This equilibrium is characterized by a strategy profile

$$(\mathbf{p}^{SP}, \mathbf{Y}^{SP}) = (p^{LR}, ..., p^{LR}, Y^{LR}, ..., Y^{LR}) \in \mathcal{S}^N$$

with

$$N > \rho^{LR} > N^{LR} \geq 1. \tag{29}$$

Figure 3A illustrates a single price equilibrium for the case of $N = 3$ and $2 < \rho^{LR} < 3$. In this case, as in the generalized Bertrand equilibrium above, two of the firms, say 1 and 2, each sell y^{LR} units at p^{LR}. The third firm, 3, sells none. The market demand is equal to $D(p^{LR}) > 2y^{LR}$. Since $N^{LR} = 2$, the rationed demand curve becomes $D^R = D(p) - 2y^{LR}$, which must lie below the average cost curve in a single price equilibrium.

As in the case of a generalized Bertrand equilibrium, the set of quantities that two of the three firms are willing to sell at p^{LR} is illustrated by points P_0, P_1, and P_2 in Figure 3A. Because the supply at p^{LR} is smaller than the demand, $D^{LR} = D(p^{LR})$, an individual firm faces a positive rationed demand curve, D^R, by raising its price above p^{LR}. However, this rationed demand is too small for the firm to make a positive profit, because demand curve D^R lies below the average cost curve, AC.

In the model of this study, this fact is incorporated in the following way. Since $\hat{N} = N^{LR}$, in the case of Figure 3A, the rationed demand becomes, by (17) and (18),

$$d^2(p_k, Y_k, \mathbf{p}^{SP}_{\sim k}, \mathbf{Y}^{SP}_{\sim k}) = \max\{0, D(p_k) - 2y^{LR}\} \text{ for } p_k > p^{LR}, \tag{30}$$

which lies below the average cost curve. If $(\mathbf{p}^{SP}, \mathbf{Y}^{SP})$ is not a Nash equilibrium, there is $(p_k, Y_k) \in \mathcal{S}$ such that $\Pi_k(p_k, Y_k, \mathbf{p}^{SP}_{\sim k}, \mathbf{Y}^{SP}_{\sim k}) > 0$. Then, by (11), $\pi(p_k, Y_k) > 0$. This implies $p_k > p^{LR}$, and that $0 \notin Y_k$. Since $i^k(p_k, \mathbf{p}^{LR}_{\sim k}) = 2$ and $\mathbb{K}^2(p_k, \mathbf{p}^{LR}_{\sim k}) = \{k\}$, $\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^{LR}_{\sim k})$ implies

$$\max(\sum_{k' \in \mathbb{K}} Y_{k'} \cap [0, d^2(p_k, Y_k, \mathbf{p}^{SP}_{\sim k}, \mathbf{Y}^{SP}_{\sim k})]) = Y_k \cap [0, D(p_k) - 2y^{LR}] \neq \phi.$$

By $0 \notin Y_k$, this implies that there is $y_k \in Y_k$ such that $0 < y_k \leq D(p_k) - 2y^{LR}$. Since $D(p_k) - 2y^{LR}$ lies below the average cost curve, this implies $\pi(p_k, Y_k) \leq 0$, a contradiction.

What I call a **dual price equilibrium**, or a dual price solution to the non-integer problem, emerges if the long-run market-to-firm ratio, ρ^{LR}, is a non-integer larger than 1, if the rationed demand curve, $D^R = D(p) - N^{LR}y^{LR}$, intersects the average cost curve, AC, and if the number of firms exceeds this ratio by more than 2 ($N > \rho^{LR} + 2$). This equilibrium is characterized by a strategy profile

$$(\mathbf{p}^{DP}, \mathbf{Y}^{DP}) = (\mathbf{p}^{LR}, \mathbf{p}^{AC}, \mathbf{Y}^{LR}, \mathbf{Y}^{AC}) \in \mathcal{S}^N$$

where

$$\begin{aligned} (\mathbf{p}^{LR}, \mathbf{Y}^{LR}) &\in \mathcal{S}^{N'},\ (\mathbf{p}^{AC}, \mathbf{Y}^{AC}) \in \mathcal{S}^{N''}, \\ N' + N'' &= N,\ N' > N^{LR}, \text{ and } N'' > 1. \end{aligned} \tag{31}$$

Figure 3B illustrates a dual price equilibrium for the case of $N = 5$ and $2 < \rho^{LR} < 3$. In this equilibrium, as in the generalized Bertrand equilibrium above, two firms, say 1 and 2, each sell y^{LR} units at p^{LR}. The third firm, 3, chooses p^{LR} but sells none. The fourth firm, 4, sells y^{AC} at p^{AC}. Finally, the fifth firm chooses p^{AC} but sells none. The profit of each firm is zero. The market demand is equal to $D(p^{LR}) > 2y^{LR}$, and the rationed demand is equal to $D(p^{AC}) - 2y^{LR} = y^{AC}$.

The set of quantities that two of the firms 1, 2, and 3 are willing to sell at p^{LR} is captured by points P_0, P_1, and P_2. That which firms 1, 2, and 3 are willing to sell is captured by points P_0 through P_3. The set of quantities that on of the firms 4 and 5 is willing to sell is captured by points Q_0, Q_1, and Q_2; that which firms 4 and 5 are willing to sell is captured by points Q_0 through Q_5.

That $(\mathbf{p}^{DP}, \mathbf{Y}^{DP})$ is supported as an equilibrium can be explained by two reasons. (i) Although an individual firm faces a positive (rationed) demand by raising its price to a level between p^{LR} and p^{AC}, the demand will be too small for the firm to make a positive profit. (ii) If it raises its price to a level above (or equal to) p^{AC}, it will have no demand. These facts can be explained in much the same way as in the cases of a single price equilibrium and a Demsetz equilibrium.

For the sake of explanation, suppose that firm k, changing its strategy, can make a positive profit, i.e., $\Pi_k(p_k, Y_k, \mathbf{p}^{DP}_{\sim k}, \mathbf{Y}^{DP}_{\sim k}) > 0$. If $p_k \le p^{AC}$, it may be demonstrated in much the same way as in the single price equilibrium that firm k's price cannot make a positive profit. Thus, it must hold that $p_k > p^{AC}$. In this case, $(p_k, \mathbf{p}^{DP}_{\sim k})^1 = p^{LR}$, $(p_k, \mathbf{p}^{DP}_{\sim k})^2 = p^{AC}$, and $(p_k, \mathbf{p}^{DP}_{\sim k})^3 = p_k$. Since $\hat{N} = 2$, it follows from (30) that the rationed demand at p^{AC} is

$$d^2(p_k, Y_k, \mathbf{p}^{DP}_{\sim k}, \mathbf{Y}^{DP}_{\sim k}) = D(p^{AC}) - 2y^{LR}. \tag{32}$$

Since the set of firms charging the second lowest price is $\mathbb{K}^2(p_k, \mathbf{p}^{DP}_{\sim k}) = \{4, 5\}\backslash\{k\}$, $\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^{DP}_{\sim k})$ implies that

$$\sum_{k \in \mathbb{K}} K_k = \{0, y^{AC}, \hat{y}\} \text{ or } \{0, y^{AC}, \hat{y}, 2y^{AC}, y^{AC} + \hat{y}, 2\hat{y}\}. \tag{33}$$

Since $y^{AC} = D(p^{AC}) - 2y^{LR}$, by (3), (33) implies

$$\delta^2(p_k, Y_k, \mathbf{p}^{DP}_{\sim k}, \mathbf{Y}^{DP}_{\sim k}) = \max_{\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^{DP}_{\sim k})} \max(\sum_{k \in \mathbb{K}} K_k \cap [0, D(p^{AC}) - 2y^{LR}]) = y^{AC}. \tag{34}$$

Thus, by Lemma 1,

$$d^3(p_k, Y_k, \mathbf{p}^{DP}_{\sim k}, \mathbf{Y}^{DP}_{\sim k}) = 0. \tag{35}$$

By this, it may be demonstrated in much the same way as above that firm k cannot make a positive profit. Thus, $(\mathbf{p}^{DP}, \mathbf{Y}^{DP})$ is a Nash equilibrium.

In a two-stage game, Perry (1984) and Canoy (1994) investigate the possibility that a natural monopoly may sell a single product at two different prices to deter entry. This study is similar to their work in demonstrating that a dual price equilibrium may result from contestability. It differs from their work in that different prices are set by different firms in the present model.

4 Theorems and Some Discussions

The main results of this study can be summarized as follows. All the proofs are given in the Appendix.

Theorem 1 (***Generalized Bertrand Equilibrium***) *Suppose that the firm-to-market ratio ρ^{LR} is a positive integer ($\rho^{LR} = N^{LR}$) and that the number of firms N is larger than the firm-to-market ratio ($N > \rho^{LR}$). Strategy profile $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is a Nash equilibrium in the price competition game. In this equilibrium, $N^{LR} = \rho^{LR}$ firms each sell y^{LR} units at p^{LR}.*

Theorem 2 (***Demsetz's Equilibrium***) *Suppose that the firm-to-market ratio ρ^{LR} is a real number between 0 and 1 ($0 < \rho^{LR} < 1$) and that the number of firms N exceeds the firm-to-market ratio at least by 1 (i.e., $N > \rho^{LR} + 1$). Strategy profile $(\mathbf{p}^{D}, \mathbf{Y}^{D})$ is a Nash equilibrium in the price competition equilibrium. In this equilibrium, only one firm sells y^{AC} units at price p^{AC}.*

Theorem 3 (***Single Price Equilibrium***) *Suppose that the firm-to-market ratio ρ^{LR} is a non-integer exceeding 1 ($\rho^{LR} \neq N^{LR}$ and $\rho^{LR} > 1$) and that the number of firms N is larger than the firm-to-market ratio ($N > \rho^{LR}$). If $u'(y + N^{LR}y^{LR}) < c(y)/y$ for all $y \geq 0$, strategy profile $(\mathbf{p}^{GB}, \mathbf{Y}^{GB})$ is a Nash equilibrium in the price competition game. In this equilibrium, N^{LR} firms each sell y^{LR} units at price p^{LR}.*

Theorem 4 (***Dual Price Equilibrium***) *Let $\mathbb{N}' = \{1, ..., N'\}$ and $\mathbb{N}'' = \{N' + 1, ..., N\}$. Suppose that the firm-to-market ratio ρ^{LR} is a non-integer exceeding 1 ($\rho^{LR} \neq N^{LR}$ and $\rho^{LR} > 1$), that the number of firms in $\mathbb{N}'$ is larger than the firm-to-market ratio ($N' > \rho^{LR}$), and that the number of firms in $\mathbb{N}''$ is larger than 1 ($N'' \geq 2$). If there is y such that $u'(y + N^{LR}y^{LR}) > c(y)/y$ for all $y \geq 0$, strategy profile $(\mathbf{p}^{DP}, \mathbf{Y}^{DP})$ is a Nash equilibrium in the price competition game. In this equilibrium, N^{LR} firms each sell y^{LR} units at price p^{LR}, and one firm sells y^{R} units at price p^{R}.*

The results of this section needs to be carefully interpreted. First of all, I do not intend to argue that the dual price equilibrium characterized by Theorem

7 is a realistic description of real world markets. In order for a dual price equilibrium to be sustainable, several conditions need to be satisfied.

First of all, in order for the price differential existing in a dual price equilibrium to be sustainable, there must exist some strong factors that prevent arbitrage. In the literature on both Bertrand competition and contestability, the presence of such factors is commonly assumed (so as to justify Bertrand's assumption that each individual firm can set a price different from other firms' prices).

The second condition is the absence of another technology that can achieve the minimum average cost q^{LR} lower than p^R at an output level z^{LR} smaller than y^R. If such a technology exists, there is a strong incentive for potential entrants to choose that technology. In that case, there may be an equilibrium in which a few large firms, adopting price p^{LR}, and many small firms, adopting price q^{LR}, coexist in an equilibrium. Such an equilibrium is analyzed elsewhere (see Yano, 2004).

While a dual price may emerge only in limited circumstances, a single price equilibrium may be more likely to emerge. In such an equilibrium, although the demand exceeds the supply, the residual demand is too small (the first rationed demand curve is located too low) so that there is no incentive for potential entrants. For example, such a situation is sometimes observed in local markets in which no new stores enter the market despite that merchandises are sold out in the existing stores by a certain time of a day.

In the same manner as in Theorem 2, uniqueness can be proved for the rest of the equilibria presented in this study. However, proofs are not given here, for they require highly complicated and lengthy arguments (see Yano, 2003, for a proof).

The results of this study are based on a process of proportional rationing. Whether or not they can be extended to a model under efficient rationing is an interesting question, particularly because it has been well known that a choice of a rationing process crucially affects an equilibrium in a two-stage quantity-price game (Kreps and Scheinkman (1983) and Davidson and Deneckere (1986)).

Appendix

In this Appendix, I will prove Propositions 1 through 3 and Theorems 1 through 4. In order to simplify the discussion below, define

$$\theta^i(\mathbf{p}, \mathbf{Y}, \mathbb{K}) = \max(\sum_{k \in \mathbb{K}} Y_k \cap [0, d^i(\mathbf{p}, \mathbf{Y})]). \tag{36}$$

Then, it follows from (3) that

$$\delta^i(\mathbf{p}, \mathbf{Y}) = \max_{\mathbb{K} \subset \mathbb{K}^i(\mathbf{p})} \theta^i(\mathbf{p}, \mathbf{Y}, \mathbb{K}). \tag{37}$$

In order to simplify notation, adopt the following:

$$d^i_*(p_k, Y_k) = d^i((p_k, \mathbf{p}^*_{\sim k}), (Y_k, \mathbf{Y}^*_{\sim k}));$$

$$\mathcal{K}^i_*(p_k, Y_k) = \mathcal{K}^i((p_k, \mathbf{p}^*_{\sim k}), (Y_k, \mathbf{Y}^*_{\sim k}));$$
$$\theta^i_*(p_k, Y_k, \mathbb{K}) = \theta^i((p_k, \mathbf{p}^*_{\sim k}), (Y_k, \mathbf{Y}^*_{\sim k}), \mathbb{K})$$
$$\delta^i_*(p_k, Y_k) = \delta^i((p_k, \mathbf{p}^*_{\sim k}), (Y_k, \mathbf{Y}^*_{\sim k})).$$

Proof of Proposition 1: $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^B, \mathbf{Y}^B)$. Since $\pi(p^*_k, Y^*_k) = \pi(c, R_+) = 0$, by (11), $\Pi^*_k = \Pi_k(p^*_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = 0$ for $k = 1, 2$. Suppose that $(\mathbf{p}^*, \mathbf{Y}^*)$ is not a Nash equilibrium. Then, by $\Pi^*_k = 0$, there is $(p_k, Y_k) \in \mathcal{S}$ such that $\Pi_k(p_k, Y_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) > 0$. By (11) and (9), this implies $\pi(p_k, Y_k) > 0$ and $\mathcal{K}^2_*(p_k, Y_k) = \{\{k\}\}$, since $\pi(p_k, Y_k) > 0$ implies $p_k > p^*_{k'} = c$ and $i^k(p_k, \mathbf{p}^*_{\sim k}) = 2$, where $\{k'\} = \{1, 2\}\backslash\{k\}$. Since $p^1(p_k, \mathbf{p}^*_{\sim k}) = c$, by (2), $d^1_*(p_k, Y_k) = D(c)$. Since $Y^*_{k'} = R_+$, by (36), $\theta^1_*(p_k, Y_k, \mathbb{K}) = \max(Y^*_{k'} \cap [0, D(c)]) = D(c)$ for any non-empty $\mathbb{K} \subset \mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k}) = \{k'\}$. Thus, by (37), $\delta^1_*(p_k, Y_k) = D(c)$. Since this implies $d^1_*(p_k, Y_k) = \delta^1_*(p_k, Y_k)$, by Lemma 1, $d^2_*(p_k Y_k) = 0$. Since $\mathcal{K}^2_*(p_k, Y_k) = \{\{k\}\}$, by (36), $Y_k \cap [0, d^2_*(p_k Y_k)] = Y_k \cap \{0\} \neq \phi$. Since this implies $0 \in Y_k$, $\pi(p_k, Y_k) = 0$, a contradiction. Thus, $(\mathbf{p}^B, \mathbf{Y}^B)$ is a Nash equilibrium.

In this equilibrium, since $\mathbb{K}^1(\mathbf{p}^*) = \{1, 2\}$ and $Y^*_1 + Y^*_2 = R_+$, $\theta^1(p^*_1, Y^*_1, \{1, 2\}) = D(c)$. This implies $\delta^1(\mathbf{p}^*, \mathbf{Y}^*) = D(c)$, which is the amount that the buyers actually purchase at price $p = c$. This proves Proposition 1.

Proof of Proposition 2: In order to prove this proposition, it is useful first to establish several lemmas.

Lemma 3 *Let $p \neq c$. Then, $(p, Y) \in \mathcal{S}$ if and only if $Y = \{y\}$ and $y \in R_+$.*

Proof. This is because $\pi = (p - c)y$ is either increasing or decreasing. ∎

Take an arbitrary strategy profile $(\mathbf{p}^*, \mathbf{Y}^*) = (p^*_1, p^*_2, Y^*_1, Y^*_2) \in \mathcal{S}^2$. Let $\Pi^*_k = \Pi_k(p^*_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k})$. The following holds.

Lemma 4 *If $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium,* $\min\{p^*_1, p^*_2\} \geq c$.

Proof. Suppose not. Then, there are k'' and k' in $\{1, 2\}$ such that $p^*_{k''} < c$ and $p^*_{k''} \leq p^*_{k'}$.

First, I will prove $\Pi^*_{k''} = 0$. Since $p^*_{k''} < c$, by (8), $\pi(p^*_{k''}, Y^*_{k''}) \leq 0$, which implies $\Pi^*_{k''} \leq 0$ by (11). By Lemma 3 and (8), $\pi(p^*_{k''}, \{0\}) = 0$, which implies $\Pi_{k''}(p^*_{k''}, \{0\}, \mathbf{p}^*_{\sim k''}, \mathbf{Y}^*_{\sim k''}) = 0$ by (11). Thus, $\Pi^*_{k''} \geq 0$, since $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. Thus, $\Pi^*_{k''} = 0$.

Suppose $p^*_{k'} \leq c$. Then, in the same way as $\Pi^*_{k''} = 0$, $\Pi^*_{k'} = 0$ can be proved. Under the hypothesis of the theorem (see Theorem 1), there is $p'_{k'} > c$ such that $D(p'_{k'}) > 0$. Let $Y'_{k'} = \{D(p'_{k'})\}$. Then, by Lemma 3, $(p'_{k'}, Y'_{k'}) \in \mathcal{S}$, and by (8), $\pi(p'_{k'}, Y'_{k'}) > 0$. I will prove $\delta^1_*(p'_{k'}, Y'_{k'}) = 0$. Suppose not. Then, by (36) and (37), $\delta^1_*(p'_{k'}, Y'_{k'}) > 0$. Then, by (10), $\mathcal{K}^1_*(p'_{k'}, Y'_{k'}) \neq \phi$. This implies, by $\mathbb{K}^1(p'_{k'}, \mathbf{p}^*_{\sim k'}) = \{k''\}$, $\mathcal{K}^1_*(p'_{k'}, Y'_{k'}) = \{\{k''\}\}$. Thus, by $\delta^1_*(p'_{k'}, Y'_{k'}) > 0$ and (36), $\theta^1_*(p'_{k'}, Y'_{k'}, \{k''\}) > 0$. Since $k'' \in \mathbb{K}^1(\mathbf{p}^*)$ and $k'' \in \mathbb{K}^1(p'_{k'}, \mathbf{p}^*_{\sim k'})$, and since $p^*_{k''} < c$ implies $Y^*_{k''} = \{y^*_{k''}\}$ by Lemma 3, by (2) and (36), $0 <$

$\theta^1_*(p'_{k'}, Y'_{k'}, \{k''\}) = \theta^1(\mathbf{p}^*, \mathbf{Y}^*, \{k''\}) = \max\{Y^*_{k''} \cap [0, D(p^*_{k''})] = y^*_{k''}$. Thus, $y^*_{k''} > 0$ and $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) \neq \phi$ by (10). Suppose $\{k''\} \in \mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*)$. Then, $\varphi^1_{k''}(\mathbf{p}^*, \mathbf{Y}^*) > 0$. This implies, by $y^*_{k''} > 0$, $\Pi^*_{k''} = \varphi^1_{k''}(\mathbf{p}^*, \mathbf{Y}^*)\pi(p^*_{k''}, Y^*_{k''}) < 0$, a contradiction to $\Pi^*_{k''} = 0$. Thus, $\{k''\} \notin \mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*)$. This implies, by $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) \neq \phi$, $\{k'\} \in \mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*)$ by (10). Thus, $i^{k'}(\mathbf{p}^*) = 1$, and $\varphi^1_{k'}(\mathbf{p}^*, \mathbf{Y}^*) > 0$. Since $\{k''\} \notin \mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*)$, $y^*_{k''} = \theta^1(\mathbf{p}^*, \mathbf{Y}^*, \{k''\}) < \delta^1(\mathbf{p}^*, \mathbf{Y}^*) = \theta^1(\mathbf{p}^*, \mathbf{Y}^*, \{k'\}) = y^*_{k'}$. Since, by $y^*_{k''} > 0$, this implies $y^*_{k'} > 0$, $\Pi^*_{k'} = \varphi^1_{k'}(\mathbf{p}^*, \mathbf{Y}^*)\pi(p^*_{k'}, Y^*_{k'}) < 0$, a contradiction to $\Pi^*_{k'} = 0$. Thus, $\delta^1_*(p'_{k'}, Y'_{k'}) = 0$. This implies, by (5), $d^2_*(p'_{k'}, Y'_{k'}) = D(p'_{k'}) > 0$. This implies, by $\mathbb{K}^2(p'_{k'}, \mathbf{p}^*_{\sim k}) = \{k'\}$ and $\{D(p'_{k'})\} = Y'_{k'}$, $\theta^2_*(p'_{k'}, Y'_{k'}, \mathbb{K}) = D(p'_{k'})$ for any $\mathbb{K} \subset \mathbb{K}^2(p'_{k'}, \mathbf{p}^*_{\sim k'}) = \{k'\}$. Thus, $\mathcal{K}^2_*(p'_{k'}, Y'_{k'}) \neq \phi$. This implies, by $\mathbb{K}^2(p'_{k'}, \mathbf{p}^*_{\sim k'}) = \{k'\}$, $\mathcal{K}^2_*(p'_{k'}, Y'_{k'}) = \{\{k'\}\}$. This implies, by $i^{k'}(p'_{k'}, \mathbf{p}^*_{\sim k'}) = 2$ and (11), $\Pi_{k'}(p'_{k'}, Y'_{k'}, \mathbf{p}^*_{\sim k'}, \mathbf{Y}^*_{\sim k'}) = \pi(p'_{k'}, Y'_{k'}) = (p'_{k'} - c)D(p'_{k'}) > 0$. This together with $\Pi^*_{k'} = 0$ contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. This establishes $p^*_{k'} > c$.

Under the hypothesis of Theorem 1, there is $p''_{k''}$ such that $c < p''_{k''} < p^*_{k'}$ and $D(p''_{k''}) > 0$. Let $Y''_{k''} = \{D(p''_{k''})\}$. Then, $(p''_{k''}, Y''_{k''}) \in \mathcal{S}$ and $\pi(p''_{k''}, Y''_{k''}) > 0$. Since $\mathbb{K}^1(p''_{k''}, \mathbf{p}^*_{\sim k''}) = \{k''\}$, by (2), $d^1_*(p''_{k''}, Y''_{k''}) = D(p''_{k''}) > 0$. This implies, by (36) and $\{D(p''_{k''})\} = Y''_{k''}$, $\theta^1_*(p''_{k''}, Y''_{k''}, \mathbb{K}) = D(p''_{k''})$ for any $\mathbb{K} \subset \mathbb{K}^1(p''_{k''}, \mathbf{p}^*_{\sim k''}) = \{k''\}$. This implies, by $\mathbb{K}^1(p''_{k''}, \mathbf{p}^*_{\sim k''}) = \{k''\}$, $\mathcal{K}^1_*(p''_{k''}, Y''_{k''}) = \{\{k''\}\}$. Since $i^{k''}(p''_{k''}, \mathbf{p}^*_{\sim k''}) = 1$, this implies, by (11) and (9), $\Pi_{k''}(p''_{k''}, Y''_{k''}, \mathbf{p}^*_{\sim k''}, \mathbf{Y}^*_{\sim k''}) = \pi(p''_{k''}, Y''_{k''}) > 0$. This together with $\Pi^*_{k''} = 0$ contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. ■

Next, I will prove the following:

Lemma 5 *If* $(\mathbf{p}^*, \mathbf{Y}^*)$ *is a Nash equilibrium,* $\max\{p^*_1, p^*_2\} \leq c$.

Proof. Suppose not. Let $p^*_{k'} \geq p^*_{k''}$. Then, $p^*_{k'} > c$. Suppose $p^*_{k''} = c$. Then, by choosing $(p''_{k''}, Y''_{k''})$ in the same way as in the proof of the previous lemma, a contradiction can be derived. Thus, $c < p^*_{k''} \leq p^*_{k'}$. This implies, by Lemma 3, $Y^*_k = \{y^*_k\}$ for $k \in \{k', k''\}$.

Next, I will prove $D(p^*_{k''}) > 0$. Suppose $D(p^*_{k''}) = 0$. Since, by the hypothesis of Theorem 1, it is again possible to choose $(p''_{k''}, Y''_{k''})$ in the same way as in the previous proof, a contradiction can be derived. Thus, $D(p^*_{k''}) > 0$.

Next, I will prove that $y^*_k > 0$ for $k \in \{k', k''\}$. Suppose $y^*_k = 0$. Then, $\pi(p^*_k, Y^*_k) = 0$ by (8). This implies, by (11), $\Pi^*_k = 0$. Take p''_k satisfying $c < p''_k < p^*_{k''}$ and $D(p''_k) > 0$. Let $Y''_k = \{D(p''_k)\}$. Then, in the same way as in the second part of the previous proof, it may be proved that $\Pi_k(p''_k, Y''_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = \pi(p''_k, Y''_k) > 0$. This together with $\Pi^*_k = 0$ contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. Thus, $y^*_k > 0$.

Suppose $p^*_{k''} < p^*_{k'}$. Let $Y''_{k''} = \{D(p^*_{k''})\}$. By $\mathbb{K}^1(\mathbf{p}^*) = \{k''\}$ and (36), $\theta^1_*(p^*_{k''}, Y''_{k''}, \{k''\}) = D(p^*_{k''}) > 0$, and $\mathcal{K}^1_*(p^*_{k''}, Y''_{k''}) = \{\{k''\}\}$. Thus, by (8),

$$\Pi_{k''}(p^*_{k''}, Y''_{k''}, \mathbf{p}^*_{\sim k''}, \mathbf{Y}^*_{\sim k''}) = (p^*_{k''} - c)D(p^*_{k''}) > 0. \tag{38}$$

Suppose $y^*_{k''} \neq D(p^*_{k''})$. If $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) = \phi$, by (11) and (9), $\Pi^*_{k''} = 0$, which together with (38) contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. Thus,

$\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) \neq \phi$. This implies, by $\mathbb{K}^1(\mathbf{p}^*) = \{k''\}$ and (10), $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) = \{\{k''\}\}$ and, by $y^*_{k''} \neq D(p^*_{k''})$, $y^*_{k''} < D(p^*_{k''})$. Thus, by (8), (11), and (9), $\Pi^*_{k''} = (p^*_{k''} - c)y^*_{k''} < p^*_{k''} - c)D(p^*_{k''})$, which together with (38) contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. This implies $y^*_{k''} = D(p^*_{k''})$. This implies, by (36) and (37), $\theta^1_*(p^*_{k''}, Y^*_{k''}, \{k''\}) = \delta^1(\mathbf{p}^*, \mathbf{Y}^*) = D(p^*_{k''})$. Since, by (2), this implies $\delta^1(\mathbf{p}^*, \mathbf{Y}^*) = d^1(\mathbf{p}^*, \mathbf{Y}^*)$, by Lemma 1, $d^2(\mathbf{p}^*, \mathbf{Y}^*) = 0$. Since $i^{k'}(\mathbf{p}^*) = 2$, this implies $y^*_{k'} = 0$. This implies, by (11) and (9), $\Pi^*_{k'} \leq \pi(p^*_{k'}, Y^*_{k'}) = 0$. Let $c < p'_{k'} < p^*_{k''}$ and $Y'_{k'} = \{D(p'_{k'})\}$. Then, $\mathbb{K}^1(p'_{k'}, \mathbf{p}^*_{\sim k}) = \{k'\}$ and $\theta^1_*(p'_{k'}, Y'_{k'}, \{k'\}) = D(p'_{k'}) > 0$. This implies $\mathcal{K}^1_*(p'_{k'}, Y'_{k'}) = \{k'\}$ and, by (8), (11) and (9), $\Pi_{k''}(p'_{k'}, Y'_{k'}, \mathbf{p}^*_{\sim k'}, \mathbf{Y}^*_{\sim k'}) = (p'_{k'} - c)D(p'_{k'}) > 0$, which together with $\Pi^*_{k'} \leq 0$ contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. Thus, $p^*_{k''} = p^*_{k'} > c$.

This implies $\mathbb{K}^1(\mathbf{p}^*) = \{k', k''\}$. Suppose $\{k', k''\} \in \mathcal{K}^1_*(p^*_1, Y^*_1)$. Then, by (2), (10) and (36), $\theta^1(\mathbf{p}^*, \mathbf{Y}^*, \{k', k''\}) = y^*_{k'} + y^*_{k''} \leq D(p^*_{k'})$. This implies, by $y^*_{k''} > 0$, $D(p^*_{k'}) > y^*_{k'}$. This implies, by (8), (11), and (9), $\pi(p^*_{k'}, \{D(p^*_{k'}\}) > \Pi^*_{k'}$. Let $p'_{k'} = p^*_{k'} - \varepsilon$, $\varepsilon > 0$, and $Y'_{k'} = \{D(p^*_{k'})\}$. Since $D(p^*_{k'}) < D(p'_{k'})$, and since $\{k'\} = \mathbb{K}^1(p'_{k'}, \mathbf{p}^*_{\sim k'})$, $\theta^1_*(p'_{k'}, Y'_{k'}, \mathbb{K}) = D(p^*_{k'})$ for any $\mathbb{K} \subset \mathbb{K}^1(p'_{k'}, \mathbf{p}^*_{\sim k'}) = \{k'\}$. Since this implies $\mathcal{K}^1_*(p'_{k'}, Y'_{k'}) = \{k'\}$, by (8), (11), and (9), $\Pi_{k'}(p'_{k'}, Y'_{k'}, \mathbf{p}^*_{\sim k'}, \mathbf{Y}^*_{\sim k'}) = \pi(p'_{k'}, Y'_{k'}) = (p^*_{k'} - \varepsilon - c)D(p^*_{k'})$. Since $\pi(p^*_{k'}, \{D(p^*_{k'}\}) > \Pi^*_{k'}$, by choosing ε small, $\Pi_{k'}(p'_{k'}, Y'_{k'}, \mathbf{p}^*_{\sim k'}, \mathbf{Y}^*_{\sim k'}) > \Pi^*_{k'}$, which contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. Thus, $\{k', k''\} \notin \mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*)$.

This implies $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) = \{\{k'\}, \{k''\}\}$. Thus, by (9), $\varphi^1_k(\mathbf{p}^*, \mathbf{Y}^*) = 1/2$ for $k = k', k''$. Thus, by (8) and (11),

$$\Pi^*_k = \frac{1}{2}\pi(p^*_k, Y^*_k) = \frac{1}{2}(p^*_k - c)y^*_k > 0. \tag{39}$$

By (36), $\mathcal{K}^1(\mathbf{p}^*, \mathbf{Y}^*) = \{\{k'\}, \{k''\}\}$ implies $\theta^1(\mathbf{p}^*, \mathbf{Y}^*, \{k\}) = y^*_k \leq D(p^*_k)$ for $k \in \{k', k\}$. Let $p'_k = p^*_k - \varepsilon$, $\varepsilon > 0$. Then, $\mathbb{K}^1(p'_k, \mathbf{p}^*_{\sim k}) = \{k\}$. Thus, since $D(p^*_k) < D(p'_k)$, by (36), $\theta^1_*(p'_k, Y^*_k, \{k\}) = y^*_k$ for any $\mathbb{K} \subset \mathbb{K}^1(p'_k, \mathbf{p}^*_{\sim k}) = \{k\}$. Since, by (10), this implies $\mathcal{K}^1_*(p'_k, Y^*_k) = \{k\}$, by (8), (11), and (9),

$$\Pi_k(p'_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = \pi(p'_k, Y^*_k) = (p^*_k - \varepsilon - c)y^*_k. \tag{40}$$

By (39) and (40), $\Pi_k(p'_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) > \Pi^*_k$ for a sufficiently small $\varepsilon > 0$, which contradicts that $(\mathbf{p}^*, \mathbf{Y}^*)$ is a Nash equilibrium. ∎

In order to prove Proposition 2, suppose that there is a Nash equilibrium $(\mathbf{p}^*, \mathbf{Y}^*)$ such that $(\mathbf{p}^*, \mathbf{Y}^*) \neq (\mathbf{p}^B, \mathbf{Y}^B)$. By Lemmas 1 and 2, $p^*_1 = p^*_2 = c$. By (1) and $c(y) = cy$, $\mathcal{Y}(c) = \{R_+\}$. Thus, by $p^*_k = c$, $(p^*_k, Y^*_k) \in \mathcal{S}$ implies $Y^*_k = R_+$ for $k = 1, 2$. This implies $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^B, \mathbf{Y}^B)$, a contradiction. This establishes Proposition 2.

Proof of Proposition 3: By setting $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^E, \mathbf{Y}^E)$, adopt the same notation as above. Since $\pi(p^*_k, Y^*_k) = 0$, by (11), $\Pi^*_k = \Pi_k(p^*_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = 0$. Thus, in order to prove the theorem, it suffices to prove that there is $(p_k, Y_k) \in \mathcal{S}$

such that $\Pi_k(p_k, Y_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) > \Pi^*_k = 0$. Under the hypothesis of the theorem, there is $p'_1 > c$ such that $D(p'_1) > \bar{y}$ and $Y'_1 = \{D(p'_1) - \bar{y}\}$. Then, since $D(c) - \bar{y} \leq \bar{y}$, $(p'_1, Y'_1) \in \mathcal{S}$. Since $\mathbb{K}^1(p'_1, \mathbf{p}^*_{-1}) = \{2\}$, and since $d^1_*(p'_1, Y'_1) = D(c)$, by (2) and (36), $\theta^1_*(p'_1, Y'_1, \{2\}) = \bar{y} < D(c)$. This implies, by $\mathbb{K}^1(p'_1, \mathbf{p}^*_{-1}) = \{2\}$, $\mathcal{K}^1_*(p'_1, Y'_1) = \{\{2\}\}$. Thus, by (37), $\delta^1_*(p'_1, Y'_1) = \bar{y}$. This implies, by (5), $d^2_*(p'_1, Y'_1) = D(p'_1) - \bar{y}$. Thus, by $\mathbb{K}^2(p'_1, \mathbf{p}^E_{-1}) = \{1\}$, $\theta^2_*(p'_1, Y'_1, \{1\}) = D(p'_1) - \bar{y} > 0$ and $\mathcal{K}^2_*(p'_1, Y'_1) = \{\{1\}\}$. Thus, by (11) and (9), $\Pi_1(p'_1, Y'_1, \mathbf{p}^*_{-1}, \mathbf{Y}^*_{-1}) = \pi(p'_1, Y'_1) = (p'_1 - c)(D(p'_1) - \bar{y}) > 0$. This complete the proof.

Proof of Theorem 1: Let $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^{GB}, \mathbf{Y}^{GB})$. Then, by the definition of (p^{LR}, Y^{LR}), $\pi(p^*_k, Y^*_k) = 0$ for any k. Suppose $(\mathbf{p}^*, \mathbf{Y}^*)$ is not a Nash equilibrium. Then, by (11) and (9), there is $k \in \mathbb{N}$ such that

$$\pi(p_k, Y_k) \geq \Pi_k(p_k, Y_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) > \Pi_k(p^*_k, Y^*_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = \Pi^*_k = 0. \quad (41)$$

This implies $p_k > p^{LP}$. By (**??**), $\mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k}) = \mathbb{N} \backslash \{k\}$. Thus, $\#\mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k}) = N - 1 \geq N^{LR} \geq 1$ by hypothesis. Thus, $\mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k}) \neq \phi$. Since $p^1(p_k, \mathbf{p}^*_{\sim k}) = p^{LR}$, by the definition of ρ^{LR}, $D(p^{LR}) = \rho^{LR} y^{LR}$. Thus, by (2),

$$d^1_*(p_k, Y_k) = \rho^{LR} y^{LR}. \quad (42)$$

Denote by nA the direct sum obtained by adding a set A n times. Since $Y^*_{k'} = Y^{LR}$ for any $k' \in \mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k})$, by (36), $\theta^1_*(p_k, Y_k, \mathbb{K}') = \max(K' Y^{LR} \cap [0, \rho^{LR} y^{LR}])$ for any $\mathbb{K}' \subset \mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k})$, where $K' = \#\mathbb{K}'$. This implies, by $N^{LR} Y^{LR} = \{0, y^{LR}, ..., N^{LR} y^{LR}\}$ and $N^{LR} \leq \#\mathbb{K}^1(p_k, p^*_{\sim k})$,

$$\delta^1_*(p_k, Y_k) = N^{LR} y^{LR} \quad (43)$$

by the definition of N^{LR}. Since $N^{LR} = \rho^{LR}$ by hypothesis, by (42) and (43), Lemma 1 implies $d^2_*(p_k, Y_k) = 0$. Thus, by $\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$ and (36), $\theta^2_*(p_k, Y_k, \mathbb{K}) = \max(Y_k \cap \{0\}) = 0$ for any $\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$. Thus, $0 \in Y_k$, which implies $\pi(p_k, Y_k) = 0$, a contradiction to (41).

Proof of Theorem 2: Suppose $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^D, \mathbf{Y}^D)$ is not a Nash equilibrium. Then, in the same manner as in the proof of Theorem 1, it is possible to find (p_k, Y_k) such that (41) holds. Since this implies $p_k > c(y_k)/y_k$, by the hypothesis of the theorem, $p_k > p^{AC}$. Thus, by (11), $\mathcal{K}^2_*(p_k, Y_k) = \{\{k\}\}$. Since $\#\mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k}) \geq N - 1 \geq 1$, and since $p^1(p_k, \mathbf{p}^*_{\sim k}) = p^{AC}$, by the definition of (p^{AC}, y^{AC}), $d^1_*(p_k, Y_k) = D(p^{AC}) = y^{AC}$. Since $Y^*_{k'} = Y^{AC}$ for any $k' \in \mathbb{K}^1(p_k, \mathbf{p}^*_{\sim k})$, and since $y^{AC} \in Y^{AC}$, by (36), $\theta^1_*(p_k, Y_k, \{k'\}) = \max(Y^{AC} \cap [0, y^{AC}]) = y^{AC}$. Since this implies, by (37), (10) and (36), $\delta^1_*(p_k, Y_k) = y^{AC}$, it holds that $d^1_*(p_k, Y_k) = \delta^1_*(p_k, Y_k)$. Thus, in the same way as in the proof of Theorem 1, $\pi(p_k, Y_k) = 0$, a contradiction to (41).

Proof of Theorem 3: Suppose that $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^{SP}, \mathbf{Y}^{SP})$ is not a Nash equilibrium. Then, in the same manner as in the proof of Theorem 1, it is possible to find (p_k, Y_k) such that (41), (42) and (43) hold. Thus, by (5),

$$d^2_*(p_k, Y_k) = \max\{0, D(p^2(p_k, \mathbf{p}^*_{\sim k})) - N^{LR} y^{LR}\}. \quad (44)$$

By (41), $p_k > p^{LR}$ and, by (11), $\mathcal{K}^2_*(p_k, Y_k) = \{\{k\}\}$. By (44), (37), (10), and (36), there is $y_k > 0$ such that $y_k \in \{Y_k \cap [0, D(p_k) - N^{LR}y^{LR}]\}$. Since $y_k < D(p_k) - N^{LR}y^{LR}$, by the hypothesis of Theorem 3, $0 > p_k y_k - c(y_k) = \pi(p_k, Y_k)$, a contradiction to (41).

Proof of Theorem 4: Let $(\mathbf{p}^*, \mathbf{Y}^*) = (\mathbf{p}^{DP}, \mathbf{Y}^{DP})$. Then, in the same manner as in the proof of Theorems 1 and 3, it is possible to find (p_k, Y_k) such that (41), (42), (43) and (44). hold. Suppose $p_k < p^{AC}$. Since (41) implies $p_k > p^{LR}$, $\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$ and $p^2(p_k, \mathbf{p}^*_{\sim k}) = p_k$. Then, by (36), $\theta^2_*(p_k, Y_k, \mathbb{K}) = \max(Y_k \cap [0, \max\{0, D(p_k) - N^{LR}y^{LR}\}])$ for any $\mathbb{K} \subset \mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$. This implies, by (10), $\mathcal{K}^2_*(p_k, Y_k) \neq \phi$. This implies that there is $y_k \in Y_k$ such that $0 \leq y_k \leq \max\{0, D(p_k) - N^{LR}y^{LR}\}$. Moreover, it implies, by $\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$, $\mathcal{K}^2_*(p_k, Y_k) = \{\{k\}\}$. This implies, by $i^k(p_k, \mathbf{p}^*_{\sim k}) = 2$ and (9), $\Pi_k(p_k, Y_k, \mathbf{p}^*_{\sim k}, \mathbf{Y}^*_{\sim k}) = \pi(p_k, Y_k)$. Since $y_k \in Y_k$ and $0 \leq y_k \leq \max\{0, D(p_k) - N^{LR}y^{LR}\}$, by the hypothesis of the theorem, $c(y_k) \geq p_k y_k$. This implies $\pi(p_k, Y_k) = p_k y_k - c(y_k) \leq 0$, which contradicts (41). Thus, $p_k \geq p^{AC}$.

Suppose $p_k = p^R$. Then, $i^k(p_k, \mathbf{p}^*_{\sim k}) = 2$. By (41), (11), and (9), there is $\mathbb{K} \in \mathcal{K}^2_*(p_k, Y_k)$ such that $k \in \mathbb{K}$. Since $p^2(p_k, \mathbf{p}^*_{\sim k}) = p^{AC}$, by (36) and (44), $\mathbb{K} \backslash \{k\} \neq \phi$ implies

$$\delta^2_*(p_k, Y_k) = \max((Y_k + K'Y^{AC}) \cap [0, D(p^{AC}) - N^{LR}y^{LR}]), \tag{45}$$

where $K' = \#\mathbb{K} - 1$. Moreover, $\mathbb{K} \backslash \{k\} = \phi$ implies

$$\delta^2_*(p_k, Y_k) = \max(Y_k \cap [0, D(p^{AC}) - N^{LR}y^{LR}]). \tag{46}$$

Suppose that $Y_k \cap [0, D(p^{AC}) - N^{LR}y^{LR}] = \phi$. Then, $Y_k > D(p^{AC}) - N^{LR}y^{LR}$. This implies $(Y_k + KY^{AC}) \cap [0, D(p^{AC}) - N^{LR}y^{LR}] = \phi$. This implies, by (45) and (46), $\mathcal{K}^2_*(p_k, Y_k) \neq \phi$, a contradiction. Thus, $Y_k \cap [0, D(p^{AC}) - N^{LR}y^{LR}] \neq \phi$. Thus, there is $y_k \in Y_k$ such that $y_k \leq D(p^{AC}) - N^{LR}y^{LR}$. This implies, by the hypothesis of the theorem, $c(y_k) \geq p_k y_k$. Thus, $\pi(p_k, Y_k) = p_k y_k - c(y_k) \leq 0$, which contradicts (41). Thus, $p_k > p^{AC}$.

This implies $i^k((p_k, \mathbf{p}^*_{\sim k}) = 3$. Since $p_k > p^R$, either $\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \mathbb{N}''$ or $\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) = \mathbb{N}'' \backslash \{k\}$. Thus, $\#\mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k}) \geq N'' - 1 \geq 1$. Thus, there is $\mathbb{K}' \subset \mathbb{K}^2(p_k, \mathbf{p}^*_{\sim k})$ such that $\mathbb{K}' \neq \phi$. Recall that, by definition, $(p^*_{k'}, Y^*_{k'}) = (p^{AC}, Y^{AC})$ for all $k' \in \mathbb{N}''$, $Y^{AC} = \{0, y^{AC}, \hat{y}\}$ with $y^{AC} < \hat{y}$, and $D(p^{AC}) - N^{LR}y^{LR} = y^{AC}$. Thus, by (36) and (44), $\theta^2_*(p_k, Y_k, , \mathbb{K}') = \max\{K'Y^{AC} \cap [0, D(p^{AC}) - N^{LR}y^{LR}] = y^{AC}$. This implies, by (10), $\mathcal{K}^2_*(p_k, Y_k) \neq \phi$ and, by (37), $\delta^2_*(p_k, Y_k) = y^R$. Since, by (44), $d^2_*(p_k, Y_k) = D(p^{AC}) - N^{LR}y^{LR} = y^R$, $\delta^2_*(p_k, Y_k) = d^2_*(p_k, Y_k)$. This implies, by Lemma 1, $d^3_*(p_k, Y_k) = 0$. Thus, by $\mathbb{K}^3(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$ and (36), $\theta^3_*(p_k, Y_k, \mathbb{K}) = \max(Y_k \cap \{0\}) = 0$ for any $\mathbb{K} \subset \mathbb{K}^3(p_k, \mathbf{p}^*_{\sim k}) = \{k\}$. Thus, $0 \in Y_k$, which implies $\pi(p_k, Y_k) = 0$, a contradiction to (41).

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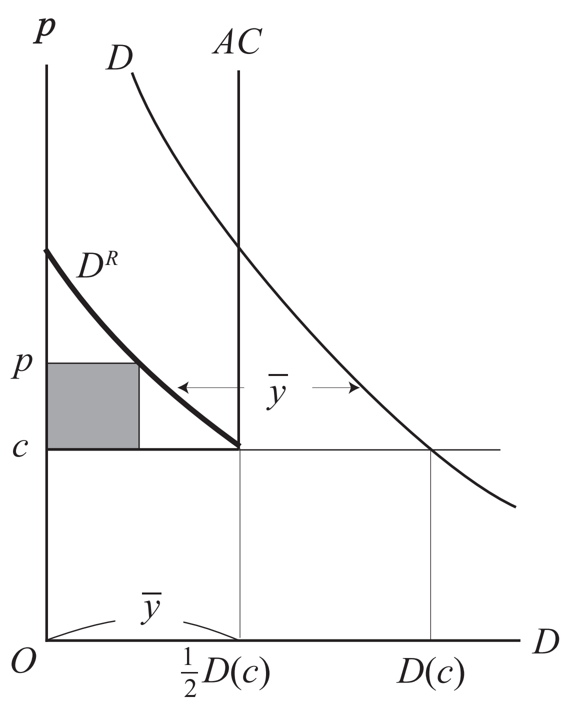


Figure 1: *Edgeworth's Bertrand Critism*

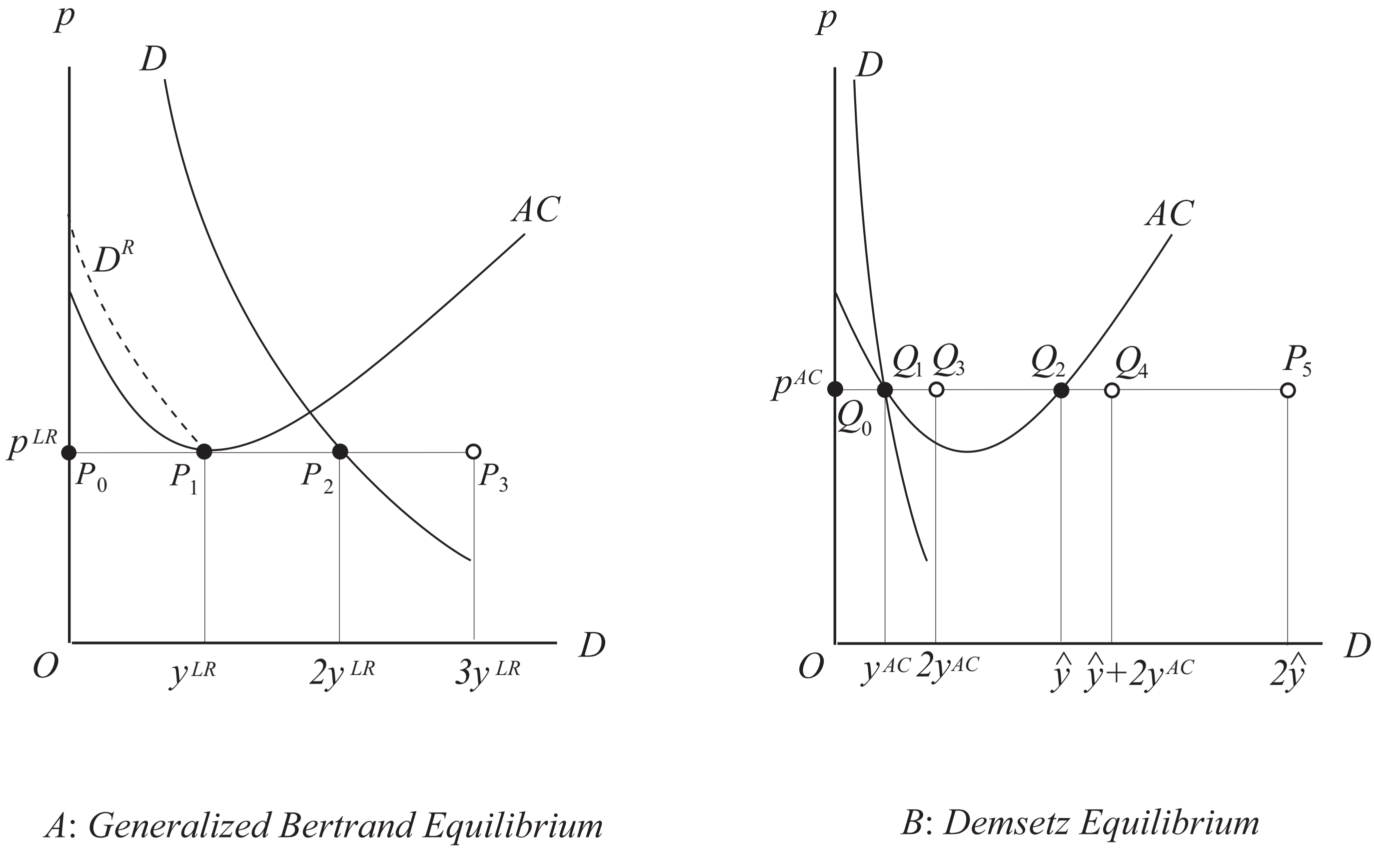
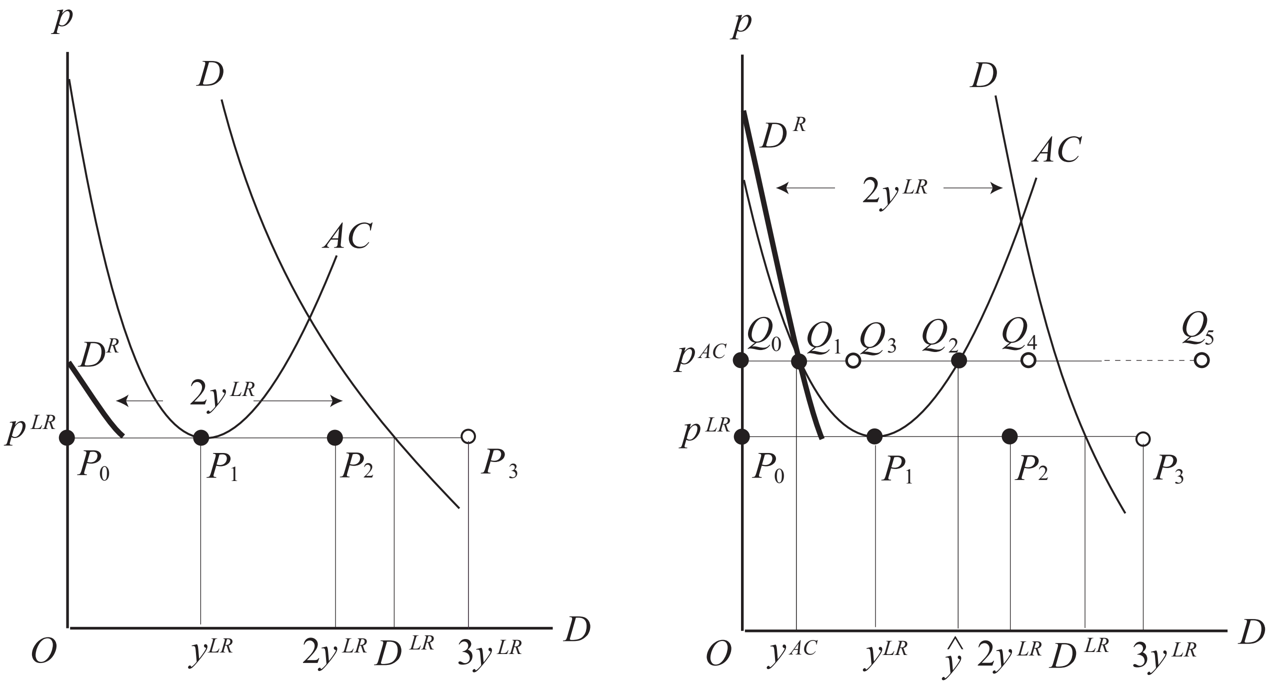


Figure 2: *Contestability*



A: *Single Price Equilibrium*

B: *Dual Price Solution*

Figure 3: *Solutions to the Non-integer Problem*